

Readings Ch. 23

Self test Ex. 23.1-1, 23.2-2

Lecture 19

Minimum Spanning Trees

Let $G = (V, E)$ be a connected undirected graph with edge weights $w(e)$ for edge e in E .

A tree is an acyclic graph.

A spanning tree is a set of edges that does not contain a cycle and every vertex v in V is an endpoint of at least one edge.

A minimum cost spanning tree (MCST) is a spanning tree A such that the sum of the weights is minimized for all possible spanning trees B .

$$w(A) = \sum_{e \in A} w(e) \leq w(B)$$

Prim's Algorithm

Define 2 arrays p such that $p[u]$ contains vertex v such that (u, v) is in E and $w(u, v)$ is minimized for all v in T adjacent to u . Priority such that $\text{priority}[u]$ contains $w(u, p[u])$.

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Prim-MST( $G=(V, E)$   $w: E \rightarrow \mathbb{R}$ )
   $T = \{\}$ 
  initialize priority queue  $Q$ 
  for all  $v$  in  $V$ :
     $\text{priority}[v] = \infty$ 
     $p[v] = \text{NIL}$ 
  Insert( $Q, v$ )
  pick arbitrary vertex  $s$  in  $V$ 
   $\text{priority}[s] = 0$ 
  while (not isEmpty( $Q$ )) do:
     $u = \text{extract-min}(Q)$ 
    if  $p[u] \neq \text{NIL}$ 
       $T = T \cup \{(p[u], u)\}$ 
    for each  $v$  in adjacency-list[ $u$ ]:
      if  $v$  in  $Q$  and  $w(u, v) < \text{priority}[v]$ :
        decrease-priority( $v, w(u, v)$ )
       $p[v] = u$ 

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