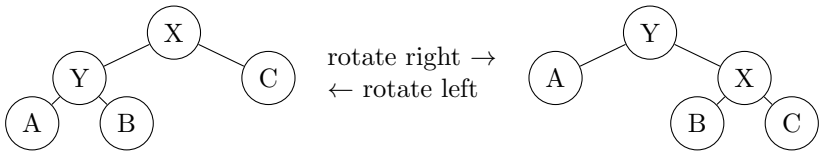
Lecture 08




```
AVL-insert(root, x):
  if root == NIL: # found insertion point
    root = TreeNode(x) # initial height = 0
  else if x.key < root.item.key:
    root.left = AVL-insert(root.left, x)
    if root.left.height > root.right.height + 1:
      root = AVL-rebalance-right(root)
    else: # no rebalancing, but height might have changed
      AVL-update-height(root)
  else if x.key > root.item.key:
    root.right = AVL-insert(root.right, x)
    if root.right.height > root.left.height + 1:
      root = AVL-rebalance-to-the-left(root)
    else:
      AVL-update-height(root)
  else: # x.key = root.item.key
    root.item = x
  return root

AVL-rebalance-to-the-left(root):
  if root.right.left.height > root.right.right.height:
    root.right = AVL-rotate-to-the-right(root.right)
  return AVL-rotate-to-the-left(root)

AVL-rebalance-to-the-right(root):
  if root.left.right.height > root.left.left.height:
    root.left = AVL-rotate-to-the-left(root.left)
  return AVL-rotate-to-the-right(root)

AVL-rotate-to-the-left(y):
  x = y.right
  y.right = x.left
  x.left = y
  AVL-update-height(x.left);
  AVL-update-height(x);
  return x

AVL-rotate-to-the-right(y):
  x = y.left
  y.left = x.right
  x.right = y
  AVL-update-height(x.right)
  AVL-update-height(x)
  return x

AVL-update-height(node):
  node.height = 1 + max(node.left.height, node.right.height)
```

No checking for special cases when references are `NIL` ... **Trick** → Define NIL node:
NIL.item = NIL
NIL.left = NIL
NIL.right = NIL
NIL.height = -1
Use just one `NIL` node for the entire tree: every node with a reference to `NIL` refers to the same `NIL`. Now, no need for lots of extra code to check for `NIL` before each access!

delete

Like `delete` on BST. After every recursive call:

(1) check if subtrees have become unbalanced

(2) rebalance with rotations

(3) update heights

```
AVL-remove(root, x):
  if root == NIL:
    pass
  else if x.key < root.item.key:
    root.left = AVL-remove(root.left, x)
    if root.right.height > root.left.height + 1:
      root = AVL-rotate-to-the-left(root)
    else:
      AVL-update-height(root)
  else if x.key > root.item.key:
    root.right = AVL-remove(root.right, x)
    if root.left.height > root.right.height + 1:
      root = AVL-rebalance-to-the-right(root)
    else:
      AVL-update-height(root)
  else:
    if root.left == NIL or root.right == NIL:
      if root.left == NIL:
        root = root.right
      else:
        root = root.left
    else:
      # Select whether to replace root.item with its predecessor or
      # its successor, depending on the heights of subtrees
      if root.left.height > root.right.height:
        root.item, root.left = AVL-remove-max(root.left)
      else:
        root.item, root.right = AVL-remove-min(root.right)
      AVL-update-height(root)
  return root

AVL-remove-min(root):
  if root.left == NIL:
    return root.item, root.right
  else:
    item, root.left = tree-remove-min(root.left)
    AVL-update-height(root)
    return item, root
```

```
AVL-remove-max(root):
  if root.right == NIL:
    return root.item, root.left
  else:
    item, root.right = tree-remove-max(root.right)
    AVL-update-height(root)
    return item, root
```

search

Unchanged.

```
AVL-search(root, k):
  if root == NIL:
    pass
  else if k < root.item.key:
    root = AVL-search(root.left, k)
  else if k > root.item.key:
    root = AVL-search(root.right, k)
  else:
    pass
  return root
```

Augmented Data Structures

Existing data structure modified to store additional information and/or perform additional operations.
In general:

(1) Choose data structure to augment

(2) Determine additional info

(3) Check additional info can be maintained in every old operation. (and at what additional cost?)

(4) Implement new operations

Ordered Sets

set = $\{144, 20, 100, 40, 17\}$
Operations `insert`, `delete`, `search`
`rank(k)` - index of k in a sorted ordering of set elements [17,20,40,100,144]
`select(rank)` $\rightarrow$