CSC343H1 Winter 2016 Assignment 3 Part 2

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1. (a) BCNF requires that the LHS of an FD be a superkey.
   i. $A^+$: AE, $A$ is not a superkey and $A \to E$ violates BCNF.
   ii. $M^+$: MEZA, $M$ is not a superkey and $M \to EZA$ violates BCNF.
   iii. $S^+$: SFZBMEA, $S$ is a superskey and $S \to FZB$ does not violate BCNF.
   iv. $SZ^+$: superset of S, $SZ$ is a superset of $S$ and $SZ \to M$ does not violate BCNF.
   v. $BF^+$: BFMEZA, $BF$ is not a superskey and $BF \to M$ violates BCNF.

   (b) i. Decompose $R$ using FD $M \to EZA$. $M^+$: MEZA, so this yields two relations: $R1 = AEZM$ and $R2 = BFMS$.
   ii. Project the FD's onto $R1 = AEZM$.

| A | E | Z | M | closure | FDs |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $A^+$: AE | $A \to E$: violates BCNF; abort the projection |

   iii. Decompose $R1$ using $A \to E$. This yields two relations: $R3 = AMZ$ and $R4 = AE$.
   iv. Project the FD's onto $R3 = AMZ$.

| A | M | Z | closure | FDs |
|---|---|---|---|---|
| 1 | 0 | 0 | $A^+$: AE | $A \to E$: nothing |
| 0 | 1 | 0 | $M^+$: MEZA | $M \to EZA$: $M$ is a superkey of $R3$ |
| 0 | 0 | 1 | $Z^+$: Z | nothing |
| | supersets of $M$ | | irrelevant | can only generate weaker FDs than what we already have |
| 1 | 0 | 0 | $AZ^+$: AZE | nothing |

   This relation satisfies BCNF.

   v. Project the FD's onto $R4 = AE$.

| A | E | closure | FDs |
|---|---|---|---|
| 1 | 0 | $A^+$: AE | $A \to E$: $A$ is a superskey of $R4$ |
| 0 | 1 | $E^+$: E | nothing |

   This relation satisfies BCNF.

   vi. Return to $R2 = BFMS$ and project the FD's onto it.

| B | F | M | S | closure | FDs |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $B^+$: B | nothing |
| 0 | 1 | 0 | 0 | $F^+$: F | nothing |
| 0 | 0 | 1 | 0 | $M^+$: MEZA | nothing |
| 0 | 0 | 0 | 1 | $S^+$: BFMS | $S \to BFM$: $S$ is a superkey of $R2$ |
| 1 | 1 | 0 | 0 | $BF^+$: BFM | $BF \to M$: violates BCNF; abort the projection |

   We must decompose $R2$ further.

   vii. Decompose $R2$ using FD $BF \to M$. This yields two relations: $R5 = BFS$ and $R6 = BFM$.
   viii. Project the FD's onto $R5 = BFS$

| B | F | S | closure | FDs |
| --- | --- | --- | --- | --- |
| 1 | 0 | 0 | $B^+$: B | nothing |
| 0 | 1 | 0 | $F^+$: F | nothing |
| 0 | 0 | 1 | $S^+$: SBF.. | $S \to BF$: $S$ is a superkey of $R5$ |
| 1 | 1 | 0 | $BF^+$: BF.. | nothing |
| | supersets of $S$ | | irrelevant | can only generate weaker FDs than what we already have |

This relation satisfies BCNF.

ix. Project the FD's onto $R6 = BFM$

| B | F | M | closure | FDs |
| --- | --- | --- | --- | --- |
| 1 | 0 | 0 | $B^+$: B | nothing |
| 0 | 1 | 0 | $F^+$: F | nothing |
| 0 | 0 | 1 | $M^+$: M.. | nothing |
| 1 | 1 | 0 | $BF^+$: BFM | $BF \to M$: $BF$ is a superkey of $R6$ |
| 0 | 1 | 1 | $FM^+$: FM.. | nothing |
| 1 | 0 | 1 | $BM^+$: BM.. | nothing |

This relation satisfies BCNF.

x. Final decomposition:

A. $R3 = AMZ$ with FD's $A \to E$ and $M \to EZA$,

B. $R4 = AE$ with FD $A \to E$,

C. $R5 = BFS$ with FD $S \to BF$,

D. $R6 = BFM$ with FD $BF \to M$

2. $R = ABCDEF$.

(a) Closures:

i. $A^+$: ADFB, $A$ is not a superkey.

ii. $ABE^+$: ABECDF, $ABE$ is a superkey.

iii. $ACDF^+$: ACDFEB, $ACDF$ is a superskey.

iv. $AD^+$: ADBF, $AD$ is not a superkey.

v. $C^+$: CD, $C$ is not a superskey.

We know that $A$ has to be a part of every key because it only appears on the $LHS$ and never on $RHS$, whereas $BCDEF$ appears on both $LHS$ and $RHS$ so we have to check them. Additionally, the only way to get $C$ is to have $A$ or $B$ or $E$ on $LHS$, but we can get $B$ if we have $A$ on $LHS$. $AC^+$: ABCDEF. Therefore, $AC$ is key. If we don't start with $E$ on $LHS$, we can never get $E$ on $RHS$. $AE^+$: ABCDEF. Therefore, $AE$ is key. The only keys are $AE$ and $AC$.

(b) i. **Step 1**: Split the $RHS$ to get our initial set of FD's, $S1$:

A. $A \rightarrow D$

B. $A \rightarrow F$

C. $ABE \rightarrow C$

D. $ABE \rightarrow D$

E. $ABE \rightarrow F$

F. $ACDF \rightarrow E$

G. $AD \rightarrow B$

H. $C \rightarrow D$

ii. **Step 2**: For each FD, try to reduce the $LHS$:

A. $A^+$: AD..; keep

B. $A^+$: ADF..; keep

C. $A^+$: ADFB, $B^+$: B, $E^+$: E, $AB^+$: ABDF, $BE^+$: BE, $AE^+$: AEDFBC; reduce to $AE \rightarrow C$

D. $A^+$: AD..; reduce to $A \rightarrow D$

E. $A^+$: ADF..; reduce to $A \rightarrow F$

F. $AC^+$: ADFBCE; reduce to $AC \rightarrow E$

G. $A^+$: ADFB..; reduce to $A \rightarrow B$

H. $C^+$: CD..; keep

Our new set of FD's, let's call it $S2$:

A. $AE \rightarrow C$

B. $A \rightarrow D$

C. $A \rightarrow F$

D. $AC \rightarrow E$

E. $A \rightarrow B$

F. $C \rightarrow D$

iii. **Step 3**: Try to eliminate each FD.

A. $AE^+_{S2-(A)}$: AEDBF. We need this FD.

B. $A^+_{S2-(B)}$: AFB. We need this FD.

C. $A^+_{S2-(C)}$: ADB. We need this FD.

D. $AC^+_{S2-(D)}$: ACDFB. We need this FD.

E. $A^+_{S2-(E)}$: ADF. We need this FD.

F. $C^+_{S2-(F)}$: C. We need this FD.

Our final set of FD's is $S2$ (listed above)

(c) i. Let's call the revised FD's $S5$:

A. $AE \to C$

B. $A \to DFB$

C. $AC \to E$

D. $C \to D$

ii. The set of relations that would result would have these attributes:
$R1(A, E, C)$, $R2(A, D, F, B)$, $R3(A, C, E)$, $R4(C, D)$

iii. Since the attributes $ACE$ occur within $R2$, we don't need to keep the relation $R3$.

iv. $AE$ and $AC$ are keys of $R$, so there is no need to add another relation that includes a key.

v. So, the final set of relations is:
$R1(A, E, C)$, $R2(A, D, F, B)$, $R4(C, D)$

(d) Redundancy: Project onto relations to find all FD's.

i. Project FD's onto $R1$:

| A | E | C | closure | FDs |
|---|---|---|---|---|
| 1 | 0 | 0 | $A^+$: ADBF | nothing |
| 0 | 1 | 0 | $E^+$: E | nothing |
| 0 | 0 | 1 | $C^+$: CD | nothing |
| 1 | 1 | 0 | $AE^+$: AEC | $AE \to C$: $AE$ is a superkey of $R1$ |
| 0 | 1 | 1 | $EC^+$: EC | nothing |
| 1 | 0 | 1 | $AC^+$: ACE | $AC \to E$: $AC$ is a superkey of $R1$ |

This relation satisfies BCNF.

ii. Project FD's onto $R2$:

| A | D | B | F | closure | FDs |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $A^+$: ADBF | $A \to BDF$: $A$ is a superkey of $R2$ |
| 0 | 1 | 0 | 0 | $D^+$: D | nothing |
| 0 | 0 | 1 | 0 | $B^+$: B | nothing |
| 0 | 0 | 0 | 1 | $F^+$: F | nothing |
| | supersets of $A$ | | | irrelevant | we already know that $A$ is superkey |
| | combinations of $B, D, F$ | | | irrelevant | nothing |

This relation satisfies BCNF.

iii. Project FD's onto $R3$:

| C | D | closure | FDs |
|---|---|---|---|
| 1 | 0 | $C^+$: CD | $C \to D$: $C$ is superkey of $R3$ |
| 0 | 1 | $D^+$: D | nothing |

This relation satisfies BCNF.

iv. Since all relations are in BCNF, this schema does not allow any redundancy.