

COL215

ASSIGNMENT 1

NAME- ANIKET GUPTA

ENTRY NO. 2019CS10327

PROBLEM 1:

PROCEDURE:

My ENTRY NO. is 2019CS10327

Thus, $N=27$

Then, $B = \{1,3,9,27\}$

For our question, the set and the operations are defined as

- B = set of all divisors of N , including 1 as well as N itself
- $a.b$ = greatest common divisor of a and b
- $a + b$ = least common multiple of a and b
- $a' = N / a$
- $1+ = 1$ {identity of $+$ operation}
- $1. = N$ {identity of $.$ operation}

For a tuple of the given form in question to be a Boolean algebra, it should satisfy all of the below given properties.

- | | |
|---------------------------------------|----------------------|
| 1.) $a + a = a . a = a$ | -- idempotent |
| 2.) $a + b = b + a$ | --commutative |
| 3.) $a . b = b . a$ | --commutative |
| 4.) $a + (b + c) = (a + b) + c$ | --associative |
| 5.) $a . (b . c) = (a . b) . c$ | --associative |
| 6.) $a . (b + c) = (a . b) + (a . c)$ | --distributive |
| 7.) $a + (b . c) = (a + b) . (a + c)$ | --distributive |
| 8.) $a . N = a ; a + 1 = a$ | --identity elements |
| 9.) $a . a' = 1 ; a + a' = N$ | --inverse/complement |

NOTE: the last two properties are in respect to the given question.

- ❖ But, for my set, not all of these properties are not satisfied. It is enough to show an example from my set which does not satisfy atleast one of

the above given properties to show that it does not form a Boolean algebra.

- ❖ For my example, I took $a=3$, which gave $a'=N/a=9$.
- ❖ Now, if it is a Boolean algebra, then it must satisfy the property 9 given above.
- ❖ But, $a.a' = \gcd(a, a') = \gcd(3,9) = 3 \neq 1$.

Thus, it does not form a Boolean algebra.