COL 215 Assignment 1

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Entry No.- 2019CS10327

PROBLEM-1

My Entry No.- 2019 CS 10327

$\Rightarrow N = 27$

$\Rightarrow B = \{1, 3, 9, 27\}$

Given, B = set of all divisors of N,

$a \cdot b$ = greatest common divisor of a and b

$a + b$ = least common multiple of a and b.

$a' = N/a$

$I_+ = 1$ (identity of + operation)

$I_\cdot = N$ (identity of $\cdot$ operation)

For a tuple $\langle B, \cdot, +, ', I_+, I_\cdot \rangle$ to form a boolean algebra, it must satisfy the following ~~postulat~~ properties.

* $a + a = a$ — idempotent
* $a \cdot a = a$ — idempotent
* $a + b = b + a$
* $a \cdot b = b \cdot a$ } — Commutative
* $a + (b + c) = (a + b) + c$
* $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ } — Associative
* $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
* $a + (b \cdot c) = (a + b) \cdot (a + c)$ } — Distributive

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* Identity of '+' operation : $x + I_+ = x$
* Identity of '·' operation : $x \cdot (I_\cdot) = x$
* Inverse rule : $a + a' = I_+$ and $a \cdot a' = I_\cdot$

From my set B, if we take $a = 3$

Then, $a' = \frac{N}{a}$ (according to question)

$\Rightarrow a' = \frac{27}{3} = 9$

Now, $a \cdot a'$ = greatest common divisor of a and a'

$\Rightarrow a \cdot a' = \gcd(3, 9)$

$\Rightarrow a \cdot a' = 3$

But this violates the inverse rule $a \cdot a' = I_\cdot$ and thus, our tuple does not form a ~~binary~~ Boolean Algebra. Note, one example is enough to prove that a given set, along with its operations, does not form a boolean algebra.

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