Building Optimal Virtual Team on Fantasy Sports

Optimization Models
BANA 7020

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Index

1.1 About Fantasy Sports:

Fantasy sports is a multibillion-dollar industry that gathers players from around the world. The competition consists in selecting virtual or fantasy teams composed by players from a pool of games. The virtual teams are ranked according to the real score achieved by the players in the team. Contestants compete for money or other prizes usually via webpages like draftkings or fanduel.

1.2 Problem Statement:

Consider a fantasy football competition in which each contestant is allowed to participate with at most one fantasy team or entry. Each player has a salary that must be paid to get the player into the entry, a projected score that is an estimation of how many points will the player achieve, and a corresponding position:

- Quarterback (QB)
- Running Back (RB)
- Wide Receiver (WR)
- Tight End (TE)
- Defense (DST)

An entry consists of 6 players that satisfy the following conditions:

- The total combined salary of the selected players is at most 50,000
- There must be at least 1 player for each position
- The sixth player is a flexible player that can be either a RB, WR, or TE

1.3 Data Overview:

The initial dataset provided consists a list of 100 players with the only information about their respective position. Every player can only play in the position defined for them.

Additionally, another level of diversity has been added to the dataset as to classify the highest level a player has played. This additional factor i.e. **Level** can take two values {International, National}.

The constrain that is imposed while creating a team now ensures that there can not be more than two players who have played International. This is to promote the diversity among team and up-bring rising talent.

2. Data Preparation:

The data given is a list of 100 player names along with their playing positions. Our aim is to create a team of 6 by selecting players based on their position, salary and the projected score such that the salary to be paid out is minimized. Projected score and the Salary variables were created using R's runif() function. The new variables created have dimensions of 100 X 1 but our decision variable $X_{i,j}$ has a dimension of 100 X 5. In order to get the matrix multiplication right, we transposed and broke down the salary, position variables according to the 5 playing positions (denoted by j). A snapshot of the final data to be input into xpress is provided below

					Position wise Salary					Level Diversity				Position wise Projected Score					Position				
	Name	Position	Salary	ProjectedScore	QB	RB	WR	TE	DST	Level	National	International	Is_International	QB	RB	WR	TE	DST	QB	RB	WR	TE	DST
1	Le'Veon	RB	9682.24	3.196264552	0	9682.24	0	0	0	National	9682.24	0	0	0	3.19626	0	0	0	0	1	0	0	0
2	David Jol	RB	6358.38	24.85154353	0	6358.38	0	0	0	National	6358.38	0	0	0	24.8515	0	0	0	0	1	0	0	0
3	Antonio	WR	10943.8	4.831502257	0	0	10943.8	0	0	National	10943.8	0	0	0	0	4.8315	0	0	0	0	1	0	0
4	Alvin Kar	RB	7316.02	17.40878057	0	7316.02	0	0	0	Internati	0	7316.024903	1	0	17.4088	0	0	0	0	1	0	0	0
5	DeAndre	WR	8276.61	5.777482547	0	0	8276.61	0	0	National	8276.61	0	0	0	0	5.77748	0	0	0	0	1	0	0
6	Michael	WR	6436.33	21.51007248	0	0	6436.33	0	0	Internati	0	6436.328399	1	0	0	21.5101	0	0	0	0	1	0	0
7	Ezekiel E	RB	13288.2	13.83503095	0	13288.2	0	0	0	Internati	0	13288.1544	1	0	13.835	0	0	0	0	1	0	0	0
8	Keenan A	WR	12194	12.85645767	0	0	12194	0	0	National	12194	0	0	0	0	12.8565	0	0	0	0	1	0	0
9	A.J. Gree	WR	5028.79	4.771657865	0	0	5028.79	0	0	Internati	0	5028.793466	1	0	0	4.77166	0	0	0	0	1	0	0
10	Tom Brad	QB	11708.6	3.346962482	11708.6	0	0	0	0	National	11708.6	0	0	3.34696	0	0	0	0	1	0	0	0	0
11	Leonard	RB	11426.5	18.08454767	0	11426.5	0	0	0	Internati	0	11426.5083	1	0	18.0845	0	0	0	0	1	0	0	0
12	Odell Be	WR	10721.8	20.90291902	0	0	10721.8	0	0	National	10721.8	0	0	0	0	20.9029	0	0	0	0	1	0	0
13	Rob Gron	TE	7570.33	24.81963841	0	0	0	7570.33	0	National	7570.33	0	0	0	0	0	24.8196	0	0	0	0	1	0
14	Adam Th	WR	13683	7.931895893	0	0	13683	0	0	Internati	0	13683.01249	1	0	0	7.9319	0	0	0	0	1	0	0
15	Kareem H	RB	5656.21	20.5340409	0	5656.21	0	0	0	Internati	0	5656.21084	1	0	20.534	0	0	0	0	1	0	0	0
16	Cam New	QB	10738.6	12.50047599	10738.6	0	0	0	0	Internati	0	10738.61806	1	12.5005	0	0	0	0	1	0	0	0	0
17	Andrew L	QB	5129.14	19.91647655	5129.14	0	0	0	0	Internati	0	5129.142673	1	19.9165	0	0	0	0	1	0	0	0	0
18	Joe Mixo	RB	8719.05	12.41437093	0	8719.05	0	0	0	Internati	0	8719.046085	1	0	12.4144	0	0	0	0	1	0	0	0
19	LeSean M	RB	11727.7	3.620250981	0	11727.7	0	0	0	Internati	0	11727.74867	1	0	3.62025	0	0	0	0	1	0	0	0
20	Julian Ed	WR	12431.5	24.62350211	0	0	12431.5	0	0	National	12431.5	0	0	0	0	24.6235	0	0	0	0	1	0	0
21	Patrick M	QB	6302.45	24.12751125	6302.45	0	0	0	0	National	6302.45	0	0	24.1275	0	0	0	0	1	0	0	0	0
22	Kenyan D	RB	8165.75	19.48602218	0	8165.75	0	0	0	Internati	0	8165.749777	1	0	19.486	0	0	0	0	1	0	0	0
23	JuJu Smit	WR	12744.4	2.908419536	0	0	12744.4	0	0	National	12744.4	0	0	0	0	2.90842	0	0	0	0	1	0	0
24	Jerick Mc	RB	13164.2	6.046068273	0	13164.2	0	0	0	National	13164.2	0	0	0	6.04607	0	0	0	0	1	0	0	0
25	Andy Dal	QB	5402	18.42463823	5402	0	0	0	0	Internati	0	5401.995225	1	18.4246	0	0	0	0	1	0	0	0	0
26	Josh Gor	WR	5016.73	15.64333642	0	0	5016.73	0	0	National	5016.73	0	0	0	0	15.6433	0	0	0	0	1	0	0
27	Mark Ing	RB	8237.03	23.33534366	0	8237.03	0	0	0	Internati	0	8237.033178	1	0	23.3353	0	0	0	0	1	0	0	0
28	Jimmy Ga	QB	13611.7	12.99707886	13611.7	0	0	0	0	National	13611.7	0	0	12.9971	0	0	0	0	1	0	0	0	0

Figure 1: shows the data manipulation for easier matrix multiplication. Notice how the salary variable has only one non-zero value in the entire row. Similarly, Projected score, Level, and Position variables have also been transposed w.r.t the Position variable

Note: Po, denoting his position in the team where o belongs to 1..5

1. Quarter Back (QB)
2. Running Back (RB)
3. Wide Receiver (WR)
4. Tight End (TE)
5. Defence (DST)

Data:

- Each player has a salary that must be paid (random salaries between 5,000 and 14,000)
- Projected score of the points that he will achieve (generated random scores between 1 and 25)

Diversity of the player, if international or not (0 indicates a National level player and 1 indicates an international level player)


projectedscore_updated_V2.txt


salary_updated_v2.txt


position_v2.txt


level_updated.txt


players_updated_final_report_data.xlsx

3. Modeling:

Variable & Data definitions:

$j \in Po = \{1:5\}$; set of player positions.

$i \in P = \{1:100\}$; set of all players

Team1 $X_{ij} = \begin{cases} 1, & \text{if player } i \text{ is selected to play at position } j \\ 0, & \text{otherwise} \end{cases}$

Team2 $Y_{ij} = \begin{cases} 1, & \text{if player } i \text{ is selected to play at position } j \\ 0, & \text{otherwise} \end{cases}$

$W_{ij} = \begin{cases} 1, & \text{if player } i \text{ is selected to play at position } j \text{ for both teams} \\ 0, & \text{otherwise} \end{cases}$

Data

Position ij : Position of player i from j positions.

S_{ij} : Salary matrix for player i at position j

P_{ij} : Projected score for player i at position j

level i : Diversity variable introduced into the data. Can be either National / International. Level of player i. 1 indicates International player while 0 indicates national player.

Objective fn : Maximize the projected score of the team

$$\sum_{j \in Po} \sum_{i \in P} P_{ij} * X_{ij} + \sum_{j \in Po} \sum_{i \in P} P_{ij} * Y_{ij}$$

Constraints :

1) Keeping the total cost of players below 50000 (team 1)

$$\sum_{i \in Po} \sum_{j \in P} S_{ij} * X_{ij} \leq 50000$$

2) Keeping the total cost of players below 50000 (team 2)

$$\sum_{i \in Po} \sum_{j \in P} S_{ij} * Y_{ij} \leq 50000$$

3) Keeping the max. no. of international level players as 2

team 1

$$\sum_{j \in Po} \sum_{i \in P} level(i) * x_{ij} \leq 2$$

team 2

$$\sum_{j \in Po} \sum_{i \in P} level(i) * y_{ij} \leq 2$$

4) Keeping the number of players in a team as 6

team 1

$$\sum_{i \in P} \sum_{j \in Po} x_{ij} = 6$$

team 2

$$\sum_{i \in P} \sum_{j \in Po} y_{ij} = 6$$

5) The second best team can be formed by changing just 1 player from the 1st team combinations.

$$\sum_{i \in P} \sum_{j \in Po} w_{ij} \leq 5$$

6) Selecting at least 1 player from each position

team 1

$$\sum_{i \in P} x_{ij} * Position\,ij \geq 1 \quad \forall j \in Po$$

team 2

$$\sum_{i \in P} y_{ij} * Position\,ij \geq 1$$

7) Multiple players can be selected from positions 1 & 5.

team 1

$$\sum_{i \in P} x_{ij} * Position\,ij = 1 \quad \forall j = 1,5$$

team 2

$$\sum_{i \in P} y_{ij} * Position\,ij = 1 \quad \forall j \in \{1,5\}$$

8) Keeping the relationships between both the teams according to w.

$$w_{ij} + 1 \geq y_{ij} + x_{ij} \quad \forall i \in P;\ j \in Po$$

9)

$$\sum_{i \in P} \sum_{j \in Po} x_{ij} + \sum_{i \in P} \sum_{j \in Po} y_{ij} \leq \sum_{i \in P} \sum_{j \in Po} w_{ij} + 1$$

Keeping w_{ij} 1 iff x_{ij} & y_{ij} are both 1.

4. Optimal Teams:

Best Team W/O diversity	Salary($)	Projected Score	Position
David Johnson	6358.4	24.9	RB
Rob Gronkowski	7570.3	24.8	TE
Royce Freeman	13574.9	24.8	RB
Patrick Mahomes II	6302.5	24.1	QB
Kendall Wright	5747.3	23.7	WR
Vikings	5361.5	12.9	DST
Total	**44914.9**	Objective	**135.2**

Best team	Salary ($)	Projected Score	Position	Level
David Johnson	6358.4	24.9	RB	National
Rob Gronkowski	7570.3	24.8	TE	National
Royce Freeman	13574.9	24.8	RB	International
Patrick Mahomes II	6302.5	24.1	QB	National
Kendall Wright	5747.3	23.7	WR	International
Vikings	5361.5	12.9	DST	National
Total	**44914.9**	Objective		**135.2**

Second Best Team	Salary($)	Projected Score	Position	Level
Julian Edelman	12431.5	24.6	WR	National
David Johnson	6358.4	24.9	RB	National
Rob Gronkowski	7570.3	24.8	TE	National
Patrick Mahomes II	6302.5	24.1	QB	National
Kendall Wright	5747.3	23.7	WR	International
Vikings	5361.5	12.9	DST	National
Total	**43771.5**	Objective		**135**

Notes:

- Introducing level as an additional diversity doesn't changes the optimal team obtained i.e initial team had only two players who appeared for international.
- 2nd Best Team could be formed by just replacing Royce Freeman (RB) with Julian Edelman (WR)
- The Teams formed here are aggressive in nature, weak defence strengthen by two Running Back players.

5. Xpress Codes:

1. Maximizing projected score only on the basis of salary constrain and position constrain.

```
model ModelName
uses "mmxprs"; !gain access to the Xpress-Optimizer solver
!optional parameters section
parameters
  PROJECTDIR='' ! for when file is added to project
end-parameters

declarations
        ! Set definition
        P=1..100
        Po = 1..5

        ! variables
        x : array(P,Po) of mpvar

        ! Data
        s : array(Po,P) of real
        p : array(Po,P) of real
        Position : array(Po,P) of integer
        level : array(P) of integer

 Objective:linctr
end-declarations

initializations from 'salary_updated.txt'
        s
end-initializations

initializations from 'projectedscore_updated.txt'
        p
end-initializations

initializations from 'position.txt'
        Position
end-initializations

initializations from 'level_updated.txt'
        level
end-initializations


forall (i in P, j in Po) do
        x(i,j) is_binary ! is_integer
```

```
end-do

! Model
Obj :=  sum(i in P,j in Po)p(j,i)*x(i,j)

sum(i in P,j in Po)s(j,i)*x(i,j) <= 50000
sum(i in P,j in Po)x(i,j)=6

forall (j in Po) do
        sum(i in P)x(i,j)*Position(j,i)>=1
end-do

forall (j in Po| j=1 or j=5 ) do
        sum(i in P)x(i,j) = 1
end-do

maximize(Obj)
! Output
writeln("Model Solved -  Optimial Objective is ",getobjval)
forall(i in P, j in Po | getsol(x(i,j) ) <> 0) do
        writeln("x",i,j," = ",getsol(x(i,j)) )
end-do
end-model
```

2. Maximizing projected score and finding two best teams on the basis of salary constrain, position constrain and international diversity.

```
model ModelName
uses "mmxprs";
parameters
  PROJECTDIR='' ! for when file is added to project
end-parameters
declarations
        ! Set definition
        P=1..100
        Po = 1..5

        ! variables
        x : array(P,Po) of mpvar
        y : array(P,Po) of mpvar
        w : array(P,Po) of mpvar

        ! Data
        s : array(P,Po) of real
        p : array(P,Po) of real
        Position : array(P,Po) of integer
        level : array(P) of integer
```

```
Objective:linctr
end-declarations
! Salary matrix
initializations from 'salary_updated_v2.txt'
        s
end-initializations

! Projected score matrix
initializations from 'projectedscore_updated_V2.txt'
        p
end-initializations

! Player position matrix
initializations from 'position_v2.txt'
        Position
end-initializations

! The  below condition is added only to include another level of diversity
initializations from 'level_updated.txt'
        level
end-initializations

! Defining the decision variable as binary for the team one selection
forall (i in P, j in Po) do
        x(i,j) is_binary ! is_integer is_free
end-do

!  Defining the decision variable as binary for the 2nd best team selection
forall (i in P, j in Po) do
        y(i,j) is_binary ! is_integer is_free
end-do

! defining W as 1 if a player is playing in the best and the 2nd best team
forall (i in P, j in Po) do
        w(i,j) is_binary ! is_integer is_free
end-do

! Model
! Maximize the Projected score of the team
Obj :=  sum(i in P,j in Po)p(i,j)*x(i,j) + sum(i in P,j in Po)p(i,j)*y(i,j)

! adding the constraint of keeping the total cost of players below 50000
sum(i in P,j in Po)s(i,j)*x(i,j)  <= 50000

! adding the constraint of keeping the total cost of players below 50000: team 2
sum(i in P,j in Po)s(i,j)*y(i,j)  <= 50000

! keeping the max number of players who play in international level as 2 : team 1
sum(i in P,j in Po)level(i)*x(i,j) <= 2
```

```
! keeping the max number of players who play in international level as 2 : team 2
sum(i in P,j in Po)level(i)*y(i,j) <= 2
! Keeping the number of players in a team as 6 : team1
sum(i in P,j in Po)x(i,j)=6
! Keeping the number of players in a team as 6 : team2
sum(i in P,j in Po)y(i,j)=6
! keeping the max number of common players between 2nd best team and the first best team as 5
sum(i in P,j in Po)w(i,j)<=5

! Selecting at least one player from each of the positions : team 1
forall (j in Po) do
        sum(i in P)x(i,j)*Position(i,j)>=1
end-do
! Multiple Players can be selected from positions 1 & 5 : team 1
forall (j in Po| j=1 or j=5 ) do
        sum(i in P)x(i,j)*Position(i,j) = 1
end-do
! Selecting at least one player from each of the positions : team 2
forall (j in Po) do
        sum(i in P)y(i,j)*Position(i,j)>=1
end-do
! Multiple Players can be selected from positions 1 & 5 : team 2
forall (j in Po| j=1 or j=5 ) do
        sum(i in P)y(i,j)*Position(i,j) = 1
end-do
! Keeping the relationships between both the teams according to W
forall (i in P, j in Po) do
        w(i,j) + 1 >=       y(i,j) + x(i,j)
end-do

maximize(Obj)
! Output
writeln("Model Solved -  Optimial Objective is ",getobjval)
forall(i in P, j in Po | getsol(x(i,j) ) <> 0) do
        writeln("x",i,j," = ",getsol(x(i,j)))
                end-do
writeln(getsol(sum(i in P,j in Po)p(i,j)*x(i,j)))

writeln("Model Solved -  Optimial Objective is ",getobjval)
forall(i in P, j in Po | getsol(y(i,j) ) <> 0) do
        writeln("y",i,j," = ",getsol(y(i,j)) )
end-do

        writeln(getsol(sum(i in P,j in Po)p(i,j)*y(i,j)))

end-model
```