Investment Decision Analysis Using Markov Decision Processes: A Case Study on Startups (New Application)

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1 Introduction

1.1 Motivation

The economic and venture capital landscape in India is characterized by conservative new angel investors. I want to get into venture capitalism and investment banking. This project explores applications of operations research and financial engineering by applying systematic approaches to assess risks and potential returns in startup investments. Using data from sources such as Preqin, this paper provides a practical use of Markov Decision Processes.

1.2 Goals

The primary goals of this project are to:

- Develop a Markov Decision Process (MDP) model for the problem.
- Maximize expected returns while minimizing associated risks (Expected Rewards).
- Provide investors/VC with an optimal policy they can leverage to make their investment decisions.

2 Problem Statement

2.1 Inputs

1. **Investment Amount:** VC specifies the amount of capital they intend to invest in USD.
2. **Startup Stage:** State/Stage identification of the potential startup VC wants to invest in.
3. **Investment Duration:** VC indicates the number of years they wish to lock their investment for.
4. **Industry Preference:** VC specify the industry they are interested in investing in such as technology, healthcare, finance, etc.

2.2 Assumptions

1. **Rewards Proportional to Valuation Increase:** This assumption implies that we are focusing on the success rate of the startup as the primary driver of rewards. In the code, I have made valuation to be rewarding on a higher state by putting the state as an exponent on a number. I expand on the Valuation calculation in the next subsection.

2. **Transition Probabilities:** Transition probabilities are estimated using historical data on startup transitions.

3. **Holding Costs:** Holding costs are calculated as x% (Rate of Interest in a bank) of the invested amount per year. In the code, I used 5%.

2.3 Calculating Startup Valuation (Uncertainty)

Ways to calculate Valuation - Outside Scope of this class but interesting expansion of this project

1. Identify similar companies in the industry and average their performance metrics, then adjust based on the startup's unique value proposition. This method provides a benchmark for valuation while considering the uncertainty inherent in comparing startups to established companies.

2. Discount future cash flows to their present value using an appropriate discount rate to estimate the startup's valuation.

3. Utilizing ARIMA/exponential smoothing to forecast future revenues and profits.

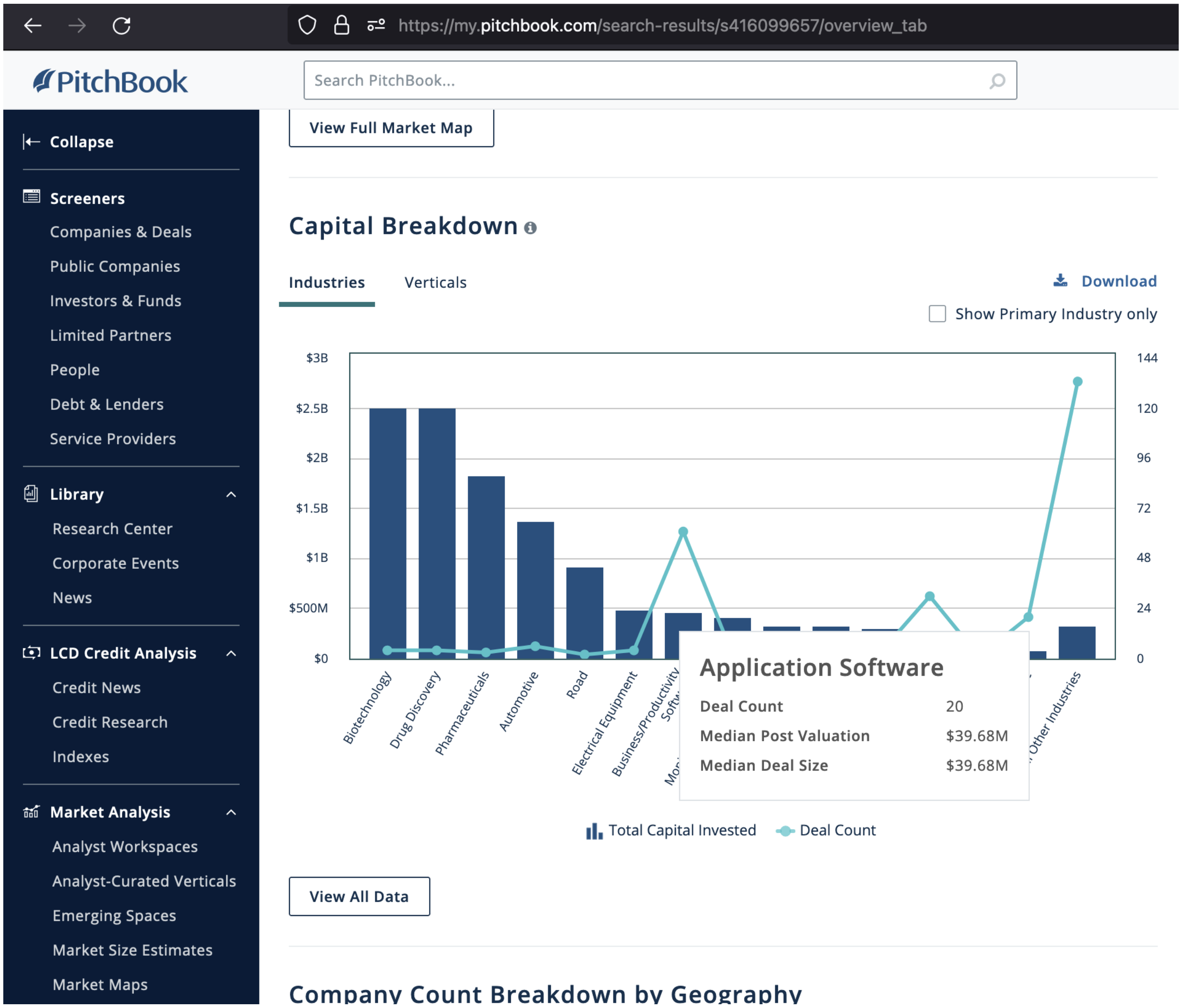


Figure 1: A way to estimate valuation. Source: `https://www.pitchbook.com/`

3 Implementation

3.1 Data Preparation

The historical data was found using the website Preqin.

Information about data

1. Dartmouth Library → Preqin → Companies & deals → Private Equity
2. This data is US based.
3. Data was exported and used for educational purposes.
4. Only essential columns including ”DEAL.DATE”, ”TARGET.COMPANY.ID”, ”DEAL.TYPES”, ”PRIMARY.INDUSTRY”, and ”INVESTORS” were retained for analysis.

5. Only rows corresponding to startup investment rounds (e.g., Seed, Series A, etc.) were retained for further analysis.

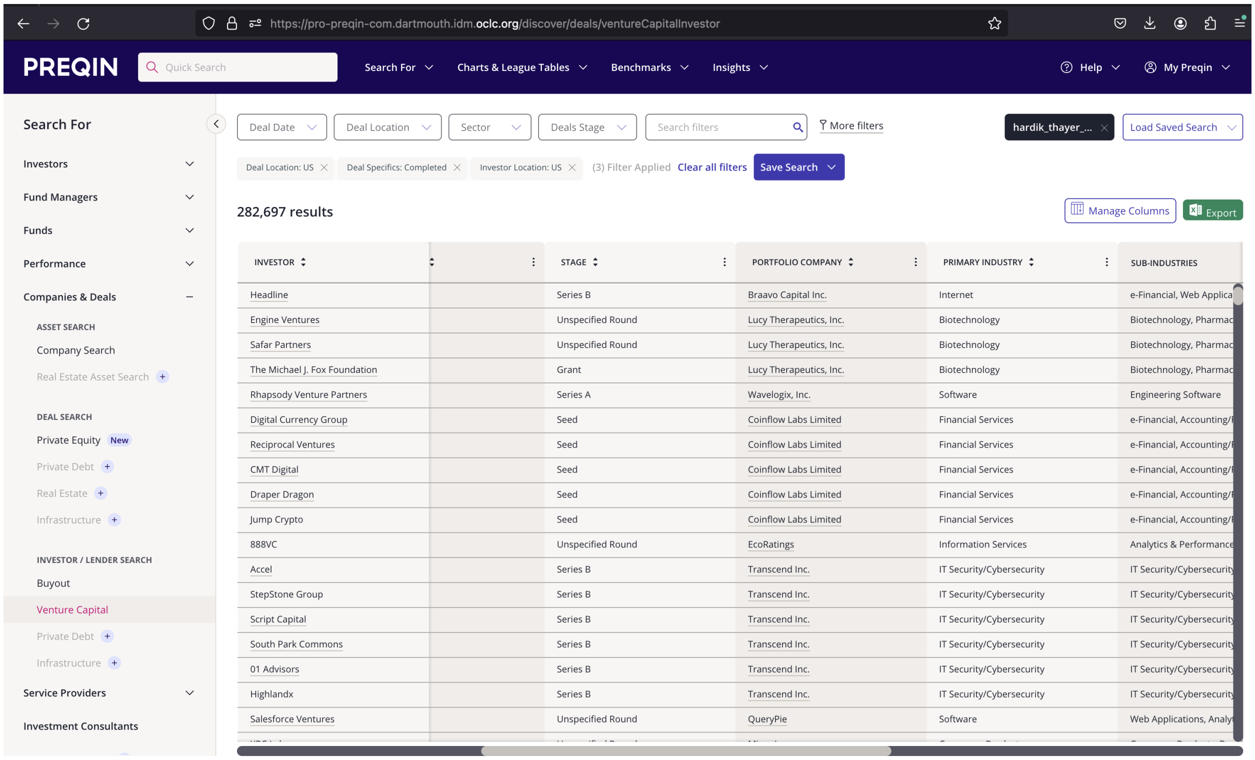
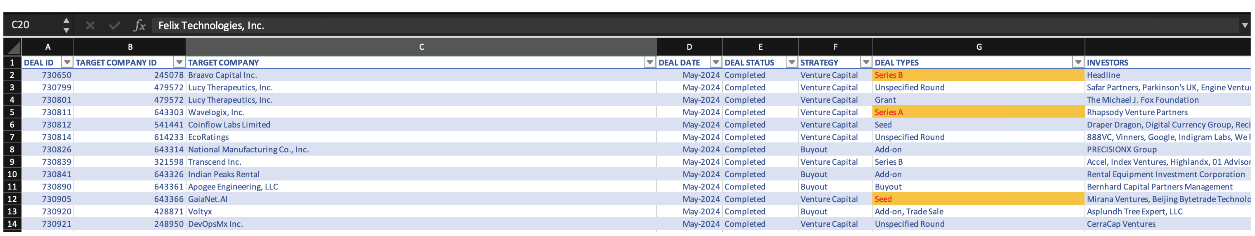


Figure 2: Source: `https://www.preqin.com/`


C20 Felix Technologies, Inc.

	DEAL ID	TARGET COMPANY ID	TARGET COMPANY	DEAL DATE	DEAL STATUS	STRATEGY	DEAL TYPES	INVESTORS
2	730650	245078	Braavo Capital Inc.	May-2024	Completed	Venture Capital	Series B	Headline
3	730799	479572	Lucy Therapeutics, Inc.	May-2024	Completed	Venture Capital	Unspecified Round	Safar Partners, Parkinson's UK, Engine Ventu
4	730801	479572	Lucy Therapeutics, Inc.	May-2024	Completed	Venture Capital	Grant	The Michael J. Fox Foundation
5	730811	643303	Wavelogix, Inc.	May-2024	Completed	Venture Capital	Series A	Rhapsody Venture Partners
6	730812	541441	Coinflow Labs Limited	May-2024	Completed	Venture Capital	Seed	Draper Dragon, Digital Currency Group, Reci
7	730814	614233	EcoRatings	May-2024	Completed	Venture Capital	Unspecified Round	888VC, Vinners, Google, Indigram Labs, We F
8	730826	643314	National Manufacturing Co., Inc.	May-2024	Completed	Buyout	Add-on	PRECISIONX Group
9	730839	321598	Transcend Inc.	May-2024	Completed	Venture Capital	Series B	Accel, Index Ventures, Highlandx, 01 Advisor
10	730841	643326	Indian Peaks Rental	May-2024	Completed	Buyout	Add-on	Rental Equipment Investment Corporation
11	730890	643361	Apogee Engineering, LLC	May-2024	Completed	Buyout	Buyout	Bernhard Capital Partners Management
12	730905	643366	GaiaNet.AI	May-2024	Completed	Venture Capital	Seed	Mirana Ventures, Beijing Bytetrade Technolo
13	730920	428871	Voltyx	May-2024	Completed	Buyout	Add-on, Trade Sale	Asplundh Tree Expert, LLC
14	730921	248950	DevOpsMx Inc.	May-2024	Completed	Venture Capital	Unspecified Round	CerraCap Ventures

Figure 3: Data File

3.2 Markov Decision Process (MDP) Model

3.2.1 States

$$S = \{\text{Seed}, \text{Series A}, \text{Series B}, \text{Series C}, \text{Series D}, \text{Series E}, \text{Series F}, \text{Series G}, \text{Series H}, \text{Series I}, \text{Series J}, \text{Series K}, \text{Series L}, \text{Pre-IPO}, \text{Venture Debt}, \text{Absorption/Exit}\}$$

3.2.2 Actions

Let A be the set of actions available to the investor:

$$A = \{\text{Invest}, \text{Hold}, \text{Exit}\}$$

3.2.3 Transition Probabilities

Let $P(s' \mid s, a)$ be the transition probability of moving from state s to state s' under action a:

$$P : S \times A \times S \to [0, 1]$$

such that:

$$\sum_{s' \in S} P(s' \mid s, a) = 1, \quad \forall s \in S, \forall a \in A$$

Calculation of transitional probabilities:

Algorithm 1 Calculate Transitional Probabilities for Startup Funding Stages

procedure ComputeTransitionProbabilities(Data, InitialStage, Stages)
  $n \leftarrow$ number of startups at InitialStage
  for each $s \in$ Stages **do**
    $count_s \leftarrow$ number of startups transitioning from InitialStage to s
  end for
  $count_{\text{none}} \leftarrow$ number of startups that did not transition anywhere
  $total \leftarrow n$
  for each $s \in \text{Stages} \cup \{\text{none}\}$ **do**
    $P_s \leftarrow \frac{count_s}{total}$
    Print P_s
  end for
end procedure

3.2.4 Rewards

Let $R(s, a)$ be the reward function representing the immediate reward received after taking action a in the state s:

$$R : S \times A \to \mathbb{R}$$

The rewards are defined as follows:

- **Investment Costs** $C_{\text{invest}}(s)$: The negative reward (cost) for investing in a startup at state s:

$$C_{\text{invest}}(s) = -\text{cost}(s)$$

- **Holding Costs** $C_{\text{hold}}(s)$: The negative reward (cost) for holding the investment at state s, calculated as 5% of the invested amount per year:

$$C_{\text{hold}}(s) = -0.05 \times \text{investment}(s)$$

- **Exit Rewards** $R_{\text{exit}}(s)$: The positive reward for exiting the investment at state s, proportional to the valuation of the startup at that stage:

$$R_{\text{exit}}(s) = \text{proportion of share} \times \text{valuation}(s)$$

3.2.5 Objective

The objective is to find the optimal policy π that maximizes the expected total reward over the lifecycle of the investment:

$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{T} R(s_t, \pi(s_t)) \mid s_0\right]$$

where s_t is the state at time t, $\pi(s_t)$ is the action taken in state s_t, and T is the investment horizon.

3.3 Backward Induction

Backward induction was used to solve the MDP and determine the optimal policy. The value function and policy were recursively calculated, considering the discount factor for future rewards.

Algorithm 2 Recursive Backward Induction for Markov Decision Processes

```
function RecursiveBackwardInduction(P, r, rterm, γ, t, S, A, memo, policy)
    if t ∈ memo then
        return memo[t]
    end if
    V ← zeros(S)                                    ▷ Initialize value function for this stage
    if t = size(P, 3) − 1 then                      ▷ Check if at the last stage
        for s = 1 to S do
            Q ← zeros(A)                            ▷ Initialize action-value function
            for a = 1 to A do
                Q[a] ← r[s, t, a] + γ ∑_{sp=1}^{S} P[s, sp, t, a] × rterm[sp]
            end for
            V[s] ← max(Q)
            policy[s, t] ← arg max(Q)
        end for
    else
        for s = 1 to S do
            Q ← zeros(A)
            for a = 1 to A do
                Q[a] ← r[s, t, a] + γ ∑_{sp=1}^{S} P[s, sp, t, a] × RecursiveBackwardInduction(P, r, rterm, γ, t+
    1, S, A, memo, policy)[sp]
            end for
            V[s] ← max(Q)
            policy[s, t] ← arg max(Q)
        end for
    end if
    memo[t] ← V
    return V
end function
```

Algorithm 3 Backward Induction Recursive

```
function BackwardInductionRecursive(P, r, rterm, γ)
    S,, T, A ← dimensions of P
    memo ←
    policy ← zeros(S, T, dtype=int)
    RecursiveBackwardInduction(P, r, rterm, γ, 0, S, A, memo, policy)
    return memo, policy
end function
```

4 Results

4.1 Transition Probabilities

The transition probabilities for the software industry were computed and normalized to reflect realistic scenarios. Below is a sample of the transition matrix for the 'Invest' action:

A snippet of the probabilities in a 30-year window:

$$\begin{bmatrix} \text{2014-05-19 01:00:00} & \text{2024-05-16 01:00:00} & \text{Energy Storage \& Batteries} & \text{Hold} & 14 & 12 & 0.0 \\ \text{2014-05-19 01:00:00} & \text{2024-05-16 01:00:00} & \text{Energy Storage \& Batteries} & \text{Hold} & 14 & 13 & 0.0 \\ \text{2014-05-19 01:00:00} & \text{2024-05-16 01:00:00} & \text{Energy Storage \& Batteries} & \text{Hold} & 14 & 14 & 0.0 \\ \text{2014-05-19 01:00:00} & \text{2024-05-16 01:00:00} & \text{Education/Training} & \text{Hold} & 0 & 0 & 0.5037593984962406 \\ \text{2014-05-19 01:00:00} & \text{2024-05-16 01:00:00} & \text{Education/Training} & \text{Hold} & 0 & 1 & 0.43609022556390975 \\ \text{2014-05-19 01:00:00} & \text{2024-05-16 01:00:00} & \text{Education/Training} & \text{Hold} & 0 & 2 & 0.06015037593984962 \end{bmatrix}$$

4.2 Optimal Policy

The optimal policy derived from the backward induction algorithm is summarized below. The policy indicates the best action (Invest, Hold, Exit) for each state over a 4-year horizon.

Input:

- **T:** 4
- **Investment Amount:** $10,000
- **Interest Rate:** 3%
- **Industry:** Software

Output:

	$t1$	$t2$	$t3$	$t4$
Seed	0	0	0	0
Series A	0	0	0	0
Series B	0	0	0	0
Series C	0	0	0	0
Series D	0	0	0	0
Series E	0	0	0	0
Series F	0	0	0	0
Series G	0	0	0	0
Series H	0	0	0	0
Series I	2	2	2	0
Series J	2	2	2	0
Series K	2	2	2	0
Series L	1	1	1	1
Pre-IPO	2	2	2	0
Venture Debt	0	0	0	0
Absorption/Exit	1	1	1	1

5 Conclusions

The MDP model developed in this project successfully identifies optimal investment strategies for startups in the given industry. Key findings include:

- Holding investments at early stages can be beneficial before making additional investments.
- The transition probabilities reflect realistic scenarios based on historical data.
- The reward function effectively balances the costs and benefits of different actions.

Potential extensions of this project could involve incorporating more sophisticated reward functions, considering additional industries, or analyzing the impact of external economic factors on investment decisions.

6 References

1. Dartmouth Libraries
2. Preqin.com
3. *Series B, C Funding - What It All Means And How It Works.* Accessed: 2023-04-17. 2015. URL: `https://www.investopedia.com/articles/personal-finance/102015/series-b-c-funding-what-it-all-means-and-how-it-works.asp`
4. Vyara Kostadinova et al. "An application of Markov chains in stock price prediction and risk portfolio optimization". In: *AIP Conference Proceedings.* Vol. 2321. 1. University of Ruse Angel Kanchev. Feb. 2021, p. 030018. DOI: `10.1063/5.0041119`. URL: `https://www.researchgate.net/publication/349617762_An_application_of_Markov_chains_in_stock_price_prediction_and_risk_portfolio_optimization`

5. Yijun Shou. “Venture Risk of Small- and Medium-Sized Sci-Tech Enterprises Based on Markov Model”. In: *Wireless Communications and Mobile Computing* 2022 (2022). Ed. by Kalidoss Rajakani. Received 6 May 2022; Revised 17 May 2022; Accepted 27 May 2022; Published 13 June 2022, Article ID 2032771. DOI: 10.1155/2022/2032771. URL: https://www.hindawi.com/journals/wcmc/2022/2032771/

6. Use of CHAT GPT-4:Made the code for cleaning code and told my transitional probability logic. Ended up making a I X S X S X A matrix, where I = Industry type

7. Use of my own HW3 code which I wrote for backward induction.