Binomial distribution — B.P.D

There are two types of distribution

- Discrete prob. distribution — Binomial, Poisson, Geometric
- Continous Prob. distribution — Normal, exponential

In B.P.D — there are n independent trial in an experiment. Let p is prob. of success & q is prob of failure in a single trial $(p+q)=1$

Let X be random variable which denote no. of success

$$\boxed{P(X=r) = {}^nC_r \, p^r \, q^{n-r}, \quad r = 0, 1, 2 - -}$$

Note; 1) n- independent trial, 2) two outcomes only (success or failure)
3) trials are finite

$P(X=0) = {}^nC_0 \, p^0 \, q^n$

$P(X=1) = {}^nC_1 \, p \cdot q^{n-1}$

$P(X=2) = {}^nC_2 \, p^2 \, q^{n-2}$

$\Rightarrow {}^nC_0 q^n, {}^nC_1 \, p \, q^{n-1}, {}^nC_2 \, p^2 q^{n-2} - - {}^nC_n \, p^n$ are coeffn of $(q+p)^n$ = binomial expansion

Recurrence Relation

$$P(r+1) = \frac{n-r}{r+1} \, \frac{p}{q} \left[P(r)\right]$$

Mean of B.P.D

$\boxed{\mu = np}$, p - prob of success

Variance

$\sigma^2 = npq$, q - prob of failure

Q1) **Comment**

For binomial distribution, mean = 6 and variance = 9

Ans) $\mu = \boxed{np = 6}$

$\sigma^2 = npq = 6q = 9$

$q = \frac{\cancel{9}\,3}{\cancel{6}\,2}$, <u>not possible</u>

Q2) A die is thrown 5 times. If getting odd number is success. Find prob. of getting at least 4 success.

Ans) $S = \{1,2,3,4,5,6\}$, fav. cases $\{1,3,5\}$

p = prob of getting odd no. $= \frac{\cancel{3}}{6} \; \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$, $p(\text{success}) = \frac{1}{2}$

$q(\text{failure}) = \frac{1}{2}$

$P(X \geq 4) = P(X=4) + P(X=5)$

$= {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5}$

$= {}^5C_4 \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5 = 5 \times \frac{1}{2^5} + \frac{1}{2^5} = \frac{1}{2^5}(5+1)$

A binomial variable X satisfies the relation $9P(X=4) = P(X=2)$ when $n=6$. Find the value of parameter p and $P(X=1)$

Ans) $P(X=r) = {}^nC_r\, p^r q^{n-r}$

$P(X=4) = {}^6C_4\, p^4 q^{6-4} = {}^6C_4\, p^4 q^2$

$P(X=2) = {}^6C_2\, p^2 q^4$

$\Rightarrow$ ATQ: $9P(X=4) = P(X=2)$

$9 \cdot {}^6C_4\, p^4 q^2 = {}^6C_2\, p^2 q^4$

$9 \cdot \frac{6!}{4!\,2!} p^2 = \frac{6!}{2!\,4!} q^2$

$9p^2 = q^2$

$9p^2 = (1-p)^2$

$9p^2 = p^2 + 1 - 2p$

$8p^2 + 2p - 1 = 0$

$8p^2 + 4p - 2p - 1 = 0$

$4p(2p+1) - 1(2p+1) = 0$, $2p+1=0$, $4p-1=0$

$p = -\frac{1}{2}$ ✗ $\boxed{p = \frac{1}{4}}$

$q = 1 - \frac{1}{4} = \frac{3}{4}$

$P(X=1) = {}^6C_1 \left(\frac{1}{4}\right)^1 \cdot \left(\frac{3}{4}\right)^{6-1}$

$= 6\ \frac{1}{4} \times \left(\frac{3}{4}\right)^5$

Q) Fit a Binomial distribution to the data **2012**

x	0	1	2	3	4
f	24	41	28	5	2

Ans) mean $\mu = np$, $\mu = \dfrac{\Sigma fx}{\Sigma f}$, $N = 100$

x	f	fx
0	24	0
1	41	41
2	28	56
3	5	15
4	2	8
	100	~~115~~ 120

$\mu = \dfrac{115}{100} = 1.15$

$n = 4$ not $n = 5$ as
$n = 0$ means no trial

$p = \dfrac{\mu}{n} = \dfrac{1.15}{4} = .2875$

$\boxed{p = 0.3}$

$\boxed{q = 0.7}$

Binomial D $= \dfrac{N}{1} (q+p)^n$ $N = \Sigma f$

$= 100\,(.7 + 0.3)^4$

12) In 800 families with 5 children each, how many families would be expected to have

1) 3 Boys, 2 girl
2) 2 Boys, 3 girls
3) No girl
4) at most 2 girl, Assume prob. of boy & girl will be equal

Ans) $N = 800$, $p(B) = \frac{1}{2}$, $p(G) = \frac{1}{2}$

(i) $P(3B, 2G) = P(\underline{r=3}) = {}^nC_r\, p^r q^{n-r}$

$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{5}{16}$

$N \cdot P(r=3) = 800 \times \frac{5}{16} = 250$ families

(ii) $P(2B, 3G) = P(r=2) = {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3$

$= 10 \times \frac{1}{2^5} = \frac{5}{16}$

(iii) No of families $= 800 \times \frac{5}{16} = 250$

(iv) no girls : all will be boy

$P(r=5) = {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 = \frac{1}{2^5}$

no. of families $= \frac{1}{2^5} \times 800 = \frac{1}{\cancel{32}_2} \times \cancel{800}^{50}$

$= 25$

(v) at most 2 girl

girl : 0, 1, 2

OR at least 3 boy : $r = 3, 4, 5$

$P = P(r=3) + P(r=4) + P(r=5)$

$= {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 + {}^5C_4 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_3 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3$

$= \left(\frac{1}{2}\right)^5 \left[{}^5C_5 + {}^5C_4 + {}^5C_3 \right]$

Q) Calculate MGF for Binomial distribution
Find mean & Variance

Poission Distribution P.D.

- limiting case of B.D
- If n is large (very) & p is very small

$n \to \infty$, $p \to 0$

$\mu = np$ = In P.D is called λ.

- $P(X=r) = \dfrac{e^{-\lambda}\lambda^{r}}{r!}$, $r = 0, 1, 2, 3 - -$

$\downarrow$ pmf λ = possison distribution parameter $\boxed{\lambda = np}$

$\left[\begin{array}{l} \cdot\ P(x) \geq 0 \\ \cdot\ \Sigma P(x) = 1 \end{array}\right]$

$$\sum_{r=0}^{\infty} P(r) = \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^{r}}{r!},$$

$$= e^{-\lambda} + e^{-\lambda}\frac{\lambda}{1!} + e^{-\lambda}\frac{\lambda^2}{2!}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + -\right]$$

$$= e^{-\lambda}.e^{\lambda} = e^0 = 1$$

its .p.d.f

mean & variance

mean: $\mu = E\{x\} = \sum_{0}^{\infty} x \cdot p(x)$

$$= \sum_{x=0}^{\infty} x \cdot \left(\frac{e^{-\lambda}.\lambda^{x}}{x!}\right) = 0 + e^{-\lambda}\lambda + e^{-\lambda}\lambda^2 + \frac{1}{2}e^{-\lambda}\lambda^3 + --$$

$$= \cancel{e^{-\lambda}(\lambda + \lambda^2 + \tfrac{1}{2}}$$

$$= e^{-\lambda}.\lambda\left(1 + \lambda^2 + \frac{\lambda^2}{2} + \frac{\lambda}{3} + -\right)$$

$$= e^{-\lambda}.\lambda.e^{\lambda} = \lambda$$

$$= \lambda$$

$$\text{variance} = \sigma^2 = E\{x^2\} - [\underbrace{E\{x\}}_{\text{mean}}]^2$$

$$= E\{x^2\} - \mu^2$$

$$= \lambda$$

In this distribution mean & variance are equal

$$\sum_{x=0}^{\infty} x.\frac{e^{-\lambda}\lambda^{x}}{x!} = 0 + e^{-\lambda}\lambda + \frac{2e^{-\lambda}\lambda^2}{2!} + \frac{3e^{-\lambda}\lambda^3}{3!} + \frac{4e^{-\lambda}\lambda^3}{4!} + --$$

$$= e^{-\lambda}\lambda\left[1 + \lambda + \frac{1}{2!}\lambda^2 + \frac{1}{3!}\lambda^3 + --\right] = e^{-\lambda}.\lambda.e^{\lambda} = \lambda$$

$$\sum_{r=0}^{\infty} P(r) = \sum_{r=0}^{\infty} \frac{e^{-\lambda} \lambda^{r}}{r!}$$

$$= e^{-\lambda} + e^{-\lambda} \cdot \lambda + e^{-\lambda} \frac{\lambda^{2}}{2!}$$

$$= e^{-\lambda} \left[1 + \lambda + \frac{\lambda^{2}}{2!} + \cdots \right] = e^{-\lambda} . e^{\lambda} = 1$$

it ~~is~~ is valid pmf

(a) show P.D is a particular limiting case of B.D, when p is very small & n is very large.

$n \to \infty$, $p \to 0$, $\boxed{np = \lambda}$

Q) In a certain factory producing cycle tyres, there is a small chance of 1 in 500 typers to be defective. The tyres are supplied in lots of 10. using p.d, calculate the approximate no. of lots containing no defective, one defective and two defective tyres, respectively in a consignment of 10,000 lots.

A) $p = \frac{1}{500}$ $\quad n = 10$

$m = \lambda = np = \frac{1}{50} = 0.02$

$$P(x) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

$r = 0, 1, 2$

$P(0) = \frac{e^{-0.02} (0.02)^{0}}{0!} = 0.9802$, no. of lots $= 10 \times 000$
$= 0.9802$
$= 9802$

$P(1) = \frac{e^{-0.02} (0.02)^{1}}{1!} = 0.019604$, " $= 196$

$P(2) = \frac{e^{-0.02} (0.02)^{2}}{2!} = 0.00019604$, $= 2$ lots

A manufacturer knows that the condenser he makes contain on an average 1% defective. He packs them in boxes of 100. what is the prob. that a box picked at random will contain 3 or more defective condensor.

A)
$$p = \frac{1}{100} = .01$$

$$\lambda = \text{mean} = np = 100 \times \frac{1}{100} = 1$$

By poisson distribution, the prob. that a box picked at random will contain 3 or more defective condenser is
$$p(r) = \frac{e^{-\lambda} \cdot \lambda^{r}}{r!}$$

Find; $P(3) + P(4) + \text{——} + P(100)$

$$= 1 - P(0) + P(1) + P(2)$$

$$= 1 - \left[\frac{e^{-1} \cdot 1^0}{0!} + \frac{e^{-1} \cdot 1^1}{1!} + \frac{e^{-1} \cdot 1^2}{2!}\right]$$

$$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2}\right] =$$

$$= 1 - \frac{5}{2e} = 0.0803$$

Q) Fit a poisson distribution to the set of observation -

x:	0	1	2	3	4
f:	122	60	15	2	1

Ans) Find mean = ? $N = 200$

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0+60+30+6+4}{200} = \frac{100}{200} = \frac{1}{2}$$

Now theoritical frequency for r success

is $N \cdot \frac{m^r e^{-m}}{r!} = \frac{200 \times (0.5)^r \cdot e^{-0.5}}{r!}$, $e^{-0.5} = 0.6065$

$r = 0, 1, 2, 3, 4$

$$P(0) = \frac{200 \cdot (0.5)^0 e^{-0.5}}{0!} =$$

$$P(1) = \frac{200 (0.5)^1 e^{-0.05}}{1!} =$$

$$P(2) = \frac{200 (0.5)^2 e^{-0.05}}{2!} =$$

$$P(3) = \frac{200 (0.5)^3 e^{-0.05}}{3!} =$$

$$P(4) = \frac{200 (0.5)^4 \cdot e^{-0.05}}{4!} =$$

Hence theoritical frequency