

COMPUTING ASSIGNMENT 4

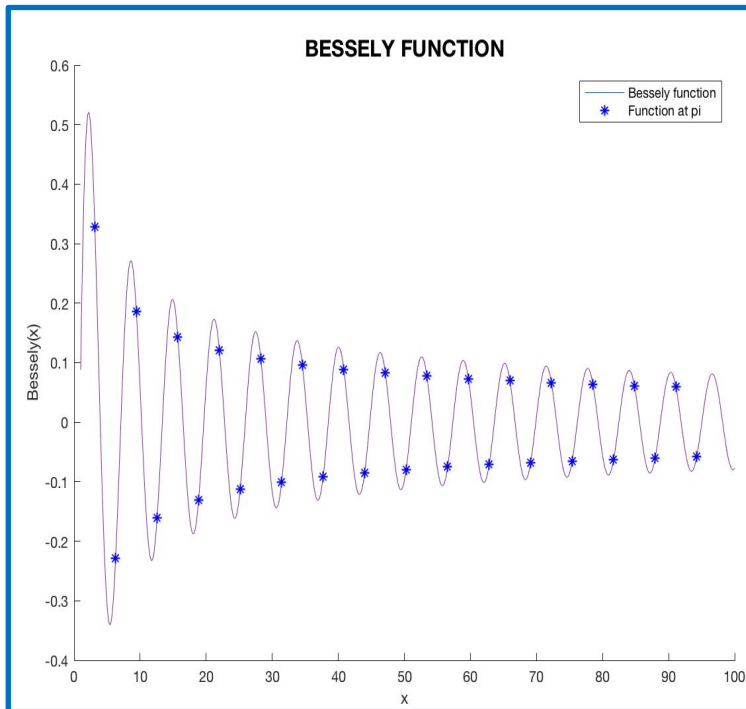


FIG A

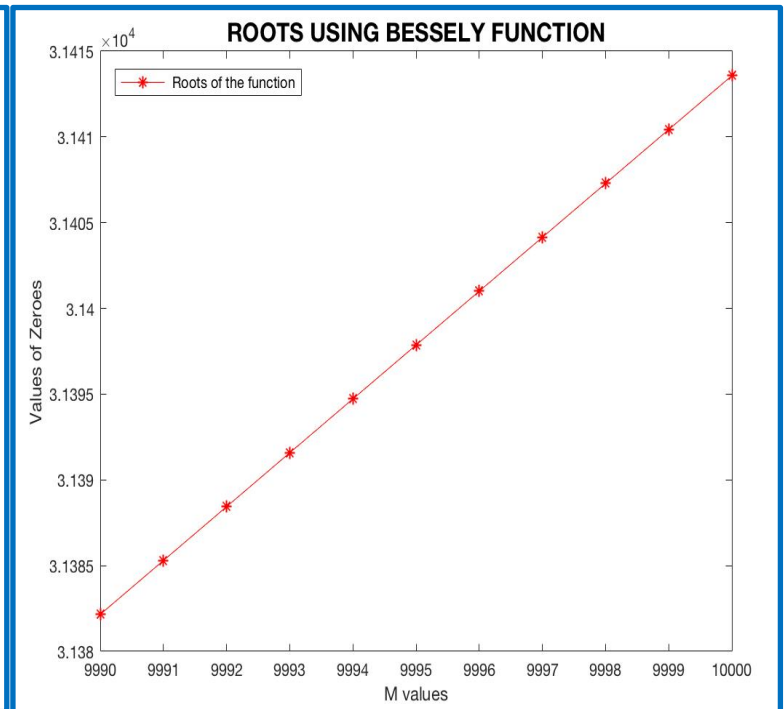


FIG B

To estimate the values for a , b we plot the Bessel function and watch out for the general pattern which gives us an impression of sine curve. The zeros that appeared in the wavelength of the graph are π apart and if plotted with the values of π with the Bessel function, it was totally clear that zeros were π apart which give us the indication to take a and b π apart i.e. $a=0$ and $b = \pi$. **Fig A** shows the graph which are used to identify the values of a and b . To get both accuracy and efficiency, I choose $M = 10000$ as this number does not take too long to compute but definitely helps us to get better and more accurate values. After computing a couple of values for TOL and comparing time and accuracy, $TOL = 0.00001$ was a number small enough to get perfect values. To maintain the efficiency and robustness of the code the maximum number of iterations used are 50 which does not take long to compute and give us a good idea of the trend in the function.

For the values of α and β ,

$\alpha = 3.141592653190592$

$\beta = -0.749997462786269$

In the equation

$$X_{\text{sub } M} = \alpha(M + \beta) + O(1/M)$$

As M approaches to infinity, $O(1/M)$ approached to 0. So as M grows, $O(1/M)$ gets smaller and can be ignored. Since it's hard to get all the 10000 roots we consider the last ten roots to observe the pattern and the difference between the different roots. By the use of polyfit function we can plot the values (**Fig B**) and I also computed the difference between the roots by individually subtracting one root from the other which clearly indicated that with every increasing root the difference between roots approaches to π and thus follow linear behavior. Therefore we compute that with the continuous increase in M , α is getting closer to π .

Hypothesis:

Zeros of the Bessel Function uses the equation:

$$X_{\text{sub } M} = \alpha(M + \beta) + O(1/M)$$

and according to all the calculations above we compute $\alpha = \pi$ and $\beta = -0.75$

CODE FOR FIG (A) TO FIND A AND B

```
clear;
yValues_Function = [];
multiples_Pi = [];
bessely_pi = [];
N = 100;
for n = 1:0.1:N
    yValues_Function = [yValues_Function bessely(0,n)];
end
% Storing the multiples of pi and the value of bessely function at pi
for n = 1:30
    n_pi = n*pi;
    multiples_Pi = [multiples_Pi n_pi];
    bessely_pi = [bessely_pi bessely(0, n_pi)];
end
% Plot of bessely function and
hold on;
plot(1:0.1:N, yValues_Function)
plot(multiples_Pi, bessely_pi, 'b*')
xlabel("x")
ylabel("Bessely(x)")
title("BESSELY FUNCTION", 'fontsize',15)
legend("Bessely function", "Function at pi")
hold off;
```

CODE FOR FIB (B) TO FIND ALPHA AND BETA

```
clear;
format long;
zeroes = [];
a = 0;
b = pi;
M = 10000;
TOL = 0.0001;

for i = 1:M
    y = Bisection(a, b, 50, TOL);
    zeroes = [zeroes y];
    a = b;
    b = b + pi;
end

zeroes_plot = zeroes([M-10:M]);

plot(M-10:M, zeroes_plot, 'r-*')
xlabel("M values")
ylabel("Values of Zeroes")
l = legend('Location', 'northwest');
legend(" Roots of the function")

p1 = polyfit(M-10:M, zeroes(M-10:M), 1)
Alpha = p1(1)
Beta = p1(2)/Alpha
dif_1 = zeroes(2) - zeroes(1)
dif_2 = zeroes(3) - zeroes(2)
dif_3 = zeroes(4) - zeroes(3)

function y = Bisection(a, b, Nmax, TOL)
    for i = 1:Nmax
```

```
y = a + ((b-a)/2);  
  
if (abs(y - 0) >= TOL)  
    if ((sign(bessely(0, a)) * sign(bessely(0, y))) == 1)  
        a = y;  
    else  
        b = y;  
    end  
else  
    break  
end  
end  
end
```
