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Computing Assignment

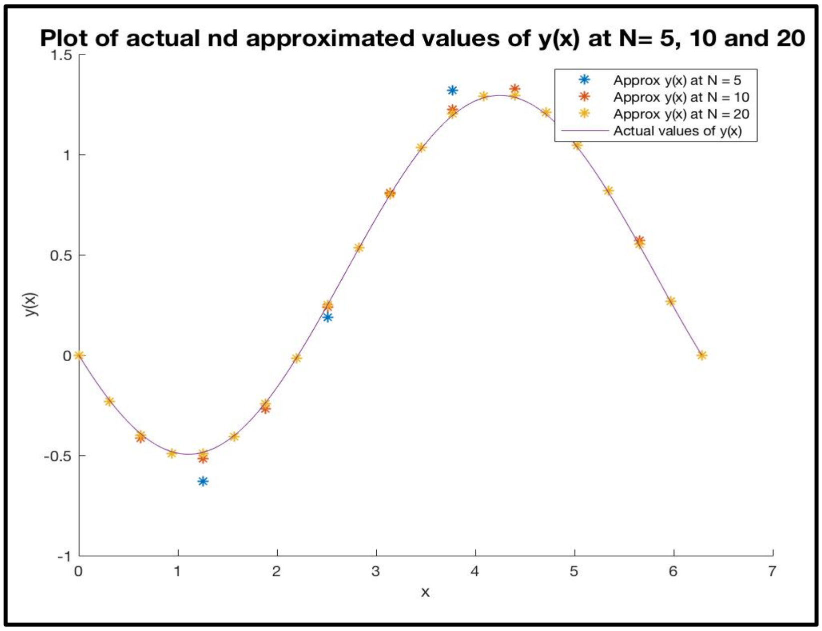

Fig 1

a). Given the following differential equation:

y''(x) + 0:5y'(x) = sin(x);

Solving this equation for y(x) using the second order formula and replacing the values for y' and Y'

$(1/h^2)*A.y + (0.5/2h)*B.y = \sin(x)$

=> $((1/h^2)*A + (0.5/2h)*B).y = \sin(x)$

=> $\mathbf{y = ((1/h^2)*A + (0.5/2h)*B)\backslash \sin(x)}$

b) The Matrix A shown in fig 2 and Matrix B shown in figure 3 are highly sparse. They seem to belong from the family of Band Matrices, or to be more specific to the Toeplitz matrix family which is a subfamily of the band matrices, with the lower as well as the upper bandwidth is equal to 1.

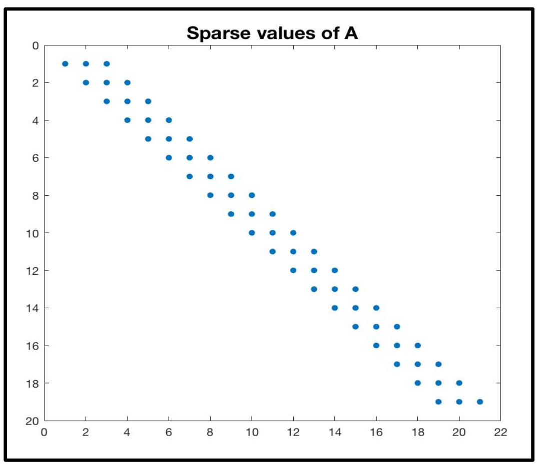

Fig 2

c) Fig 1. represents the plot of the actual values of y(x) and the approximated values for the given values of n = 5, 10 and 20. It is clearly visible that at n = 5 the data points are a little away from the true values but as the number of n increases the data points get closer to the actual values of the function. Therefore, as the value of N increases, the approximation is getting better.

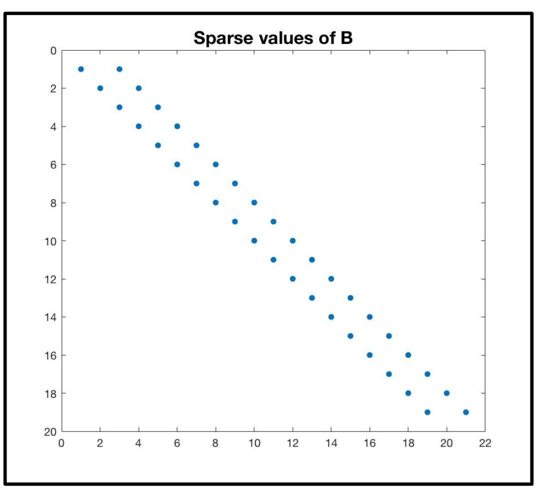

Fig 3

Code

```
n_values = [5, 10, 20];

func = @(x) 0.4*(1-cos(x))-0.8*(sin(x));
figure;
hold on;

for j = 1:3
    n = n_values(j);

    % Creating the matrix B:
    B_first_column = zeros(n-1,1);
    B_first_column(1) = -1;

    B_last_column = zeros(n-1,1);
    B_last_column(n-1) = 1;

    B_temp = full(gallery('tridiag',n-1, -1, 0, 1));
    B = [B_first_column B_temp B_last_column];

    % Creating the matrix A:
    A_first_column = zeros(n-1,1);
    A_first_column(1) = 1;

    A_last_column = zeros(n-1,1);
    A_last_column(n-1) = 1;

    A_temp = full(gallery('tridiag',n-1, 1, -2, 1));
    A = [A_first_column A_temp A_last_column];

    % creating the grid:
    x = zeros(n+1,1);
    for i = 1:n
    x(i + 1) = 0 + ((2*pi)/n)*i;
    end

    % creating the actual y values graph:
    actual_y = zeros(n+1,1);
    for i = 1:n
        k = x(i);
        actual_y(i) = func(k);
    end

    % The difference in the grid
    h = 2*pi/n;

    sin_x = ones(n-1,1);
    for i = 1:n-1
        sin_x(i) = sin(i*h);
    end

    % Approximate values using the differential equation
    coefficients = (1/(h^2))*A + (1/(4*h))* B;
    y_approx = coefficients\sin_x

    plot(x, y_approx, '*')
end

x = 0:pi/100:2*pi;
```

```
plot(x,func(x))
xlabel("x")
ylabel("y(x)")
title("Plot of actual nd approximated values of y(x) at N= 5, 10 and 20 ", 'fontsize', 16)
legend("Approx y(x) at N = 5", "Approx y(x) at N = 10", "Approx y(x) at N = 20", "Actual values
of y(x)")
hold off;

figure
spy(A)
xlabel(" ")
title("Sparse values of A",'fontsize', 16)

figure
spy(B)
xlabel(" ")
title("Sparse values of B", 'fontsize', 16)
```