Please ensure that you include your full name and student number *on your submission.*

Your submission ***must be saved as a "pdf"*** *document and* ***have the filename "lastname_studentid_a2.pdf"*** *(using your last name and student number)*

Your submission ***must have been created using Microsoft Word, Google Docs, or LaTeX****.*

Do NOT compress your submissions *into a "zip" archive.*

Late assignments will not be accepted *and will* ***receive a mark of 0****.*

Submissions ***written by hand****,* ***compressed into an archive*** *or* ***submitted in the wrong format*** *(i.e., not "pdf") will also be penalized and* ***may receive a mark of 0****.*

The due date for this assignment is Saturday, February 15, 2020, by 11:00pm.

1. Determine whether the following arguments are valid or invalid. If it is valid, then you must state the rules of inference used to prove validity, but if it is not then you must provide an example to outline precisely why it is invalid. (n.b., You do not need quantified predicate logic expressions to complete this problem.) (12 marks

    a. If you get regular maintenance on your car, it won't break down. If you care about your car, you make sure it gets regular maintenance. Riley's car broke down. Therefore, he did not care about his car.

    b. If Jordan rides his bike in the winter, he ruins his bike. If Jordan ruins his bike and it is spring, he gets his bike fixed. It is spring but Jordan does not get his bike fixed. Therefore, Jordan did not ride his bike in the winter.

    c. If my office floor is wet, or my office floor is dirty, it is because I wore my boots to school. I wore my boots to school if it is raining outside or if it is cold outside. It is not raining outside, but it is cold. Therefore, my office floor is wet.

2. Starting from the six numbered premises below (which are assumed to be true) and using only the rules of inference (including the instantiation and generalization rules) and the logical equivalences (as both were presented in class), show that $\exists x\, E(x)$. Make sure that you include both the rule and the line number(s) to which that rule is applied. (6 marks)

    a. $\forall x \neg B(x) \rightarrow \neg F(x)$
    b. $\forall x D(x) \vee F(x)$
    c. $\exists x D(x) \vee C(x)$
    d. $\forall x A(x) \rightarrow C(x)$
    e. $\exists x\big(B(x) \vee C(x)\big) \rightarrow E(x)$
    f. $\forall x \neg D(x) \wedge \neg A(x)$

3. Prove, by indirect proof, that if n is an integer and 3n + 3 is odd, then n is even. Show all your work. (4 marks)

4. Prove, using a proof by contradiction, that if n is an integer and $n^2 - 3$ is an odd number for $n \geq 2$, then n is an even number. Show all your work. (4 marks)

5. The definition of a rational number is a number that can be written with the form a/b with the fraction a/b being in lowest form. Prove that $\sqrt{27}$ is an irrational number using a proof by contradiction. You MUST use the approach described in class (and on the supplemental material on cuLearn) and your solution MUST include a lemma demonstrating that if a^2 is divisible by 3 then a is divisible by 3. Hint: reduce $\sqrt{27}$ to the product of two numbers and recall that the product of two rational numbers is a rational number. (8 marks)

6. Use a proof by induction to show that the sum $1 + 3 + 9 + 27 + \cdots + 3^{n-1} = \frac{3^n - 1}{2}$. (6 marks)

7. Assume you are visiting another country where everything you buy requires exact change, but the country only has \$2 and \$3 coins. Use a proof by induction to show that for any product that costs \$2 or more you can always make exact change. (6 marks)