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ASSINGMENT #3
COMP-1805 B
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1) We need to find the all integers that lie in the interval 23,522 and are divisible by 4.

   The numbers are: {24,28,32…………….520}

   Number of terms that are divisible by 4, we can find them by using sequence and series.

   $a = 24$, $a_n=520$
   $d = 4$
   $a_n = a +(n-1)\, d$
   $520 = 24 +(n-1)4$
   $520 = 24 + 4n -4$
   $520 = 20 + 4n$
   $4n = 500$
   $n = 125$
   So, there are 125 integers that are divisible by 4.

   First number that is divisible by 4 between 23 and 522 is 24 and the last number is 520.

   So, the sum is:

   Sum= $\sum_{6}^{130} 4i$

   $= \sum_{6}^{130} 4i$
   $= 4\sum_{6}^{130} i$
   $=4(125)\,(6+130)/2$
   $=34000$

2) The sum of all the integers from 16 to 611. We can find the sum by finding the sum from 1 to 611 and subtracting the sum from 1 to 15 from that.

   $= \sum_{1}^{611} i - \sum_{1}^{15} i$
   $= 611(612)/2 – 15(16)/2$
   $= 186846$
   The sum of all the integers between 16 to 611 is 186846.

   For finding sum from 16 to 611 that are not divisible by either 6 or 21, we need to find the sum of integers from 16 to 611 that is divisible by 6 or by 21. Then subtract the sum from the total sum of integer from 16 to 611. We also have to add back the common integers that are common multiples of 6 or 21.

   Find the total number of integers between 16 to 611 that are divisible by 6. The numbers are {18,24, 30…….606}.

   $a = 18$, $a_n=606$
   $d = 6$
   $a_n = a +(n-1) d$
   $606 = 18 +(n-1)6$
   $606 = 18 + 6n -6$
   $600 = 12 + 6n$
   $6n = 594$
   $n = 99$
   So, there are 99 integers that are divisible by 6.

   The sum of numbers between 16 and 611 that are divisible by 6 is:

   $= \sum_{1}^{101} 6i - \sum_{1}^{2} 6i$
   $= 6(101(102)/2) – 6(2(3)/2)$
   $= 30888$

   Find the total number of integers between 16 to 611 that are divisible by 21. The numbers are {21, 42………609}.

   $a = 21$, $a_n=609$

$d = 21$
$a_n = a + (n-1)\ d$
$609 = 21 + (n-1)21$
$606 = 21 + 21n - 21$
$606 = 21n$
$n = 29$

So, there are 29 integers that are divisible by 21.

The sum of numbers between 16 and 611 that are divisible by 21 is:

$= \sum_{1}^{29} 21i$
$= 21(29(30)/2)$
$= 9135$

Find the total number of integers that are common in division of 6 and 21 that lies in the range of 16, 611:
The total numbers are {42, 84………588)

$a = 42$, $a_n = 588$
$d = 42$
$a_n = a + (n-1)\ d$
$588 = 42 + (n-1)42$
$588 = 42 + 42n - 42$
$n = 14$
So, the total number of integers are 14.

The sum of numbers is:

$= (\sum_{1}^{14} 42i)$
$= 42(14(15)/2)$
$= 4410$

The required Sum according to the question is:
= 186846 – (30888 + 9135) + 4410
= 151233

3) Construct membership tables for each of the following expressions:

(a) $p - (\overline{q \cap r})$

p	q	r	$q \cap r$	$(\overline{q \cap r})$	p-$(\overline{q \cap r})$
1	1	1	1	0	1
1	1	0	0	1	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	0	0
0	1	0	0	1	0
0	0	1	0	1	0
0	0	0	0	1	0

(b) $(\overline{p \cup \bar{r}}) \cup (q - r)$

p	q	r	$\bar{r}$	q-r	p$\cup\bar{r}$	$(\overline{p \cup \bar{r}})$	$(\overline{p \cup \bar{r}}) \cup (q - r)$
1	1	1	0	0	1	0	0
1	1	0	1	1	1	0	1
1	0	1	0	0	1	0	0
1	0	0	1	0	1	0	0
0	1	1	0	0	0	1	1
0	1	0	1	1	1	0	1
0	0	1	0	0	0	1	1
0	0	0	1	0	1	0	0

(c) $((\bar{q} \cup r) - p) \cup (\bar{p} \cap r)$

P	q	r	$\bar{p}$	$\bar{q}$	$\bar{q} \cup r$	$(\bar{q} \cup r) - p$	$\bar{p} \cap r$	$((\bar{q} \cup r) - p) \cup (\bar{p} \cap r)$
1	1	1	0	0	1	0	0	0
1	1	0	0	0	0	0	0	0
1	0	1	0	1	1	0	0	0
1	0	0	0	1	1	0	0	0
0	1	1	1	0	1	1	1	1
0	1	0	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1
0	0	0	1	1	1	1	0	1

(d) $((p \cup r) - ((p \cap \bar{r}) - (q \cap r)))$

p	q	r	$\bar{r}$	$p \cup r$	$p \cap \bar{r}$	$q \cap r$	$((p \cap \bar{r}) - (q \cap r))$	$((p \cup r) - ((p \cap \bar{r}) - (q \cap r)))$
1	1	1	0	1	0	1	0	1
1	1	0	1	1	1	0	1	0
1	0	1	0	1	0	0	0	1
1	0	0	1	1	1	0	1	0
0	1	1	0	1	0	1	0	1
0	1	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	0	1	0	0	0	0	0

4) The membership table for $((Y \cup Z) \cap (\bar{X} \cup Z)) - (Y \cap Z)$.

				$Y \cup Z$	$\bar{X} \cup Z$	$(Y \cup Z) \cap (\bar{X} \cup Z)$	$(Y \cap Z)$	$((Y \cup Z) \cap (\bar{X} \cup Z)) - (Y \cap Z)$
				1	1	1	1	0
				1	0	0	0	0
				1	1	1	0	1
				0	0	0	0	0
				1	1	1	1	0
				1	1	1	0	1
				1	1	1	0	1
				0	1	0	0	0

The membership table for $(Z \cup (Y \cap \bar{X})) \cap ((\overline{(Y \cap Z) \cup X}) \cup ((\overline{Y \cap Z}) \cap X))$.

Let a = $\overline{(Y \cap Z) \cup X)}$

Let b = $(((\overline{(Y \cap Z) \cup X}) \cup ((\overline{Y \cap Z}) \cap \mathrm{X}))$

Let c = $(Z \cup (Y \cap \bar{X})) \cap (((\overline{(Y \cap Z) \cup X}) \cup ((\overline{Y \cap Z}) \cap \mathrm{X}))$

X	Y	Z	$\bar{X}$	$Y \cap \bar{X}$	$Z \cup (Y \cap \bar{X})$	$(Y \cap Z)$	$(\overline{Y \cap Z})$	$(\overline{Y \cap Z}) \cap \mathrm{X})$	$(Y \cap Z) \cup \mathrm{X}$	a	b	c
1	1	1	0	0	1	1	0	0	1	0	0	0
1	1	0	0	0	0	0	1	1	1	0	1	0
1	0	1	0	0	1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	1	1	1	0	1	0
0	1	1	1	1	1	1	0	0	1	0	0	0
0	1	0	1	1	1	0	1	0	0	1	1	1
0	0	1	1	0	1	0	1	0	0	1	1	1
0	0	0	1	0	0	0	1	0	0	1	1	0

So, membership table for both expressions are equal.
Hence proved.

5)

1) $((Y \cup Z) \cap (\bar{X} \cup Z)) - (Y \cap Z)$
2) $Z \cup (Y \cap \bar{X}) - (Y \cap Z)$ (By Distributive,1)
3) $((Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z})$ (By Difference Equivalence,2)

Let p = $((Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z})$

1) $(Z \cup (Y \cap \bar{X})) \cap (((\overline{(Y \cap Z) \cup X}) \cup ((\overline{Y \cap Z}) \cap \mathrm{X}))$
2) $(Z \cup (Y \cap \bar{X})) \cap ((\overline{Y \cap Z}) \cap \bar{X}) \cup ((\overline{Y \cap Z}) \cap \mathrm{X}))$ (By De Morgan's law,1)
3) $(Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z}) \cap (\bar{X} \cup \mathrm{X})$ (By Distributive,2)
4) $(Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z}) \cap \mathrm{U}$ (By Complement,3)
5) $(Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z})$ (By Identity, 4)

Let q = $(Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z})$

p = $((Z \cup (Y \cap \bar{X})) \cap (\overline{Y \cap Z})$ (Proved above)

Hence, p and q are equivalent expressions.
Hence proved

6) To Prove: $(r \cap (r \cup p)) \cap (\overline{r \cup (\overline{p \cup q})})$ represents empty set.

a. $(r \cap (r \cup p)) \cap (\overline{r \cup (\overline{p \cup q})})$
b. $r \cap (\overline{r \cup (\overline{p \cup q})})$ **(By Absorption of a)**
c. $r \cap (\bar{r} \cap (\overline{\overline{p \cup q}}))$ **(By De Morgan's law of b)**
d. $r \cap (\bar{r} \cap (p \cup q))$ **(By Complementation of c)**
e. $(r \cap \bar{r}) \cap (p \cup q)$ **(By Associative of d)**
f. $\emptyset \cap (p \cup q)$ **(By Complement of e)**
g. $\emptyset$ **(By Domination of f)**

Hence, the given expression represents the empty set.