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Assignment #4

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Ans 1)

A) 7 and 9 are cut vertices of the graph after removing vertex 4 as they will disconnect vertex 0 and vertex 6 respectively.

B) The sub – graph G' is G' = (V',E')
Where V' is the set {7,8,11} and E' is the set {7-8,8-11,7-11}

This graph is a complete graph hence belongs to the set K_n and has a value of 3.

Matrix for graph G'

	7	8	11
7	0	1	1
8	1	0	1
11	1	1	0

Ans 2)

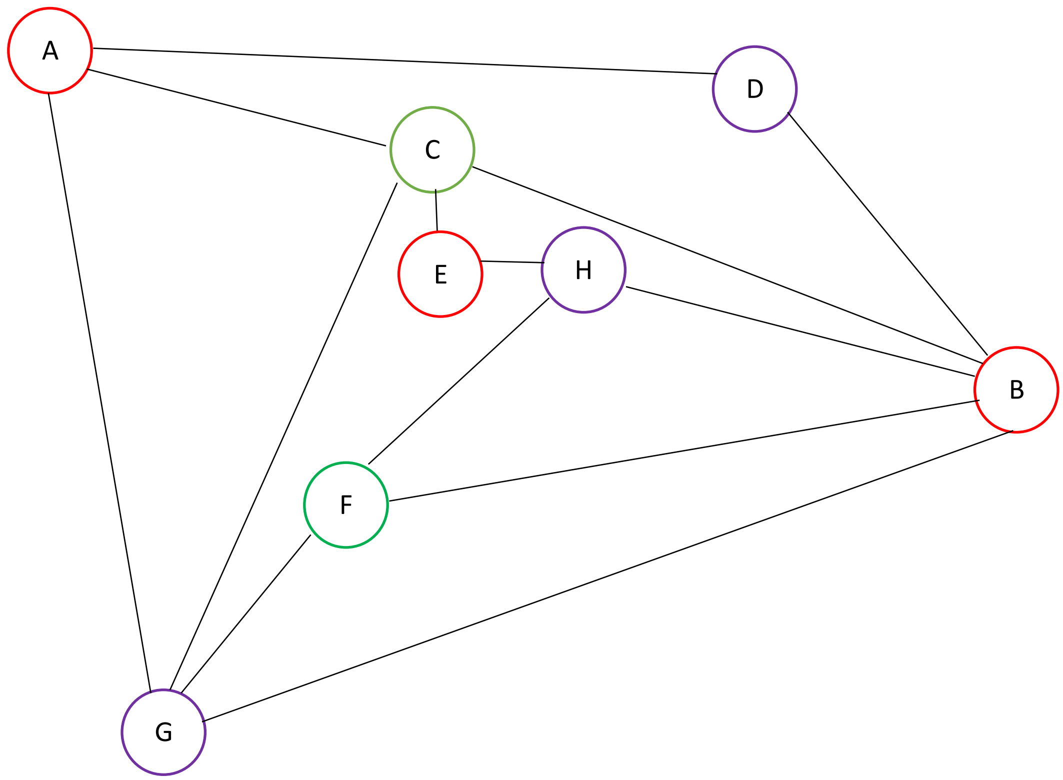
a)

0: -> 2 ->3
1: -> 0 -> 3
2: -> 0 -> 4 -> 6
3: -> 5 -> 6 -> 7
4: -> 2 -> 7
5: -> 6 -> 7
6: -> 5 -> 7
7: -----

b) 0 -> 2 -> 4 -> 7 -> 6 -> 5 -> 3

c) 0 -> 2 -> 3 -> 4 -> 6 -> 5 -> 7

Ans 3) Graph of the matrix is:



Ans 4)

The chromatic number for the above graph is 3. (SHOWN ABOVE)
Let’s take a sub – graph of the above graph, G’ = {B, F, H}
Now this sub – graph belongs to the set K_n, as all the vertices are connected with every other vertex.
As we know in a complete graph every vertex has a different colour hence the minimum chromatic number of a complete graph is the number of vertices.
Vertices in G’ = 3, hence minimum chromatic number = 3
Now as G’ is a sub – graph of G (the above graph), G can’t have chromatic number less than 3

Ans 5)

For each value of “c”, the loop “d” runs “c” times.
For each value of “a”, the loop “c” runs “a” times.

As there is only 1 function in the loop “d”, for each value of “c”, the function will run **1* ” c”** times.

If the loop “c” runs from (1, a+1), then it will have values from 1 to a
And the number of times loop “d” will run = $\sum_{c=1}^{a} \boldsymbol{c}$ times

for each value of “a”, the function “c” will run **1* ”a”** times.
If the loop “a” runs from (1, n+1), then it will have values from 1 to n
And the number of times loop “c” will run $\sum_{a=1}^{n} \boldsymbol{a}$ times
And if loop “c” runs $\sum_{a=1}^{n} a$ times, then loop “d” will run $\sum_{a=1}^{n} \sum_{c=1}^{a} \boldsymbol{c}$ times

Every time "a" loop runs, a foo() function runs
Hence for n runs, foo() function runs $\sum_{a=1}^{n} \mathbf{1}$ times

Total foo() runs $= \sum_{a=1}^{n} \sum_{c=1}^{a} c \quad + \quad \sum_{a=1}^{n} 1$

Solving:

1. $\sum_{a=1}^{n} \sum_{c=1}^{a} c \quad + \sum_{a=1}^{n} 1$
2. $\sum_{a=1}^{n} c(c+1)/2 \quad + \sum_{a=1}^{n} 1$ using $\sum_{i=1}^{n} i = n(n+1)/2$
3. $\sum_{a=1}^{n} (c^2 + c)/2 \quad + \sum_{a=1}^{n} 1$ using math
4. $\frac{1}{2} \sum_{a=1}^{n} c^2 + c \quad + \sum_{a=1}^{n} 1$ using $\sum_{i=1}^{n} 2i = 2 \sum_{i=1}^{n} i$
5. $\frac{1}{2} (\sum_{a=1}^{n} c^2 + \sum_{a=1}^{n} c) + \sum_{a=1}^{n} 1$ using $\sum_{i=1}^{n} i + b = \sum_{i=1}^{n} i + \sum_{i=1}^{n} b$
6. $\frac{1}{2} (\frac{n(n+1)(2n+1)}{6} + \sum_{a=1}^{n} c) \quad + \sum_{a=1}^{n} 1$ using $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
7. $\frac{1}{2} (\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}) + \sum_{a=1}^{n} 1$ using $\sum_{i=1}^{n} i = n(n+1)/2$
8. $\frac{1}{2} (\frac{n(n+1)}{2} (\frac{2n+1}{3} + 1)) + \sum_{a=1}^{n} 1$ using math
9. $\frac{1}{2} (\frac{n(n+1)}{2} (\frac{2n+4}{3})) + \sum_{a=1}^{n} 1$ using math
10. $\frac{1}{2} (\frac{2n^3+6n^2+4n}{6}) + \sum_{a=1}^{n} 1$ using math
11. $\frac{1}{2} (\frac{n^3+3n^2+2n}{3}) + \sum_{a=1}^{n} 1$ using math
12. $(\frac{n^3+3n^2+2n}{6}) + n$ using $\sum_{i=1}^{n} k = nk$
13. $\frac{n^3+3n^2+8n}{6}$ using math

$\frac{n^3+3n^2+2n}{6}$ is the closed form of the above sigma notation

For a value of n =2000, the function foo() would be called $\frac{2000^3+3*2000^2+2*2000}{6}$ = 1,335,334,000 times

Function bar()

It is called at 2 places (Loop "b" and Loop "c")

For loop "c" – each time "c" runs, function bar () is called 2 times.

Value of "c" goes from 1 to a, hence function bar() will be called $\sum_{c=1}^{a} \mathbf{2}$ times

The loop "c" runs "a" times

The loop "a" goes from 1 to n, hence loop "c" will be called $\sum_{a=1}^{n} \boldsymbol{a}$ times

So, the function bar() will be called $\sum_{a=1}^{n} (\sum_{c=1}^{a} \mathbf{2})$ times.

Solving :

1. $\sum_{a=1}^{n} (\sum_{c=1}^{a} 2)$
2. $\sum_{a=1}^{n} 2a$ using $\sum_{i=1}^{n} k = nk$
3. $2 \sum_{a=1}^{n} a$ using $\sum_{i=1}^{n} 2i = 2 \sum_{i=1}^{n} i$
4. $2 * \frac{n(n+1)}{2}$ using $\sum_{i=1}^{n} i = n(n+1)/2$
5. $2 * \frac{n(n+1)}{2}$ using math - equation 1

For loop "b", bar() is called once every time it is called, so for value from 1 to n, the function will be called $\sum_{b=1}^{n} \mathbf{1}$ times

The loop "b" is called "a" times

The loop "a" goes from 1 to n, hence loop "b" will be called $\sum_{a=1}^{n} \boldsymbol{b}$ times

Hence the function bar() will be called $\sum_{a=1}^{n}(\sum_{b=1}^{n} \mathbf{1})$ times

Solving:

1. $\sum_{a=1}^{n}(\sum_{b=1}^{n} 1)$
2. $\sum_{a=1}^{n} n$ using $\sum_{i=1}^{n} k = nk$
3. $n * n$ using $\sum_{i=1}^{n} k = nk$
4. n^2 - equation 2

Adding equation 1 and equation 2

1. $2 * \frac{n(n+1)}{2} + n^2$
2. $n^2 + n + n^2$ using math
3. $2n^2 + n$ by adding

$2n^2 + n$ is the closed from of the above sigma notations

For a value of n =2000, the function bar() would be called $2 * 2000^2 + 2000$ = 8,002,000 times

Ans 6)

a) $3n^2 – 3n + 9$ is O(n)

Proof that $3n^2 – 3n + 9$ is NOT O(n) by contradiction.

1. $\exists c\ \exists k\ \forall n \geq k\ (3n^2 – 3n + 9 \leq cn)$
2. $\exists k\ \forall n \geq k\ (3n^2 – 3n + 9 \leq an)$ by Existential Instantiation,1
3. $\forall n \geq b\ (3n^2 – 3n + 9 \leq an)$ by Existential Instantiation, 2
4. $\forall n \geq b\ (3n^2 – 3n < 3n^2 – 3n + 9)$ by math
5. $\forall n \geq b\ (3n^2 – 3n < an)$ by transitivity, 3,4
6. $\forall n \geq b\ (3n^2 < an + 3n)$ by adding 3n on both sides
7. $\forall n \geq b\ (3n < a + 3)$ by dividing both sides by n
8. $(3(a+3) < a + 3)$ by universal Instantiation,7
9. FALSE Since $3k \nless k$

Hence proved by contradiction

b) $n\,(3n + 4logn - 2)$ is $\Theta(n^2)$

$n\,(3n + 4logn - 2) = 3n^2 + 4n\,logn - 2n$

When $n \geq 1$, $3n^2 + 4n \log n - 2n < 3n^2 + 4n^2 - 2n^2 = 5n^2$

is $O(n^2)$ w/ k = 1, c = 5

When $n \geq 1$, $3n^2 + 4n \log n - 2n > 3n^2$

is $\Omega(n^2)$ w/ k = 1, c =3

c) $n^2\,(3logn+2)$ is $O(n^3)$

$n^2\,(3logn+2) = 3n^2\,logn + 2n^2$

When $n \geq 1$, $3n^2\,logn + 2n^2 < 3n^3 + 2n^3 = 5n^3$

is $O(n^3)$ w/ k = 1, c = 5

d) $n^3 + 5n^2 + \frac{\log n}{n}$ is $O\,(n^3)$

When $n \geq 1$, $n^3 + 5n^2 + \frac{\log n}{n} < n^3 + 5n^3 + \frac{n*n*n*n}{n} = n^3 + 5n^3 + n^3 = 7n^3$

is $O(n^3)$ w/ k = 1, c = 7

e) $3n^2 - 4n - 5n\,logn$ is $\Omega\,(n^2)$

When $n \geq 1$, $3n^2 - 4n - 5n\,logn \geq 3n^2$

is $\Omega(n^2)$ w/ k = 1, c =3

f) $n^5 - 7n^3 + 5n^2\,logn$ is $\Omega\,(n^3)$

When $n \geq 1$, $n^5 - 7n^3 + 5n^2\,logn \geq n^5 \geq n^3$

$n^5 - 7n^3 + 5n^2\,logn \geq n^3$

is $\Omega(n^3)$ w/ k = 1, c =1