***Please ensure that you include your full name and student number** on your submission.*

*Your submission **must be saved as a "pdf"** document and **have the filename "lastname_studentid_a1.pdf"** (using your last name and student number)*

*Your submission **must have been created using Microsoft Word, Google Docs, or LaTeX**.*

***Do NOT compress your submissions** into a "zip" archive.*

***Late assignments will not be accepted** and will **receive a mark of 0**.*

*Submissions **written by hand**, **compressed into an archive** or **submitted in the wrong format** (i.e., not "pdf") will also be penalized and **may receive a mark of 0**.*

The due date for this assignment is Saturday, January 25, 2020, by 11:00pm.

1. Let p be the proposition "I learned to play a musical instrument", q be the proposition "I can read music", and r be the proposition "I own a guitar". Translate the following expressions into English. (1.5 marks)

   a. $p \wedge \neg r$

   I learned to play a musical instrument but I do not own a guitar.

   [Alt] I learned to play a musical instrument and I do not own a guitar.

   b. $p \leftrightarrow q$

   I learned to play a musical instrument if and only if I can read music.

   $(q \wedge r) \rightarrow p$

   If I can read music and I own a guitar, then I must have learned to play a musical instrument.

   [Alt] If I can read music and I own a guitar, then I have learned to play a musical instrument.

2. Translate the following English expressions into logical statements. You must explicitly state what the atomic propositions are (e.g., "Let p be proposition ...") and then show their logical relation. (1.5 marks)

   a. If there are maple trees and it is spring we can make maple syrup.

Let m be the proposition “there are maple trees”.

Let s be the proposition “it is spring”.

Let y be the proposition "we can make maple syrup".

$$(m \land s) \rightarrow y$$

b. You are never bored if you have a cellphone or a good book.

Let b be the proposition "you are bored".

Let c be the proposition "you have a cellphone".

Let g the be proposition "you have a good book".

$$(c \lor g) \rightarrow \neg b$$

c. I have airmiles but I have not flown in an airplane.

Let a be the proposition "I have airmiles."

Let f be the proposition "I have flown in an airplane."

$$(a \land \neg f)$$

3. Determine which of the following are True and demonstrate why or why not by performing a reduction. (1.5 marks)

a. 4 > 3 and 0 < 1 and 6 < 8.

$$(4 > 3) \land (0 < 1) \land (6 < 8)$$

$$(True \land True) \land True$$

$$True \ \land True$$

$$True$$

b. If 2 < 3 or 2 > 7 then 3 < 4.

$$\big((2 < 3) \lor (2 > 7)\big) \rightarrow (3 < 4)$$

$$(True \lor False) \rightarrow True$$

$$True \ \rightarrow True$$

$$True$$

c. 2 + 2 = 4 if and only if 6 = 3 or 5 < 2.

$$(2+2=4) \leftrightarrow \big((6=3) \vee (5<2)\big)$$

$$(4=4) *\!\leftrightarrow (False \vee False)$$

$$True \leftrightarrow (False \vee False)$$

$$True \ \leftrightarrow False$$

$$False$$

**optional*

4. Using only the ¬ and the ∧ operators, find a logical expression that is equivalent to $(p \vee \neg r) \leftrightarrow \ \neg q$. For this question, you do not need to specify "how" you found the equivalent expression because you will show both techniques in questions 5 and 6 below. (0.5 marks)

$$\neg(\neg(\neg p \wedge r) \wedge q) \wedge \neg\big(\neg q \wedge (\neg p \wedge r)\big)$$

5. Prove that the expression you found for question 4 above is equivalent to the expression $(p \vee \neg r) \leftrightarrow \neg q$ by using only truth tables. Show all your work and do not skip any steps (i.e., ensure you that you include a new column for every single operation). (4.0 marks)

$$\neg(\neg(\neg p \wedge r) \wedge q) \wedge \neg(\neg q \wedge (\neg p \wedge r))$$

p	q	r	$\neg$r	p $\vee$ $\neg$r	$\neg$q	(p $\vee$ $\neg$r) $\leftrightarrow$ $\neg$q
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	T	F	T	T	F	F
F	F	T	F	F	T	F
F	F	F	T	T	T	T

p	q	r	$\neg$p	$\neg$p $\wedge$ r	$\neg$ ($\neg$p $\wedge$ r)		$\neg$ ($\neg$p $\wedge$ r) $\wedge$ q		$\neg$ ($\neg$ ($\neg$p $\wedge$ r) $\wedge$ q)
T	T	T	F	F	T		T		F
T	T	F	F	F	T		T		F
T	F	T	F	F	T		F		T
T	F	F	F	F	T		F		T
F	T	T	T	T	F		F		T
F	T	F	T	F	T		T		F
F	F	T	T	T	F		F		T
F	F	F	T	F	T		F		T

p	q	r	$\neg$p	$\neg$p$\wedge$ r	$\neg$q	$\neg$q$\wedge$($\neg$p$\wedge$ r)		$\neg$($\neg$q$\wedge$($\neg$p$\wedge$ r))	$\neg$($\neg$ ($\neg$p $\wedge$ r) $\wedge$ q) $\wedge$ $\neg$($\neg$q$\wedge$($\neg$p$\wedge$ r))
T	T	T	F	F	F	F		T	F
T	T	F	F	F	F	F		T	F
T	F	T	F	F	T	F		T	T
T	F	F	F	F	T	F		T	T
F	T	T	T	T	F	F		T	T
F	T	F	T	F	F	F		T	F
F	F	T	T	T	T	T		F	F
F	F	F	T	F	T	F		T	T

6. Prove that the expression you found for question 4 above is equivalent to the expression $(p \lor \neg r) \leftrightarrow \neg q$ by using only the logical equivalences. Show all your work and do not skip any steps. (4.0 marks)

$((p \lor \neg r) \rightarrow \neg q) \land (\neg q \rightarrow (p \lor \neg r))$	biconditional equivalence	(1)
$(\neg(p \lor \neg r) \lor \neg q) \land (\neg q \rightarrow (p \lor \neg r))$	implication equivalence	(2)
$(\neg(p \lor \neg r) \lor \neg q) \land (\neg\neg q \lor (p \lor \neg r))$	implication equivalence	
$(\neg(p \lor \neg r) \lor \neg q) \land (q \lor (p \lor \neg r))$	double negation	(3)
$((\neg p \land \neg\neg r) \lor \neg q) \land (q \lor (p \lor \neg r))$	DeMorgan's	(4)
$((\neg p \land r) \lor \neg q) \land (q \lor (p \lor \neg r))$	double negation	(5)
$((\neg p \land r) \lor \neg q) \land (q \lor \neg(\neg p \land \neg\neg r))$	DeMorgan's	(6)
$((\neg p \land r) \lor \neg q) \land (q \lor \neg(\neg p \land r))$	double negation	(7)
$((\neg p \land r) \lor \neg q) \land \neg(\neg q \land \neg\neg(\neg p \land r))$	DeMorgan's	(8)
$((\neg p \land r) \lor \neg q) \land \neg(\neg q \land (\neg p \land r))$	double negation	(9)
$\neg(\neg(\neg p \land r) \land \neg\neg q) \land \neg(\neg q \land (\neg p \land r))$	DeMorgan's	(10)
$\neg(\neg(\neg p \land r) \land q) \land \neg(\neg q \land (\neg p \land r))$	double negation	(11)

n.b., the Double Negation step in the process should be considered optional.

7. Determine if the following expressions are tautologies, contradictions, or contingencies by using truth tables. Show all your work. (10.0 marks)

a. $\neg(p \lor q) \lor \neg(p \leftrightarrow q)$

p	q	p ∨ q	¬ (p ∨ q)	$(p \leftrightarrow q)$	$\neg(p \leftrightarrow q)$		$\neg(p \lor q) \lor \neg(p \leftrightarrow q)$	
T	T	T	F	T	F		F	
T	F	T	F	F	T		T	
F	T	T	F	F	T		T	
F	F	F	T	T	F		T	

"Contingency"

b. $(p \rightarrow \neg q) \rightarrow \neg(p \land \neg q)$

"Contingency"

p	q	$\neg q$	$(p \rightarrow \neg q)$	$(p \land \neg q)$	$\neg(p \land \neg q)$		$(p \rightarrow \neg q) \rightarrow \neg(p \land \neg q)$	
T	T	F	F	F	T		T	
T	F	T	T	T	F		F	
F	T	F	T	F	T		T	
F	F	T	T	F	T		T	

c. $((q \rightarrow \neg p) \vee (q \wedge p)) \rightarrow (p \wedge \neg p)$ "Contradiction"

q	p	$\neg$p	$(q \rightarrow \neg p)$	$(q \wedge p)$	$(q \rightarrow \neg p) \vee (q \wedge p)$	$(p \wedge \neg p)$	$((q \rightarrow \neg p) \vee (q \wedge p)) \rightarrow (p \wedge \neg p)$	
T	T	F	F	T	T	F	F	
T	F	T	T	F	T	F	F	
F	T	F	T	F	T	F	F	
F	F	T	T	F	T	F	F	

8. Determine if the following expression is a tautology, a contradiction, or a contingency by using the logical equivalences. You may not use a truth table. Show all your work, labelling each rule used, and do not skip any steps. (4 marks)

a. $(a \rightarrow \neg(b \vee c)) \wedge ((\neg b \rightarrow c) \wedge a)$ *"Contradiction"*

$(a \rightarrow \neg(b \vee c)) \wedge ((\neg b \rightarrow c) \wedge a)$		(1)
$(\neg a \vee \neg(b \vee c)) \wedge ((\neg b \rightarrow c) \wedge a)$	implication equivalence	(2)
$(\neg a \vee (\neg b \wedge \neg c)) \wedge ((\neg b \rightarrow c) \wedge a)$	DeMorgan's law	(3)
$(\neg a \vee (\neg b \wedge \neg c)) \wedge (a \wedge (\neg b \rightarrow c))$	commutitivity	(4)
$(\neg a \vee (\neg b \wedge \neg c)) \wedge (a \wedge (\neg\neg b \vee c))$	implication equivalence	(5)
$(\neg a \vee (\neg b \wedge \neg c)) \wedge (a \wedge (b \vee c))$	double negation	(6)
$(\neg a \wedge (a \wedge (b \vee c))) \vee ((\neg b \wedge \neg c) \wedge (a \wedge (b \vee c)))$	distribution	(7)
$((\neg a \wedge a) \wedge (b \vee c)) \vee ((\neg b \wedge \neg c) \wedge (a \wedge (b \vee c)))$	associativity	(8)
$(F \wedge (b \vee c)) \vee ((\neg b \wedge \neg c) \wedge (a \wedge (b \vee c)))$	negation	(9)
$F \vee ((\neg b \wedge \neg c) \wedge (a \wedge (b \vee c)))$	domination	(10)
$((\neg b \wedge \neg c) \wedge (a \wedge (b \vee c)))$	negation	(11)
$(\neg b \wedge \neg c \wedge a) \wedge (b \vee c)$	associativity	(12)
$((\neg b \wedge \neg c \wedge a) \wedge b) \vee ((\neg b \wedge \neg c \wedge a) \wedge c)$	distribution	(13)
$(\neg b \wedge \neg c \wedge a \wedge b) \vee (\neg b \wedge \neg c \wedge a \wedge c)$	associativity	(14)
$(\neg b \wedge b \wedge \neg c \wedge a) \vee (\neg b \wedge a \wedge \neg c \wedge c)$	commutativity	(15)
$((\neg b \wedge b) \wedge (\neg c \wedge a)) \vee ((\neg b \wedge a) \wedge (\neg c \wedge c))$	associativity	(16)
$(F \wedge (\neg c \wedge a)) \vee ((\neg b \wedge a) \wedge F)$	negation	(17)
$F \vee F$	domination	(18)
F	identity	(19)

Or branch at line 6:

$(\neg a v(\neg b\text{^}\neg c))\text{^}(a\text{^}(bvc))$		(6)
$(\neg a v\neg(bvc))\text{^}(a\text{^}(bvc))$	by DeMorgan's	(7)
$\neg(a\text{^}(bvc))\text{^}(a\text{^}(bvc))$	by DeMorgan's	(8)

False by negation