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Assignment 1
COMP 1805 B
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Ans (1) p = "I learned to play a musical instrument."
q = "I can read music."
r = "I own a guitar."

(a) $p \land \neg r$: "I learned to play a musical instrument **and** I do **not** own the guitar."

(b) $p \leftrightarrow q$: "I learned to play a musical instrument **If and only If** I can read music."

OR

"I can read music **If and only If** I learned to play music."

(c) $(q \land r) \rightarrow$ p: "**If** I can read music and I own a guitar **Then** I learned to play a musical instrument."

Ans (2) (a) Let "t" be the proposition "There are maple trees."

Let "s" be the proposition "it is spring."

Let "m" be the proportion "We can make maple syrup."

The answer to this question is: $(t \land s) \rightarrow m$

(b) Let "b" be the proposition "You are never bored."
Let "c" be the proposition "You have a cellphone."
Let "g" be the proposition "a good book."

The answer to this question is: $b \rightarrow (c \lor g)$

(c) Let "a" be the proposition "I have airmiles."
Let "f" be the proposition "I have flown in an aeroplane."

The answer to this question is: $(a \land \neg f)$

Ans (3) (a) $4>3$ and $0<1$ and $6<8$.

Let “a” be $4>3$.
Let “b” be $0<1$.
Let “c” be $6<8$.

$4>3$ = True
$0<1$ = True
$6<8$ = True

“And” means “conjunction”
Therefore, $(a \wedge b \wedge c)$ = True ∧ True ∧ True = True
The answer is “**True**”

(b) If $2<3$ or $2>7$ then $3<4$.

Let “a” be $2<3$.
Let “b” be $2>7$.
Let “c” be $3<4$.

a = $2<3$ = False
b = $2>7$ = True
c = $3<4$ = True

According to the question
If “a” or “b” then “c”.

Which can be reduced to $(a \vee b) \rightarrow c$,
= (False ∨ True) → True
= True → True (False ∨ True is always True)
= Ture (Implication of True → True is always True)

The answer is **True**.

(c) $2+2=4$ if and only if $6=3$ or $5<2$.

Let “a” be $2+2=4$.
Let “b” be $6=3$.
Let “c” be $5<2$.

a = $2+2=4$ = True
b = $6=3$ = False
c = $5<2$ = False

According to the question:
“a” if and only if “b” or “c”

Which can be reduced to a ↔ (b ∨ c),
= True ↔ (False ∨ False)
= True ↔ False (False disjunction with False is False)
= False (True biconditional with False is False)

The answer is **False**.

Ans (4) The answer to the question is:

(¬ (q ∧ p) ∧ ¬ (q ∧ ¬ r)) ∧ ¬ (¬ q ∧ (¬ p ∧ r))

Ans (5)

The Truth table for the expression: $(p \vee \neg r)$ ↔ ¬q

p	q	r	¬ r	¬ q	p ∨ ¬ r	(p ∨ ¬ r) ↔ ¬ q
T	T	T	F	F	T	F
T	T	F	T	F	T	F
T	F	T	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	T	F	T	F	T	F
F	F	T	F	T	F	F
F	F	F	T	T	T	T

The Truth table for the expression: (¬ (q ∧ p) ∧ ¬ (q ∧ ¬ r)) ∧ ¬ (¬ q ∧ (¬ p ∧ r))

q ∧ p	¬ (q ∧ p) (X)	q ∧ ¬ r	¬ (q ∧ ¬ r) (Y)	X ∧ Y	¬ p	(¬ p ∧ r)	¬ q ∧ (¬ p ∧ r). (Z)	¬ Z	(X∧ Y) ∧¬ Z
T	F	F	T	F	F	F	F	T	F
T	F	T	F	F	F	F	F	T	F
F	T	F	T	T	F	F	F	T	T
F	T	F	T	T	F	F	F	T	T
F	T	F	T	T	T	T	F	T	T
F	T	T	F	F	T	F	F	T	F
F	T	F	T	T	T	T	T	F	F
F	T	F	T	T	T	F	F	T	T

The result of both the Truth Tables are same, hence the expression $(p \vee \neg r)$ ↔ ¬q is equal to the expression (¬ (q ∧ p) ∧ ¬ (q ∧ ¬ r)) ∧ ¬ (¬ q ∧ (¬ p ∧ r)).

Ans (6) The expression is: $(p \vee \neg r) \leftrightarrow \neg q$

The result expression is: $(\neg (q \wedge p) \wedge \neg (q \wedge \neg r)) \wedge \neg (\neg q \wedge (\neg p \wedge r))$

According to the question.

$= (p \vee \neg r) \leftrightarrow \neg q$

$= ((p \vee \neg r) \rightarrow \neg q) \wedge (\neg q \rightarrow (p \vee \neg r))$ (Biconditional Equivalence)

$=(\neg (p \vee \neg r) \vee (\neg q)) \wedge (\neg (\neg q) \vee (p \vee \neg r))$ (Implication Equivalence)

$=((\neg p \wedge \neg(\neg r)) \vee \neg q) \wedge (\neg (\neg q) \vee (p \vee \neg r))$ (De morgan's law)

$= ((\neg p \wedge r) \vee \neg q) \wedge (\neg (\neg q) \vee (p \vee \neg r))$ (Double negation)

$= ((\neg p \wedge r) \vee \neg q) \wedge (q \vee (p \vee \neg r))$ (Double Negation)

$= (\neg q \vee (\neg p \wedge r)) \wedge (q \vee (p \vee \neg r))$ (Commutative)

$= ((\neg q \vee \neg p) \wedge (\neg q \vee r)) \wedge (q \vee (p \vee \neg r))$ (Distributive)

$= \{(\neg [\neg [\neg q] \wedge \neg (\neg p)]) \wedge \neg (\neg [\neg q] \wedge \neg r)\} \wedge \neg (\neg q \wedge \neg (p \vee \neg r))$

(Inverse of De Morgan's Law)

$= \{(\neg [q \wedge p]) \wedge \neg (q \wedge \neg r)\} \wedge \neg (\neg q \wedge \neg (p \vee \neg r))$

(Double Negation)

$= \{(\neg [q \wedge p]) \wedge \neg (q \wedge \neg r)\} \wedge \neg \{\neg q \wedge (\neg p \wedge \neg (\neg r))\}$

(De Morgan's law)

$= \{(\neg [q \wedge p]) \wedge \neg (q \wedge \neg r)\} \wedge \neg \{\neg q \wedge (\neg p \wedge r)\}$

(Double Negation)

Hence, the expression $(p \vee \neg r) \leftrightarrow \neg q$ has been proved equal to $(\neg (q \wedge p) \wedge \neg (q \wedge \neg r)) \wedge \neg (\neg q \wedge (\neg p \wedge r))$.

Ans (7) (a) $\neg (p \vee q) \vee \neg (p \leftrightarrow q)$

p	q	p ∨ q	¬ (p ∨ q) (X)	p ↔ q	¬ (p ↔ q) (Y)	X ∨ Y
T	T	T	F	T	F	F
T	F	T	F	F	T	T
F	T	T	F	F	T	T
F	F	F	T	T	F	T

Hence the above expression is the **contingency.**

(b) $(p \rightarrow \neg q) \rightarrow \neg(p \wedge \neg q)$

p	q	¬q	p→¬q **(X)**	p ∧ ¬ q	¬ (p∧¬ q) **(Y)**	X → Y
T	T	F	F	F	T	T
T	F	T	T	T	F	F
F	T	F	T	F	T	T
F	F	T	T	F	T	T

Hence the above expression is the **contingency.**

(c) $((q \rightarrow \neg p) \vee (q \wedge p)) \rightarrow (p \wedge \neg p)$

p	q	¬ p	(q → ¬p) (X)	(q ∧ p) (Y)	(X ∨ Y) (Z)	p ∧ ¬ p (A)	Z → A
T	T	F	F	T	T	F	F
T	F	F	T	F	T	F	F
F	T	T	T	F	T	F	F
F	F	T	T	F	T	F	F

Hence the above expression is the **Contradiction.**

Ans (8) $(a \rightarrow \neg(b \vee c)) \wedge ((\neg b \rightarrow c) \wedge a)$

$= (\neg a \vee \neg (b \vee c)) \wedge \{(\neg(\neg b) \vee c) \wedge a\}$ (Implication equivalence)

$= (\neg a \vee \neg (b \vee c)) \wedge ((b \vee c) \wedge a)$ (double negation)

$= \neg (a \wedge (b \vee c)) \wedge ((b \vee c) \wedge a)$ (inverse of De Morgan's Law)

$= \neg (a \wedge (b \vee c)) \wedge (a \wedge (b \vee c))$ (Commutative)

$=$ False (Negation)

Hence the expression above is always **False.**
Hence it is contradiction.