Please ensure that you ***include your name and student number*** *on your submission.*

Your submission ***must be created using Microsoft Word, Google Docs, or LaTeX****.*

Your submission ***must be saved as a "pdf"*** *document and* ***have the filename "lastname_studentid_a3.pdf"*** *(using your last name and student number)*

Do not compress your submission *into a "zip" file.*

Late assignments will not be accepted *and will receive a mark of 0.*

Submissions ***written by hand****,* ***compressed into an archive****, or submitted in the* ***wrong format*** *(i.e., not "pdf") will be penalized and* ***may receive a mark of 0.***

The due date for this assignment is March 7, 2020, by 11:00pm.

1. What is the sum of all integers between 23 and 522 (inclusive) that are divisible by 4? You must use Sigma notation and the rules discussed in class to solve this problem and you must show all your work. (4 marks)

The first number in that range divisible by 4 is 24 = 6(4), and the last number is 520 = 130(4)

$$\sum_{i=6}^{130} 4i$$
$$= 4\sum_{i=6}^{130} i$$
$$= 4\left(\sum_{i=1}^{130} i - \sum_{i=1}^{5} i\right)$$
$$= 4\left(\frac{130(131)}{2} - \frac{5(6)}{2}\right)$$
$$= 34\,000$$

2. What is the sum of all integers between 16 and 611 (inclusive) that are not divisible by either 6 or 21? You must use Sigma notation and the rules discussed in class to solve this problem and you must show all your work. (6 marks)

We will sum the numbers from 16 to 611, then subtract those divisible by 6 and 21. Since 6 and 21 have a common factor of 3, we must add all numbers divisible by 3 back in.

First number in the range divisible by 6: $18 = 6(3)$
Last number in the range divisible by 6: $606 = 6(101)$

First number in the range divisible by 21: $21 = 21(1)$
Last number in the range divisible by 21: $609 = 21(29)$

First number in the range divisible by 3: $18 = 3(6)$
Last number in the range divisible by 3: $609 = 3(203)$

$$\sum_{i=16}^{611} i - \sum_{i=3}^{101} 6i - \sum_{i=1}^{29} 21i + \sum_{i=6}^{203} 3i$$

$$= \left(\sum_{i=1}^{611} i - \sum_{i=1}^{15} i\right) - 6\sum_{i=3}^{101} i - 21\sum_{i=1}^{29} i + 3\sum_{i=6}^{203} i$$

$$= \left(\frac{611(612)}{2} - \frac{15(16)}{2}\right) - 6\sum_{i=3}^{101} i - 21\sum_{i=1}^{29} i + 3\sum_{i=6}^{203} i$$

$$= \left(\frac{611(612)}{2} - \frac{15(16)}{2}\right) - 6\left(\sum_{i=1}^{101} i - \sum_{i=1}^{2} i\right) - 21\sum_{i=1}^{29} i + 3\left(\sum_{i=1}^{203} i - \sum_{i=1}^{5} i\right)$$

$$= \left(\frac{611(612)}{2} - \frac{15(16)}{2}\right) - 6\left(\frac{101(102)}{2} - \frac{2(3)}{2}\right) - 21\frac{29(30)}{2} + 3\left(\frac{203(204)}{2} - \frac{5(6)}{2}\right)$$

$$= 208\,896$$

3. Construct membership tables for each of the following expressions:

a. $p - (\overline{q \cap r})$

p	q	r	$q \cap r$	$\overline{(q \cap r)}$	$p - \overline{(q \cap r)}$
1	1	1	1	0	1
1	1	0	0	1	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	0	0
0	1	0	0	1	0
0	0	1	0	1	0
0	0	0	0	1	0

b. $\overline{(p \cup \overline{r})} \cup (q - r)$

p	q	r	$\overline{r}$	$(p \cup \overline{r})$	$\overline{(p \cup \overline{r})}$	$q - r$	$\overline{(p \cup \overline{r})} \cup (q - r)$
1	1	1	0	1	0	0	0
1	1	0	1	1	0	1	1
1	0	1	0	1	0	0	0
1	0	0	1	1	0	0	0
0	1	1	0	0	1	0	1
0	1	0	1	1	0	1	1
0	0	1	0	0	1	0	1
0	0	0	1	1	0	0	0

c. $\big((\overline{q} \cup r) - p\big) \cup (\overline{p} \cap r)$

p	q	r	$\overline{q}$	$(\overline{q} \cup r)$	$(\overline{q} \cup r) - p$	$\overline{p}$	$\overline{p} \cap r$	$\big((\overline{q} \cup r) - p\big) \cup (\overline{p} \cap r)$
1	1	1	0	1	0	0	0	0
1	1	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0
1	0	0	1	1	0	0	0	0
0	1	1	0	1	1	1	1	1
0	1	0	0	0	0	1	0	0
0	0	1	1	1	1	1	1	1
0	0	0	1	1	1	1	0	1

d. $\Big((p \cup r) - \big((p \cap \overline{r}) - (q \cap r)\big)\Big)$

p	q	r	$\overline{r}$	$p \cap \overline{r}$	$q \cap r$	$(p \cap \overline{r}) - (q \cap r)$	$(p \cup r)$	$\Big((p \cup r) - \big((p \cap \overline{r}) - (q \cap r)\big)\Big)$
1	1	1	0	0	1	0	1	1
1	1	0	1	1	0	1	1	0
1	0	1	0	0	0	0	1	1
1	0	0	1	1	0	1	1	0
0	1	1	0	0	1	0	1	1
0	1	0	1	0	0	0	0	0
0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	0

(5 marks each)

4. Use membership tables (i.e., no set identities or Venn diagrams) to demonstrate that

$$\left((Y \cup Z) \cap \left(\overline{X} \cup Z\right)\right) - (Y \cap Z)$$

and

$$\left(Z \cup \left(Y \cap \overline{X}\right)\right) \cap \left(\overline{\left((Y \cap Z) \cup X\right)} \cup \left(\left(\overline{Y \cap Z}\right) \cap X\right)\right)$$

are equivalent expressions. (5 marks)

X	Y	Z	$Y \cup Z$	$\overline{X}$	$\overline{X} \cup Z$	$\left((Y \cup Z) \cap \left(\overline{X} \cup Z\right)\right)$	$Y \cap Z$	$\left((Y \cup Z) \cap \left(\overline{X} \cup Z\right)\right) - (Y \cap Z)$
1	1	1	1	0	1	1	1	0
1	1	0	1	0	0	0	0	0
1	0	1	1	0	1	1	0	1
1	0	0	0	0	1	0	0	0
0	1	1	1	1	1	1	1	0
0	1	0	1	1	1	1	0	1
0	0	1	1	1	1	1	0	1
0	0	0	0	1	0	0	0	0

X	Y	Z	$Y \cap Z$	$(Y \cap Z) \cup X$	$\overline{(Y \cap Z) \cup X}$	$\overline{Y \cap Z}$	$\left(\overline{Y \cap Z}\right) \cap X$	$\overline{((Y \cap Z) \cup X)} \cup \left(\left(\overline{Y \cap Z}\right) \cap X\right)$
1	1	1	1	1	0	0	0	0
1	1	0	0	1	0	1	1	1
1	0	1	0	1	0	1	1	1
1	0	0	0	1	0	1	1	1
0	1	1	1	1	0	0	0	0
0	1	0	0	0	1	1	0	1
0	0	1	0	0	1	1	0	1
0	0	0	0	0	1	1	0	1

X	Y	Z	$\overline{X}$	$Y \cap \overline{X}$	$\left(Z \cup \left(Y \cap \overline{X}\right)\right)$	$\overline{((Y \cap Z) \cup X)} \cup \left(\left(\overline{Y \cap Z}\right) \cap X\right)$	$\left(Z \cup \left(Y \cap \overline{X}\right)\right) \cap \left(\overline{((Y \cap Z) \cup X)} \cup \left(\left(\overline{Y \cap Z}\right) \cap X\right)\right)$
1	1	1	0	0	1	0	0
1	1	0	0	0	0	1	0
1	0	1	0	0	1	1	1
1	0	0	0	0	0	1	0
0	1	1	1	1	1	0	0
0	1	0	1	1	1	1	1
0	0	1	1	0	1	1	1
0	0	0	1	0	0	1	0

5. Use set identities (i.e., no membership tables or Venn diagrams) to demonstrate that

$$\left((Y \cup Z) \cap (\overline{X} \cup Z)\right) - (Y \cap Z)$$

and

$$\left(Z \cup (Y \cap \overline{X})\right) \cap \left(\overline{((Y \cap Z) \cup X)} \cup \left((\overline{Y \cap Z}) \cap \mathrm{X}\right)\right)$$

are equivalent expressions. Since these are the same expressions presented in question 4, you should use your membership table in question 4 to verify your answer. (5 marks)

$$\left(Z \cup (Y \cap \overline{X})\right) \cap \left(\overline{((Y \cap Z) \cup X)} \cup \left((\overline{Y \cap Z}) \cap \mathrm{X}\right)\right)$$

$\left(Z \cup (Y \cap \overline{X})\right) \cap \left((\overline{(Y \cap Z)} \cap \bar{X}) \cup \left((\overline{Y \cap Z}) \cap \mathrm{X}\right)\right)$ by DeMorgan's Law

$\left(Z \cup (Y \cap \overline{X})\right) \cap \left(\overline{(Y \cap Z)} \cap (\bar{X} \cup \mathrm{X})\right)$ by Distributivity

$\left(Z \cup (Y \cap \overline{X})\right) \cap \left(\overline{(Y \cap Z)} \cap universe\right)$ by Complement

$\left(Z \cup (Y \cap \overline{X})\right) \cap \overline{(Y \cap Z)}$ by Identity

$\left(Z \cup (Y \cap \overline{X})\right) - (Y \cap Z)$ by Difference Equivalence

$\left(Z \cup (Y \cap \overline{X})\right) - (Y \cap Z)$ by Difference Equivalence

$\left((Y \cup Z) \cap (\overline{X} \cup Z)\right) - (Y \cap Z)$ by Distributivity

6. Prove that the expression:

$$\big(r \cap (r \cup p)\big) \cap \overline{(r \ \cup \overline{(p \cup q)})}$$

actually represents the empty set.

(4 marks)

$$\big(r \cap (r \cup p)\big) \cap \overline{(r \ \cup \overline{(p \cup q)})}$$

$\big(r \cap (r \cup p)\big) \cap \left(\bar{r} \cap \overline{\overline{(p \cup q)}}\right)$ by DeMorgan's Law

$\big(r \cap (r \cup p)\big) \cap (\bar{r} \cap (p \cup q))$ by Double Complement

$\big(r \cap (r \cup p)\big) \cap (\bar{r} \cap (p \cup q))$ by Double Complement

$(r) \cap (\bar{r} \cap (p \cup q))$ by Absorption

$(\emptyset \cap (p \cup q))$ by Complement

$\emptyset$ by Domination