- Gravity

- when the object is very far from the Earth we consider Object in the gravitation and when at the surface of the Earth we consider as Gravity.

- The gravity differs from place to place on the Earth it's maximum value in at the pole which is 9.83 m/s² and at the Equator it is 9.78 mis² and the center of the Earth in 0 (zero).

- When the object falls due to gravity on the surface the velocity is continuous increasing but the rate at conic the velocity increased remains constant.

- let us consider the Earth has mass 'M' and the Object of mass 'm' it at the height 'n' from the surface of the Earth then the graviton force between Earth and the object is

$$f = \frac{GMm}{(R+H)}$$

- The force is due to the Earth on the object

$$mg = \frac{GMm}{(R+H)^2}$$

$$g=\frac{GM}{(R+H)^2}$$

- At the surface of the earth(R+H) is approximately is equal to R. then

  the gravity becomes

$$g=\frac{GM}{R^2}$$

$$g \propto \frac{1}{R^2}$$

- Variation of gravity with altitude (Height)

- Let us consider the mass of the Earth is 'M' and 'R' is the radius of the Earth. An Object in at height ‘H’ from the surface of the Earth. Then from the Relation of the gravity at the surface of the earth is

g=$\frac{Gm}{R^2}$ → eqn(i)

Now for the height ‘H’ the gravity will be

g’=$\frac{Gm}{(R+H)^2}$

Now ,dividing the eqn (i) by eqn (ii)

$$\frac{g}{g'}=\frac{\frac{Gm}{R^2}}{\frac{Gm}{(R+H)^2}}$$

$$\frac{g}{g'}=\frac{Gm}{R^2}\times\frac{(R+H)^2}{Gm}$$

$$\frac{g}{g'}=\frac{(R+H)^2}{R^2}$$

$$\frac{g}{g'}=\frac{R^2(1+\frac{H}{R})^2}{R^2}$$

$$\frac{g}{g'}=(\frac{1+H}{R})^2$$

$$g'=\frac{g}{(1+\frac{H}{R})^2}$$

$$g' = g(1 + \frac{H}{R})^{-2}$$

Now,

Using the binomial expansion and neglecting higher power.

Then,

$$g' = (\frac{1-2h}{r})^2$$

→ Conditions

(i).at the surface of the earth

(ii).if H is greater than 1

(iii).if g' is less than g (g'<g)

Hence, if the height increases gravity decreases.

For example;

1. Calculate the value of gravity at the height 100 km and 500 km above the surface of the earth.

   Solution;

$$g' = g(1 - \frac{2 \times 100000}{6400000}$$

$$g' = 9.8(1 - \frac{200000}{6400000}$$

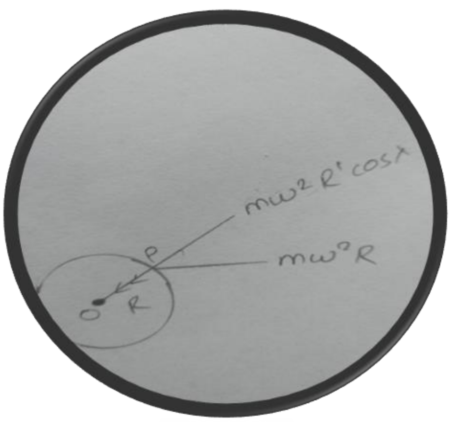
$$g' = 9.49m\backslash s^2$$

For 500 km

$$g' = 9.8(1 - \frac{2 \times 500000}{6400000})$$

$$g' = 9.8(1 - \frac{10}{64})$$

$$g' = 8.26m\backslash s^2$$




The net force towards the centre of the centre of the earth will be

mg'=mg'-$mw^2rcos$ λ

$$g' = g - w^2r'cos\lambda \rightarrow (i)$$

From the triangle OBP

$$cos\lambda = \frac{r'}{r}$$

$$r' = rcos\lambda \rightarrow (ii)$$

Now,

Substituting the value in r' in eqn(i)

$$g' = g - w^2 r' cos\lambda$$

At pole

λ = 90°

-g'=g-$w^2 r \times 0$

$$g' = g$$

At equator

$$\lambda = 0$$

$$g' = g - w^2 r$$

Conclusion: the gravity at equator is less than gravity at the pole.

- Gravitational field

  → The reason around the Earth up to which the Effect of gravity can be experienced in known as gravitational field

- Gravitational field intensity.

  → The Force Experience by the mass in the gravitational field is known as Gravitational field intensity

  $$I=\frac{Gm}{R^2}$$

  At the surface of the Earth the gravitational Field intensity will be

  $\mathbf{I=\frac{Gm}{R^2}}$ which is numerically equal to acceleration due to gravity.

- Gravitational potential energy

→ The amount of work done on bring an object of mass 'M' from infinite to a certain point is known as gravitational potential.

- Let us consider, the earth has mass ‘M’. here, we have to calculate the work done on bringing the object of mass ‘M’ from infinite to a certain point P.
- At any instant of time the object is at point ‘A’ and it moves through a small distance ’dx’.
- Now the small work done on moving object from A to B will be

  dw = fdx; Here the force is gravitational force between object and the earth which is F=$\frac{GMm}{x^2}$

  Dw=fdx → eqn (i)

  F=$\frac{GMm}{x^2}$ →eqn (ii)

  Now,

  Replacing the value of force to eqn (ii) to eqn (i)

  DW=$\frac{GMm}{x^2}$ dx

  Rule to do integration

$$\text{If } \int x dx = \frac{x^2}{2}$$

$$\text{If } \int dx = x$$

Now,

The total work done can be obtain by integration the above eqn

∫dw=∫$\frac{GMm}{x^2}dx$

$$\int dw = \int_{\infty}^{r} \frac{GMm}{x^2} \times dx$$

$$w = GMm \int_{\infty}^{r} x^{-2} \times dx$$

$$w = -GMm[\frac{x^{-2+1}}{-2+1}]_{\infty}^{r}$$

$$w = -GMm[\frac{1}{x}]_{\infty}^{r}$$

$$w = -GMm[\frac{1}{r} - \frac{1}{r}]^{0}$$

$$w = -\frac{GMm}{r}$$

Here,

The work done is gravitational potential Energy and Hence,

U = $\frac{-GMm}{r}$

Here, the negative sing indicates the attractive force between earth and object.

- Gravitational potential (v)

The work done on bringing a unit mass from infinite to a certain point is known as gravitational potential (V).

At any instant of time the object is at point ‘A’ and moves to point ‘B’ through a small displacement ‘dx’ then the small work done will be

$$dw = f \times dx$$

$$dw = \frac{GM}{x^2} dx$$

Now

The total work can be obtain by integrating the above eqn

$$\int dw = \int \frac{GM}{x^2} \times dx$$

$$w = GM \int_{\infty}^{r} x^{-2} \times dx$$

$$-GM[\frac{1}{x}]_0^r$$

$$-GM[\frac{1}{r} - \frac{1}{r}]^{\infty}$$

$$w = -\frac{GM}{r}$$

- Escape velocity

The velocity with which the body must be projected upward so that it overcomes the gravitational pool and moved into space in known as escape velocity

Consider the Earth has mass 'm' in to be mass 'M’ Object of projected From surface of the Earth to infinite when the objects move upward it gains the kinetic Energy which is

$$K.E.=\frac{1}{2}mv^2 \rightarrow (i)$$

At any instant of time the object reaches at point 'A' and moves through small displacement $dx$ and reaches at point 'B' then the work done from A to B will be

$$dw = f \times dx$$

Here the force is gravitational force between Earth and the object then

$$dw = \frac{GMm}{x^2} \times dx$$

Now,

For the total work done, Integrating on both side

$$\int dw = \int_r^\infty \frac{GMm}{r^2} \times dx$$

$$w = GMm[\frac{x^{-2+1}}{-2+1}]_r^\infty$$

$$w = -GMm[\frac{1}{\infty} - \frac{1}{r}]$$

$$w = \frac{GMm}{r} \rightarrow \text{(ii)}$$

From work energy theorem

Work done=charge in kinetic energy

$$\frac{GMm}{r} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$v = \sqrt{\frac{2GM}{r} \times \frac{r}{r}}$$

$$v = \sqrt{\frac{2GM}{r} \times r}$$

$$v = \sqrt{2Gr}$$

Here, the escape velocity doesn't depend on mass of the object but it depends on gravity and radius of the planet.

For example;

What will be the escape velocity of the earth? If the gravity of the earth is 9.8m\$s^2$and radius of the earth is 6400 km.

Solution;

Gravity of earth = 9.8m\$s^2$

Radius =6400 km

$=64 \times 10^5 m$

$=6.4 \times 10^6 m$

Now,

$$v = \sqrt{2gr}$$

$$= \sqrt{2 \times 9.8 \times 6.4 \times 10 \times 10^6}$$

$$= 11200 m \backslash s$$

$$= 11.2 kg \backslash s$$