Exponential Distribution & Application of Central Limit Theorem

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Overview

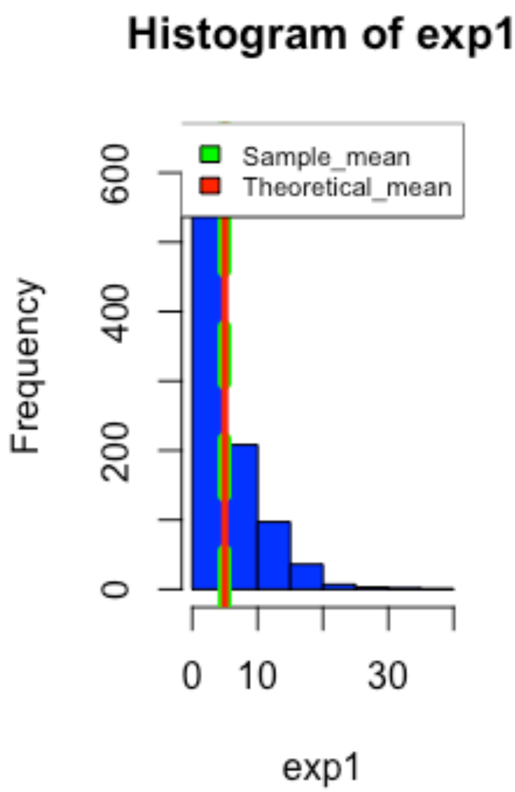
This report will investigate the exponential distribution in R and compare it with the Central Limit Theorem. Exponential distribution with lambda=0.2 will be used for the purpose with simulation count of 1000.To validate the central limit theorem, average of 40 exponentials wil be taken.

Exponential random variable simulation

Firstly, random variable with exponential distribution has been simulated (based on theoretical & sample central measures). This has been used to calculate the required confidence intervals.

```
lambda =0.2
par(mfrow=c(1,2))
exp1= rexp(1000,lambda)
hist(exp1, col = "blue", ylab = "Frequency")
theoretical_mean = 1/lambda
sample_mean = mean(exp1)
sample_sd=sd(exp1)
theoretical_sd=1/lambda
abline(v=sample_mean, col="green", lwd=6, lty=2);
abline(v=theoretical_mean, col="red", lwd=3);
legend("topright", c("Sample_mean", "Theoretical_mean"), fill=c("green", "red
"),cex=0.7)

cat("Sample variance of exponential distribution:", sample_sd^2,"\n")

## Sample variance of exponential distribution: 24.19949

cat("Sample two-sided 95% confidence interval:",c((sample_mean-1.96*sample_sd
),(sample_mean+1.96*sample_sd)),"\n")

## Sample two-sided 95% confidence interval: -4.690749 14.5929

cat("Population variance of exponential distribution:", theoretical_sd^2,"\n"
)

## Population variance of exponential distribution: 25
```

```
cat ("Population two-sided 95% confidence interval",c((theoretical_mean-1.96*
theoretical_sd),(theoretical_mean+1.96*theoretical_sd)),"\n")

## Population two-sided 95% confidence interval -4.8 14.8

cat("\n")

summary(exp1)

##      Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
##  0.00604  1.42745  3.41498  4.95107  6.98622 39.34666

cat("\n")
```

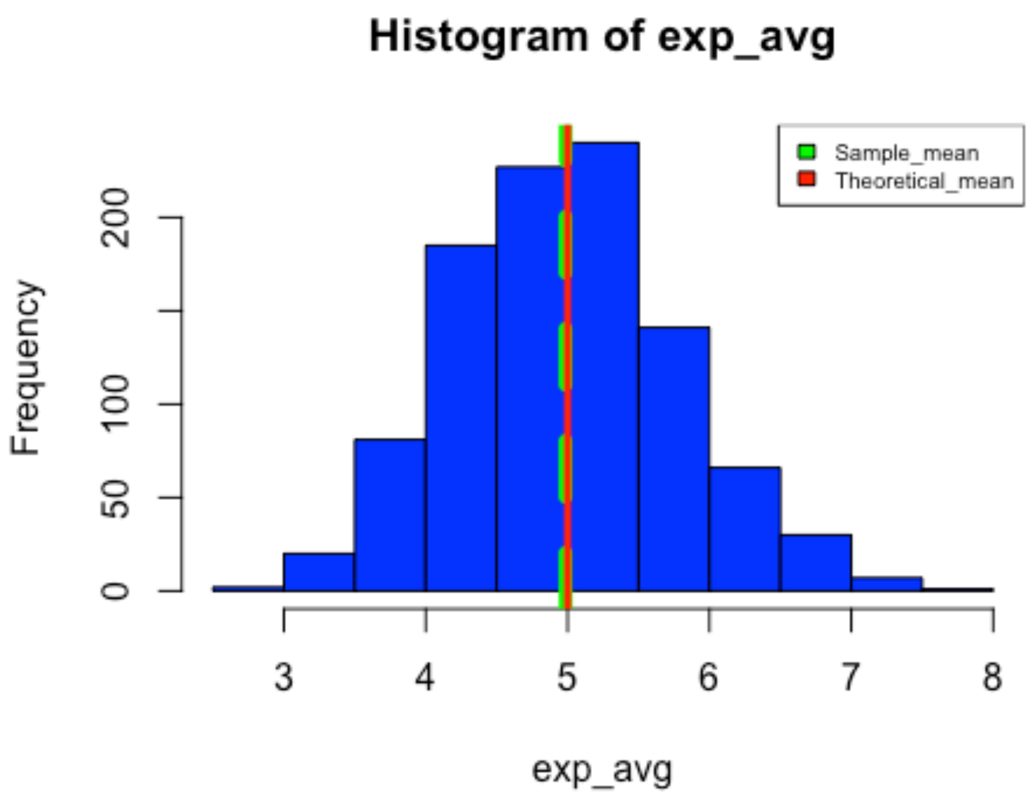


Exponential random variables mean simulation

Secondly, mean of 40 random variables with exponential distribution has been simulated (based on theoretical & sample central measures). This should tend towards normal random variable as per central limit theorem. This has been validated & required confidence intervals have been calculated.

```
exp_avg =vector(mode="numeric", length=1000)
 for (i in 1:1000)
 { exp_avg[i]= mean(rexp(40,lambda))
```

```
}

hist(exp_avg, col = "blue", ylab = "Frequency")
theoretical_mean1 = 1/lambda
sample_mean1 = mean(exp_avg)
sample_sd1=sd(exp_avg)
theoretical_sd1=1/(lambda* (40^0.5))
abline(v=sample_mean1, col="green", lwd=6, lty=2);
abline(v=theoretical_mean1, col="red", lwd=3);
legend("topright", c("Sample_mean", "Theoretical_mean"), fill=c("green", "re
d"),cex=0.6)
```




```
cat("Sample variance of averaged exponential distribution:", sample_sd1^2,"\n
")

## Sample variance of averaged exponential distribution: 0.6359177

cat("Sample two-sided 95% confidence interval:",c((sample_mean1-1.96*sample_s
d1),(sample_mean1+1.96*sample_sd1)),"\n")

## Sample two-sided 95% confidence interval: 3.422852 6.548834

cat("Population variance of averaged exponential distribution:", theoretical_
sd1^2,"\n")
```

```
## Population variance of averaged exponential distribution: 0.625

cat ("Population two-sided 95% confidence interval",c((theoretical_mean1-1.96
*theoretical_sd1),(theoretical_mean1+1.96*theoretical_sd1)),"\n")

## Population two-sided 95% confidence interval 3.450484 6.549516

cat("\n")

summary(exp_avg)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##   2.659   4.424   4.978   4.986   5.486   7.836
```