Assignment 2

Aerodynamics II | AE612A
Due: Apr. 28, 2023 | Total marks: 20

Instructions:

1. Use a programming language of your choice to solve these problems.
2. It is helpful to make subroutines for normal shock relations – you will need to use these repeatedly. For example, you can code for one that, given the upstream Mach number, returns all ratios of flow variables across a normal shock and the Mach number downstream.
3. For oblique shocks, you can exploit the fact that $M_{1,n} = M_1 \sin\beta$ and $M_{2,n}= M_2 \sin(\beta-\theta)$ and use the normal shock subroutine.
4. When M_1 and θ are known for an oblique shock, you can predict β using a Newton or Secant method, or simply using an interpolation approach (*e.g.*, using M_1, create an array for β ranging from $(\mu_1, \theta_{\max})$ and compute an array of θ for the same. Use any linear interpolation function to find the value of β given θ from these arrays).

Important:

1. Submit all codes you have used along with the assignment.
2. Soft copy is preferred.
3. If you do not know something, you can always take my help. DO NOT COPY!

Questions:

1. **Diamond aerofoil**: Consider a diamond aerofoil with a nose angle of $2\epsilon = 5°$ placed in a supersonic flow of $M_\infty = 3$. Assume that the freestream velocity, pressure and density and aerofoil chord are all unity with appropriate dimensions. For convenience, the different flow regions are denoted by subscripts 1 to 4, with $M_1 = M_\infty$ and M_2, the Mach number on the fore part of the aerofoil M_3, in the aft portion and M_4 in the wake. The incidence angle is given by α.

(a) Find the expressions for the flow angle with respect to the horizontal in the fore and aft regions (referred to as δ_2 and δ_3) based on α and ϵ. Assume that α is such that the waves and flow remain attached and δ is *positive* if the freestream is turned onto itself. (1 mark)

(b) At $\alpha = 1°$, $2.5°$ and $5°$, sketch the prominent flow features. Include the flow angles with respect to the horizontal, δ_2 and δ_3. (1 mark)

(c) What is the maximum incidence angle α_{max} beyond which the shock wave at the leading edge becomes detached? (1 mark)

(d) Compare the variation of $-C_p$ with chordwise distance, x, on the upper and lower surfaces for the incidence angles mentioned above (same plot). (2 marks)

(e) Compute the angle the slip stream in the wake makes with the horizontal (*i.e.*, δ_4) for each of these cases. (Hint: Note that pressure and velocity direction across a slip line should align, so one approach would be to make a guess for δ_4, use that to compute flow deflection from 3 to 4 and predict p_4 and see if it matches for the suction and pressure sides.) (2 marks)

(f) Plot the variation of C_L with α in the range $(0, \alpha_{\text{max}})$ in steps of $1°$. Compare with predictions from thin aerofoil theory in the same plot. (3 marks)

2. **Laval nozzle**: Consider a Convergent-Divergent nozzle design with streamwise variation of its radius given by $r = 1 - c(1 - x/x_0)^2 + c(1 - x/x_0)^3$, where x is the streamwise location and x_0 is the length of the nozzle and $c = 3$ (all quantities in dimensionless form based on the exit radius). The inflow stagnation pressure and temperature are $p_0 = 10$ bar and $T_0 = 300$ K.Answer the following:

   (a) What is the streamwise location and area of the throat? (1 mark)

   (b) Assuming isentropic flow in the nozzle, plot the variation of the Mach number and pressure as functions of x. Submit your plots along with the codes used. (Hint: If you are using the Newton-Raphson approach, use a starting guess of $M = 0.1$ or $M = 2$ in regions where you expect the flow to be subsonic or supersonic, respectively.) (2 marks)

   (c) Compare the variation of pressure and Mach number (two graphs) when (i) $M_{\text{throat}} = 0.95$, (ii) there is a normal shock at $x = 2$, (iii) at $x = 2.5$, (iv) and at nozzle exit. Based on (iv), infer what is the range of ambient pressures for which the flow is over-expanded? (4 marks)

   (d) For the ambient pressure being atmospheric pressure at ground level, compute the flow features downstream (wave angle, flow angle, Mach number, pressure and stagnation pressure) of the nozzle and up to one shock cell if possible. Sketch a rough figure showing the angles and flow directions. (3 marks)