IB Diploma Programme

Mathematics: Applications and Interpretation Standard Level

Internal Assessment Exploration

jhx864 – May 2021

Title – Finding the volume of a bottle using Calculus

Figure 1 – Coca-Cola bottle[1]

INTRODUCTION - Coca Cola is a famous sweetened carbonated beverage which was developed by John S. Pemberton in 1886. The Coca Cola Company is not only famous for the taste of their soft drink but also how the drink went through an evolution of its bottle change. My father told me how through the years, the beverage went through several changes for its bottle shape; first, when the coke was initially found, the shape of its bottle was 'Hutchinson bottle' which was a straight and tall bottle. However, the company thought that the bottle was not distinctive enough and the brand's bottles were becoming easily confused with 'copycat' brands. After several years and changes, the company came up with a unique bottle design for Coca-Cola - the famous 'Contour bottle' which was easily distinguishable amongst its competitors and easy to use shape.[2]

Upon hearing this interesting business story, for mainly two reasons - the taste and the attractive easy to use the shape of the bottle - Coca Cola is my favourite soft drink. And this got me interested how the volume of such an interesting shape could be calculated through the graphics designing ways. It is surely possible to find out volume by conventional ways – such as Archimedes method or just filling the bottle with water and then calculating the volume of water that was filled. But it is uncommon to find volume of daily life objects using complex mathematics. Such process, however, only exist on industry level scale. Since I aim to pursue business management on university level and also aim to become a passionate freelance graphic designer, by blending both of my interests and using these skills I will try to find the volume of a daily life object which would allow me to achieve the industry level procedure as this exploration's procedure is mainly used in industries and commercial companies to decide product designs from scratch and make the final outcome in 3D or to take a real life object's and find its volume virtually to improve its shape and other attributes in the most profitable manner. This exploration would important to me as a passionate graphic designer as this exploration would polish my graphic designing skills since the creation or the replication of 2D/3D illustrations of real-life objects is one of the most important skills in graphic designing. Moreover, I believe that the integration of mathematics and technology is important in today's world and hence this exploration is worthy of investigation.

[1] "Coca Cola - Soft Drink." *FMCG-Coca Cola 200 ML Glass Bottle*. Web. 10 Dec. 2020. <http://otc.in/index.php/products/coca-cola-200-ml-glass-bottle.html>.
[2] Staff, Journey. "Coca-Cola Bottle Evolution: Contour Bottle History." *The Coca-Cola Company*. Web. 10 Dec. 2020. <https://www.coca-colaindia.com/stories/contour-bottle-history>.

AIM – I am to find the volume of a Coca Cola bottle by utilising Calculus and technology which forms my title **"To Find the Volume of a Contour Shaped Coca-Cola bottle utilising Calculus"**

PROCEDURE – I shall use the skills I acquired in calculus classes to find the volume of an irregular solid, in this case, a contour shaped bottle. Even though this exploration would require mathematical knowledge that is beyond what I learnt in my mathematics class; I will understand the concepts of the concepts of calculus a step ahead to perform this exploration. Firstly, I will use my expertise (which developed in the recent months of COVID-19 lockdown) in the software named *Adobe Illustrator* which a graphic designing software used by graphic designers and companies to process 2D or 3D illustrations to rationalise each step by splitting the bottle into sections. Along with this *Desmos* (a free-to-use online tool including graphing and scientific calculator[3]) and *WebPlotDigitzer* (a web-based tool used to extract data from plots, images, maps[4]) will be used to plot and extract points for each sections of the bottle. Next, equations will be made using *GeoGebra* (a dynamic mathematical software[5]) from the data extracted/plotted the equation which seems the most promising would be finalised. Lastly, to determine the reliability of the exploration, three tests would be done which include both – visual and mathematical tests. First, a 3D model of the equations derived would be made to visually compare the results. Then the R^2 value (R^2 or the coefficient determination is a statistical measure which tells how close the data is to the regression line.[6] The closer the value of the R^2 is to '1,' the stronger and reliable the overall data is. The farther the value of R^2 is from '1,' the weaker and unreliable the overall data is) of the equations and the percentage error (the difference between estimated value and the actual value in comparison to the actual value and is expressed as a percentage.[7]) of the actual volume of the bottle and volume calculated would be checked.

[3] "About Us." *Desmos*. Web. 10 Dec. 2020. <https://www.desmos.com/about>.
[4] Rohatgi, Ankit. "WebPlotDigitizer - Extract Data from Plots, Images, and Maps." *WebPlotDigitizer - Extract Data from Plots, Images, and Maps.* 20 Oct. 2013. Web. 11 Dec. 2020. <https://automeris.io/WebPlotDigitizer/>.
[5] Hohenwarter, Markus. "About GeoGebra." GeoGebra. 2001. Web. 11 Dec. 2020. <https://www.geogebra.org/about>.
[6] Minitab Blog Editor. "Regression Analysis: How Do I Interpret R-squared and Assess the Goodness-of-Fit?" *The Minitab Blog.* 30 May 2013. Web. 12 Dec. 2020. <https://blog.minitab.com/blog/adventures-in-statistics-2/regression-analysis-how-do-i-interpret-r-squared-and-assess-the-goodness-of-fit#:~:text=R%2Dsquared%20is%20a%20statistical,multiple%20determination%20for%20multiple%20regression.&text=100%25%20indicates%20that%20the%20model,response%20data%20around%20its%20mean.>.
[7] "Percent Error - Definition, Formula, and Solved Examples." *BYJUS*. BYJU'S, 06 Jan. 2020. Web. 12 Dec. 2020. <https://byjus.com/maths/percent-error/#:~:text=Percent%20error%20is%20the%20difference,relative%20error%20multiplied%20by%20100.>.

SPLITTING UP INTO DIFFERENT SECTIONS –

Figure 2 – measuring the height of the bottle with an inch tape bottle and ruler

Since the shape of the contour plastic bottle is irregular and complex, it is important to divide the bottle into different sections to make this investigation's calculation part easier. The bottle would be divided into 5 different parts. To make the division possible, I bought a Coca-Cola bottle, emptied it, and measured its height. For this exploration, the cap of the bottle was removed for several reasons. The shape of the cap is a bevelled-up shape and the liquid inside the bottle never touches the cap. In accordance to that, the inclusion of the bottle cap could've hampered the final result hence so the removal of cap was necessary.

Since I didn't have proper tools to measure the height of the bottle, which would a long ruler in this case which would allow me to measure the height in one go. So, I improvised to measure the height. A straight flat thin object was placed on top of the bottle horizontally which would allow me to measure the bottle's height. Then, the height was measured with the help of an inch tape.

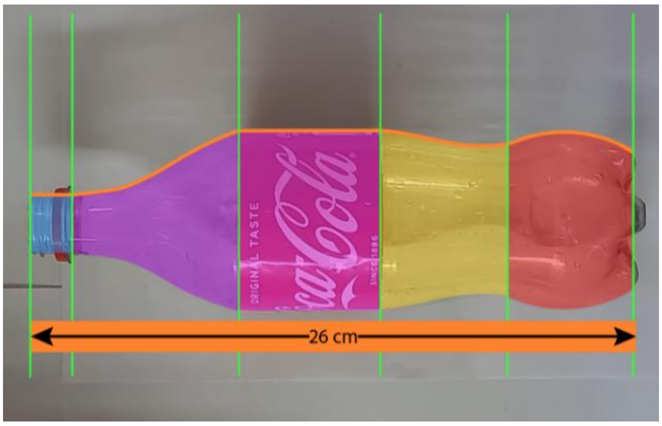


Figure 3 – bottle with 26cm rectangle and shades that separate bottle in 5 sections with outline

The height was measured to be 26cm. In accordance with this, a rectangle of 26cm height was made in *Illustrator* and an image of the Coke bottle without the cap was imported into the software. The bottle was calibrated to the size of that rectangle. This calibration was done so that it would allow me to export image to a graph that would be suitable to give proper domain values and equations for the volume.

The bottle was then divided into 5 parts, the blue, the purple, the pink, the yellow and the red part. This part was possible by creating the outline of the bottle with the help of pen tool for blue, purple, pink and yellow parts and ellipse tool for the red part. After the outline, the sections highlighted with colour shades and vertical lines were added for the ease in seeing domains for each section.

EXTRACTING THE POINTS -

The upper shades were kept and the shades below the line were removed. This was done so that the image should be suitable to put on a graph and determine its domain and plot points accordingly. This image was then exported to *Desmos* to align the bottle image with sections to a graph.

The image in *Desmos* was aligned in such a manner that the horizontal line, that was used to separate the shades, was starting at coordinates (0,0) and the end part of the outline was at *x*-coordinate of 26. This ensured the calibration of the bottle with the graph – leaving less room for errors.

The bottle image with the graph was then imported to *WebPlotDigitizer* which allowed me to plot points all along the outline of the bottle that I made in *Illustrator*. All the five sections were separately analysed in the software and points all along the outline was then made.

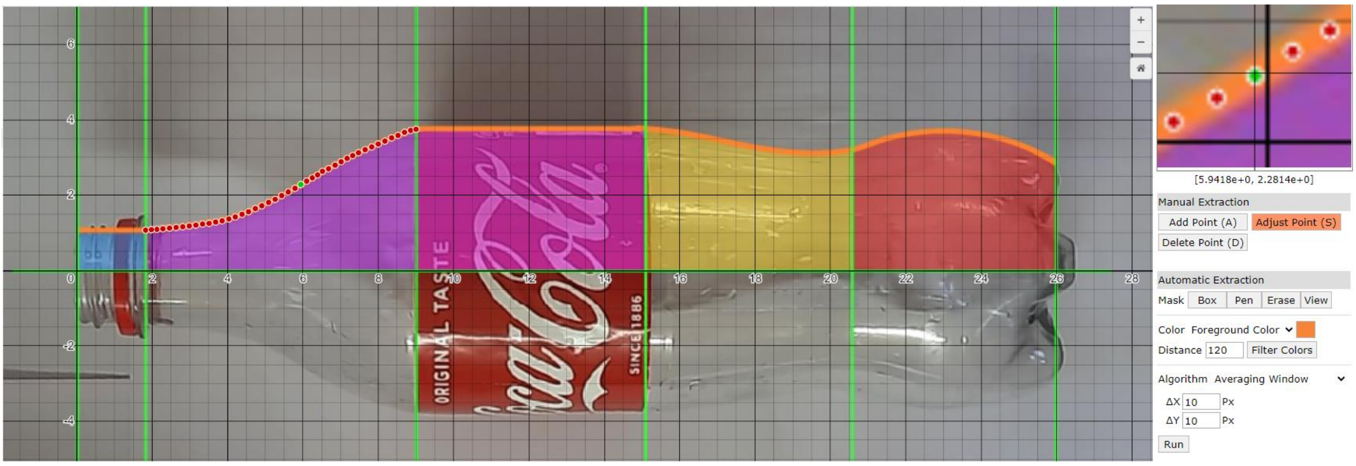


Figure 4 – image of bottle with Desmos graph and the points being extracted in WebPlotDigitizer

The coordinate data for five different sections (see appendix for raw data) was then imported to *GeoGebra* which allowed me to derive function from the coordinates I obtained from *WebPlotDigitizer*.

SHORTLISTING AND FINALISING FUNCTIONS –

To see which function would suit the best the extracted data from the GeoGebra for each part, multiple functions needs to be tested to see which would be closest to the data. The main requirement for a perfect function is that its R^2 value must be high, which means closer to 1. A high R^2 value would imply that the function is closer to the data. However, the high R^2 value is not enough since the function's graph should visually be perfect as well as it must go through all or most of the data.

First, the functions would be shortlisted for the purple part –

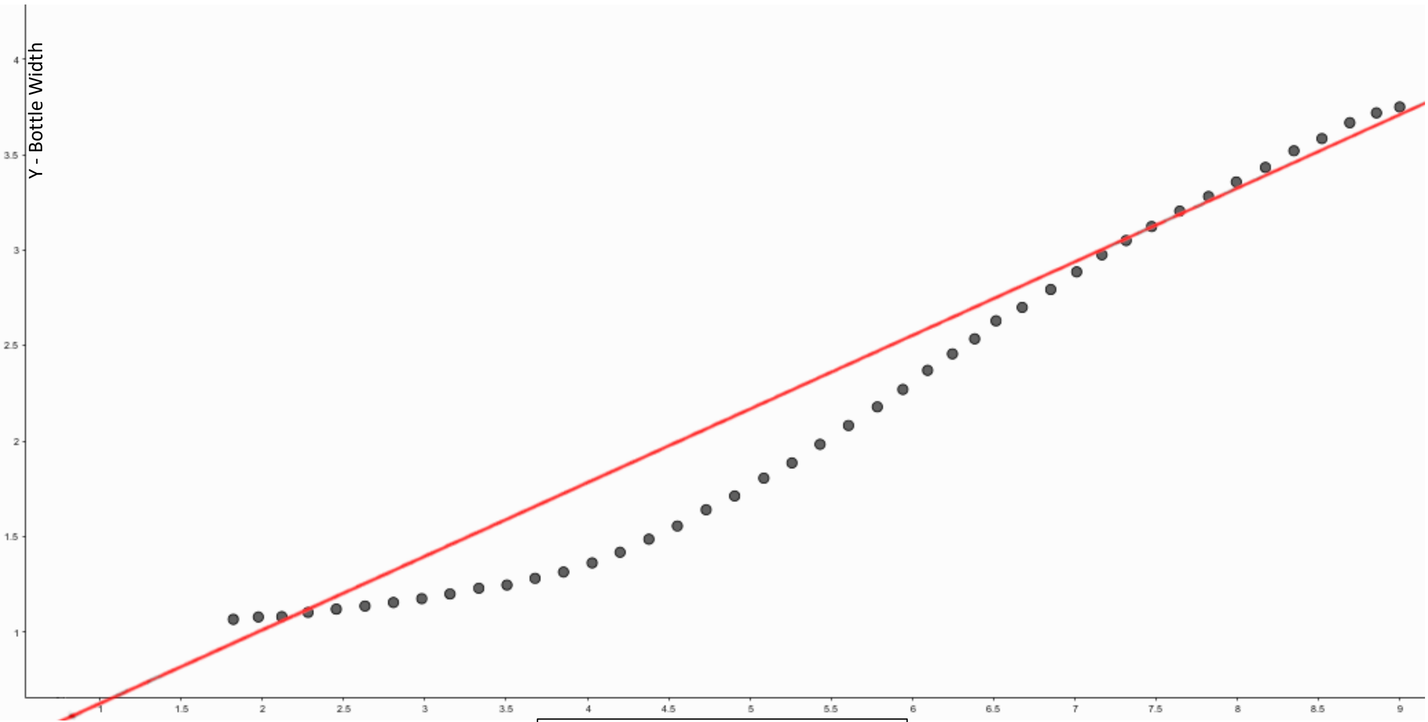


Figure 5 - graph of $y = 0.42x - 0.11$

The first trials were done with the linear equation which has a relatively high R^2 value of 0.9649 however, the shape of the function's graph can be seen not properly covering the data plotted which rejects the selection of this function.

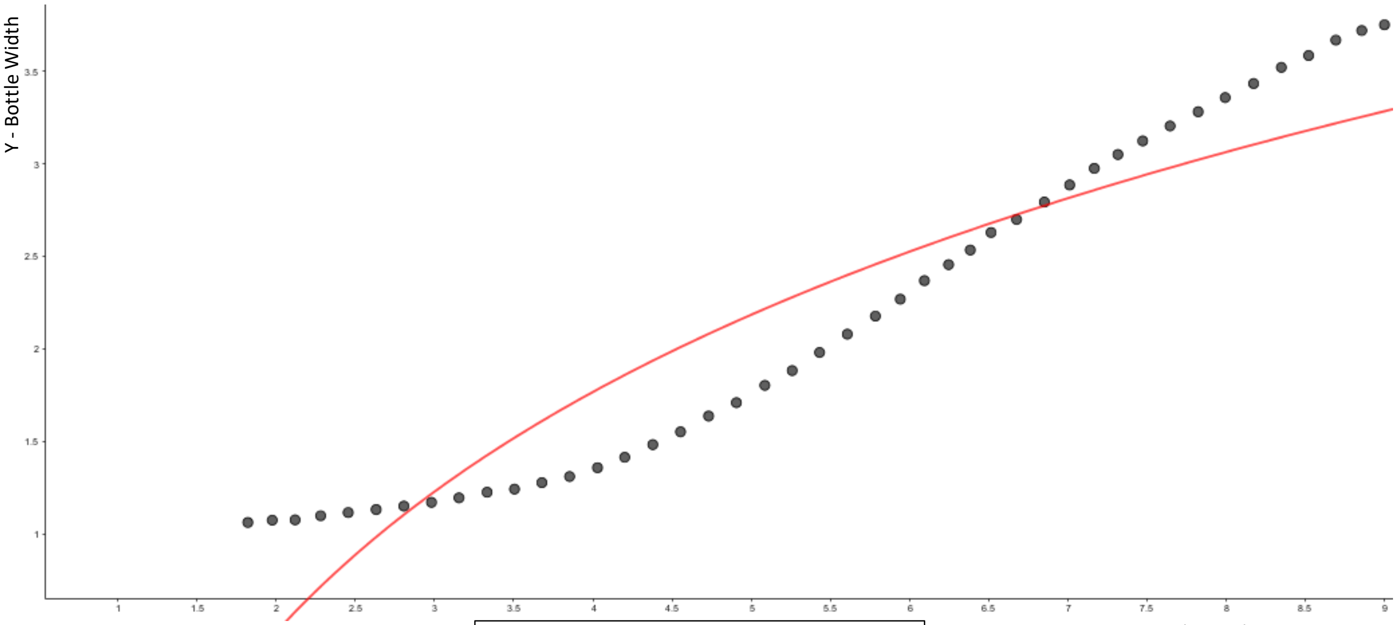


Figure 6 - graph of $-0.82 + 1.87\ \mathrm{In}(x)$

The next function was a logarithmic function which doesn't has a shape which traces the plotted data properly and moreover has the weakest R^2 value of 0.8584 out of all the functions that were tested with the data plotted.

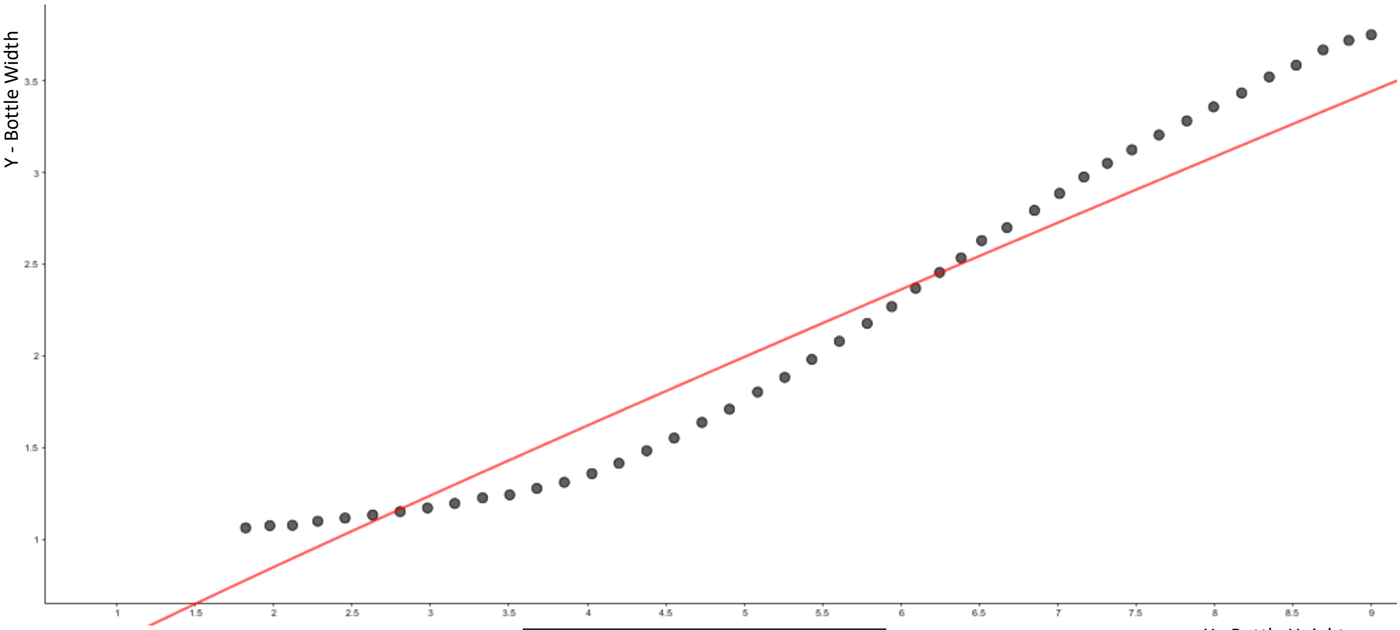


Figure 7 - graph of $y = 0.45x^{0.93}$

The next function was a power function which has a R^2 value of 0.9462 which is greater than the log function but lower than the linear function which also rejects this selection of the function and proves that there is more scope to the trial of the appropriate function.

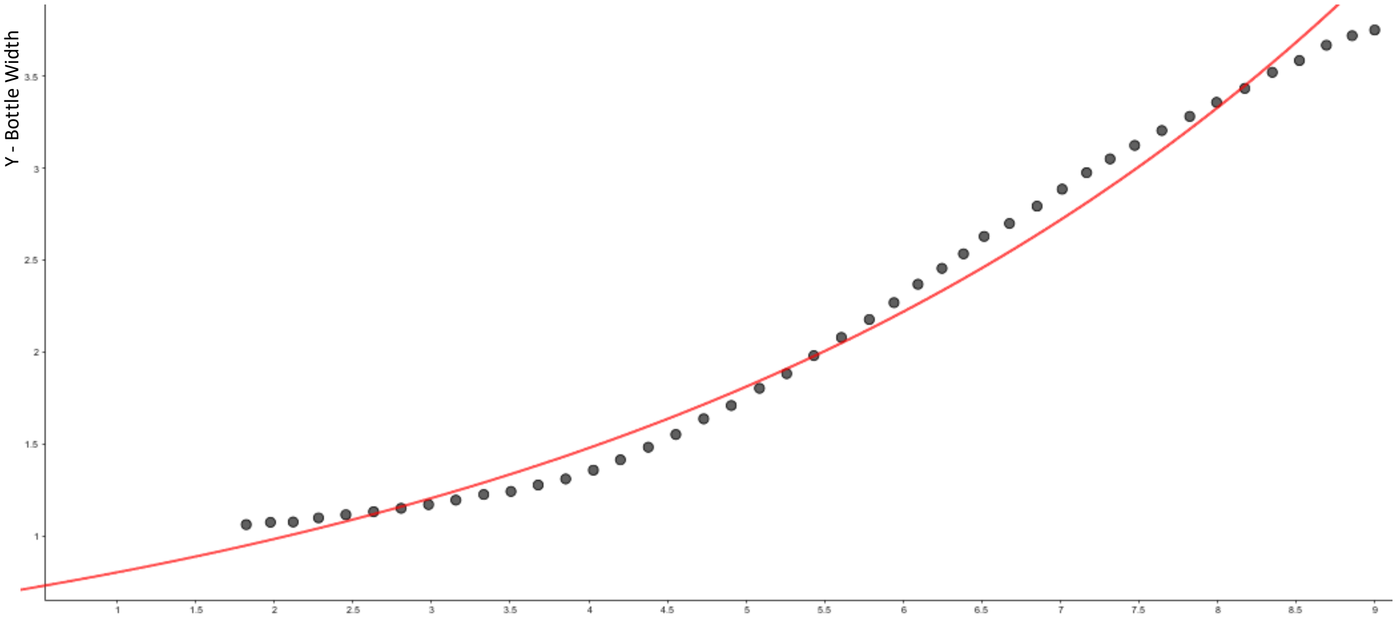


Figure 8 - graph of $y = 0.66\ e^{0.2x}$

The next function tried was an exponential function which has the highest R^2 value so far, of 0.9838, and also the function's graph covers the data relatively more compared to the previous if cheeked visually. However, there's more room to check for a better function.

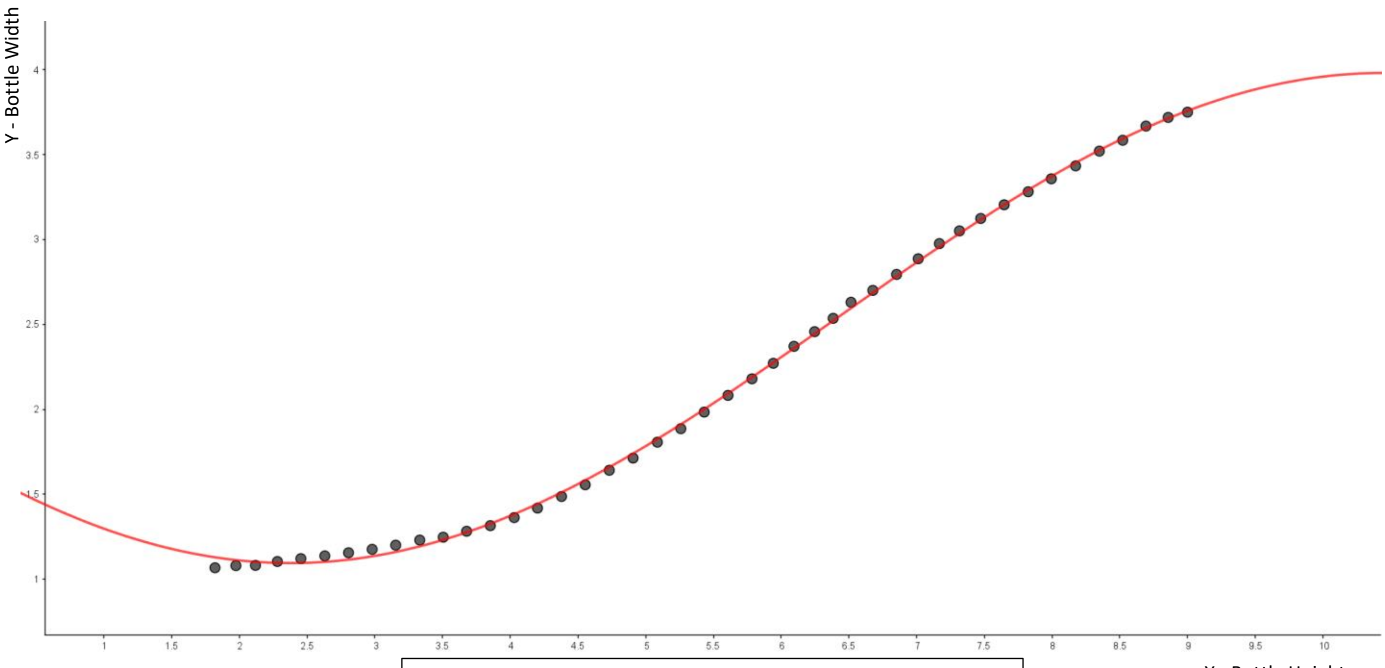


Figure 9 - graph of $y = 2.5379 + 1.4431sin(0.3894x - 2.4966)$

The last function tested was a sin function 4 and has the highest R^2 value of 0.9995 and if checked visually, the graph of the function covers the data plotted very closely and better than all the functions that were tested previously. The summary of this trials is summarised in the table below.

Table 1 - Shortlisting functions for the purple section		
Function Type	Function	R^2 Value
Linear	$y = 0.42x - 0.11$	0.9649
Log	$y = -0.82 + 1.87\ \text{In}(x)$	0.8584
Power	$y = 0.45x^{0.93}$	0.9462
Exponential	$y = 0.66\ e^{0.2x}$	0.9838
Sine	$y = 2.54 + 1.44sin(0.389x - 2.497)$	0.9994

As it can be seen from the table that after testing out multiple functions possible for the data extracted, the sin function is the most suitable function as it has the highest R^2 Value of 0.9994. So, the function finalised for the **purple part** is $y = 2.5379 + 1.4431sin(0.3894x - 2.4966)$.

Similar trial method was done for other non-linear parts which are the **red** and **yellow** part and its table can be seen in the Appendix.

Function finalised for the **yellow** part is $y = 3.44 - 0.33\sin(0.65x + 1.39)$

Function finalised for the **red** part is $\mathrm{y} = -0.09x^2 + 4.28x - 45.47$

Function for the **blue** part is a linear function $\mathrm{y} = 0x + 1.07$

Function for the **pink** part is a linear function as well $\mathrm{y} = 0x + 3.76$

Calculating the volume -

To find the volume of each of the sections, the theory of 'Volume of Revolution' was applied. The theory of 'Volume of Revolution' uses the definite integral to find the volume of a solid that is obtained by revolving a plane region about a horizontal or vertical line that does not pass through the plane. This type of solid will be made up of one of three types of elements – disks, washers, or cylindrical shells.[8] For this exploration however, I determined that the disks method will be suited best hence that theory will only be applied and hence the five functions were individually solved with the formula of 'Volume of Revolution.'

$$Volume\ of\ Revolution\ for\ disk\ method = \pi \int_a^b (f(x))^2 dx$$

Volume of the blue part –

$$\pi \int_0^{1.58908} (0x + 1.07)^2 dx$$

$= 1.81934\pi$

Volume of the purple part –

$$\pi \int_{1.58908}^{9.01711} (2.5379 + 1.4431 sin(0.3894x - 2.4966))^2 dx$$

$= 47.43784443\pi$

[8] "Calculus." *Volumes of Solids of Revolution.* Web. 16 Dec. 2020. <https://www.cliffsnotes.com/study-guides/calculus/calculus/applications-of-the-definite-integral/volumes-of-solids-of-revolution>.

Volume of the pink part –

$$\pi\int_{9.01711}^{15.166}(0x+3.76)^2dx$$

$$= 86.9305\pi$$

Volume of the yellow part –

$$\pi\int_{15.166}^{20.5722}(3.44-0.33\sin(0.65x+1.39))^2dx$$

= 61.24309257π

Volume of the red part –

$$\pi\int_{20.5722}^{26}(-0.09x^2+4.28\text{x}-45.47)^2dx$$

$$= 145.4925846\pi$$

Total Volume of the bottle = Volume of the blue part + purple part + pink part + yellow part + red part

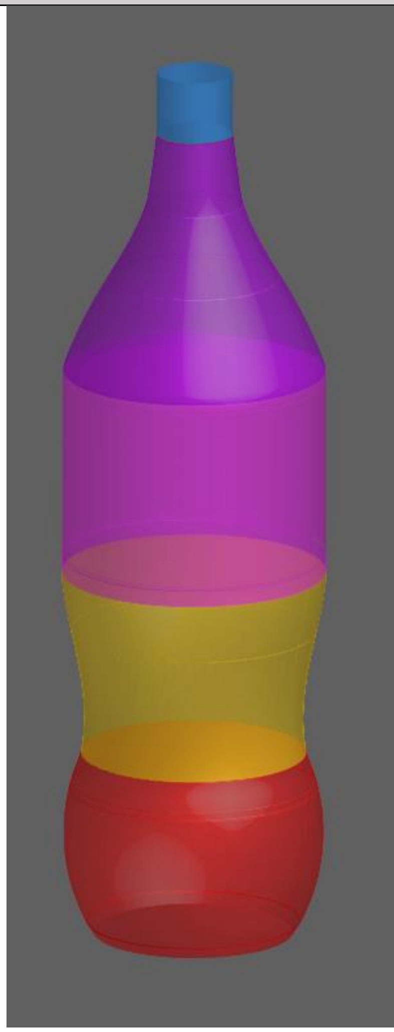
So, $1.81934\pi + 47.43784443\pi + 86.9305\pi + 61.24309257\pi + 145.4925846\pi$

Total volume of the bottle calculated = 1077.325513 cm^3 or 1077 cm^3 when rounded off to whole number.

CROSS CHECKING THE RESULTS –

Visual accuracy –

During these calculations, the graphs all the five functions that were to be translated 360° were compiled together, and a 3D model of the Coca Cola bottle was made to see the faithfulness of the exploration. The visual comparison can be seen in the Table 2.

Table 2 - Visual Comparison of the bottles	
Coca Cola bottle	3D bottle made from the equations
	

After making the 3D illustration/model of the Coca-Cola bottle with the equations, the exploration did go in the right track since the 3D illustration looks similar to the Coca-Cola bottle that I used to do the exploration on.

Mathematical accuracy –

The actual volume of the bottle purchased was 900ml and the volume calculated in this investigation was 1077 cm^3. which is obviously more than the actual volume. So, the percentage error (ε), which is calculated via $\left|\frac{Approximate\ Value - Exact/Actual\ Value}{Exact/Actual\ Value}\right| x\ 100\%$, is $\left|\frac{1077 - 900}{900}\right| x\ 100\%$ = 19.66% or 20% when rounded off to whole number. The relatively low percentage error of 20% prove the exploration mathematically as well (along with visually) that the exploration went in the right direction and the errors were minimal during this investigation

CONCLUSION – The exploration did fulfil my aim of finding the volume of the Coca-Cola bottle. Through this exploration, I learnt to use softwares like *WebPlotDigitizer* and *GeoGebra* which expanded my knowledge on the integration of Mathematics and modern-day technology. Prior to this exploration, my knowledge was only limited to a GDC. But after this exploration I learnt how different softwares can be used to understand the application of Mathematics in real life. In addition to this, the reason why this exploration was important because the graphic designing industry is an essential part in the modern day successful commercial companies. Companies use this process to visualise how their final products would look like before they are even made. Just like how the exploration was based on making 3D model of a real-life object. The similar process is used on industry level to visualise how their goods and products would look like from rough models drawn in real-life. This can be later used by companies to manipulate the values of the modelled figure to either make the good bigger, smaller, etc. Moreover, this whole exploration is much step towards the topic of optimisation where companies minimise wastes, costs and increase profit margins. While I do understand the fact the main limitation of the exploration which is the 20% percentage error which implies that the calculated volume is greater than the actual volume. But it is worth noting that the process of this exploration is the same worldwide. As I discussed before, the replication of real-life object in a virtual world is a key aspect in graphic designing. The 3D model in the Table 2 showed that I successfully made a replica of a real-life bottle from the equations derived. Even though the minute details like body texture and neck rings are not represented in the 3D model, but it must be noted that these details do not add up to the final volume of space that the liquid occupies.

Table 3 – Limitations of this exploration		
Limitations	**Why to fix it?**	**How to fix it?**
The main limitation of this exploration would be the error caused by the distortion effect by the mobile phone camera.[9]	Due to this effect, the measurements of the bottle can be disrupted. Hence there are chances that the entire calculation can go wrong due to this minor visual change.	Even though I took the image of the bottle (with my smart phone) from a distance to reduce the effect of distortion, but the final image that was used in the exploration still had some effect of distortion which made the straight parts of the bottle a bit curved. I wanted to take the image from greater distance, but the image started to become blurry. Hence to fix this issue a DSLR camera would be needed that can capture images from a distance without dropping its quality. This way, the distortion effect can be minimised to the max.
Another limitation of this investigation would be that the base of the bottle during the modelling was a flat base. This was done because then the calculation and the modelling part would become too complicated and the calculation and modelling could've gone completely haywired.	The final value of the volume was hampered and hence, even though minutely changed but an increased volume value was calculated. The actual volume of the bottle was 750 cm^3 but volume that was calculate through Volume of Revolution and 3D modelling was 1080 cm^3	For the modelling part, a better and heavy processing software can be used known as *Blender* which can model complex 3D figures. For the mathematical part, deeper knowledge of calculus would be required along with a better mathematical software that can derive functions from 3D figures.

[9] Mansurov, Nasim. "What Is Lens Distortion?" *Photography Life*. 05 July 2020. Web. 20 Dec. 2020. <https://photographylife.com/what-is-distortion>.

BIBLIOGRAPHY –

Digital Image –

- "Coca Cola - Soft Drink." *FMCG-Coca Cola 200 ML Glass Bottle*. Web. 10 Dec. 2020. <http://otc.in/index.php/products/coca-cola-200-ml-glass-bottle.html>.

Websites –

- Staff, Journey. "Coca-Cola Bottle Evolution: Contour Bottle History." *The Coca-Cola Company*. Web. 10 Dec. 2020. <https://www.coca-colaindia.com/stories/contour-bottle-history>.
- "About Us." *Desmos*. Web. 10 Dec. 2020. <https://www.desmos.com/about>.
- Rohatgi, Ankit. "WebPlotDigitizer - Extract Data from Plots, Images, and Maps." *WebPlotDigitizer - Extract Data from Plots, Images, and Maps.* 20 Oct. 2013. Web. 11 Dec. 2020. <https://automeris.io/WebPlotDigitizer/>.
- Hohenwarter, Markus. "About GeoGebra." GeoGebra. 2001. Web. 11 Dec. 2020. <https://www.geogebra.org/about>.
- Minitab Blog Editor. "Regression Analysis: How Do I Interpret R-squared and Assess the Goodness-of-Fit?" *The Minitab Blog.* 30 May 2013. Web. 12 Dec. 2020. <https://blog.minitab.com/blog/adventures-in-statistics-2/regression-analysis-how-do-i-interpret-r-squared-and-assess-the-goodness-of-fit#:~:text=R%2Dsquared%20is%20a%20statistical,multiple%20determination%20for%20multiple%20regression.&text=100%25%20indicates%20that%20the%20model,response%20data%20around%20its%20mean.>.
- "Percent Error - Definition, Formula, and Solved Examples." *BYJUS*. BYJU'S, 06 Jan. 2020. Web. 12 Dec. 2020. <https://byjus.com/maths/percent-error/#:~:text=Percent%20error%20is%20the%20difference,relative%20error%20multiplied%20by%20100.>.
- "Calculus." *Volumes of Solids of Revolution.* Web. 16 Dec. 2020. <https://www.cliffsnotes.com/study-guides/calculus/calculus/applications-of-the-definite-integral/volumes-of-solids-of-revolution>.
- Mansurov, Nasim. "What Is Lens Distortion?" *Photography Life*. 05 July 2020. Web. 20 Dec. 2020. <https://photographylife.com/what-is-distortion>.

APPENDIX –

Table 1 - Shortlisting functions for the yellow section		
Function Type	Function	R^2 Value
Linear	$y = -0.05x + 4.71$	0.1388
Log	$y = 7.08 - 1.15\ In(x)$	0.1181
Power	$y = 10.64x^{-0.36}$	0.1088
Polynomial	$y = -0.09x^2 + 4.28x - 45.47$	0.9983

Table 2 - Shortlisting functions for the red section		
Function Type	Function	R^2 Value
Linear	$y = -0.13x + 5.69$	0.8933
Log	$y = -10.09 - 2.34\ In(x)$	0.9128
Power	$y = 24.24x^{-0.69}$	0.9239
Sine	$y = 3.44 - 0.33\ sin(0.65x + 1.39)$	0.9993

Table 5 – Coordinates of outline of blue part	
x	**y**
0.146220796	1.075586297
0.321426763	1.07614321
0.49663273	1.074691764
0.671838696	1.071451254
0.857062504	1.075755106
1.027559902	1.076429348
1.197456597	1.075008875
1.372662563	1.078202056
1.545213894	1.07925225
1.67904636	1.077839194
1.818522172	1.077686258

1.5890842	1.081278335

Table 6 – Coordinates of outline of purple part	
x	**y**
1.5890842	1.081278335
2.280548027	1.103718016
2.454237059	1.120870884
2.63095996	1.136874021
2.806165927	1.155491549
2.980613427	1.175672448
3.15435528	1.200115736
3.331783827	1.230139559
3.504767214	1.246983367
3.6777506	1.28186102
3.852956567	1.314990643
4.029125516	1.36263428
4.201145919	1.418456889
4.378574467	1.48698698
4.553780433	1.555879934
4.731208981	1.640992425
4.906414947	1.713100405
5.086066075	1.806465031
5.259049461	1.88614941
5.432032847	1.983766182
5.607238814	2.082384895
5.784667361	2.179849486
5.941761639	2.270999684
6.094277396	2.371013672
6.246922721	2.457256951
6.515472998	2.630523435
6.67662501	2.700555699
6.851830977	2.794536821
7.01147888	2.886848779
7.16758366	2.976033214
7.316308916	3.050932972
7.473015768	3.124137967

7.645999154	3.204823918
7.823427701	3.281496917
7.994188507	3.357588746
8.173839634	3.433535874
8.349045601	3.520622627
8.521323342	3.584707543
8.693026457	3.667860637
8.856882858	3.719252258
9.000059964	3.750220945
1.82034588	1.067220969
1.974691744	1.079598166
6.38379744	2.536022966

Table 7 – Coordinates of outline of pink part	
x	**y**
9.017113597	3.756559114
9.15267864	3.768658173
9.295112963	3.7706689
9.450357367	3.7745913
9.626321801	3.76831898
9.801527768	3.76831898
9.978326516	3.76831898
10.1519397	3.76831898
10.32714567	3.76831898
10.50386857	3.770789894
10.6775576	3.76831898
10.85276357	3.764327504
11.0310034	3.769649472
11.2031755	3.769770426
11.37838147	3.765053227
11.55737977	3.767748769
11.7287934	3.76831898
11.89762824	3.76831898
12.07920534	3.769770426
12.2544113	3.76831898
12.42810033	3.769649472
12.60482324	3.764327504

12.7800292	3.764327504
12.9544767	3.768889191
13.13044114	3.76831898
13.3056471	3.76831898
13.48161154	3.770029612
13.65605904	3.767956118
13.831265	3.766141811
14.00965653	3.764327504
14.18167694	3.764690365
14.3568829	3.769407564
14.53512274	3.765848066
14.70729484	3.767956118
14.91071362	3.776705098
15.1660356	3.776474683

Table 8 – Coordinates of outline of yellow part	
x	**y**
15.1660356	3.747411047
15.41303162	3.728977685
15.5962015	3.708222008
15.77140746	3.692256103
15.94183509	3.66271918
16.1218194	3.634923991
16.29702536	3.604443627
16.46904577	3.570915227
16.6474373	3.549591068
16.82264326	3.497382214
16.99784923	3.458735739
17.1730552	3.422287168
17.34826116	3.388903913
17.52665269	3.348190855
17.6986731	3.317057341
17.87387906	3.280045471
18.05386338	3.254790312
18.224291	3.214730406
18.39949696	3.20094167
18.59063075	3.166106968

18.76583671	3.142883834
18.93467155	3.134247731
19.11624865	3.125466483
19.29145461	3.113854916
19.46347502	3.109500579
19.64186655	3.109500579
19.81707251	3.11240347
19.99227848	3.125466483
20.16748445	3.138529497
20.34269041	3.157398293
20.46214903	3.173364198
20.56741681	3.206708629
15.09453472	3.774554635

Table 9 – Coordinates of outline of red part	
x	**y**
20.74088579	3.240130709
20.90812785	3.293108484
21.09129773	3.358423549
21.26650369	3.422287168
21.43533853	3.473238102
21.61691563	3.527482437
21.79212159	3.568883203
21.964142	3.600452151
22.14253353	3.634923991
22.31773949	3.64943845
22.49294546	3.669105542
22.66815143	3.692981826
22.84335739	3.700239056
23.02015614	3.700239056

23.19376933	3.699513333
23.36897529	3.688627489
23.5489596	3.665114065
23.71938723	3.647987004
23.89459319	3.628392484
24.07617029	3.594864084
24.24500513	3.565980311
24.41224718	3.519991594
24.59541706	3.472223818
24.77062303	3.418658553
24.94105065	3.360165284
25.12103496	3.290931315
25.29624093	3.221261912
25.46826133	3.135844322
25.64665286	3.050156601
25.82185883	2.945628306
25.9942381	2.832900723
20.57701836	3.184251442