Skill estimation of the top 50 PGU Smash Ultimate Professionals using Stan, a simple continuous variable model

In [1]:
```
from platform import python_version
print(python_version())

import pystan
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

```
3.9.6
```

In [2]:
```
# needed to run multiple chains
import multiprocessing
multiprocessing.set_start_method("fork")
```

We'll use "stan" to define the model; this will compile an inference engine which can then perform Monte Carlo (and some other) approximate inference procedures to give estimates.

We'll describe the (prior) distribution of skill levels as Gaussian, and then model the probability of Player A winning over Player B as a logistic function of the two players' skill differences, with a scaling coefficient (which we can set or try to learn):

Parse data from SQLite database: https://github.com/smashdata/ThePlayerDatabase

In [3]:
```
import sqlite3 as sq
import ast
import parse as data

con = sq.connect("ultimate_player_database/ultimate_player_database.db")
cur = con.cursor()
```

In [4]:
```
# ********************** DATA DICTIONARIES ********************** #

# playerids = set( int(player_id) )
# playerid_to_placement = dict( int(player_id), int(placement) )
# placement_to_playerid = dict( int(placement), int(player_id) )
# playerid_to_name = dict( int(player_id), str(player_tag) )
# name_to_playerid = dict( str(player_tag), int(player_id) )
# matches = [ int(player_1_id), int(player_2_id), int(winner_id) ]
# n = len(playerids)
```

player_ids and player_id_to_placement

In [5]:
```
playerid_to_placement = dict()
placement_to_playerid = dict()

request = cur.execute("select by_id from ranking_seasons limit 1;")
playerid_to_placement_str = ast.literal_eval(request.fetchall()[0][0])

inconsistent_data_playerid = {6905,270293,160103,24835,58557}

change = 0;
for pl_id,place in playerid_to_placement_str.items() :
    if int(pl_id) not in inconsistent_data_playerid :
        playerid_to_placement[int(pl_id)] = place - change
        placement_to_playerid[place - change] = int(pl_id)
    else :
        # hard coded due to inconsistencies in data
        if place != 51:
            change += 1

playerids = {player_id for player_id in playerid_to_placement}
n = len(playerids)
all_player_id_str = ' or '.join(f'player_id = {p_id}' for p_id in playerids)
```

adjust player_ids to correlate to 1-n

In [6]:
```
playerid_to_adjustedid = dict()
adjustedid_to_playerid = dict()

for i,player_id in enumerate(playerids) :
    playerid_to_adjustedid[player_id] = i + 1
    adjustedid_to_playerid[i + 1] = player_id
```

player_id_to_name

In [7]:
```
playerid_to_name = dict()
name_to_playerid = dict()

request = cur.execute(f"select player_id,tag from players where {all_player_id
players = request.fetchall()
for player_id,tag in players :
    playerid_to_name[int(player_id)] = tag
    name_to_playerid[tag] = int(player_id)
```

matches: every match played between any 2 players from the top 50 players

In [8]:
```
all_p1_ids_str = ' or '.join(f'p1_id = {p_id}' for p_id in playerids)
all_p2_ids_str = ' or '.join(f'p2_id = {p_id}' for p_id in playerids)

request = cur.execute(f'select p1_id, p2_id,winner_id from sets where ({all_p1
matches = request.fetchall()
matches = [ (int(p1), int(p2), int(win)) for p1,p2,win in matches]
print(f'Total number of matches: {len(matches)}')
```

```
Total number of matches: 2976
```

StanModel Template

In [9]:

```
skill_model = """
data {
  int<lower=1> N;             # Total number of players
  int<lower=1> E;             # number of games
  real<lower=0> scale;        # scale value for probability computation
  int<lower=0,upper=1> win[E]; # PA wins vs PB
  int PA[E];                  # player info between each game
  int PB[E];                  #
}
parameters {
  vector[N] skill;           # skill values for each player
}

model{
  for (i in 1:N){ skill[i]~normal(0,3); }
  for (i in 1:E){
    win[i] ~ bernoulli_logit( (scale)*(skill[PA[i]]-skill[PB[i]]) );
  }   # win probability is a logit function of skill difference
}
"""
```

Compile model

In [10]:

```
import pickle
try:
    sm = pickle.load(open('skill_model.pkl', 'rb'))
except:
    sm = pystan.StanModel(model_code = skill_model)
    with open('skill_model.pkl', 'wb') as f: pickle.dump(sm, f)
```

Evaluate matches and set win, pA, pB lists

In [11]:

```
win = [1*(w == p1) for p1,_,w in matches]
pA =  [playerid_to_adjustedid[p1] for p1,_,_ in matches]
pB =  [playerid_to_adjustedid[p2] for _,p2,_ in matches]
```

Perform MCMC on model and take the empirical average over the samples to get the mean estimate for every player's skill level

In [12]:

```
scales = [i/10 for i in range(1,10)]
skill_levels = []

for scale in scales :
    skill_data = {
        'N': n,
        'E': len(matches),
        'scale': scale,
        'win': win,
        'PA': pA,
```

```
    'PB': pB
}

fit = sm.sampling(data=skill_data, iter=10000, chains=4)
samples = fit.extract()

skill_level = [(adjustedid_to_playerid[i+1], skill) for i,skill in enumera
skill_level.sort(key = lambda x : -x[1])

skill_levels.append(skill_level)
```

```
Gradient evaluation took 0.000775 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.75 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000907 seconds
1000 transitions using 10 leapfrog steps per transition would take 9.07 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.001082 seconds
1000 transitions using 10 leapfrog steps per transition would take 10.82 secon
ds.
Adjust your expectations accordingly!



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1000 transitions using 10 leapfrog steps per transition would take 10.03 secon
ds.
Adjust your expectations accordingly!


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               49.0172 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 34.3123 seconds (Warm-up)
               16.6417 seconds (Sampling)
               50.954 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 33.3824 seconds (Warm-up)
               17.7854 seconds (Sampling)
               51.1678 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 34.601 seconds (Warm-up)
               16.7082 seconds (Sampling)
               51.3091 seconds (Total)


Gradient evaluation took 0.000858 seconds
1000 transitions using 10 leapfrog steps per transition would take 8.58 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.001078 seconds
1000 transitions using 10 leapfrog steps per transition would take 10.78 secon
ds.
Adjust your expectations accordingly!



Gradient evaluation took 0.000918 seconds
1000 transitions using 10 leapfrog steps per transition would take 9.18 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000978 seconds
1000 transitions using 10 leapfrog steps per transition would take 9.78 second
s.
```

```
Adjust your expectations accordingly!


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               18.2089 seconds (Sampling)
               45.8769 seconds (Total)

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               17.5669 seconds (Sampling)
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               17.233 seconds (Sampling)
               46.9755 seconds (Total)


Gradient evaluation took 0.00098 seconds
1000 transitions using 10 leapfrog steps per transition would take 9.8 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000721 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.21 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000789 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.89 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.001034 seconds
1000 transitions using 10 leapfrog steps per transition would take 10.34 secon
ds.
Adjust your expectations accordingly!


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               16.9476 seconds (Sampling)
               43.8564 seconds (Total)

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               17.8223 seconds (Sampling)
               45.0339 seconds (Total)

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               17.0358 seconds (Sampling)
               45.5464 seconds (Total)


Gradient evaluation took 0.001303 seconds
1000 transitions using 10 leapfrog steps per transition would take 13.03 secon
ds.
Adjust your expectations accordingly!



Gradient evaluation took 0.001191 seconds

1000 transitions using 10 leapfrog steps per transition would take 11.91 secon
ds.
Adjust your expectations accordingly!
```

```
Gradient evaluation took 0.001162 seconds
1000 transitions using 10 leapfrog steps per transition would take 11.62 secon
ds.
Adjust your expectations accordingly!



Gradient evaluation took 0.001369 seconds
1000 transitions using 10 leapfrog steps per transition would take 13.69 secon
ds.
Adjust your expectations accordingly!


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                18.6294 seconds (Sampling)
                44.9403 seconds (Total)

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 Elapsed Time: 26.1861 seconds (Warm-up)
               18.8925 seconds (Sampling)
               45.0785 seconds (Total)

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               20.5429 seconds (Sampling)
               45.982 seconds (Total)

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 Elapsed Time: 25.9069 seconds (Warm-up)
               32.0121 seconds (Sampling)
               57.919 seconds (Total)



Gradient evaluation took 0.000765 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.65 second
s.
Adjust your expectations accordingly!


Gradient evaluation took 0.000754 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.54 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000722 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.22 second
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Adjust your expectations accordingly!



Gradient evaluation took 0.000722 seconds
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Adjust your expectations accordingly!


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               17.5761 seconds (Sampling)
               42.6868 seconds (Total)

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               17.4536 seconds (Sampling)
               42.7258 seconds (Total)

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 Elapsed Time: 25.0348 seconds (Warm-up)
               18.8501 seconds (Sampling)
               43.8849 seconds (Total)

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 Elapsed Time: 26.7775 seconds (Warm-up)
               20.7999 seconds (Sampling)
               47.5774 seconds (Total)
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Adjust your expectations accordingly!



Gradient evaluation took 0.000821 seconds
1000 transitions using 10 leapfrog steps per transition would take 8.21 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000737 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.37 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000861 seconds
1000 transitions using 10 leapfrog steps per transition would take 8.61 second
s.
Adjust your expectations accordingly!


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Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 26.3455 seconds (Warm-up)
               18.1933 seconds (Sampling)
               44.5388 seconds (Total)

Iteration: 9000 / 10000 [ 90%]  (Sampling)
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Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 26.5279 seconds (Warm-up)
               26.955 seconds (Sampling)
               53.483 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 26.1434 seconds (Warm-up)
               29.5855 seconds (Sampling)
               55.7288 seconds (Total)

Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 26.705 seconds (Warm-up)
               36.1391 seconds (Sampling)
               62.8441 seconds (Total)


Gradient evaluation took 0.000747 seconds

1000 transitions using 10 leapfrog steps per transition would take 7.47 second
s.
Adjust your expectations accordingly!


Gradient evaluation took 0.000733 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.33 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000734 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.34 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000734 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.34 second
s.
```

```
Adjust your expectations accordingly!


Iteration:    1 / 10000 [  0%]  (Warmup)
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Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 28.4305 seconds (Warm-up)
               20.9756 seconds (Sampling)
               49.4061 seconds (Total)

Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 27.9645 seconds (Warm-up)
               27.0826 seconds (Sampling)
               55.047 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)
```

```
 Elapsed Time: 29.4557 seconds (Warm-up)
               29.6565 seconds (Sampling)
               59.1123 seconds (Total)

Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 27.6739 seconds (Warm-up)
               42.1461 seconds (Sampling)
               69.8201 seconds (Total)


Gradient evaluation took 0.000953 seconds
1000 transitions using 10 leapfrog steps per transition would take 9.53 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000889 seconds
1000 transitions using 10 leapfrog steps per transition would take 8.89 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000752 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.52 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.0014 seconds
1000 transitions using 10 leapfrog steps per transition would take 14 seconds.
Adjust your expectations accordingly!


Iteration:    1 / 10000 [  0%]  (Warmup)
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Iteration: 5001 / 10000 [ 50%]  (Sampling)
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Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 31.5781 seconds (Warm-up)
               31.5897 seconds (Sampling)
               63.1678 seconds (Total)

Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 31.9472 seconds (Warm-up)
               35.175 seconds (Sampling)
               67.1222 seconds (Total)

Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 31.6051 seconds (Warm-up)
               42.0778 seconds (Sampling)
               73.683 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 32.0281 seconds (Warm-up)
               50.0325 seconds (Sampling)
               82.0606 seconds (Total)


Gradient evaluation took 0.000771 seconds
1000 transitions using 10 leapfrog steps per transition would take 7.71 second
s.
Adjust your expectations accordingly!



Gradient evaluation took 0.000834 seconds
1000 transitions using 10 leapfrog steps per transition would take 8.34 second
s.
Adjust your expectations accordingly!
```

```
Gradient evaluation took 0.001154 seconds
1000 transitions using 10 leapfrog steps per transition would take 11.54 secon
ds.
Adjust your expectations accordingly!



Gradient evaluation took 0.001196 seconds
1000 transitions using 10 leapfrog steps per transition would take 11.96 secon
ds.
Adjust your expectations accordingly!


Iteration:    1 / 10000 [  0%]  (Warmup)
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Iteration: 7000 / 10000 [ 70%]  (Sampling)
Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 7000 / 10000 [ 70%]  (Sampling)
Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 38.7165 seconds (Warm-up)
               32.8707 seconds (Sampling)
               71.5872 seconds (Total)
```

```
Iteration: 8000 / 10000 [ 80%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 36.9745 seconds (Warm-up)
               38.4573 seconds (Sampling)
               75.4318 seconds (Total)

Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 9000 / 10000 [ 90%]  (Sampling)
Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 38.2327 seconds (Warm-up)
               48.5357 seconds (Sampling)
               86.7684 seconds (Total)

Iteration: 10000 / 10000 [100%]  (Sampling)

 Elapsed Time: 36.1955 seconds (Warm-up)
               61.0011 seconds (Sampling)
               97.1967 seconds (Total)
```

Helper Functions for printing

In [17]:

```python
import math

def printResults(skill_levels) :
    mse = 0
    for i in range(n) :
        playerid,_ = skill_levels[i]
        mse += (i+1 - playerid_to_placement[playerid])**2
    mse /= n
    print(f"\nMEAN SQUARED ERROR: {mse}\n")

    print('{:3} {:<12}  {:<16}  {:<15}\n'.format(" ", "ACTUAL", "PREDICTED", "
    for i,(playerid,skill) in enumerate(skill_levels) :
        acutal = playerid_to_name[placement_to_playerid[i+1]]
        predicted = playerid_to_name[playerid]
        print('{:3} {:<12}  {:<16}  {:<15}'.format(i+1, acutal, predicted + f'

    return mse

def printErrorBiasForHigherPlacements(skill_levels) :
    mse = 0
    alpha = 1
    for i in range(n) :
        playerid,_ = skill_levels[i]
        mse += (i+1 - playerid_to_placement[playerid])**2 * math.exp(alpha*(n+
    mse /= n
    print(f"\nMEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: {mse}

    return mse

def printMSEByScale(mse, mse_bias, scales) :

    print('{:3} {:<12}  {:<16}  {:<15}\n'.format(" ", "scale", "MSE", "MSE_Bia
```

```
    for mse,mse_b,scale in zip(mse, mse_bias, scales) :
        print('{:3} {:<12}  {:<16}  {:<15}'.format(" ", scale, mse, mse_b))
```

Results

In [14]:

```
mse = []
mse_bias = []

for scale, skill_level in zip(scales, skill_levels) :
    print(f'MCMC Sampling with 10,000 iterations with 4 chains on a StanModel

    mse.append(printResults(skill_level))
    mse_bias.append(printErrorBiasForHigherPlacements(skill_level))

    print('----------------------------------------------------------------------
```

```
MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale l
evel: 0.1

MEAN SQUARED ERROR: 113.76
```

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	10.979739409030808
2	Tweek	Tweek (2)	8.820777237980005
3	Marss	Shuton (5)	7.224228786438038
4	Samsora	zackray (12)	6.261066748523263
5	Shuton	ProtoBanham (23)	6.249699229614205
6	Ally	Glutonny (14)	6.217147782940904
7	Dabuz	Marss (3)	5.735728106279453
8	Nairo	Nairo (8)	5.7143598008812875
9	VoiD	Samsora (4)	5.392393794476569
10	Light	Light (10)	5.140584921084673
11	Cosmos	Tea (15)	3.845478185373747
12	zackray	Dabuz (7)	3.219915948812649
13	Myran	Salem (22)	2.159916004407288
14	Glutonny	Nietono (44)	1.9554359011249922
15	Tea	ESAM (16)	1.899785474449577
16	ESAM	Ally (6)	1.8536821584584344
17	MVD	Kameme (19)	1.7479262378894214
18	Rivers	Cosmos (11)	1.4595485455244295
19	Kameme	Rivers (18)	1.2178593443746368
20	Raito	LeoN (34)	0.7680140898415182
21	justy	VoiD (9)	0.4044067886316225
22	Salem	Lea (30)	0.3114757996243297
23	ProtoBanham	Dark Wizzy (31)	-0.030839847147495318
24	WaDi	Kurama (36)	-0.24719365996420675
25	Sinji	Myran (13)	-0.8449449087359239
26	NAKAT	Abadango (39)	-1.0593167221041837
27	MuteAce	Tsu (43)	-1.0597414288510965
28	Puppeh	Ryuga (33)	-1.1692000842765824
29	Stroder Ame	WaDi (24)	-1.5125866004716848
30	Lea	T (46)	-1.5457653736815387
31	Dark Wizzy	Secretary (32)	-1.6841121957543608
32	Secretary	Marcus (42)	-1.960333457758064
33	Ryuga	Sinji (25)	-2.1843884368225424
34	LeoN	justy (21)	-2.285595061972718
35	Mr.R	Stroder Ame (29)	-2.628612899010608
36	Kurama	MVD (17)	-2.681468967557393
37	Mr. E	Mr. E (37)	-2.6866702588530575
38	Goblin	Puppeh (28)	-3.0813425004840074
39	Abadango	Captain L (49)	-3.5368231075886505
40	ScAtt	Fatality (47)	-3.8687346462485817
41	Umeki	MuteAce (27)	-3.948724922729132
42	Marcus	ScAtt (40)	-4.151260193373242
43	Tsu	Umeki (41)	-4.187723686029087
44	Nietono	Mr.R (35)	-4.188714096139262
45	Frozen	NAKAT (26)	-4.234479038290618
46	T	ZD (50)	-4.335142527369656
47	Fatality	Goblin (38)	-5.33042612273482
48	Suarez	Raito (20)	-6.547301106719237
49	Captain L	Frozen (45)	-8.624953491959129
50	ZD	Suarez (48)	-8.87899940659451

```
MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 180.65289712674385
```

```
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--------

MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale l
evel: 0.2

MEAN SQUARED ERROR: 109.36
```

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	7.152761364250594
2	Tweek	Tweek (2)	5.744949793593025
3	Marss	Shuton (5)	5.0442512606050105
4	Samsora	ProtoBanham (23)	4.76039760894299
5	Shuton	zackray (12)	4.61368511057346
6	Ally	Glutonny (14)	4.474759727080356
7	Dabuz	Nairo (8)	4.212414651039047
8	Nairo	Marss (3)	4.046456598957886
9	VoiD	Samsora (4)	3.972917275209029
10	Light	Light (10)	3.562965842139682
11	Cosmos	Tea (15)	3.085067875587834
12	zackray	Dabuz (7)	2.2596031349024575
13	Myran	Nietono (44)	1.9831838973968239
14	Glutonny	Ally (6)	1.7003310387497705
15	Tea	Kameme (19)	1.6488839752775333
16	ESAM	Salem (22)	1.4530174924189936
17	MVD	ESAM (16)	1.4156015395835544
18	Rivers	Cosmos (11)	1.2463247670866275
19	Kameme	Rivers (18)	0.8526691730019058
20	Raito	VoiD (9)	0.7533113673092295
21	justy	Lea (30)	0.7213706019535717
22	Salem	LeoN (34)	0.396432927931427
23	ProtoBanham	Dark Wizzy (31)	0.21039827456927132
24	WaDi	Kurama (36)	0.10551397496987854
25	Sinji	Abadango (39)	-0.23975320363826022
26	NAKAT	Tsu (43)	-0.27087470897022553
27	MuteAce	Myran (13)	-0.37946204810514583
28	Puppeh	T (46)	-0.5549855886530025
29	Stroder Ame	WaDi (24)	-0.9388184290485401
30	Lea	Ryuga (33)	-1.445458779357731
31	Dark Wizzy	MVD (17)	-1.5148678929580988
32	Secretary	justy (21)	-1.5342435597463961
33	Ryuga	Marcus (42)	-1.5537968557788457
34	LeoN	Secretary (32)	-1.7163620771475923
35	Mr.R	Sinji (25)	-1.820946957303965
36	Kurama	Stroder Ame (29)	-1.9152310748907855
37	Mr. E	Mr. E (37)	-1.9793957465046712
38	Goblin	Puppeh (28)	-2.3406825377350473
39	Abadango	MuteAce (27)	-2.368321265904996
40	ScAtt	Umeki (41)	-2.4176071031326525
41	Umeki	Fatality (47)	-2.9749894254385865
42	Marcus	ZD (50)	-3.1920305434083374
43	Tsu	Mr.R (35)	-3.2109834688267065
44	Nietono	Goblin (38)	-3.423071105262377
45	Frozen	ScAtt (40)	-3.6400783893201742
46	T	NAKAT (26)	-3.64024116999197
47	Fatality	Raito (20)	-3.9232026942065845
48	Suarez	Captain L (49)	-3.9795190144864887
49	Captain L	Suarez (48)	-6.1090906552703625
50	ZD	Frozen (45)	-7.996403286224477

```
MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 171.1471931049876

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MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale l
evel: 0.3

MEAN SQUARED ERROR: 109.56
```

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	5.106565778662765
2	Tweek	Tweek (2)	4.101689616482888
3	Marss	Shuton (5)	3.7033681245687213
4	Samsora	ProtoBanham (23)	3.580347110525753
5	Shuton	zackray (12)	3.4359536943859785
6	Ally	Glutonny (14)	3.2796865913007336
7	Dabuz	Nairo (8)	3.1040072908078846
8	Nairo	Marss (3)	2.951956528082662
9	VoiD	Samsora (4)	2.931947483123424
10	Light	Light (10)	2.5863157698254486
11	Cosmos	Tea (15)	2.339141765698015
12	zackray	Dabuz (7)	1.6494729156447978
13	Myran	Nietono (44)	1.6346467796540598
14	Glutonny	Kameme (19)	1.3243701089692945
15	Tea	Ally (6)	1.3199899744609596
16	ESAM	ESAM (16)	1.0570591432430594
17	MVD	Salem (22)	1.0471284589062786
18	Rivers	Cosmos (11)	0.9592905169087675
19	Kameme	Lea (30)	0.7075764119681116
20	Raito	VoiD (9)	0.6575136892232316
21	justy	Rivers (18)	0.5970877213974327
22	Salem	LeoN (34)	0.2435575849338195
23	ProtoBanham	Dark Wizzy (31)	0.2008967212134101
24	WaDi	Kurama (36)	0.15604411161167958
25	Sinji	Abadango (39)	-0.011909279015859637
26	NAKAT	Tsu (43)	-0.035357792238156015
27	MuteAce	Myran (13)	-0.22598739700800494
28	Puppeh	T (46)	-0.23259996608758174
29	Stroder Ame	WaDi (24)	-0.6747453462982815
30	Lea	MVD (17)	-1.030089768699379
31	Dark Wizzy	justy (21)	-1.1269868630038735
32	Secretary	Marcus (42)	-1.1976589513304954
33	Ryuga	Stroder Ame (29)	-1.3807055468812224
34	LeoN	Ryuga (33)	-1.3856946980639266
35	Mr.R	Sinji (25)	-1.4231659869729207
36	Kurama	Secretary (32)	-1.4455507797288105
37	Mr. E	Mr. E (37)	-1.4874291550516572
38	Goblin	Umeki (41)	-1.5930030863207645
39	Abadango	MuteAce (27)	-1.6298467728905384
40	ScAtt	Puppeh (28)	-1.756866661591838
41	Umeki	Fatality (47)	-2.2412421868339547
42	Marcus	ZD (50)	-2.364293903732143
43	Tsu	Goblin (38)	-2.427799597736325
44	Nietono	Mr.R (35)	-2.435066746512953
45	Frozen	Raito (20)	-2.660565809856121
46	T	ScAtt (40)	-2.8479589541535106
47	Fatality	NAKAT (26)	-2.873892679189704

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 48 Suarez        Captain L (49)    -3.5081838972052415
 49 Captain L     Suarez (48)       -4.451964147983954
 50 ZD            Frozen (45)       -6.630454382981157

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 170.67117115204704

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MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale l
evel: 0.4

MEAN SQUARED ERROR: 113.2

    ACTUAL        PREDICTED         SKILL

  1 MkLeo         MkLeo (1)         3.962979026616765
  2 Tweek         Tweek (2)         3.1875505902825005
  3 Marss         Shuton (5)        2.913647223975868
  4 Samsora       ProtoBanham (23)  2.8430601615432085
  5 Shuton        zackray (12)      2.7206559395132857
  6 Ally          Glutonny (14)     2.584530955175557
  7 Dabuz         Nairo (8)         2.446648788188236
  8 Nairo         Marss (3)         2.3183143845642533
  9 VoiD          Samsora (4)       2.3107121863197317
 10 Light         Light (10)        2.0327745657720158
 11 Cosmos        Tea (15)          1.8718757168864155
 12 zackray       Nietono (44)      1.3698983452362943
 13 Myran         Dabuz (7)         1.3048535717268162
 14 Glutonny      Kameme (19)       1.1059001691076447
 15 Tea           Ally (6)          1.086736881864433
 16 ESAM          ESAM (16)         0.8540793810106189
 17 MVD           Salem (22)        0.8275024384870587
 18 Rivers        Cosmos (11)       0.7810320543225966
 19 Kameme        Lea (30)          0.643443350149947
 20 Raito         VoiD (9)          0.5675058107951976
 21 justy         Rivers (18)       0.4768568990079621
 22 Salem         LeoN (34)         0.19930669772420664
 23 ProtoBanham   Dark Wizzy (31)   0.19504366359154893
 24 WaDi          Kurama (36)       0.168689373058287
 25 Sinji         Tsu (43)          0.06060456559845363
 26 NAKAT         Abadango (39)     0.06037790436387337
 27 MuteAce       T (46)            -0.09879402832497543
 28 Puppeh        Myran (13)        -0.14161559648265437
 29 Stroder Ame   WaDi (24)         -0.48880182003946604
 30 Lea           MVD (17)          -0.7505823620120075
 31 Dark Wizzy    justy (21)        -0.8491102441635727
 32 Secretary     Marcus (42)       -0.9209607005011301
 33 Ryuga         Stroder Ame (29)  -1.0381837487418275
 34 LeoN          Sinji (25)        -1.1156936251024265
 35 Mr.R          Mr. E (37)        -1.1488427798681315
 36 Kurama        Umeki (41)        -1.1511119822593725
 37 Mr. E         Secretary (32)    -1.1624508582485524
 38 Goblin        Ryuga (33)        -1.179996499404393
 39 Abadango      MuteAce (27)      -1.206042978493296
 40 ScAtt         Puppeh (28)       -1.3480011993437204
 41 Umeki         Fatality (47)     -1.742667245355798
 42 Marcus        ZD (50)           -1.8127042018940196
 43 Tsu           Goblin (38)       -1.8428955134021394
 44 Nietono       Mr.R (35)         -1.9065551638779419
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 45 Frozen        Raito (20)        -1.9639057655066408
 46 T             ScAtt (40)        -2.2587025186587217
 47 Fatality      NAKAT (26)        -2.296035945706626
 48 Suarez        Captain L (49)    -2.9723491383275404
 49 Captain L     Suarez (48)       -3.431039777483951
 50 ZD            Frozen (45)       -5.478180649903033

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 175.65266193560356

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MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale l
evel: 0.5

MEAN SQUARED ERROR: 113.48

    ACTUAL        PREDICTED         SKILL

  1 MkLeo         MkLeo (1)         3.2201291697964884
  2 Tweek         Tweek (2)         2.594243999859888
  3 Marss         Shuton (5)        2.3908936856110694
  4 Samsora       ProtoBanham (23)  2.3485603864167532
  5 Shuton        zackray (12)      2.2411621253415555
  6 Ally          Glutonny (14)     2.115474636865826
  7 Dabuz         Nairo (8)         2.003743352934233
  8 Nairo         Marss (3)         1.8997275287545181
  9 VoiD          Samsora (4)       1.8936245874521613
 10 Light         Light (10)        1.6662043132503916
 11 Cosmos        Tea (15)          1.5516371169646908
 12 zackray       Nietono (44)      1.1644305480502974
 13 Myran         Dabuz (7)         1.075991951949985
 14 Glutonny      Kameme (19)       0.9302073784388558
 15 Tea           Ally (6)          0.9062647725749297
 16 ESAM          ESAM (16)         0.7059516208979129
 17 MVD           Salem (22)        0.6862290616985751
 18 Rivers        Cosmos (11)       0.6525746286061626
 19 Kameme        Lea (30)          0.569688542827844
 20 Raito         VoiD (9)          0.49193692266101136
 21 justy         Rivers (18)       0.40069537878505973
 22 Salem         Dark Wizzy (31)   0.1773985785435086
 23 ProtoBanham   LeoN (34)         0.16800313699748695
 24 WaDi          Kurama (36)       0.16385733683014547
 25 Sinji         Abadango (39)     0.09332436904062955
 26 NAKAT         Tsu (43)          0.0886187065710175
 27 MuteAce       T (46)            -0.03920383159837061
 28 Puppeh        Myran (13)        -0.10122910290848582
 29 Stroder Ame   WaDi (24)         -0.38105905789730204
 30 Lea           MVD (17)          -0.5933360798160604
 31 Dark Wizzy    justy (21)        -0.6794489188666304
 32 Secretary     Marcus (42)       -0.7502527576969438
 33 Ryuga         Stroder Ame (29)  -0.8333533839884627
 34 LeoN          Umeki (41)        -0.8957726364219605
 35 Mr.R          Sinji (25)        -0.9018479934079129
 36 Kurama        Mr. E (37)        -0.9289280746286275
 37 Mr. E         Secretary (32)    -0.9524372210089008
 38 Goblin        MuteAce (27)      -0.9655450862753842
 39 Abadango      Ryuga (33)        -1.0284225901165978
 40 ScAtt         Puppeh (28)       -1.0919322661670152
 41 Umeki         Fatality (47)     -1.4234543291995676
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42	Marcus	ZD (50)	-1.4684599648137353
43	Tsu	Goblin (38)	-1.4848340496277572
44	Nietono	Mr.R (35)	-1.5430443617960594
45	Frozen	Raito (20)	-1.5581680496801975
46	T	ScAtt (40)	-1.8495472886806612
47	Fatality	NAKAT (26)	-1.898533984614107
48	Suarez	Captain L (49)	-2.5031064199451554
49	Captain L	Suarez (48)	-2.777600672490494
50	ZD	Frozen (45)	-4.643978182140836

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 176.0966176608784

MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale l
evel: 0.6

MEAN SQUARED ERROR: 112.68

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	2.7093471827623317
2	Tweek	Tweek (2)	2.1793122053826517
3	Marss	Shuton (5)	2.015495110574019
4	Samsora	ProtoBanham (23)	1.9807168047041785
5	Shuton	zackray (12)	1.8920083478439607
6	Ally	Glutonny (14)	1.7849298271797833
7	Dabuz	Nairo (8)	1.6887341296630836
8	Nairo	Marss (3)	1.5990685998590703
9	VoiD	Samsora (4)	1.5943248267345693
10	Light	Light (10)	1.3995881619618191
11	Cosmos	Tea (15)	1.3135253381419634
12	zackray	Nietono (44)	0.9907667678892506
13	Myran	Dabuz (7)	0.9036846910376795
14	Glutonny	Kameme (19)	0.7971562025760005
15	Tea	Ally (6)	0.7634762855267481
16	ESAM	ESAM (16)	0.597108690943673
17	MVD	Salem (22)	0.5803550696157151
18	Rivers	Cosmos (11)	0.5527767897348634
19	Kameme	Lea (30)	0.4974343909981988
20	Raito	VoiD (9)	0.4221238736478186
21	justy	Rivers (18)	0.3367499941298221
22	Salem	Dark Wizzy (31)	0.15053476203619215
23	ProtoBanham	LeoN (34)	0.140759484245712
24	WaDi	Kurama (36)	0.13311651356028034
25	Sinji	Abadango (39)	0.08951238806027761
26	NAKAT	Tsu (43)	0.08876753418130802
27	MuteAce	T (46)	-0.015165744075557347
28	Puppeh	Myran (13)	-0.08451027849892229
29	Stroder Ame	WaDi (24)	-0.320216564218067
30	Lea	MVD (17)	-0.49290769577611593
31	Dark Wizzy	justy (21)	-0.5714006166742674
32	Secretary	Marcus (42)	-0.6309010490917052
33	Ryuga	Stroder Ame (29)	-0.6980333912163975
34	LeoN	Umeki (41)	-0.7363589558603784
35	Mr.R	Sinji (25)	-0.7665253797571397
36	Kurama	Mr. E (37)	-0.7842848163115314
37	Mr. E	MuteAce (27)	-0.8043750892708826
38	Goblin	Secretary (32)	-0.8230138043773081

39	Abadango	Ryuga (33)	−0.877889681693354
40	ScAtt	Puppeh (28)	−0.9237653240075713
41	Umeki	Fatality (47)	−1.2034299233027603
42	Marcus	ZD (50)	−1.234052913529722
43	Tsu	Goblin (38)	−1.2453904621735983
44	Nietono	Raito (20)	−1.2881928279526924
45	Frozen	Mr.R (35)	−1.3098182164976788
46	T	ScAtt (40)	−1.5744167480783535
47	Fatality	NAKAT (26)	−1.6133536145446563
48	Suarez	Captain L (49)	−2.1724031689837004
49	Captain L	Suarez (48)	−2.335253394725979
50	ZD	Frozen (45)	−4.002910150250495

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 174.4411280910711

MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale level: 0.7

MEAN SQUARED ERROR: 112.76

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	2.3133921126873886
2	Tweek	Tweek (2)	1.860671372543586
3	Marss	Shuton (5)	1.7238832373812831
4	Samsora	ProtoBanham (23)	1.693745935631141
5	Shuton	zackray (12)	1.6191388925399581
6	Ally	Glutonny (14)	1.5222487162568157
7	Dabuz	Nairo (8)	1.437618057388941
8	Nairo	Marss (3)	1.3625864926286964
9	VoiD	Samsora (4)	1.357230686752678
10	Light	Light (10)	1.1919053407372655
11	Cosmos	Tea (15)	1.117845369951226
12	zackray	Nietono (44)	0.8391837002804756
13	Myran	Dabuz (7)	0.7638454131233033
14	Glutonny	Kameme (19)	0.6761622131630924
15	Tea	Ally (6)	0.6502669300287186
16	ESAM	ESAM (16)	0.4990672634900424
17	MVD	Salem (22)	0.4751664674619442
18	Rivers	Cosmos (11)	0.46023710376450006
19	Kameme	Lea (30)	0.4179940879151505
20	Raito	VoiD (9)	0.3500902705902501
21	justy	Rivers (18)	0.2671595520951485
22	Salem	Dark Wizzy (31)	0.11572398575296795
23	ProtoBanham	Kurama (36)	0.1022482937966432
24	WaDi	LeoN (34)	0.1017226687981238
25	Sinji	Abadango (39)	0.06751077130534693
26	NAKAT	Tsu (43)	0.06688851726136744
27	MuteAce	T (46)	−0.024793152290094444
28	Puppeh	Myran (13)	−0.08926709955355079
29	Stroder Ame	WaDi (24)	−0.291867387164128
30	Lea	MVD (17)	−0.4413943279681102
31	Dark Wizzy	justy (21)	−0.5122543804822322
32	Secretary	Marcus (42)	−0.566940553461282
33	Ryuga	Stroder Ame (29)	−0.6213153258616165
34	LeoN	Umeki (41)	−0.6520140668893362
35	Mr.R	Sinji (25)	−0.6892940203274367

36	Kurama	Mr. E (37)	-0.6984870179084641
37	Mr. E	MuteAce (27)	-0.7041522406694835
38	Goblin	Secretary (32)	-0.7379455650317482
39	Abadango	Ryuga (33)	-0.7953123296620712
40	ScAtt	Puppeh (28)	-0.8143322771964394
41	Umeki	Fatality (47)	-1.0544601109829574
42	Marcus	ZD (50)	-1.079797765821234
43	Tsu	Goblin (38)	-1.0870693788129968
44	Nietono	Raito (20)	-1.1234368556967493
45	Frozen	Mr.R (35)	-1.1487874618667755
46	T	ScAtt (40)	-1.3783500777721893
47	Fatality	NAKAT (26)	-1.4127864431095674
48	Suarez	Captain L (49)	-1.9119900627696464
49	Captain L	Suarez (48)	-2.0379231472907495
50	ZD	Frozen (45)	-3.536355105773172

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 174.5259795068224

MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale level: 0.8

MEAN SQUARED ERROR: 112.56

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	2.0560405835222806
2	Tweek	Tweek (2)	1.6548532013982953
3	Marss	Shuton (5)	1.5389181699964622
4	Samsora	ProtoBanham (23)	1.516307724557869
5	Shuton	zackray (12)	1.448811283853132
6	Ally	Glutonny (14)	1.360352052798821
7	Dabuz	Nairo (8)	1.2860337298968554
8	Nairo	Marss (3)	1.2209315773021778
9	VoiD	Samsora (4)	1.2172004355788282
10	Light	Light (10)	1.0708201433845457
11	Cosmos	Tea (15)	1.007815397995126
12	zackray	Nietono (44)	0.766840185318614
13	Myran	Dabuz (7)	0.6930291178512453
14	Glutonny	Kameme (19)	0.6199080738499168
15	Tea	Ally (6)	0.595582163146622
16	ESAM	ESAM (16)	0.46181568103339676
17	MVD	Salem (22)	0.4434269147425176
18	Rivers	Cosmos (11)	0.4284466088006434
19	Kameme	Lea (30)	0.3983923285616227
20	Raito	VoiD (9)	0.3352810867376731
21	justy	Rivers (18)	0.2590892693867286
22	Salem	Dark Wizzy (31)	0.12375262578321568
23	ProtoBanham	Kurama (36)	0.11723316500735363
24	WaDi	LeoN (34)	0.11263031817532655
25	Sinji	Abadango (39)	0.08859450975869734
26	NAKAT	Tsu (43)	0.08688663265635177
27	MuteAce	T (46)	0.009562727404560354
28	Puppeh	Myran (13)	-0.05331344864486149
29	Stroder Ame	WaDi (24)	-0.23126161202501275
30	Lea	MVD (17)	-0.3630450561504546
31	Dark Wizzy	justy (21)	-0.42398589893750166
32	Secretary	Marcus (42)	-0.4741289661914474

33	Ryuga	Stroder Ame (29)	−0.518013845615441
34	LeoN	Umeki (41)	−0.5437907510522526
35	Mr.R	Sinji (25)	−0.5808109115934088
36	Kurama	Mr. E (37)	−0.5900457103541985
37	Mr. E	MuteAce (27)	−0.5946396781727944
38	Goblin	Secretary (32)	−0.6237298798154239
39	Abadango	Puppeh (28)	−0.6912905537548005
40	ScAtt	Ryuga (33)	−0.6936380245157377
41	Umeki	Fatality (47)	−0.9048103739592949
42	Marcus	ZD (50)	−0.9270144521178015
43	Tsu	Goblin (38)	−0.9312319301682433
44	Nietono	Raito (20)	−0.9584566191341822
45	Frozen	Mr.R (35)	−0.9857126524520027
46	T	ScAtt (40)	−1.1912733158423394
47	Fatality	NAKAT (26)	−1.2270031309546707
48	Suarez	Captain L (49)	−1.6854721479040446
49	Captain L	Suarez (48)	−1.7642038657853292
50	ZD	Frozen (45)	−3.115868324234071

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 174.16997112151066

MCMC Sampling with 10,000 iterations with 4 chains on a StanModel with scale level: 0.9

MEAN SQUARED ERROR: 112.56

	ACTUAL	PREDICTED	SKILL
1	MkLeo	MkLeo (1)	1.7938646329402186
2	Tweek	Tweek (2)	1.437362817643228
3	Marss	Shuton (5)	1.3329605995806175
4	Samsora	ProtoBanham (23)	1.313611659061951
5	Shuton	zackray (12)	1.2515754267548902
6	Ally	Glutonny (14)	1.1739917883767363
7	Dabuz	Nairo (8)	1.1117059958110191
8	Nairo	Marss (3)	1.0494829426913987
9	VoiD	Samsora (4)	1.0475229632216896
10	Light	Light (10)	0.9171197375116339
11	Cosmos	Tea (15)	0.8592275068655377
12	zackray	Nietono (44)	0.6481049750826796
13	Myran	Dabuz (7)	0.5830036653172651
14	Glutonny	Kameme (19)	0.5142162040823848
15	Tea	Ally (6)	0.4972957357197236
16	ESAM	ESAM (16)	0.37616486311353875
17	MVD	Salem (22)	0.3603439845813855
18	Rivers	Cosmos (11)	0.34789894933381144
19	Kameme	Lea (30)	0.3157457087822692
20	Raito	VoiD (9)	0.2606004855605129
21	justy	Rivers (18)	0.19464097428555857
22	Salem	Dark Wizzy (31)	0.07619568237488457
23	ProtoBanham	Kurama (36)	0.06846845402525165
24	WaDi	LeoN (34)	0.06362089646343001
25	Sinji	Abadango (39)	0.04253150794611341
26	NAKAT	Tsu (43)	0.041100661562689024
27	MuteAce	T (46)	−0.02938749920981263
28	Puppeh	Myran (13)	−0.08222071173849872
29	Stroder Ame	WaDi (24)	−0.24396499856319398

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30 Lea           MVD (17)          -0.3569390118323179
31 Dark Wizzy    justy (21)        -0.41628995773204286
32 Secretary     Marcus (42)       -0.45985699703395605
33 Ryuga         Stroder Ame (29)  -0.5021626945268053
34 LeoN          Umeki (41)        -0.5191115334240114
35 Mr.R          Sinji (25)        -0.5571071178087226
36 Kurama        Mr. E (37)        -0.5627589026928002
37 Mr. E         MuteAce (27)      -0.5662134865522609
38 Goblin        Secretary (32)    -0.6025361541280349
39 Abadango      Puppeh (28)       -0.6528768844870569
40 ScAtt         Ryuga (33)        -0.6564930399237657
41 Umeki         Fatality (47)     -0.8405727029273806
42 Marcus        ZD (50)           -0.8641433569535735
43 Tsu           Goblin (38)       -0.8658456830319671
44 Nietono       Raito (20)        -0.8911974701807636
45 Frozen        Mr.R (35)         -0.9157327685936784
46 T             ScAtt (40)        -1.0958204552312563
47 Fatality      NAKAT (26)        -1.1322847341614668
48 Suarez        Captain L (49)    -1.5404455789802571
49 Captain L     Suarez (48)       -1.6113862454484196
50 ZD            Frozen (45)       -2.826196499157645

MEAN SQUARED ERROR WITH BIAS FOR CORRECT HIGHER PLACEMENT: 174.16997112151066

--------------------------------------------------------------------------------
--------
```

In [20]:

```
printMSEByScale(mse, mse_bias, scales)

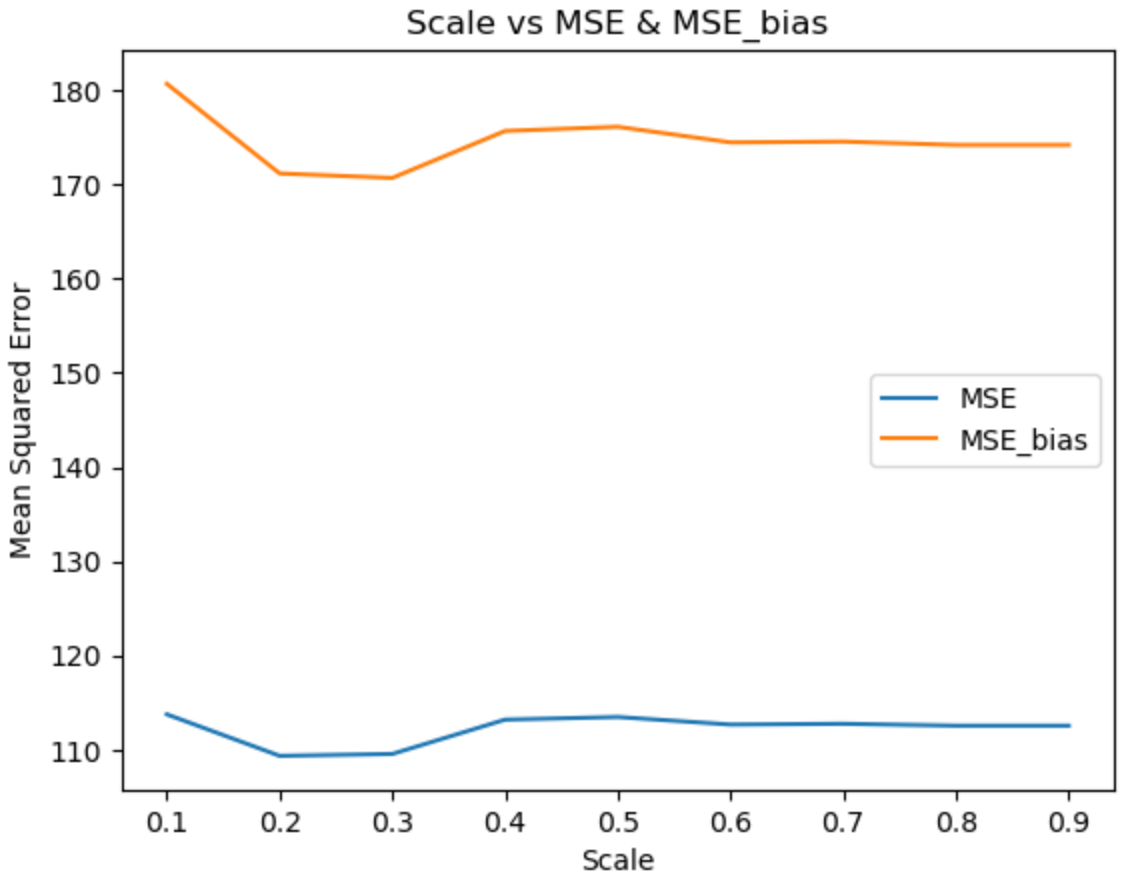
plt.plot(scales, mse, label = 'MSE')
plt.plot(scales, mse_bias, label = 'MSE_bias')
plt.xlabel('Scale')
plt.ylabel('Mean Squared Error')
plt.title('Scale vs MSE & MSE_bias')
plt.legend()
```

```
   scale         MSE               MSE_Bias

   0.1           113.76            180.65289712674385
   0.2           109.36            171.1471931049876
   0.3           109.56            170.67117115204704
   0.4           113.2             175.65266193560356
   0.5           113.48            176.0966176608784
   0.6           112.68            174.4411280910711
   0.7           112.76            174.5259795068224
   0.8           112.56            174.16997112151066
   0.9           112.56            174.16997112151066
```

Out[20]:

```
<matplotlib.legend.Legend at 0x7fba14745a60>
```



In [16]:
```
print(f'Best scale: {scales[np.argmin(mse)]} with MSE of {min(mse)}')
print(f'Best scale for more correct higher placements: {scales[np.argmin(mse_b
```

```
Best scale: 0.2 with MSE of 109.36
Best scale for more correct higher placements: 0.3 with MSE of 170.67117115204
704
```