CONTROL SYSTEM - I

⇒ CONTROL SYSTEM :-

A set of functional blocks which generates a desired controlled output with respect to a particular input.

⇒ TYPES OF CONTROL SYSTEM :-

i) Open loop control system (Non-feedback)
ii). Closed loop control system. (feedback control system).

1) OPEN LOOP CONTROL SYSTEM :⇒

A system in which output is dependent on input but controlling action ~~or input~~ is totally independent of the desired output, is called an open loop system.

The main components of open loop systems are;

Here, input is applied to the controller (i.e. filter. amplifier etc. depends upon the system) which generates the signal required to control the process which is to be controlled. Process is giving out the necessary desired controlled output c(t).

EXAMPLES. :⇒

i) Automatic washing machine is the example of open loop systems. In the machine, the operating time is set manually. After the completion of set time the machine will stops, with the result we may or may not get

⇛ CLOSED LOOP SYSTEM :-

A system in which output is dependent on input ~~but~~ and controlling action is also dependent on the desired output, is called closed loop system. To have dependency, such system uses the feedback property.

In such system, output or part of the output is feedback to the input for comparision with the reference input applied to it.

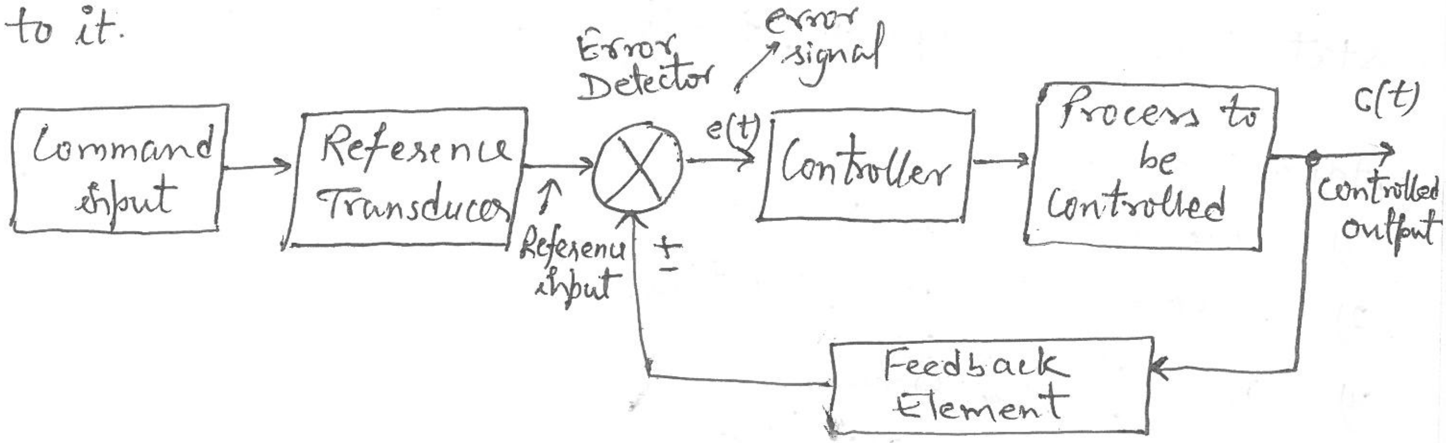
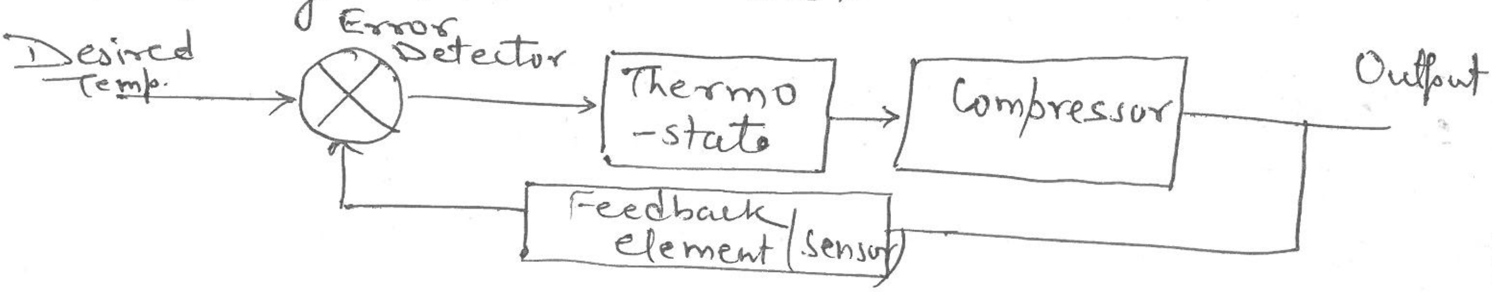


Fig: Representation of closed loop control system

Examples :-

1) Air-Conditioners (A.C.) are provided with thermostat. The actual room temperature compared with the desired temperature, an error signal is produced. On the basis of this error signal, the thermostat turns ON or OFF the compressor. The block diagram is shown below:-



2) Human Being. If a person wants to reach for a book on the table, closed loop system can be represented as shown on next page:-

5. More stable.	5. Less stable.
6. Simple to construct and cheap.	6. Complicated to design and hence costly.
7. Bandwidth is small.	7. Bandwidth is large.
8. Error detector is absent.	8. Error detector is present.

⇒ Examples of open Loop system :-

i) Sprinkler used to water a Lawn :-

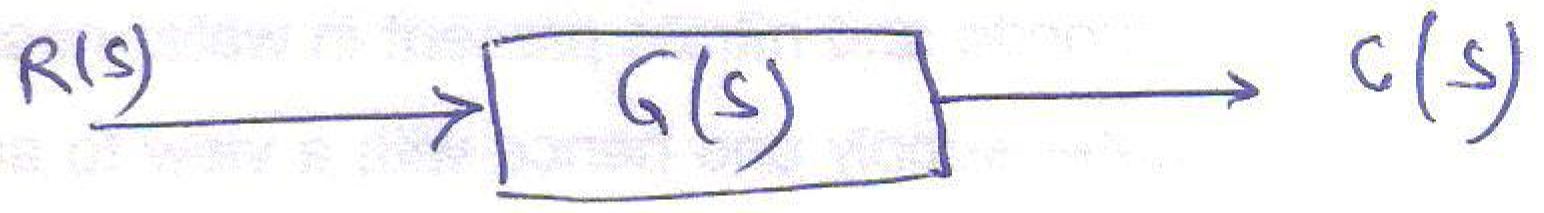
The system is adjusted to water a given area by opening the water valve and observing the resulting pattern.

2) Automatic Toaster system :-

In this system, quality of toast depends upon the time for which the toast is heated. Depending on the time setting, bread is simply heated in this system.

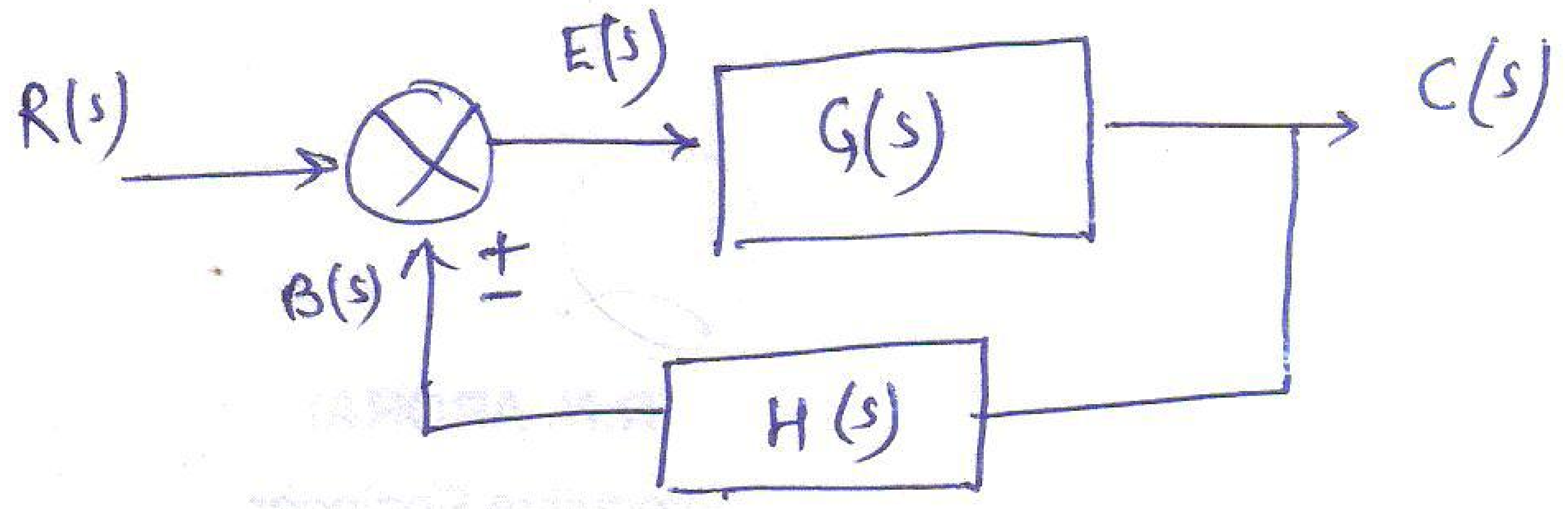
TRANSFER FUNCTION

It is defined as the ratio of Laplace transform of the output to the Laplace transform of input with all initial conditions are zero.



Here transfer function $G(s) = \frac{C(s)}{R(s)}$ is called open loop transfer function.

ii) For closed loop system;



Here, $C(s) = E(s) \cdot G(s)$

$$= \left[R(s) \pm B(s)\right] G(s)$$

or $C(s) = R(s) \cdot G(s) \pm B(s) \cdot G(s)$

$$= R(s)\, G(s) \pm C(s) \cdot H(s) \cdot G(s)$$

or $C(s)\left[1 \mp H(s)\, G(s)\right] = R(s) \cdot G(s)$

or T.F. $\Rightarrow$ $\boxed{\frac{C(s)}{R(s)} = \frac{\cancel{R(s)}\, G(s)}{1 \mp G(s)\, H(s)}}$

POLES AND ZEROS OF A TRANSFER FUNCTION

Consider $$G(s) = \frac{50(s+3)(s^2+9)}{s(s+2)(s+4)^2}$$

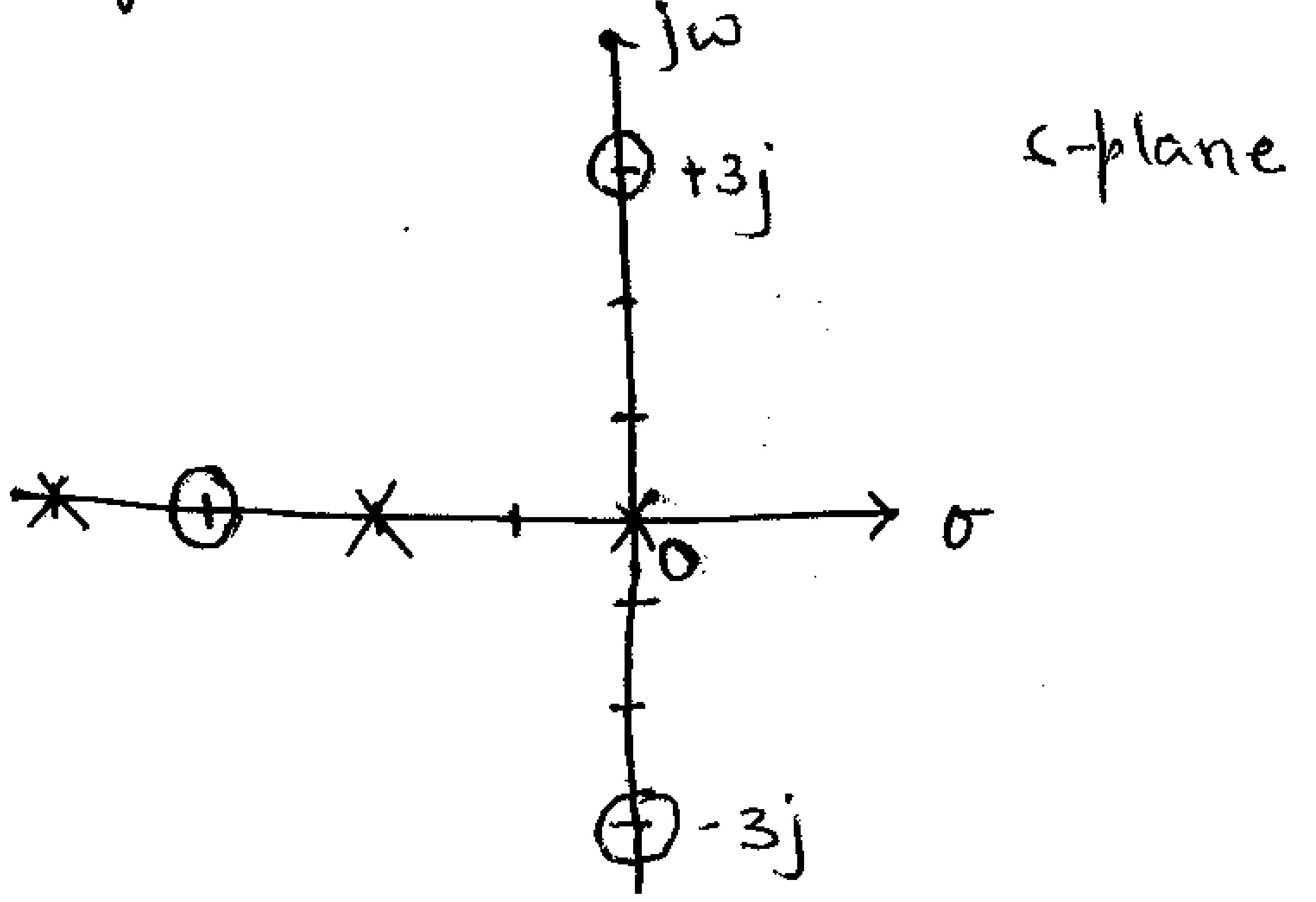
⇒ The poles of G(s) are those values of 's' which make G(s) tend to infinity.

The above T.F. has simple poles at $s=0$, $s=-2$ and multiple poles at $s=-4$ i.e. pole of order 2.

⇒ The zeros of G(s) are those values of 's' which make G(s) tends to zero.

The above T.F. has simple zero at $s=-3$. and complex pair of zeros at $s=\pm 3j$

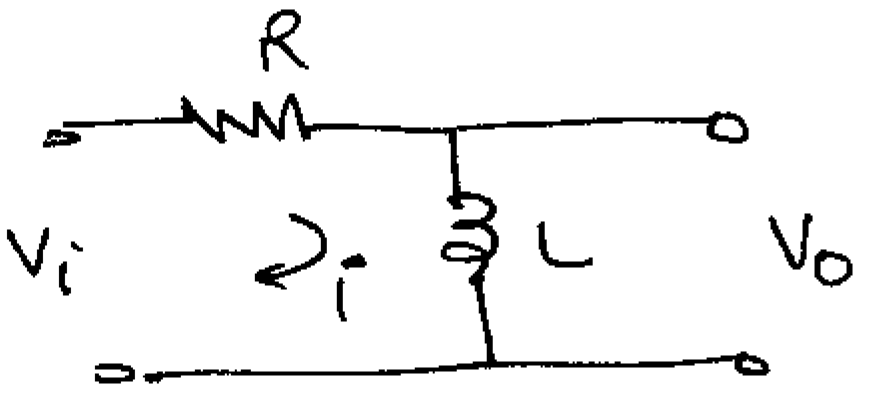
The pole is represented by 'x' and zero by 'O'. The pole zero plot of above transfer function is shown below :-



Steps for calculating transfer function :-

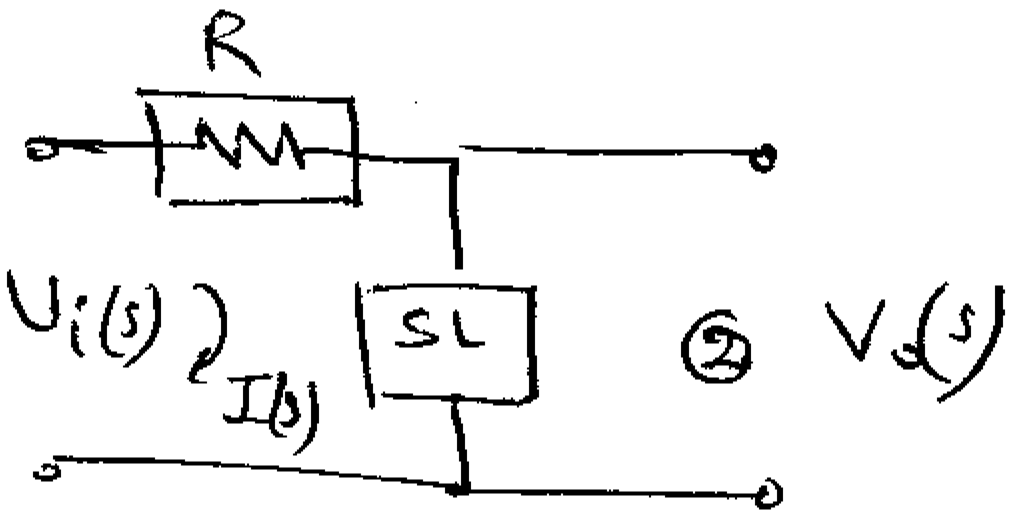
1) Convert the given network in s-domain.
2) Apply KVL / KCL to to find out the ratio of output to input in Laplace domain.

Q-1) Find the transfer function of the given network:



Soln:- Firstly convert the given network in s-domain;

apply KVL in loop (1);



$$V_i(s) - I(s) \cdot R - SL \cdot I(s) = 0$$

or $$V_i(s) = (R + SL)\, I(s) \qquad -(1)$$

apply KVL in loop (2)

$$V_o(s) - I(s) \cdot SL = 0$$

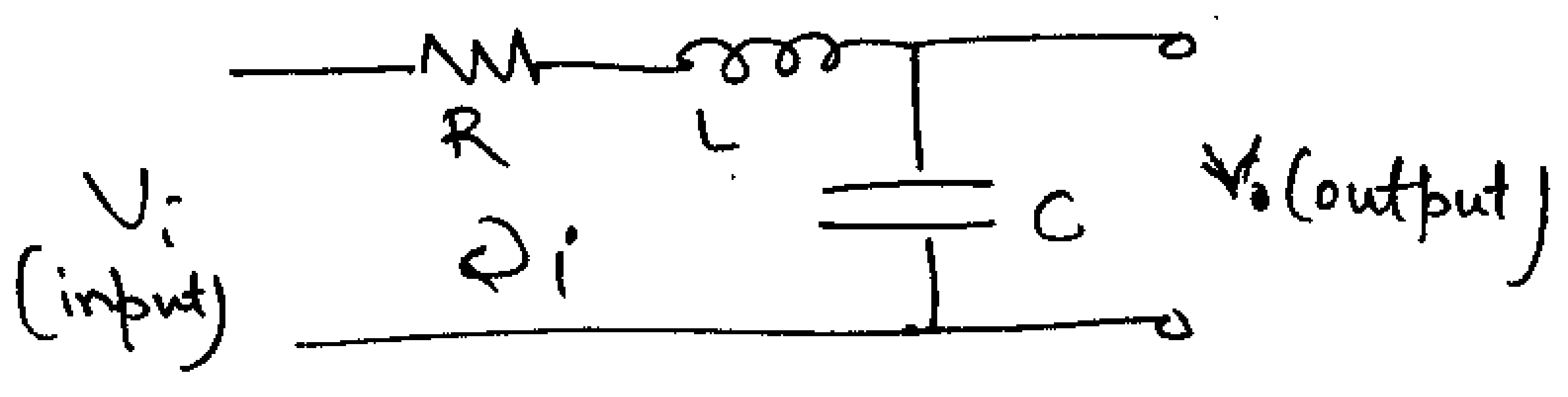
or $$V_o(s) = SL \cdot I(s) \qquad -(2)$$

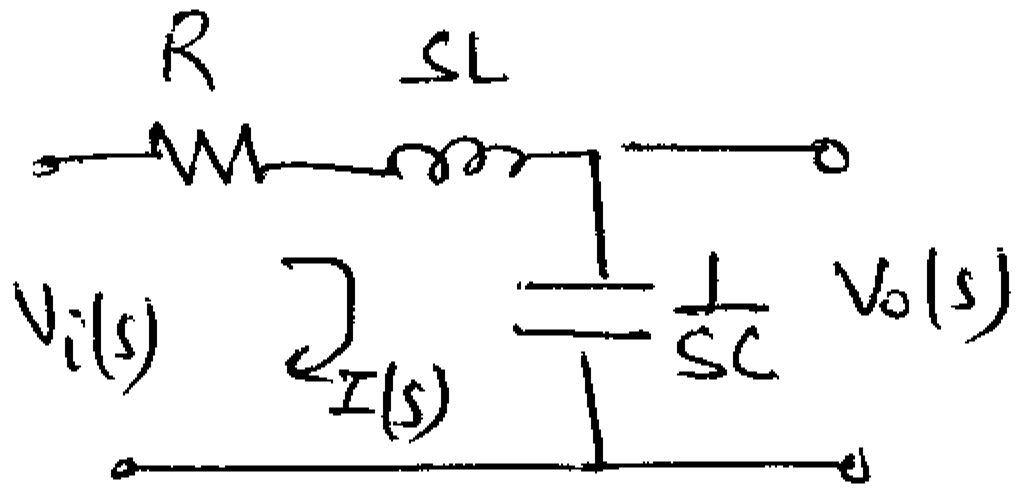
Divide eqn. (2) by eqn. (1), we get.

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{SL}{R + SL}}$$

Ans

Q-2) Determine transfer function of the given network;





Soln:- Apply KVL in loop (1),

$$V_i(s) - \left(R + SL + \frac{1}{SC}\right) I(s) = 0$$

or $$V_i(s) = \left[\frac{RSC + S^2LC + 1}{SC}\right] I(s) \qquad -(1)$$

Apply KVL in loop (2), we get.

$$V_o(s) - I(s)\cdot\frac{1}{sC} = 0$$

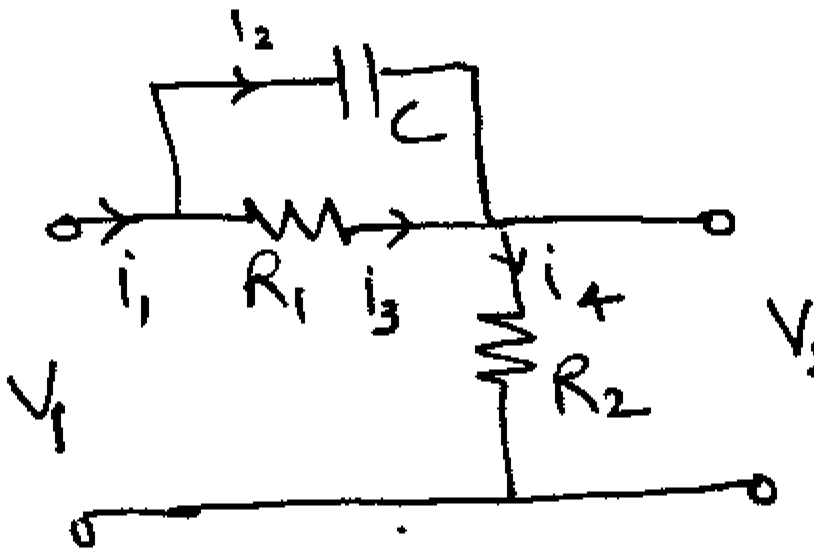
$$\text{or} \quad V_o(s) = \frac{1}{sC}\cdot I(s) \quad -(2)$$

Divide eqⁿ. (2) by eqⁿ. (1), we get

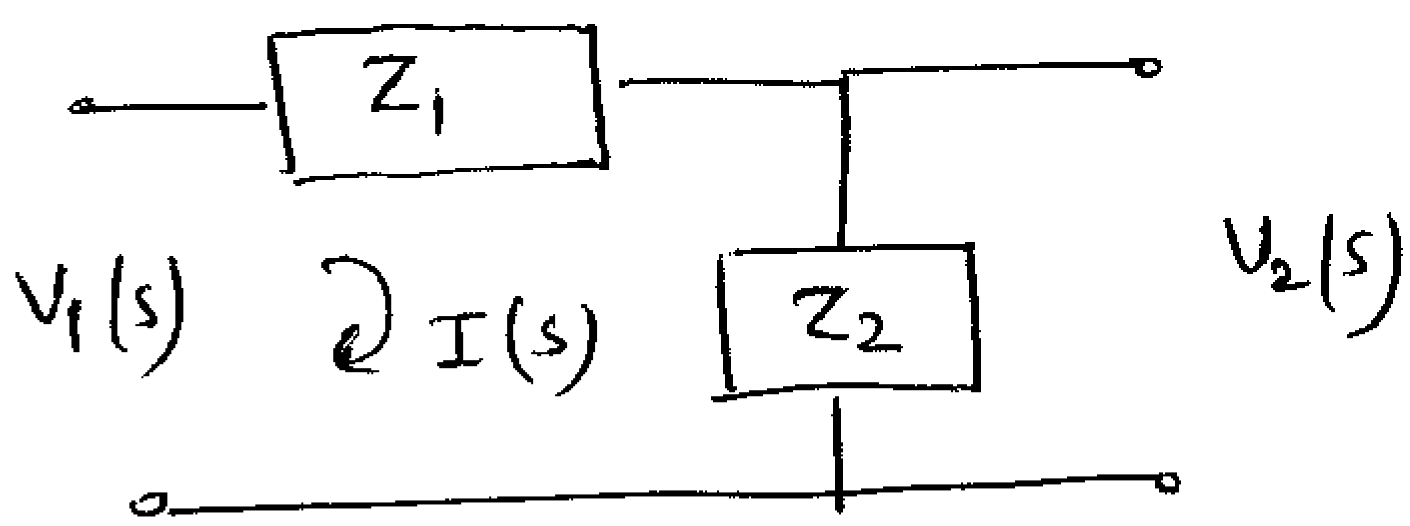
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + sRC + 1}}$$

Ans

Q-3. Calculate the transfer function of given network:



⇒



We can solve such type of networks by considering equivalent impedance of each particular branch. and then convert this into the equivalent network for which transfer function $= \dfrac{Z_2}{Z_1+Z_2}$ (Direct formula)

i.e. Apply KVL in loop (1):

$$V_1(s) - Z_1 I(s) - Z_2 I(s) = 0$$

$$\text{or} \quad V_1(s) = (Z_1 + Z_2)\, I(s) \quad -(1)$$

Apply KVL in loop (2);

$$V_2(s) - I(s)\, Z_2 = 0 \quad \text{or} \quad V_2(s) = Z_2 I(s) \quad (2)$$

Divide eqⁿ (2) by eqⁿ (1), we get

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{Z_2}{Z_1 + Z_2}}$$

In the given network;

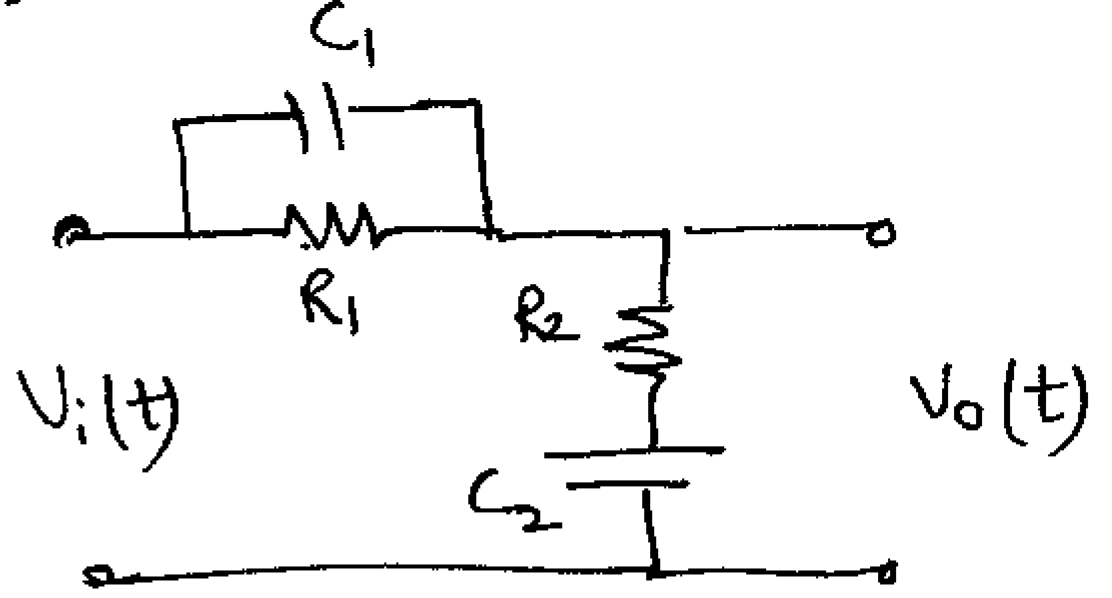
$$Z_1 = R_1 || C = \frac{R_1 \cdot \frac{1}{SC}}{R_1 + \frac{1}{SC}} = \frac{R_1}{SR_1C + 1}$$

and $Z_2 = R_2$

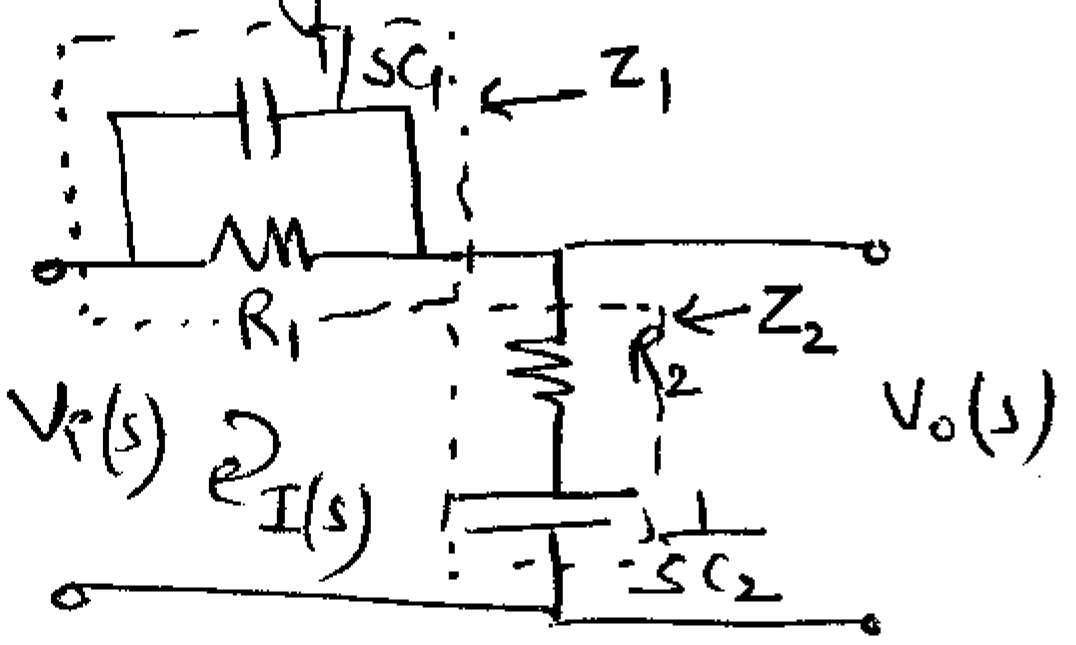
$$\therefore \frac{V_2(S)}{V_1(S)} = \frac{R_1/(SR_1C+1)}{R_2 + \frac{R_1}{SR_1C+1}}$$

or $$\boxed{\frac{V_2(S)}{V_1(S)} = \frac{R_1}{SR_1R_2C + R_1 + R_2}}$$ Ans

Q-4) Calculate transfer function for the given network;



$\Rightarrow$



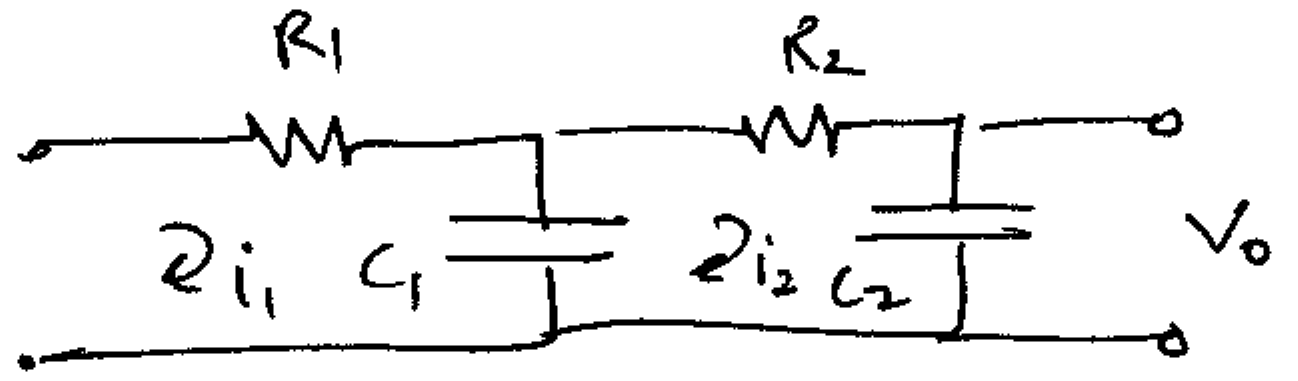
Soln; Here, $$Z_1 = R_1 || C_1 = \frac{R_1 \cdot \frac{1}{SC_1}}{R_1 + \frac{1}{SC_1}} = \frac{R_1}{1+SR_1C_1}$$

and $$Z_2 = R_2 + \frac{1}{SC_2} = \frac{1+SR_2C_2}{SC_2}$$

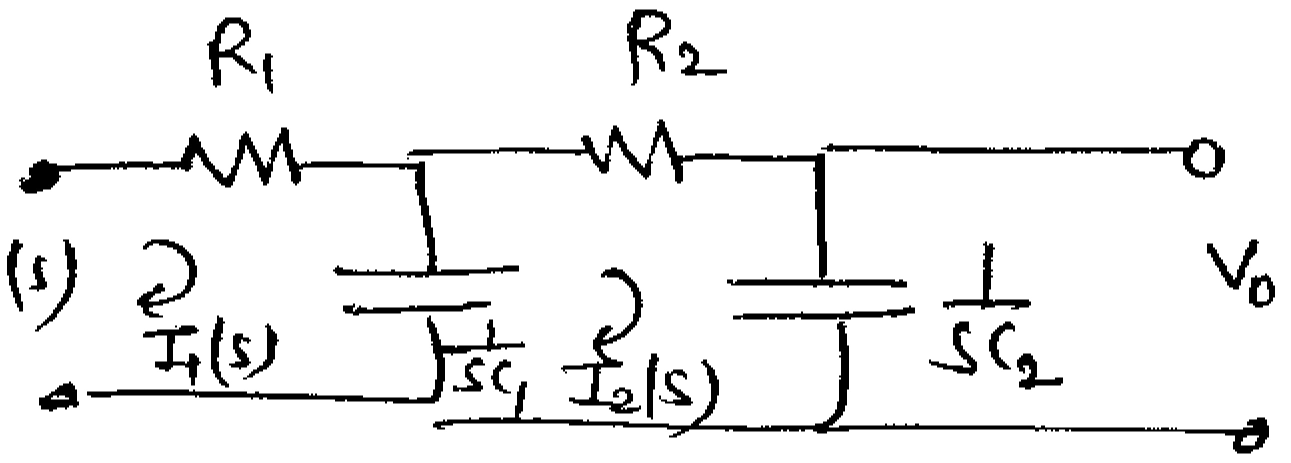
Now, $$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{1+SR_2C_2}{SC_2}}{\frac{R_1}{1+SR_1C_1} + \frac{1+SR_2C_2}{SC_2}}$$

or $$\frac{V_o(s)}{V_i(s)} = \frac{(1+SR_1C_1)(1+SR_2C_2)}{(1+SR_1C_1)(1+SR_2C_2) + SR_1C_2}$$ Ans

Q- Calculate the transfer function of the given network;



Soln;- Since, here more than 2 loops, therefore direct formula will not apply. Here, we derive the transfer function using basic KVL.



Apply KVL in loop (1),

$$V_i(s) = \left(R_1 + \frac{1}{sC_1}\right) I_1(s) - \frac{1}{sC_1} I_2(s) \quad -(1)$$

Apply KVL in loop (2),

$$0 = \left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_1}\right) I_2(s) - \frac{1}{sC_1} I_1(s)$$

or $$\left[\frac{s^2C_1C_2R_2 + sC_1 + sC_2}{s^2C_1C_2}\right] I_2(s) = \frac{1}{sC_1} I_1(s)$$

or $$I_1(s) = \left[\frac{sC_1C_2R_2 + C_1 + C_2}{C_2}\right] I_2(s) \quad -(2)$$

Apply the value of $I_1(s)$ in eqn. (1), we get;

$$V_i(s) = \left[\frac{R_1sC_1 + 1}{sC_1}\right]\left[\frac{sC_1C_2R_2 + C_1 + C_2}{C_2}\right] I_2(s) - \frac{1}{sC_1} I_2(s)$$

$$= I_2(s)\left[\frac{s^2C_1^2C_2R_1R_2 + sC_1^2R_1 + sC_1C_2R_1 + sC_1C_2R_2 + C_1 + C_2 - C_1}{sC_1C_2}\right]$$

$$V_i(s) = I_2(s)\left[\frac{s^2C_1C_2R_1R_2 + sC_1R_1 + sC_2R_1 + sC_2R_2 + 1}{sC_2}\right] \quad -(3)$$

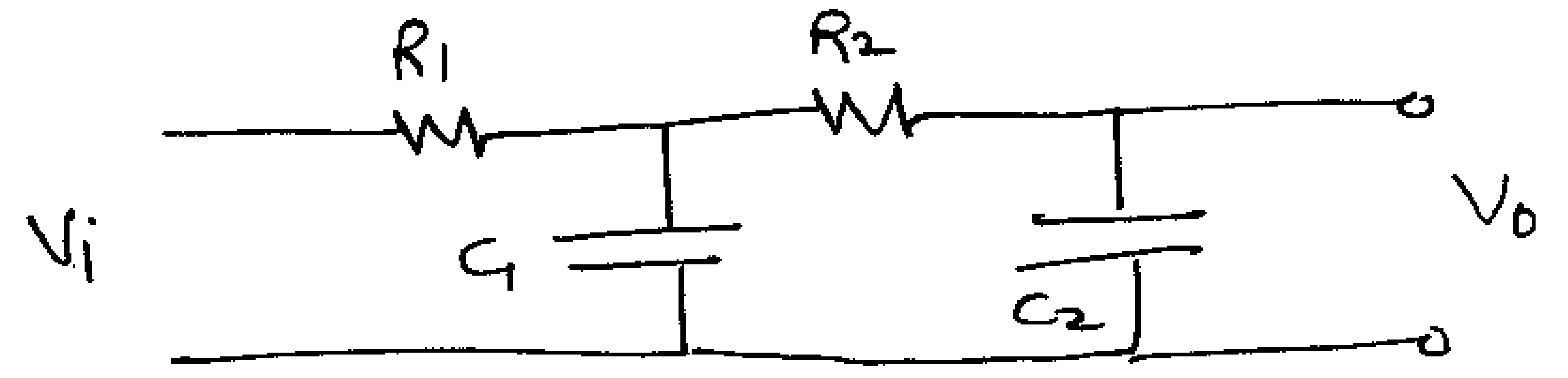
Apply KVL in loop (3), we get $V_o(s) = \frac{1}{sC_2} I_2(s) \quad -(4)$

Divide eqn. (4) by eqn. (3), we get

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2C_1C_2R_1R_2 + s(C_1R_1 + C_2R_1 + C_2R_2) + 1}}$$

TRANSFER FUNCTION CALCULATION USING BLOCK DIAGRAM REDUCTION :⇒

Consider the last question;

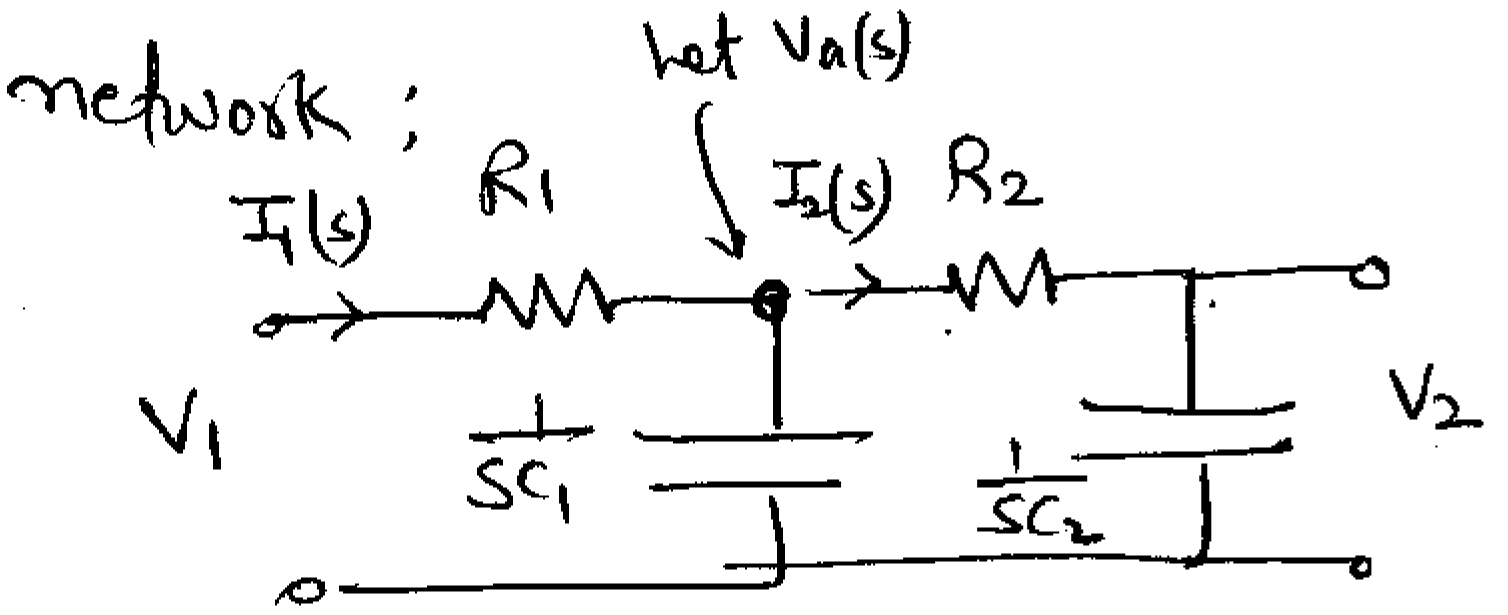


Firstly, we draw the block diagram from the given network, after that using block diagram reduction technique, we calculate the transfer function.

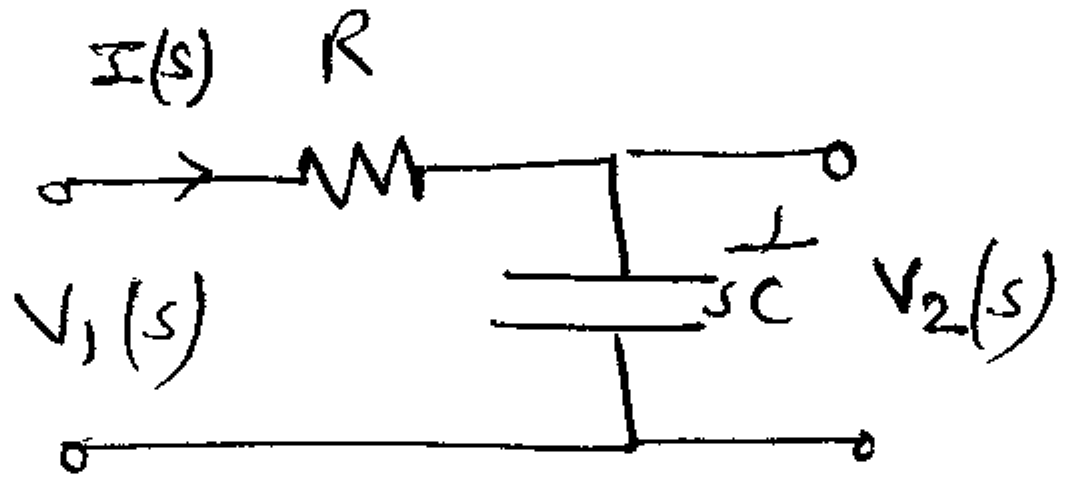
Step-1. Write current equation for all series (horizontal) branch.

Step-2 Write voltage equation for all parallel (vertical) branch.

All these two steps are applied to the Laplace form of given network;



Example (1)



Example (2)

Example (1):

$$I_1(s) = \frac{V_1(s) - V_a(s)}{R_1}$$

$$V_a(s) = \frac{1}{sC_1}(I_1 - I_2)$$

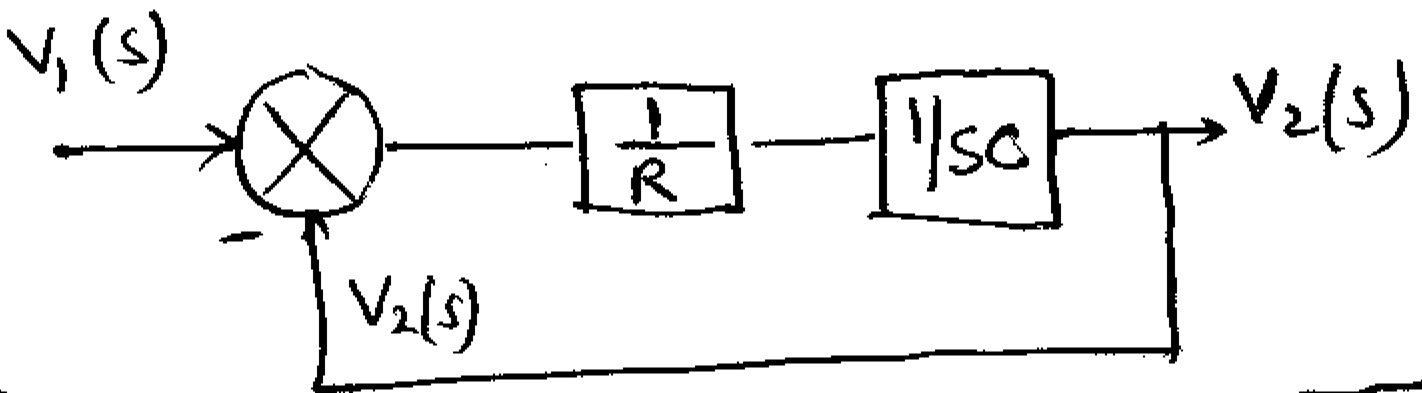
$$I_2(s) = \frac{V_a(s) - V_2(s)}{R_2}$$

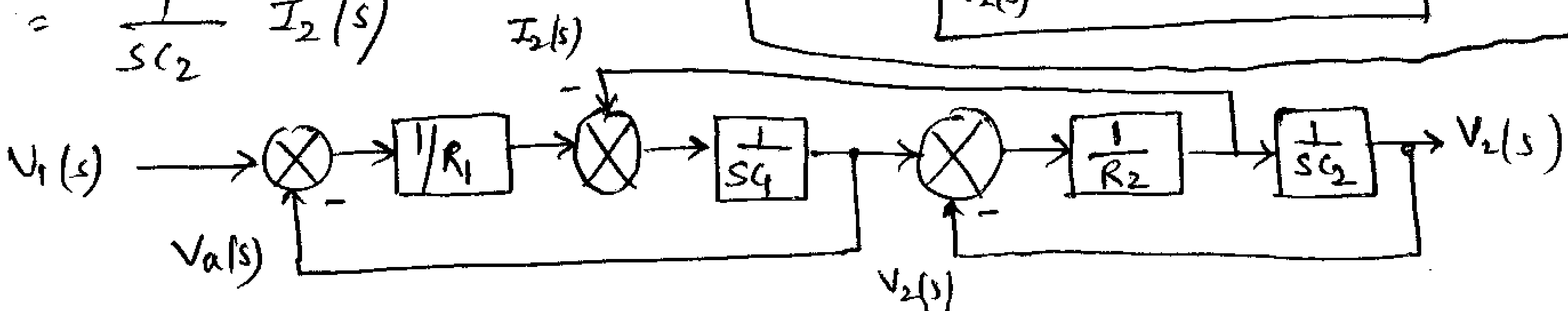
$$V_2(s) = \frac{1}{sC_2} I_2(s)$$

Example (2):

$$I(s) = \frac{V_1(s) - V_2(s)}{R}$$

$$V_2(s) = \frac{1}{sC} I(s)$$

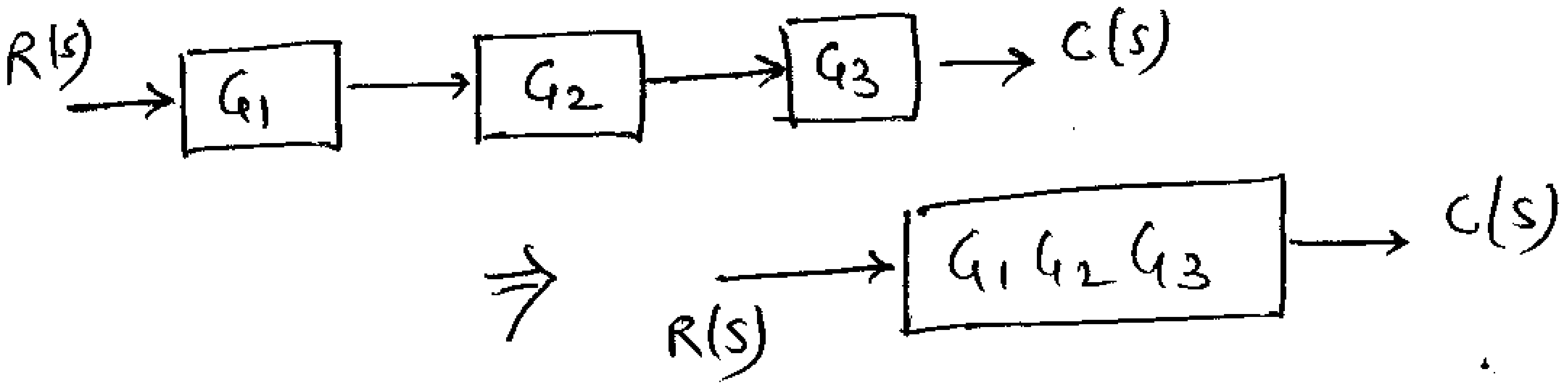




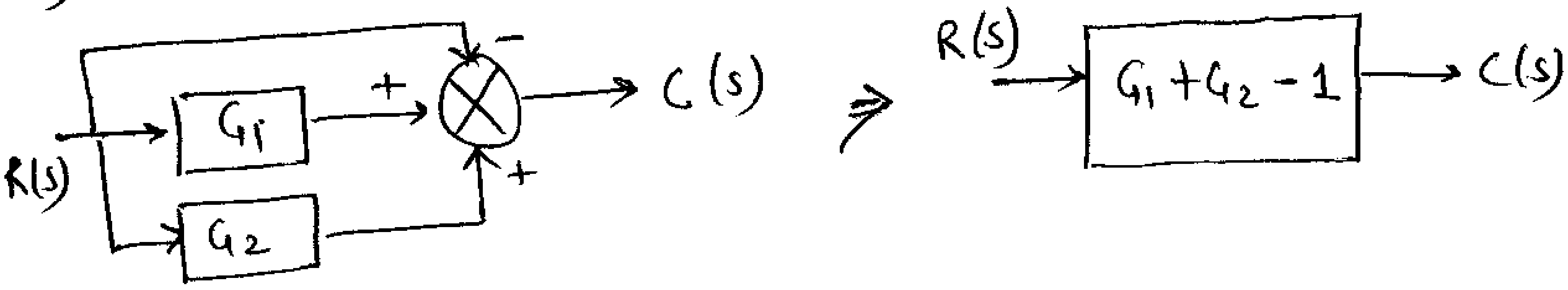
BLOCK DIAGRAM REDUCTION

Once block diagram is formed, then the transfer function can be calculate using block diagram reduction algebra.
There are certain rules associated with block reduction techniques :-

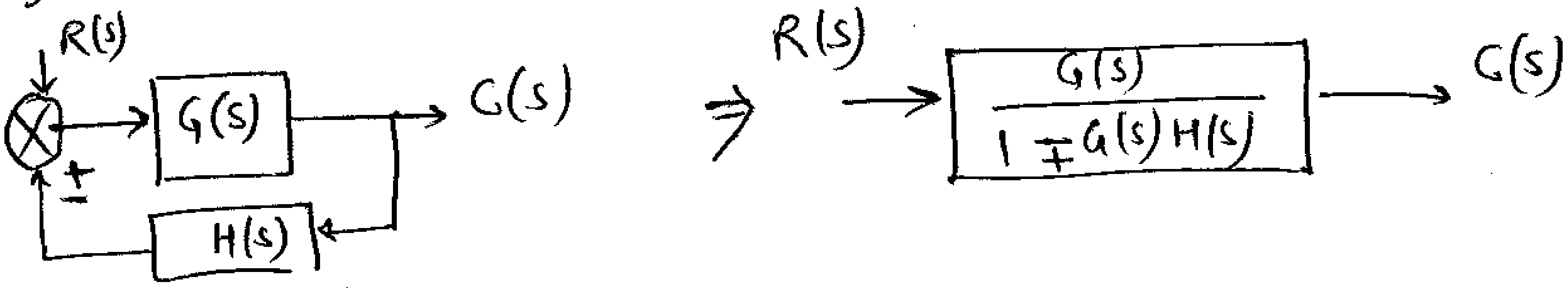
1) Blocks in Cascade | series



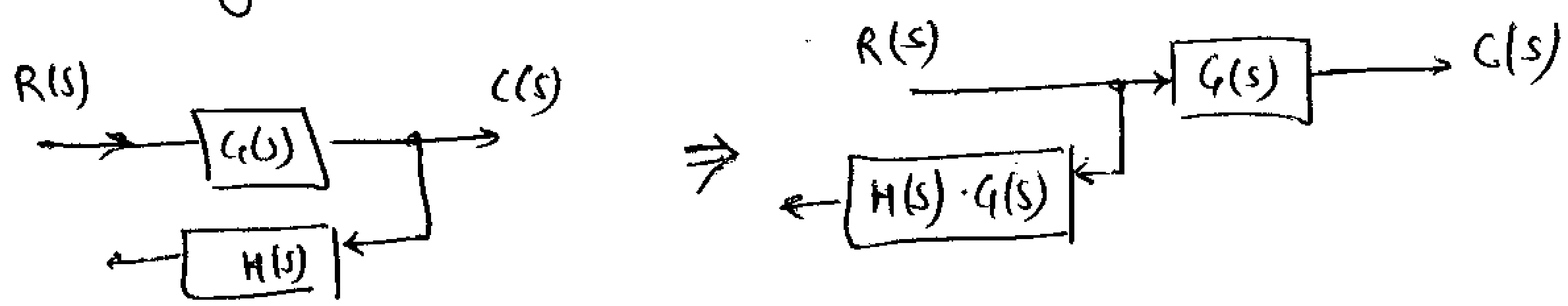
2) Blocks in Parallel :-



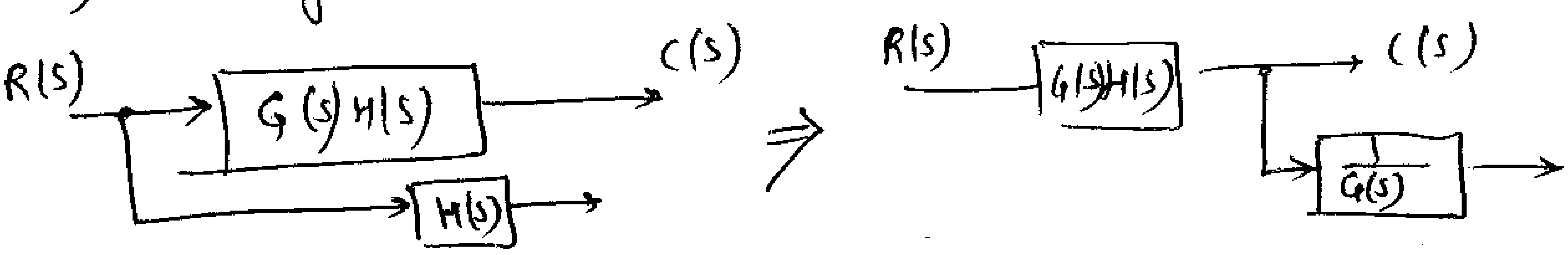
3) Feedback path Rule :-



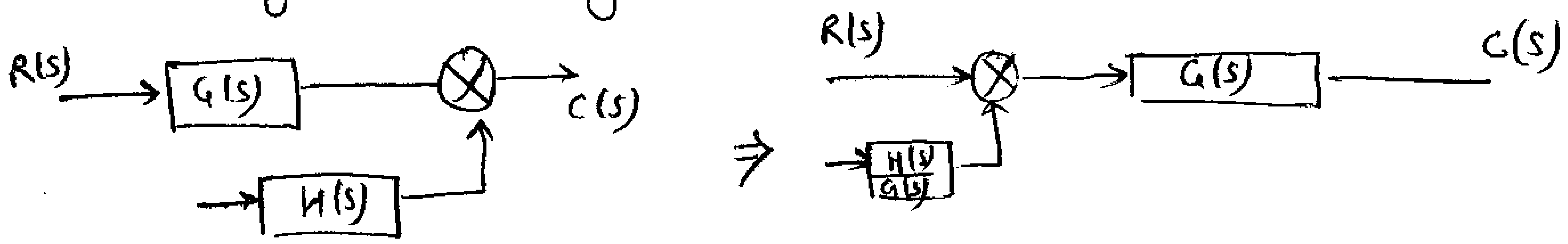
4) Moving a take off point before a block



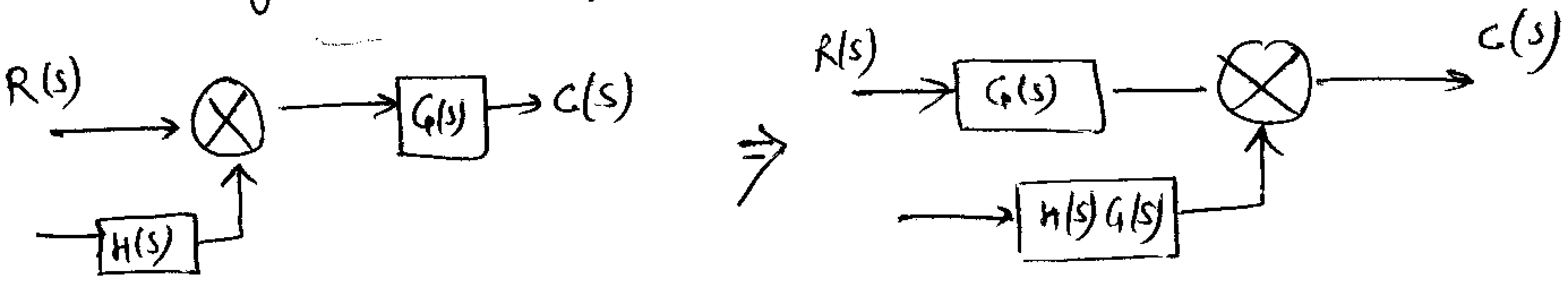
5) Moving a take off point after a block



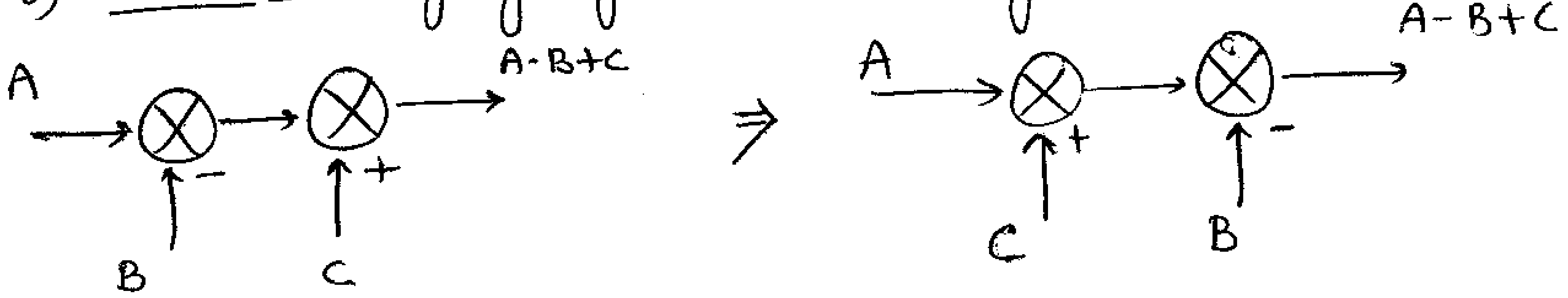
(6) Moving a summing point before a block



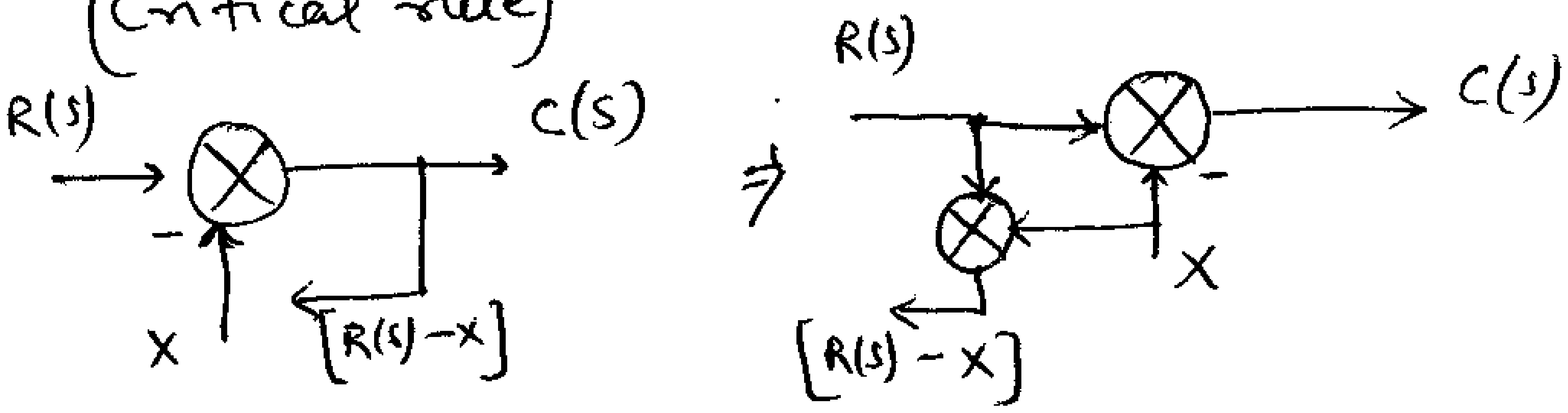
(7) Moving a summing point after a block



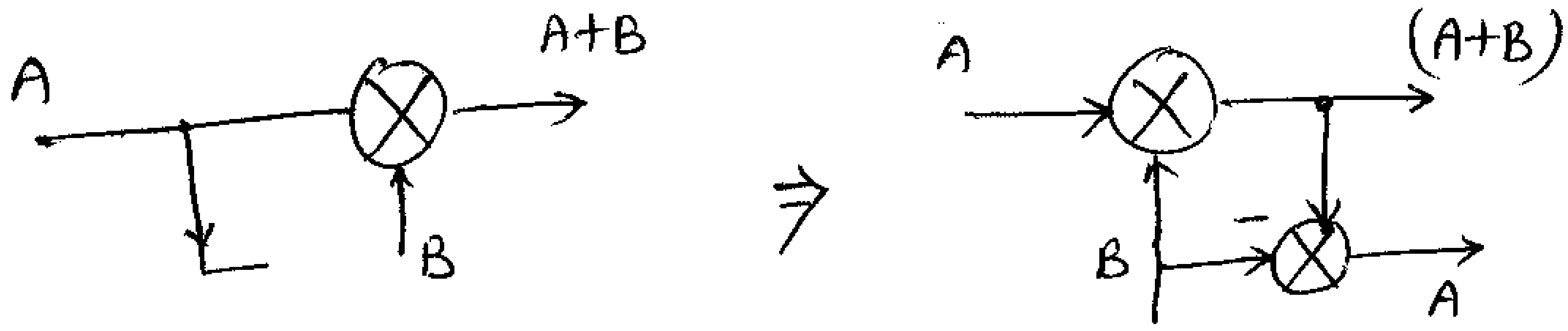
(8) Interchanging of two summing points



(9) Moving a take off point before summing point
(Critical rule)

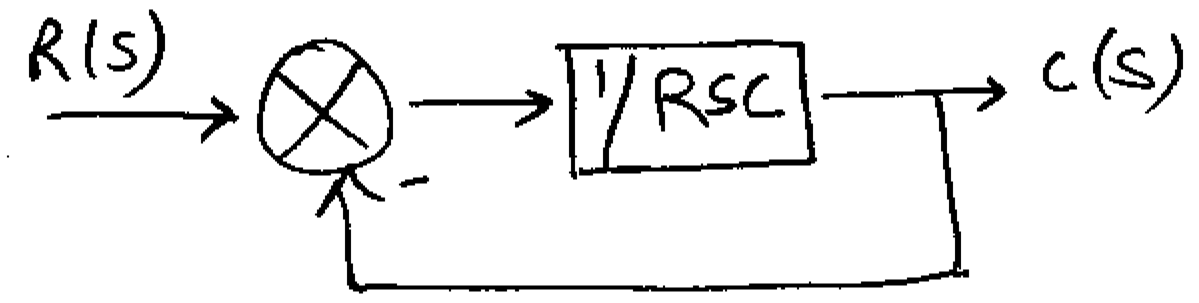


(10) Moving a takeoff point after summing point



Q. Determine the transfer function for the given block diagram;

i)

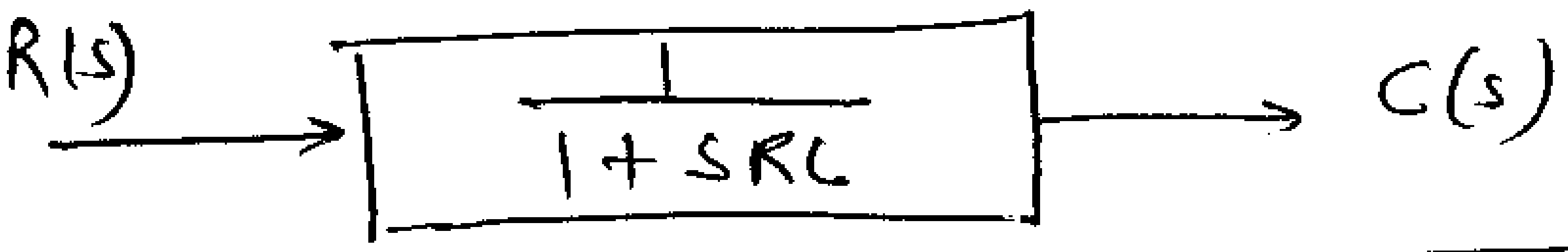


Applying the feedback path rule;

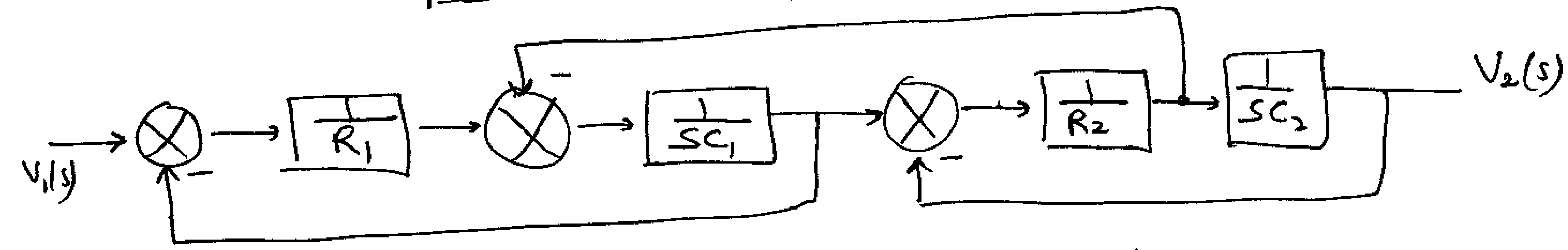
$$G(s) = \frac{1}{RSC}, \quad H(s) = 1$$

Hence, $$\frac{C(s)}{R(s)} = \frac{\frac{1}{RSC}}{1 + \frac{1}{RSC}\cdot(1)} = \frac{1}{1 + RSC}$$

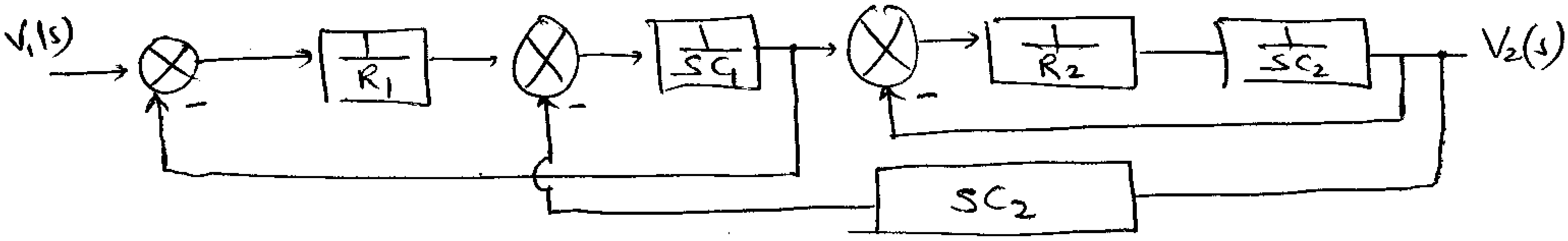
Hence,



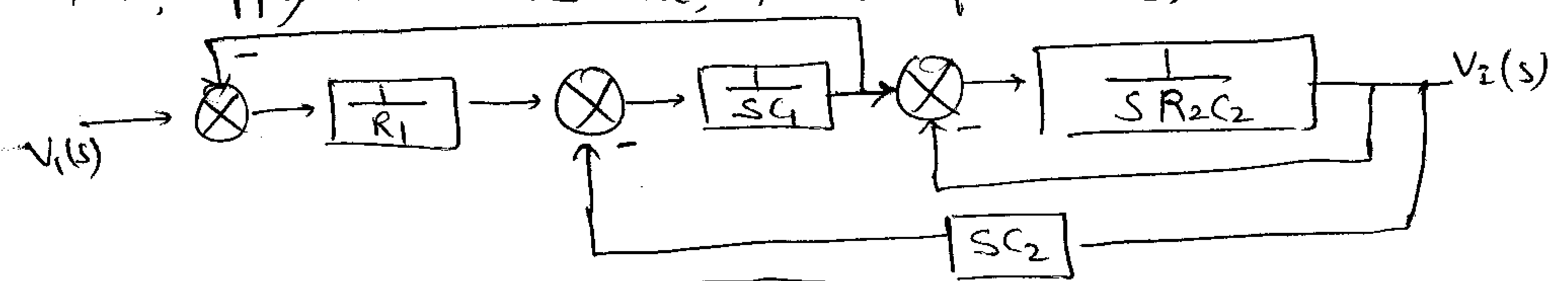
ii)

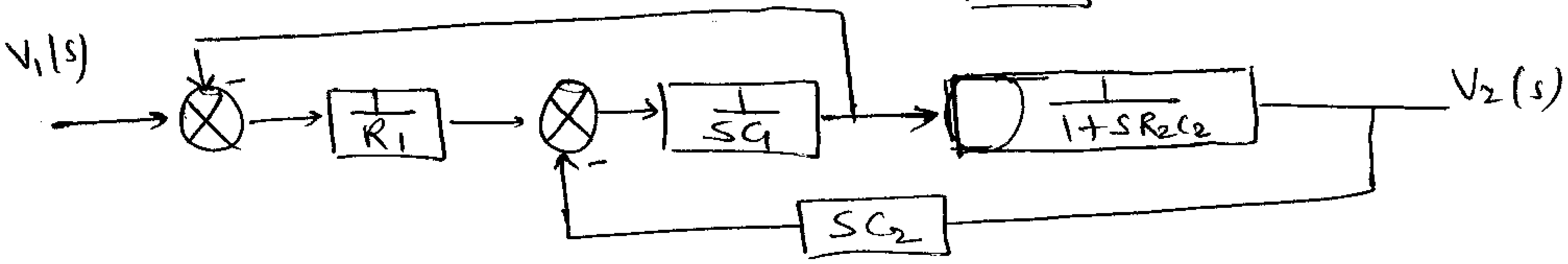


Here, we can Apply the series rule but take off point creates a problem, so firstly move this point.

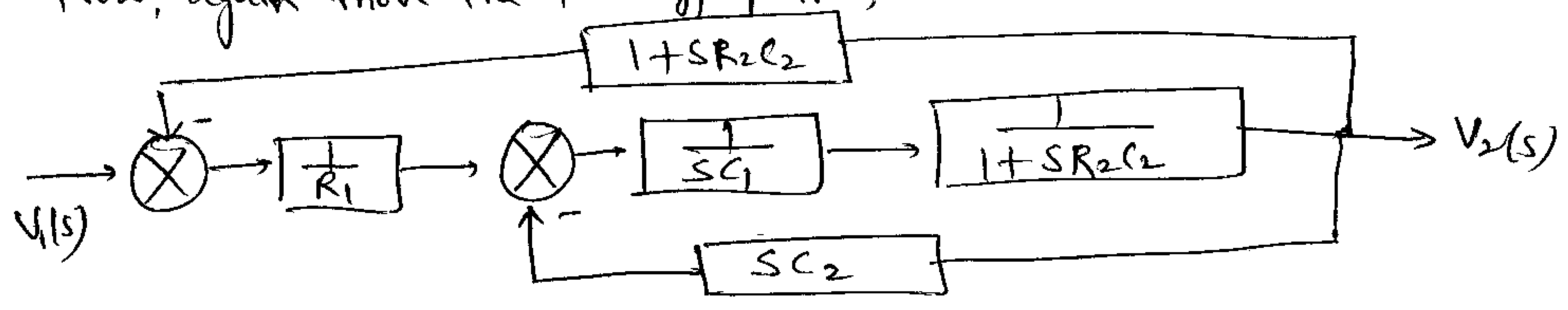


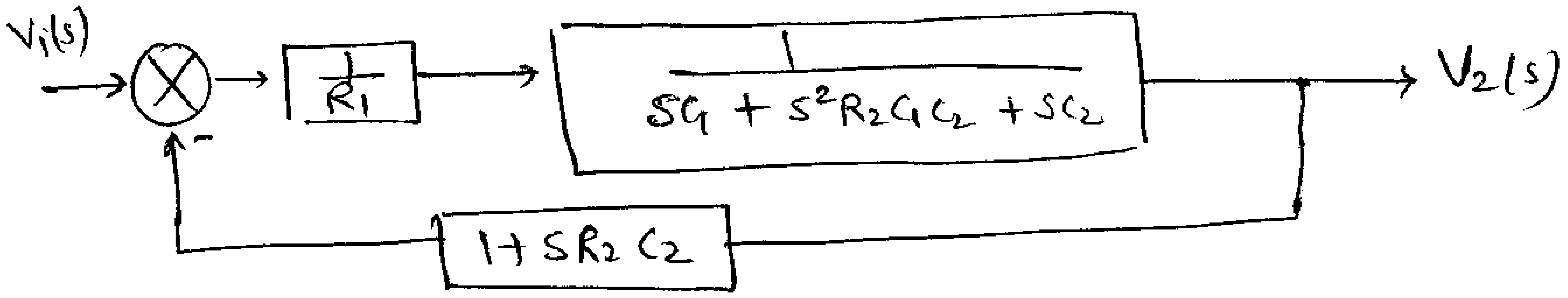
Now, apply the series rule, feedback path rule;

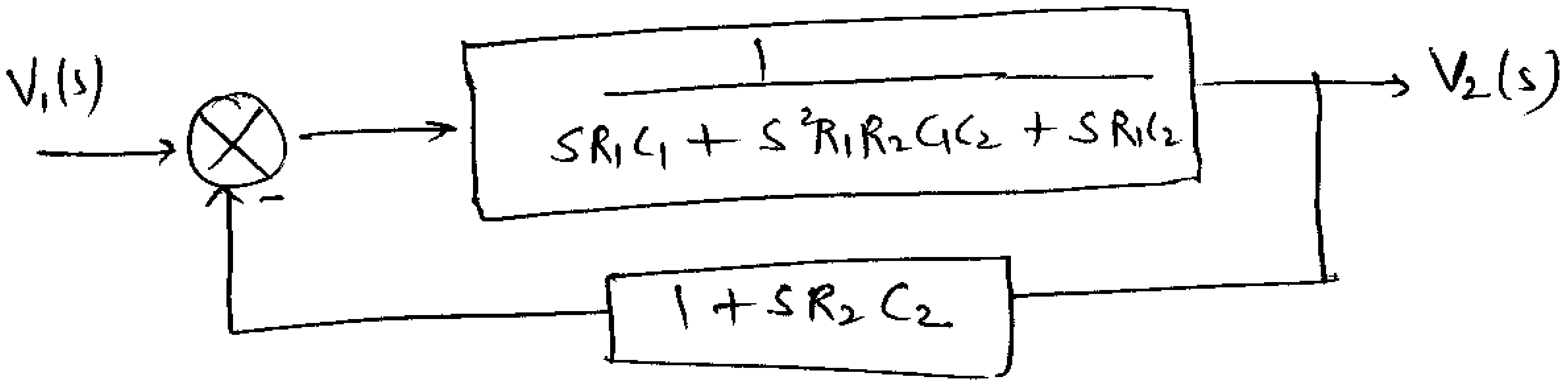


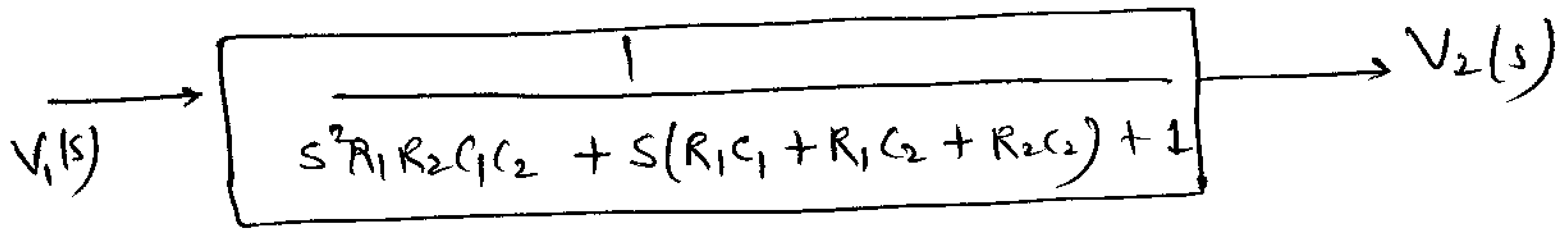


Now, again move the take off point,



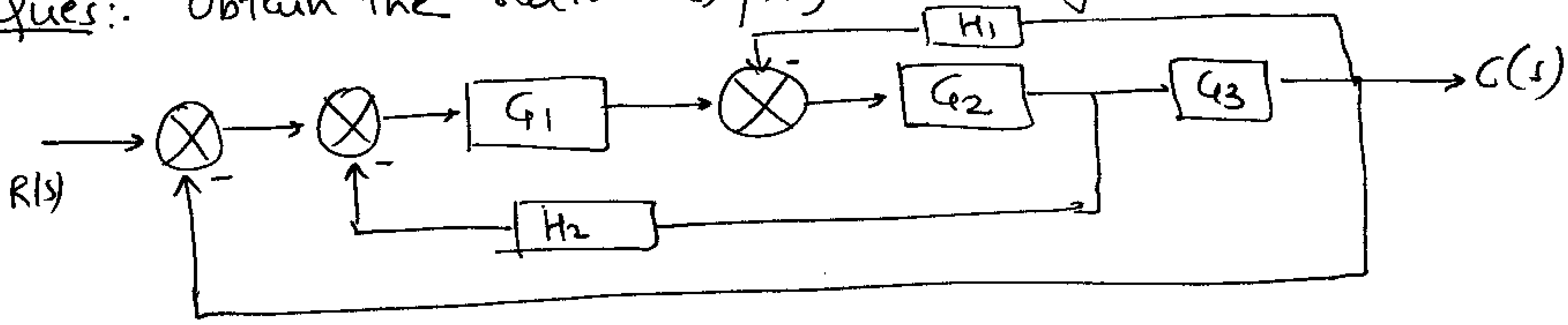




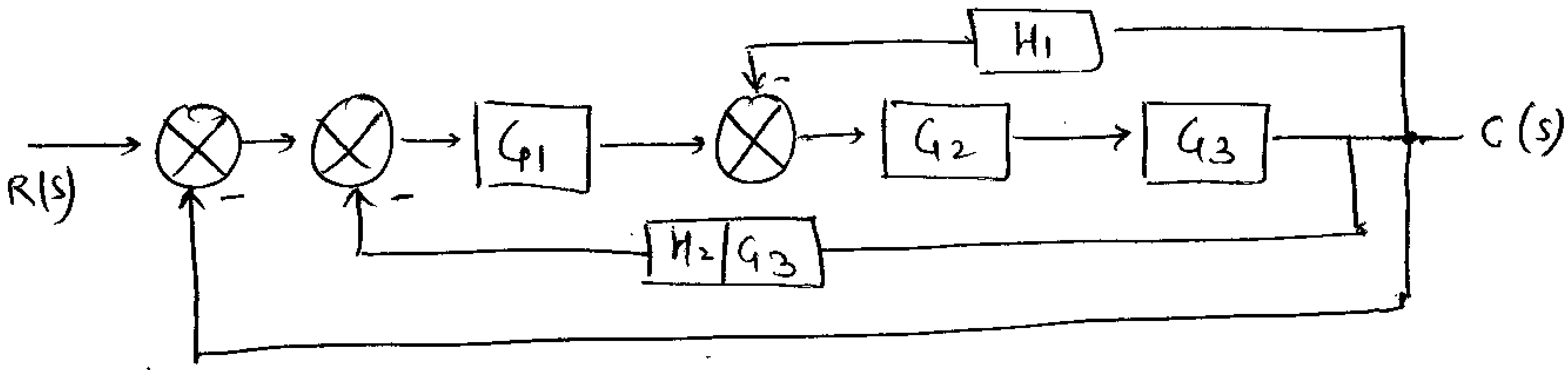


Hence, $$\frac{V_2(s)}{V_1(s)} = \frac{1}{s^2R_1R_2C_1C_2 + s(R_1C_1+R_1C_2+R_2C_2)+1}$$ Ans

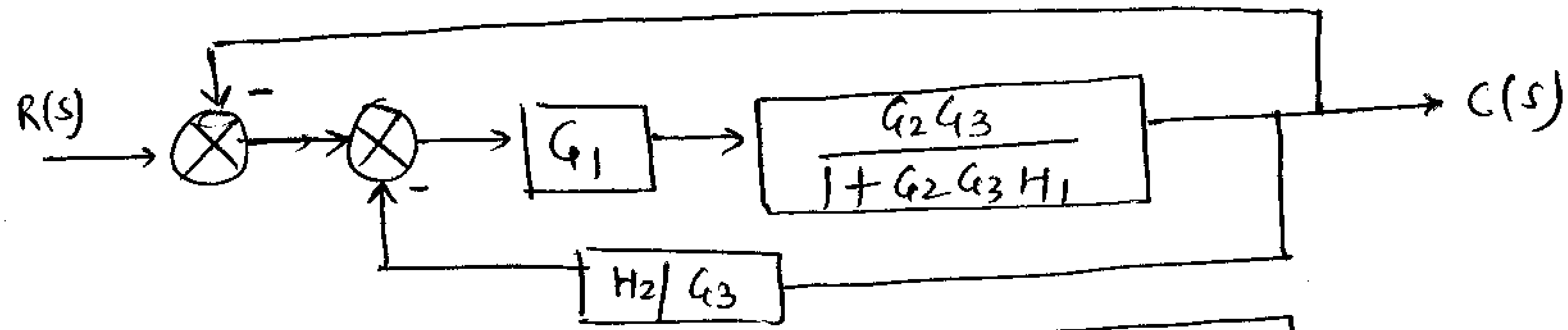
<u>Ques</u>:. Obtain the ratio $C(s)/R(s)$ for the given system;



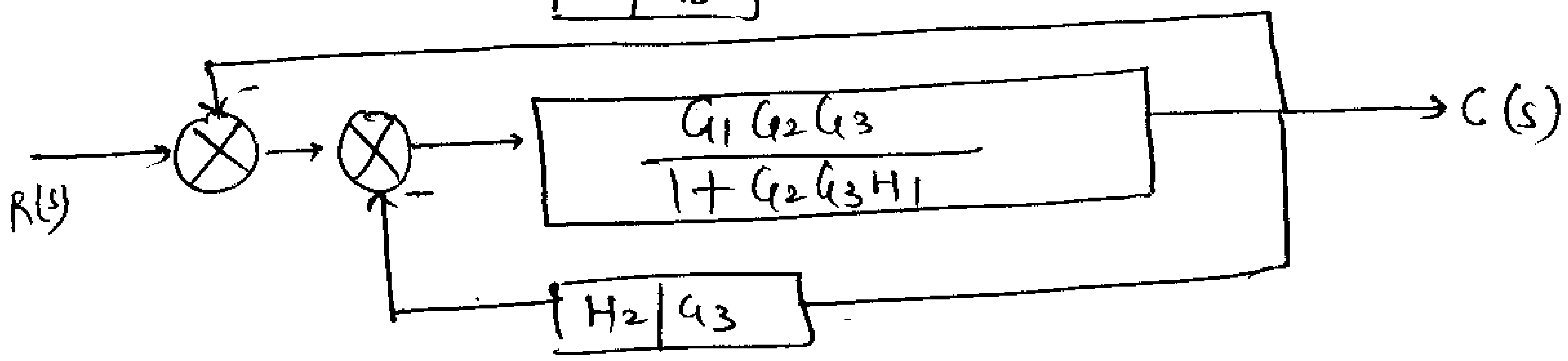
1)



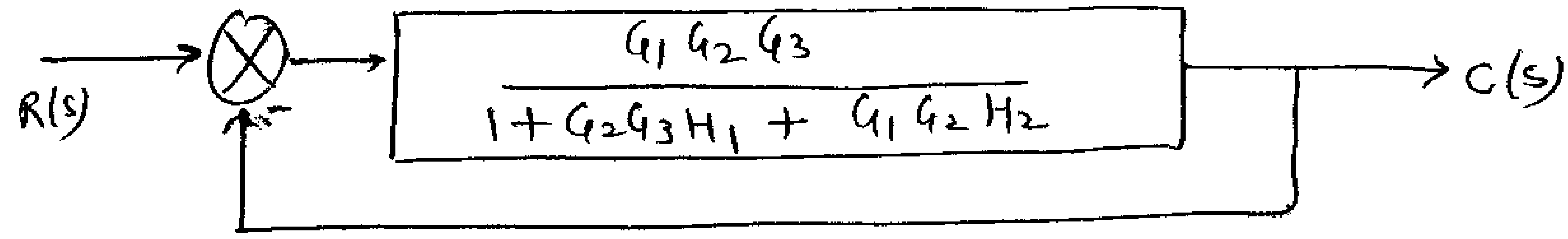
2)



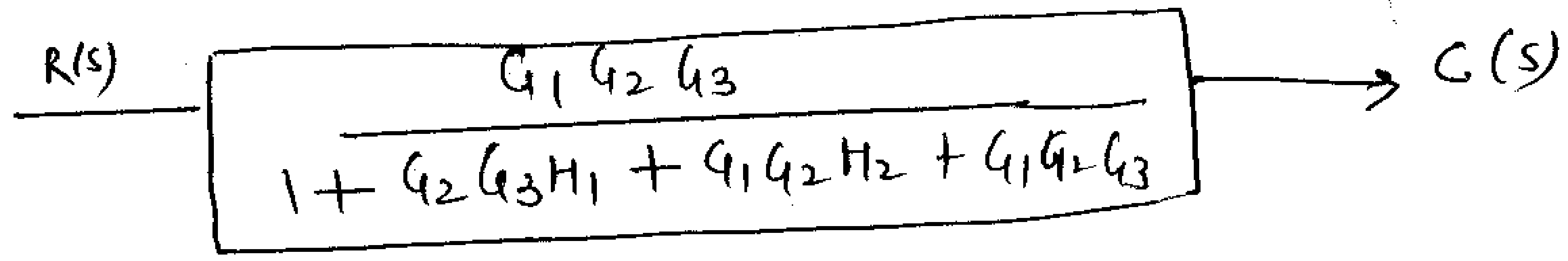
3)



4)



5)



Hence, $$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1+G_2G_3H_1 + G_1G_2H_2 + G_1G_2G_3}$$ Ans

TRANSFER FUNCTION DETERMINATION USING SIGNAL FLOW GRAPH ⇒

The technique to determine transfer function using block diagram reduction is time consuming and complex. Thus, a simple method was developed by S. J. Mason which is known as signal flow graph. This method is applicable to the linear systems.

Construction of SFG :-

a) From given equations;

Consider the following set of equations:

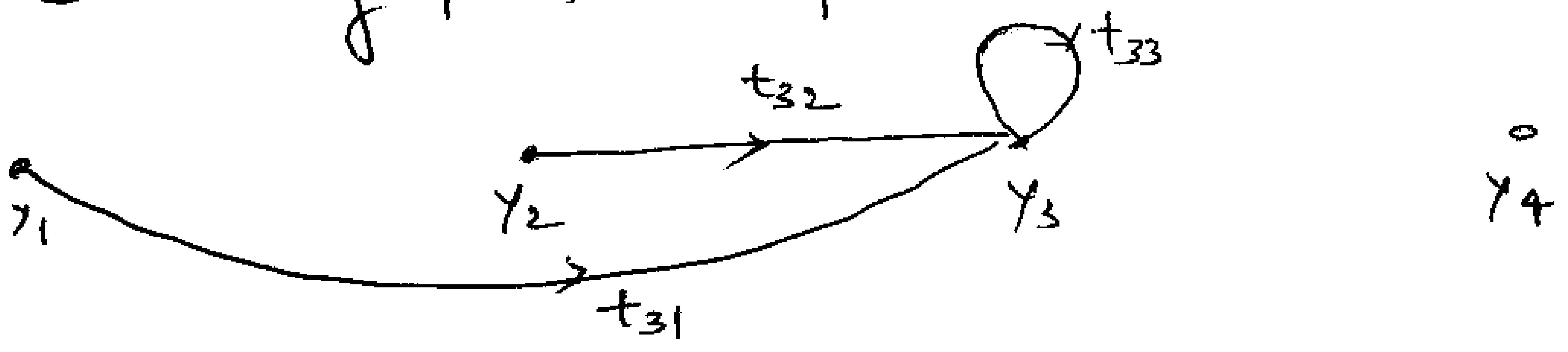
$$y_2 = t_{21} y_1 + t_{23} y_3$$
$$y_3 = t_{32} y_2 + t_{33} y_3 + t_{31} y_1$$
$$y_4 = t_{43} y_3 + t_{42} y_2 + t_{41} y_1$$

where y_1 is input and y_4 is output.

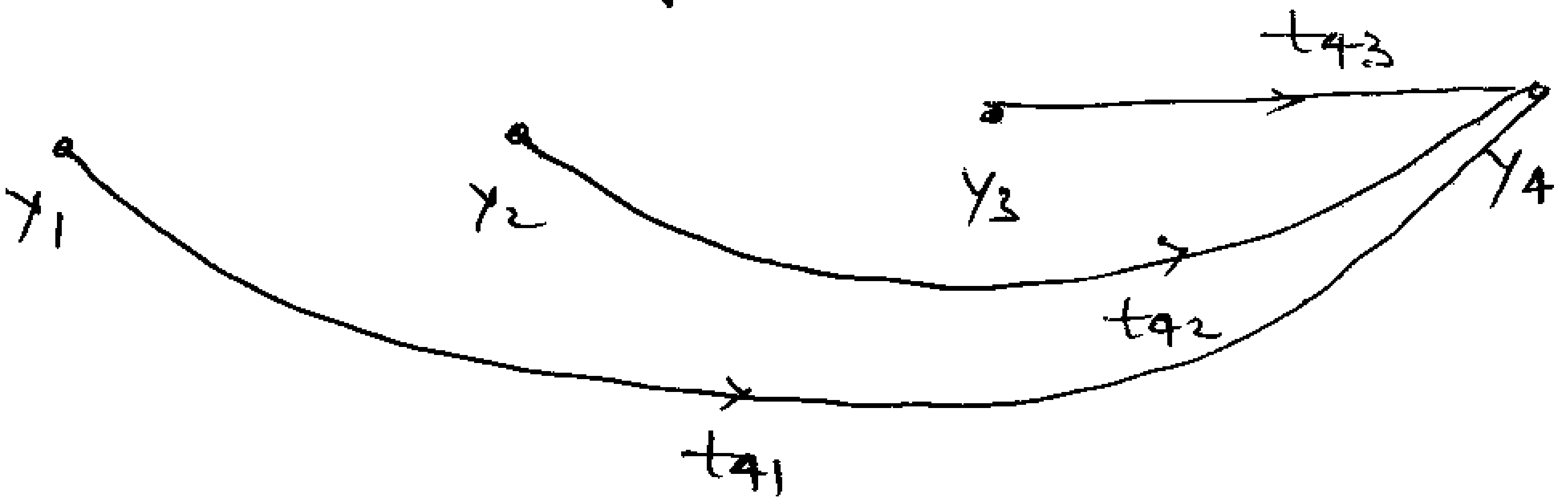
Step 1: Draw all the nodes.

Step 2 Draw the graph for I equation

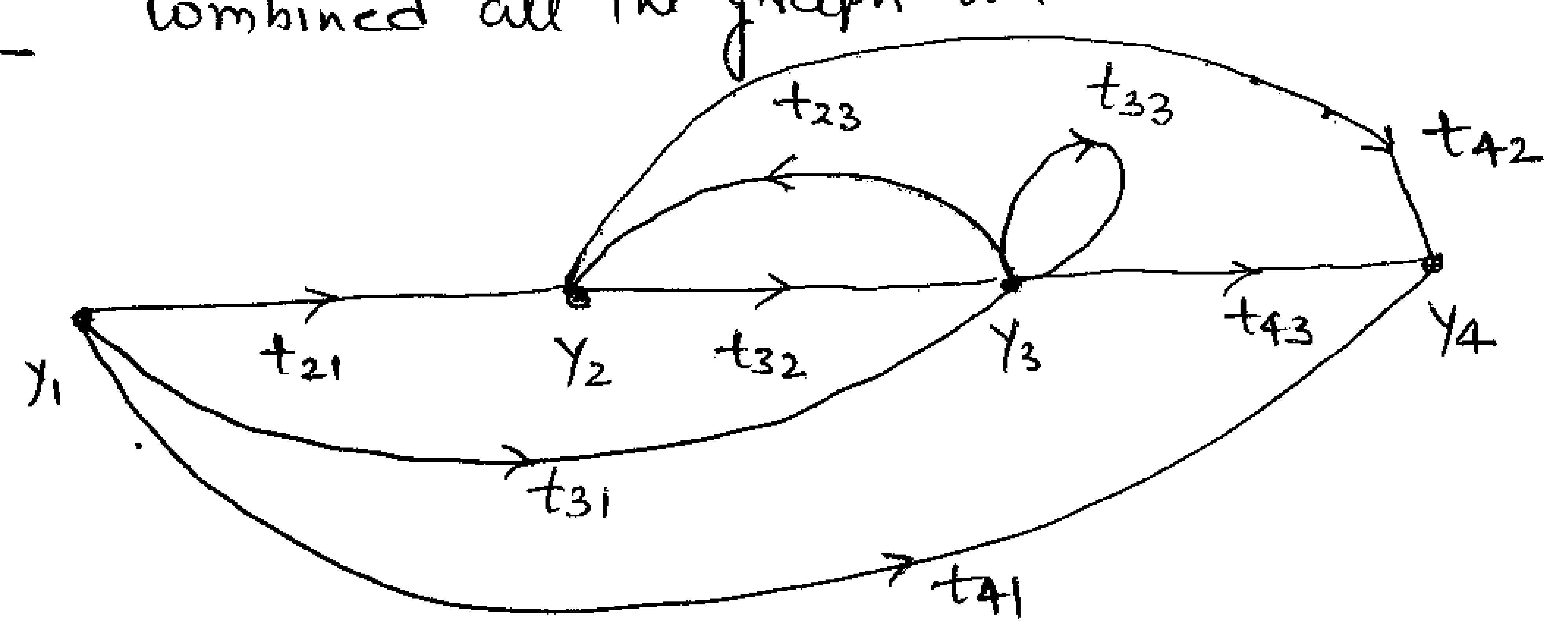
Step 3. Draw the graph for II equation;



Step 4 Draw the graph for III equation



Step 5 Combined all the graph and draw the complete SFG.



b) From differential equations :-

Consider the following differential equation

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = x$$

or $y''' + 3y'' + 5y' + 2y = x$

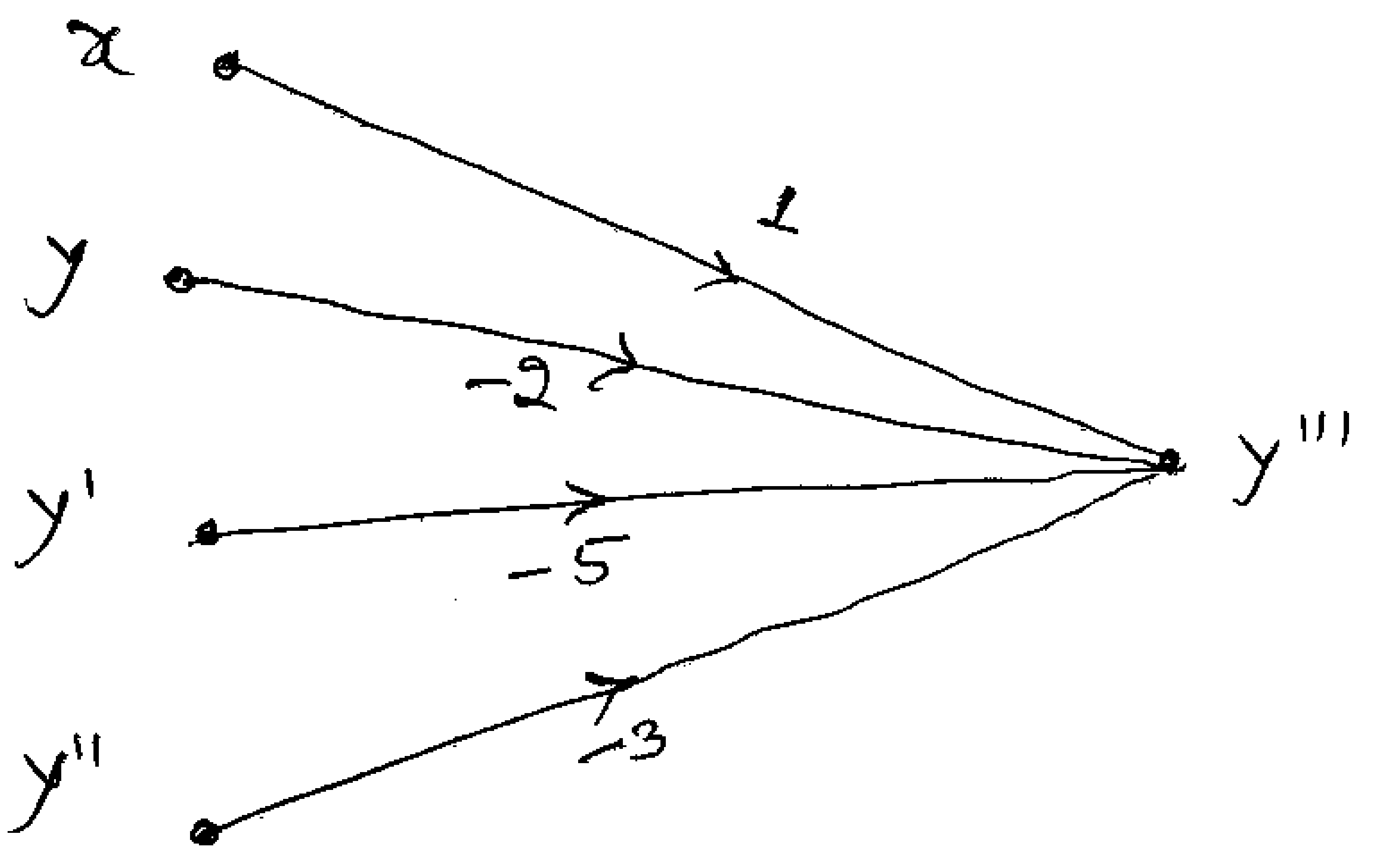
Step 1 Solve the equation for highest order

$$y''' = x - [3y'' + 5y' + 2y]$$

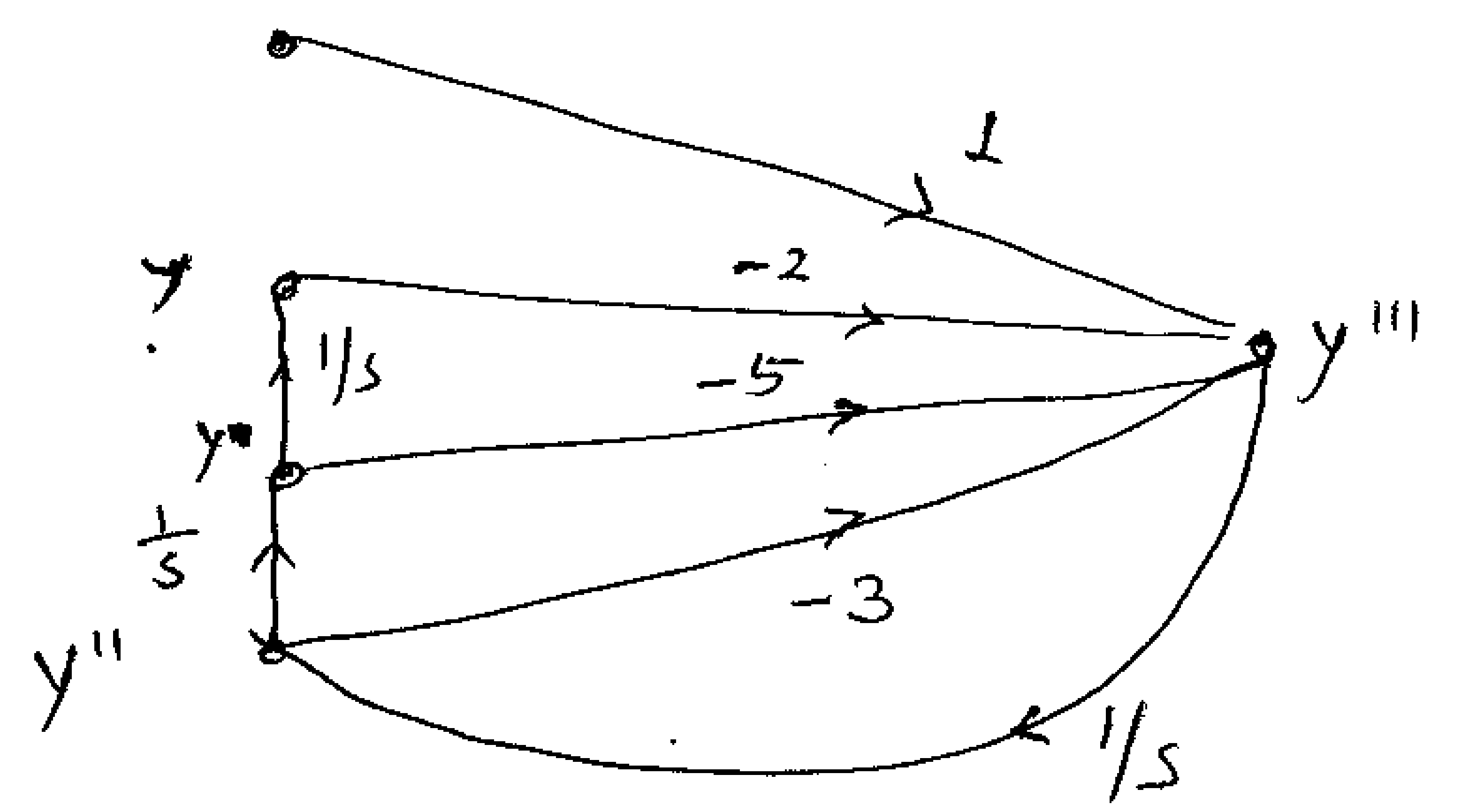
or $y''' = x - 3y'' - 5y' - 2y$

Step 2 Consider the LHS term as dependent variable and RHS terms as independent variables.

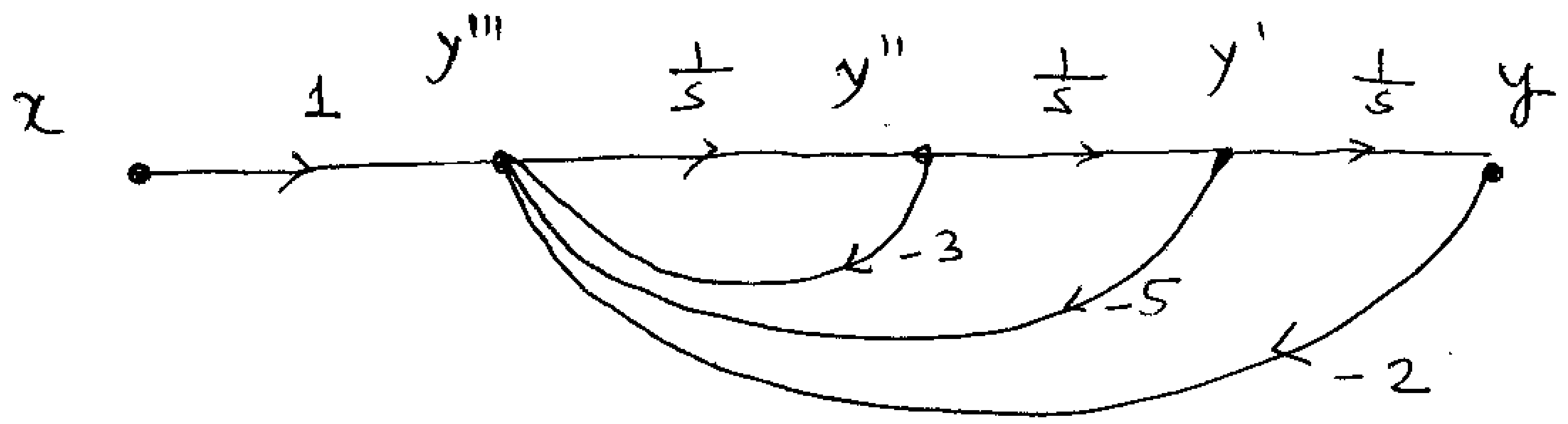
Thus, the constructed graph for this;



Step 3. Connect the nodes of highest order derivative to the node whose order is lower than this and so on. Also apply transmittance of $\frac{1}{s}$ as shown below;



Step 4 Redraw the SFG by considering x as input and y as output.



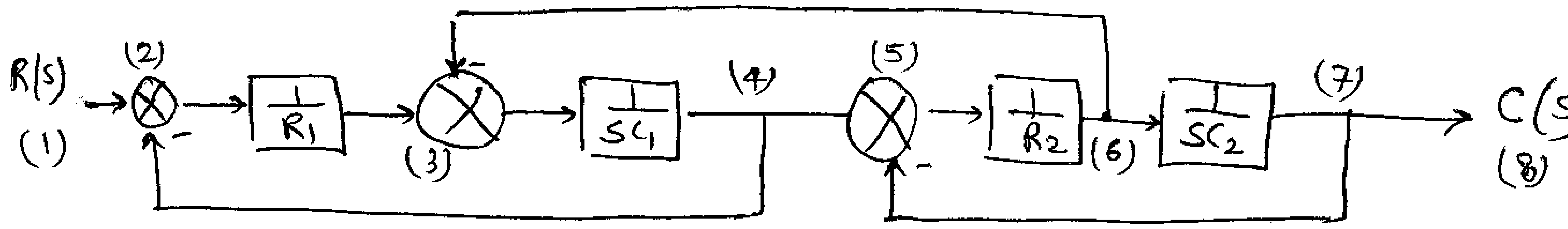
Complete SFG form by differential equation

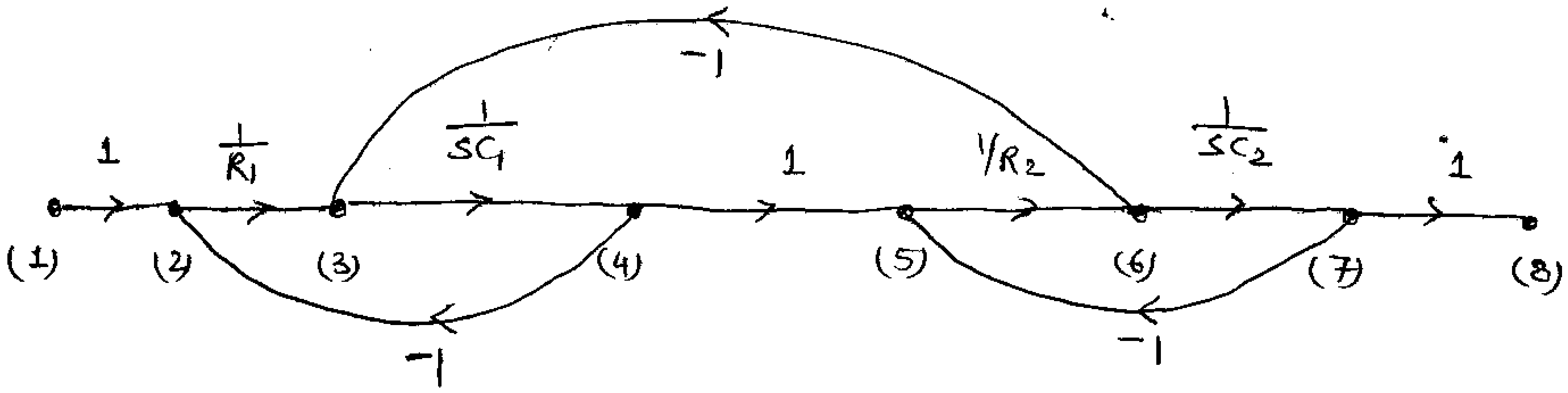
c) From Block Diagram :-

Step :- 1) Mark all variables, summing points, and take off point with input and output with number (1, 2, 3. -).

2) Connect all these nodes by a branch having transmittance according to the given block diagram.

Consider the SFG for the following block diagram:





Some definitions related to Signal Flow graph :-

1) Input Node :- A node which has only one or more outgoing branches is called an input node. e.g. node (1)

2) Output Node :- A node which has only one or more incoming branches is the output node. e.g. node (8).

3) Mixed Node :- A node having incoming and outgoing branches is known as mixed node. e.g. node (2), (3), (4), (5), (6).

4) Transmittance :- It is also called transfer function and normally written on the branch near the arrow. e.g. $\left(\frac{1}{SC_2}\right)$, $\left(\frac{1}{R_1}\right)$

5) Forward Path :- It is a path which starts from the input node and ends at output node and along which no node is

repeated more than once.

For e.g., there is only one forward path; (1, 2, 3, 4, 5, 6, 7, 8)

(6) Loop :- A path which starts and terminates on the same node and along which no other node is repeated more than once, is called Loop. For e.g. (2, 3, 4), (5, 6, 7), (3, 4, 5, 6).

7) Self Loop :- A path that starts and terminates on the same node is called self loop.

8) Path gain :- The products of the branch gain along the path is called path gain. for e.g. $g_1(1,2,3,4,5,6,7,8) = \frac{1}{s^2 R_1 R_2 C_1 C_2}$

9) Loop gain :- The gain of the loop (product of the branch gain along the whole loop) is called loop gain.

For example; $L_1(2,3,4) = -\frac{1}{sR_1C_1}$

$$L_2(5,6,7) = -\frac{1}{sR_2C_2}$$

$$L_3(3,4,5,6) = -\frac{1}{sR_2C_1}$$

10) Non-touching loops :- The loops which does not have any common node is called as non-touching loops.

→ MASON'S GAIN FORMULA :⇒

The transfer function of the given network using SFG technique is find out with the help of Mason's Gain Formula i.e.

$$\text{Transfer function } T = \frac{\sum g_K \Delta_K}{\Delta}$$

where K = no. of forward paths

g_K = gain of the K^{th} forward path

$\Delta = 1 - $ [Sum of all individual loops] + [sum of all gain products of two non-touching loops] − [sum of all gain products of three non-touching loops] + - - -

Δ_K = the part of Δ that does not touch the k^{th} forward path.

For the above SFG, $K = 1$

$$g_1(1,2,3,4,5,6,7,8) = \frac{1}{s^2R_1R_2C_1C_2}, \quad \Delta_1 = 1 - 0 = 1$$

$$L_1(2,3,4) = -\frac{1}{sR_1C_1}, \quad L_2(3,4,5,6) = -\frac{1}{sR_2C_1}$$

$$L_3(5,6,7) = -\frac{1}{sR_2C_2}$$

Thus, $$\Delta = 1 - [L_1 + L_2 + L_3]$$

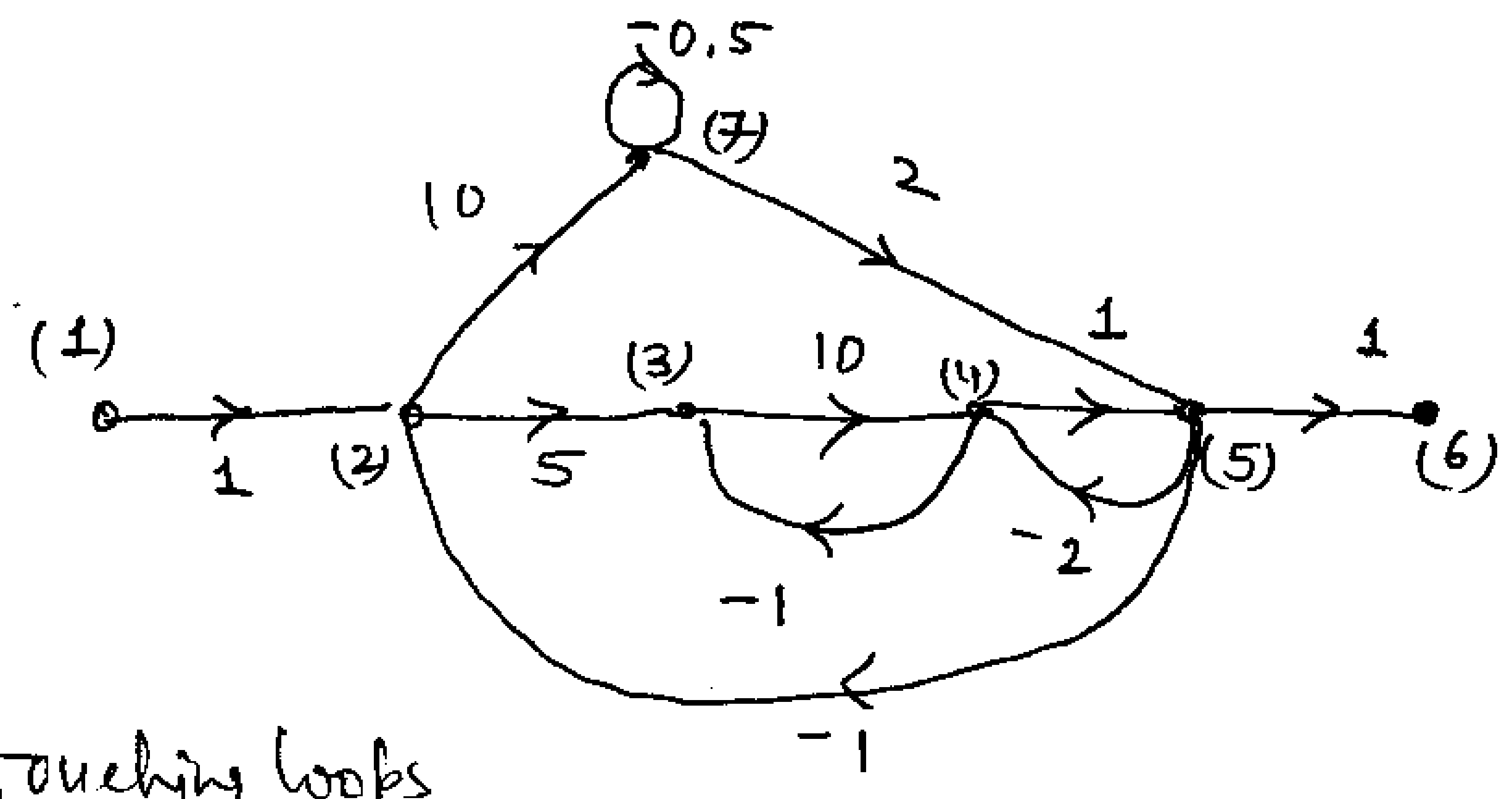
$$= 1 + \left[\frac{1}{sR_1C_1} + \frac{1}{sR_2C_1} + \frac{1}{sR_2C_2}\right]$$

or $$\Delta = \left[\frac{sR_2C_2 + sR_1C_2 + sR_1C_1 + s^2R_1R_2C_1C_2}{s^2R_1R_2C_1C_2}\right]$$

Thus, $$T = \frac{g_1\Delta_1}{\Delta} = \frac{\left[\frac{1}{s^2R_1R_2C_1C_2}\right]\cdot 1}{\Delta}$$

or $$\boxed{T = \frac{1}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}}$$ Ans

<u>Ques</u>:- Compute the T.F. using signal flow graph;



Here, $K = 2$,

$g_1(1,2,3,4,5,6) = 50$

$g_2(1,2,7,5,6) = 20$

$L_1(7) = -0.5$

$L_2(3,4) = -10$

$L_3(4,5) = -2$

$L_4(2,3,4,5) = -50$

$L_5(2,7,5) = -20$

<u>Non touching loops</u>

$L_1L_2(3,4,7) = 5$

$L_1L_3(4,5,7) = 1$

$L_1L_4(2,3,4,5,7) = 25$

$L_2L_5(2,3,4,5,7) = 200$

$$\Delta_1 = 1 - (L_1) = 1 - (-0.5) = 1.5$$

$$\Delta_2 = 1 - (L_2) = 1 - (-10) = 11$$

$$TF = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$= \frac{50(1.5) + 20(11)}{1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_2 + L_1 L_3 + L_1 L_4 + L_2 L_5)}$$

$$= \frac{75 + 220}{1 + (0.5 + 10 + 2 + 50 + 20) + (5 + 1 + 25 + 200)}$$

$$= \frac{295}{31\cancel{4}.5} = \underline{\underline{0.94}} \quad \text{or} \quad \underline{\underline{0.937}}. \quad \underline{\underline{\text{Ans.}}}$$

Mechanical systems elements, equations of mechanical systems

Contents:

- Introduction about mechanical system.
- How to draw mechanical equivalent network from mechanical network.
- Describe differential equation of equivalent mechanical network.
- Find transfer function from differential equation

MECHANICAL SYSTEM TO ELECTRICAL SYSTEM

There are two types of mechanical system,

a) Translational systems

b) Rotational systems.

a) Translational Systems :⇒

The motion takes place along the straight line is known as translational motion and the system is called Translational systems.

There are three types of forces that resists motion.

1) Inertia Force :- This is due to the mass body 'M' and is given as $\boxed{F_M(t) = M\frac{d^2x(t)}{dt^2}}$

2) Damping Force : It is due to the damper or piston and is given as ; $\boxed{F_D(t) = B\frac{dx(t)}{dt}}$

3) Spring Force :- It is due to the spring and is given as;

$$\boxed{F_K(t) = K\,x(t)}$$

D' ALEMBERT PRINCIPLE :⇒

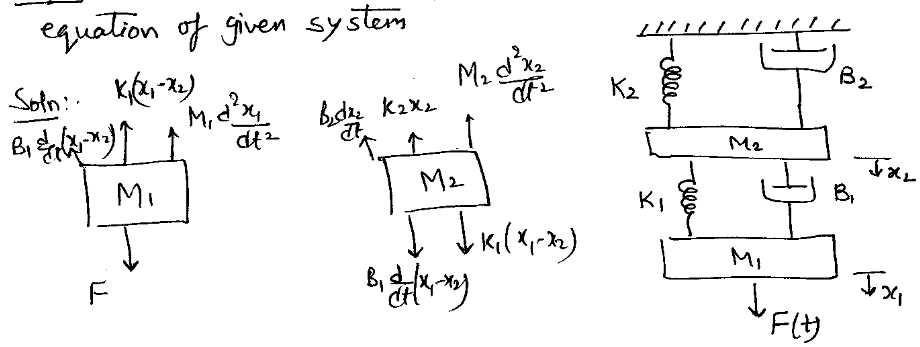
This states that, "For any body, the algebric sum of externally applied forces and the forces resisting motion in any given direction is zero".

Procedure for writing differential equation of mechanical system

1) Assume system is in equilibrium.
2) Assume that the system is given same arbitary displacement if number of distributing forces are present.
3) Draw the free body diagram of forces exerted on each mass in the system.
4) Apply Newton's Law of motion to each diagram, using the convention that any force acting in the direction of assume displacement is positive.
5) Rearrange the equations in suitable form.

Ques:- Draw the free body diagram and write the differential equation of given system

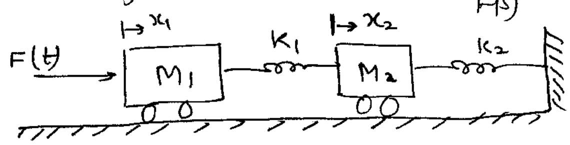
Soln:-



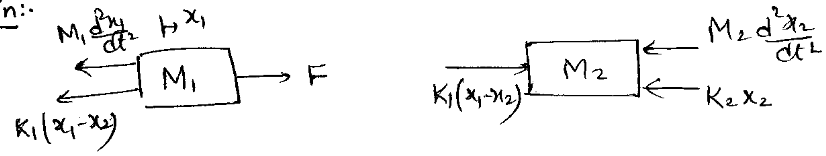
$$F = M_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1-x_2) + K_1(x_1-x_2)$$

$$B_1\frac{d}{dt}(x_1-x_2) + K_1(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + K_2x_2 + B_2\frac{dx_2}{dt}$$

Ques:- Write the differential equations describing the dynamics of the system shown in figure and find the ratio $\frac{X_2(s)}{F(s)}$.



Soln:-



Hence $\quad F = M_1 \frac{d^2x_1}{dt^2} + K_1(x_1 - x_2) \quad -(1)$

and $\quad K_1(x_1 - x_2) = M_2 \frac{d^2x_2}{dt^2} + K_2 X_2 . \quad -(2)$

Taking Laplace transform of above equations;

$$F(s) = M_1 s^2 X_1(s) + K_1 X_1(s) - K_1 X_2(s) \quad -(3)$$

$$K_1[X_1(s) - X_2(s)] = M_2 s^2 X_2(s) + K_2 X_2(s)$$

or $$X_1(s) = \left[\frac{s^2 M_2 + K_2 + K_1}{K_1}\right] X_2(s)$$

put the value of $X_1(s)$ in Eqn.(1), we get

$$\frac{F(s)}{X_2(s)} = [s^2 M_1 + K_1]\left[\frac{s^2 M_2 + K_1 + K_2}{K_1}\right] - K_1$$

or $$\frac{F(s)}{X_2(s)} = \frac{(s^2 M_1 + K_1)(s^2 M_2 + K_1 + K_2) - K_1^2}{K_1}$$

or $$\frac{X_2(s)}{F(s)} = \frac{K_1}{(s^2 M_1 + K_1)(s^2 M_2 + K_1 + K_2) - K_1^2}$$ Ans

Questioned asked in University Examinations:

a) Write the differential equation governing the behavior of the mechanical system shown in Fig. 1 and draw its mechanical equivalent diagram. **(5-Marks,2009-10**

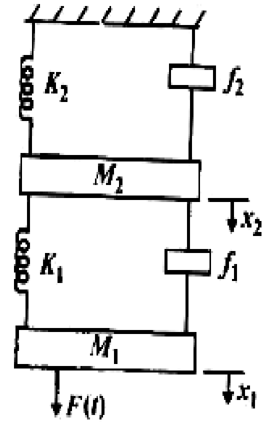


Fig. 1

b) Obtain the transfer function from the network shown in Fig. 2. **(5-Marks, 2012-13)**

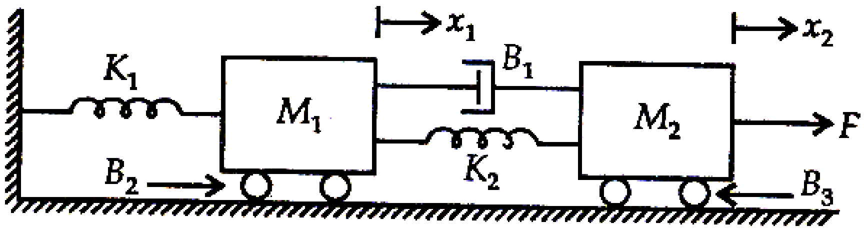


Fig. 2

Modelling of Physical systems: electrical networks (F-V Analogy)

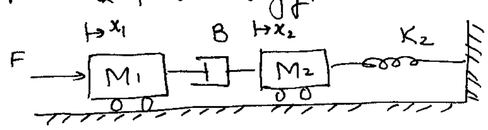
Contents:

- Numerical Problems based on Mechanical system to Electrical System conversion using Force-Voltage Analogy.

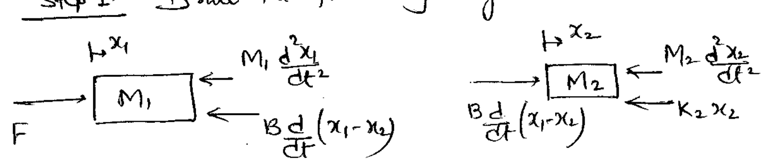
Analogous Systems

Mechanical System	Electrical System	
Translational System	F-V Analogous	F-I analogous
Force (F)	V	I
Mass (M)	L	C
Stiffness (K)	$\frac{1}{C}$	$\frac{1}{L}$
Damping Coeff (B)	R	$\frac{1}{R}$
Displacement (x)	q (charge)	ϕ (Flux)

Ques:- Draw the electrical analogous circuit of the system. Use F-V & f-i analogy.



Soln: Step-1. Draw the free body diagram.



Thus;
$$F = M_1\frac{d^2x_1}{dt^2} + B\frac{d}{dt}(x_1-x_2) \quad -(1)$$

$$B\frac{d}{dt}(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + K_2 x_2 \quad -(2)$$

Step-2. For F-V analogy, convert the equation into its equivalent voltage system;

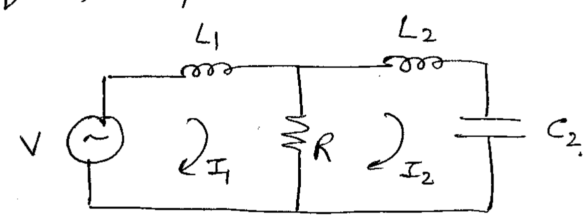
$$V = L_1 \frac{d^2 q_1}{dt^2} + R\left(\frac{d}{dt} q_1 - \frac{d}{dt} q_2\right)$$

or $$V = L_1 \frac{dI_1}{dt} + R(I_1 - I_2) \quad -(3)$$

Also, $$R \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + \frac{1}{C_2}(q_2)$$

or $$R(I_1 - I_2) = L_2 \frac{dI_2}{dt} + \frac{1}{C_2}\int I_2\, dt \quad -(4)$$

From eqn. (3), & eqn. (4), now we draw the circuit;



Equivalent electrical network (F-V analogy).

<u>Step 3.</u> <u>For F-i analogy</u>

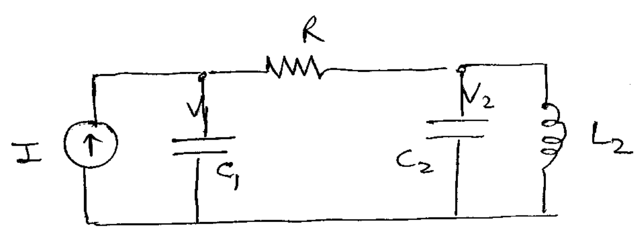
Convert the equation (1) and (2), into its equivalent current system;

i.e. $$I = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{R}\frac{d}{dt}(\phi_1 - \phi_2)$$

or $$I = C_1 \frac{dV_1}{dt} + \frac{1}{R}(V_1 - V_2) \quad -(3)$$

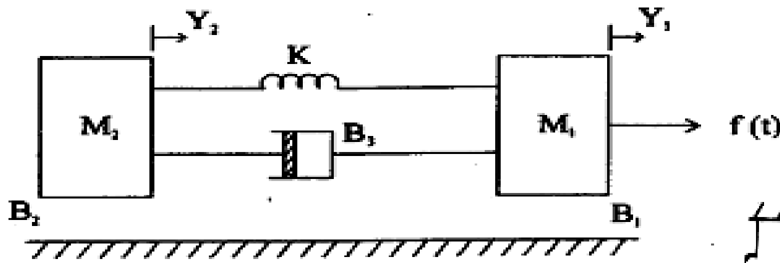
and $$\frac{1}{R}\frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2 \phi_2}{dt^2} + \frac{1}{L_2}\phi_2$$

or $$\frac{1}{R}(V_1 - V_2) = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\, dt$$



Questioned asked in University Examinations:

a) Find the transfer function $Y_1(s)$ / F(s) in the given figure: **(5-Marks, 2013-14).**



b) Obtain the analogous electrical network based on force voltage (f-v) analogy. **(5-Marks, 2009-10)(10-Marks, 2010-11)**

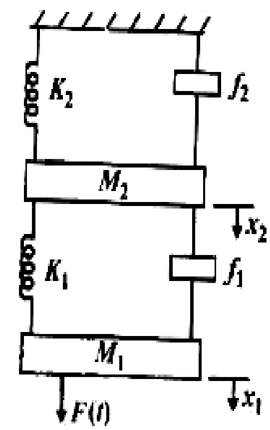


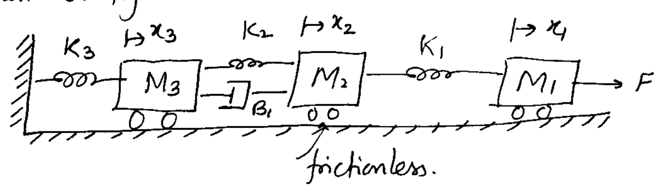
Fig. 1

Modelling of Physical systems: electrical networks (F-I Analogy)

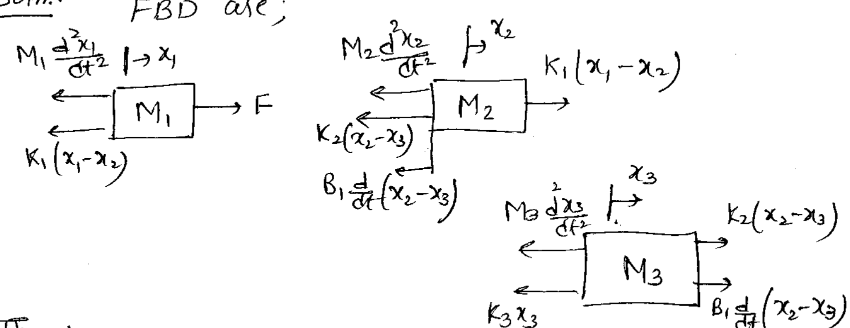
Contents:

1) Numerical Problems based on Mechanical system to Electrical System conversion using Force-Current Analogy.

Ques:- Draw the f-v and f-i analogy of the network shown in figure.



Soln:- FBD are;



Thus;

$$F = M_1\frac{d^2x_1}{dt^2} + K_1(x_1 - x_2) \quad -(1)$$

$$K_1(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + K_2(x_2-x_3) + B_1\frac{d}{dt}(x_2-x_3) \quad -(2)$$

$$K_2(x_2-x_3) + B_1\frac{d}{dt}(x_2-x_3) = M_3\frac{d^2x_3}{dt^2} + K_3x_3 \quad -(3)$$

For F-V analogy ;

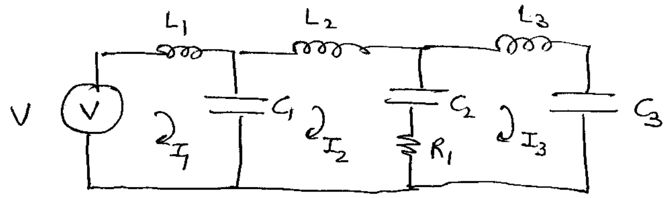
$$V = L_1\frac{d^2q_1}{dt^2} + \frac{1}{C_1}(q_1-q_2)$$

or $$V = L_1\frac{dI_1}{dt} + \frac{1}{C_1}\int(I_1-I_2)\,dt \quad -(4)$$

and $\cancel{K_1}$ $$\frac{1}{C_1}(q_1-q_2) = L_2\frac{d^2q_2}{dt^2} + \frac{1}{C_2}(q_2-q_3) + R_1\frac{d}{dt}(q_2-q_3)$$

or $$\frac{1}{C_1}\int(I_1-I_2)\,dt = L_2\frac{dI_2}{dt} + \frac{1}{C_2}\int(I_2-I_3)\,dt + R_1(I_2-I_3) \quad -(5)$$

and $\frac{1}{C_2}\int(I_2-I_3)dt + R_1(I_2-I_3) = L_3\frac{dI_3}{dt} + \frac{1}{C_3}\int I_3\,dt$



__For F-i analogy :-__

from eqⁿ.(1), $I = C_1\frac{d^2\phi_1}{dt^2} + \frac{1}{L_1}(\phi_1-\phi_2)$

or $I = C_1\frac{dV_1}{dt} + \frac{1}{L_1}\int(V_1-V_2)\,dt$ — (7)

from eqⁿ.(2), $\frac{1}{L_1}(\phi_1-\phi_2) = C_2\frac{d^2\phi_2}{dt^2} + \frac{1}{L_2}(\phi_2-\phi_3) + \frac{1}{R_1}\frac{d}{dt}(\phi_2-\phi_3)$

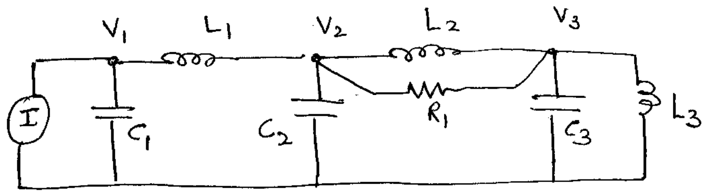
or $\frac{1}{L_1}\int(V_1-V_2)\,dt = C_2\frac{dV_2}{dt} + \frac{1}{L_2}\int(V_2-V_3)dt + \frac{1}{R_1}(V_2-V_3)$ — (8)

from eqⁿ.(3);

$\frac{1}{L_2}(\phi_2-\phi_3) + \frac{1}{R_1}\frac{d}{dt}(\phi_2-\phi_3) = C_3\frac{d^2\phi_3}{dt^2} + \frac{1}{L_3}\phi_3$

or $\frac{1}{L_2}\int(V_2-V_3)\,dt + \frac{1}{R_1}(V_2-V_3) = C_3\frac{dV_3}{dt} + \frac{1}{L_3}\int V_3\,dt$ — (9)

From eqⁿ. (7), (8) & (9), we draw the f-i based network;



Equivalent electrical network based on f-i analogy.

Questioned asked in University Examinations:

a) Obtain the analogous electrical network based on force current (f-i) analogy.
(5-Marks, 2009-10)(10-Marks, 2010-11)

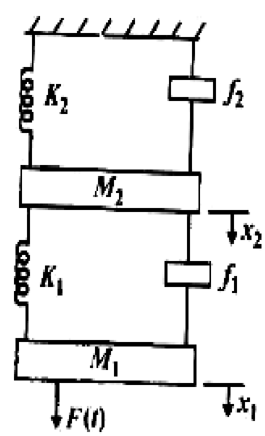


Fig. 1