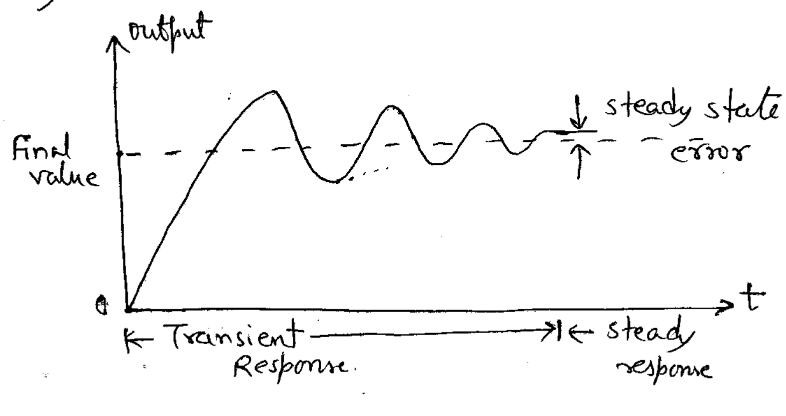
Any system containing energy storing element like capacitor, inductor, mass and inertia etc. If the energy state of the system is disturbed then its takes a certain time to change from one state to another state. This time is known as Transient Time and the values of current and voltages during this period is called transient response.

Depending upon the parameters of the system, these transient may have oscillations which may be either sustained or decaying in nature.

The time response of a control system is divided into two parts: a) Transient Response (b) Steady state Response.



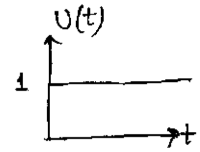
From figure, the transient response is the part of response which goes to zero as time increases and steady state response is the part of the response after transient has died.

TEST INPUT SIGNALS :⇒

1) Unit Step Function :-

It is defined as;

$$U(t) = \begin{bmatrix} 1 & t \geq 0 \\ 0 & t < 0 \end{bmatrix}$$

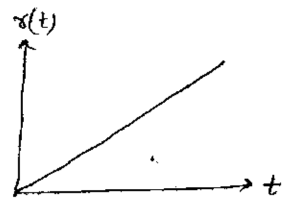


Laplace of $U(t) = \frac{1}{s}$

2) Ramp Function :-

It is defined as;

$$r(t) = \begin{bmatrix} t & t \geq 0 \\ 0 & t < 0 \end{bmatrix}$$

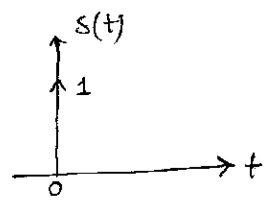


Laplace of $r(t) = \frac{1}{s^2}$

3) Impulse Function :-

It is defined as;

$$\delta(t) = \begin{bmatrix} 1 & t = 0 \\ 0 & t \neq 0 \end{bmatrix}$$

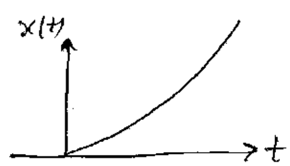


Laplace of $\delta(t) = 1$

4) Parabolic Function :-

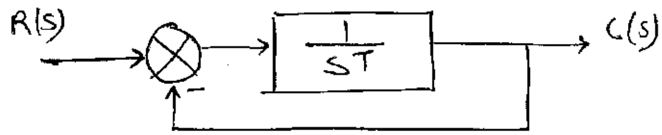
It is defined as;

$$x(t) = \begin{bmatrix} \frac{t^2}{2} & t > 0 \\ 0 & t < 0 \end{bmatrix}$$



Laplace of parabolic $\Rightarrow \frac{1}{s^3}$

TIME RESPONSE OF FIRST ORDER SYSTEM



For I order system, $G(s) = \frac{1}{ST}$, $H(s) = 1$

Thus, the transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{1/ST}{1+\frac{1}{ST}\cdot 1}$$

or $$\boxed{\frac{C(s)}{R(s)} = \frac{1}{1+ST}}$$

1) Response of first order system with unit step input or unit step response of first order system

For unit step response, $R(s) = \frac{1}{S}$

$$\therefore \quad C(s) = \frac{R(s)}{1+ST} = \frac{1}{S(1+ST)}$$

Taking the partial fraction of above function, i.e.

$$\frac{1}{S(1+ST)} = \frac{A}{S} + \frac{B}{1+ST} \quad -(1)$$

$$\frac{1}{S(1+ST)} = \frac{A(1+ST)+BS}{S(1+ST)}$$

or $$1 = A(1+ST) + BS.$$

put $S=0$; $1 = A$

put $S = -\frac{1}{T}$; $1 = B\left(-\frac{1}{T}\right)$ or $B = -T$

Substitute the value of A and B in eq. (1),

$$\frac{1}{s(1+sT)} = \frac{1}{s} + \frac{(-T)}{1+sT}$$

Therefore; $$C(s) = \frac{1}{s} - \frac{1}{s+1/T}$$

Taking inverse Laplace of above function;

$$\boxed{c(t) = \left(1 - e^{-t/T}\right)u(t)}$$

This gives the unit step response of first order system.

when $t = T$, $c(t) = 1 - e^{-1} = 0.632$ or 63.2%

where T is the time constant and it is defined as the time required for the signal to attain 63.2% of final or steady state value.

This time constant indicates how fast the system reaches the final value. For smaller time constant, system response is faster and vice-versa.

$$\text{error} \Rightarrow e(t) = -c(t) + r(t)$$
$$= -\left(1 - e^{-t/T}\right)u(t) + u(t)$$

or $$\boxed{e(t) = e^{-t/T}\, u(t)}$$

Steady state error $$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} e^{-t/T} u(t)$$

or $$\boxed{e_{ss} = 0}$$

2) Response of first order system with unit ramp function :-

For unit ramp function $R(s) = \frac{1}{s^2}$

Therefore; $$C(s) = \frac{R(s)}{1+sT} = \frac{1}{s^2(1+sT)}$$

Taking partial fraction of the function;

$$\frac{1}{s^2(1+sT)} = \frac{As+B}{s^2} + \frac{C}{1+sT} \quad \text{— (1)}$$

$$\text{or} \quad 1 = (As+B)(1+sT) + Cs^2$$

$$\text{or} \quad 1 = As + s^2AT + B + sBT + Cs^2$$

Compare s^2 terms; $0 = AT + C$

" s terms; $0 = A + BT$

" s^0 terms; $1 = B$

$$\therefore \quad A = -T \quad \text{and} \quad C = T^2$$

Put the values of A, B, and C in eqn. (1);

$$\frac{1}{s^2(1+sT)} = \frac{-Ts+1}{s^2} + \frac{T^2}{1+sT}$$

$$\text{or} \quad C(s) = \frac{1}{s^2} - \frac{T}{s} + T\left(\frac{1}{s+\frac{1}{T}}\right)$$

Taking inverse Laplace, we get

$$c(t) = \cancel{t\,u(t)} - Tu(t) + T\cdot e^{-t/T}u(t)$$

$$\text{or} \quad \boxed{c(t) = \left[t - T + Te^{-t/T}\right]u(t)}$$

This is the ramp response of first order system.

$$\text{Error} \Rightarrow e(t) = -c(t) + r(t)$$

$$= -\left[t - T + Te^{-t/T}\right]u(t) + t\,u(t)$$

$$\text{or} \quad e(t) = T\left(-e^{-t/T} + 1\right)u(t)$$

$$\text{steady state error} \quad e_{ss} = \lim_{t\to\infty} e(t)$$

$$\text{or} \quad \boxed{e_{ss} = T}$$

3) Response of first order system with impulse function or impulse response of first order system

For impulse response $R(s) = 1$ i.e. $r(t) = \delta(t)$

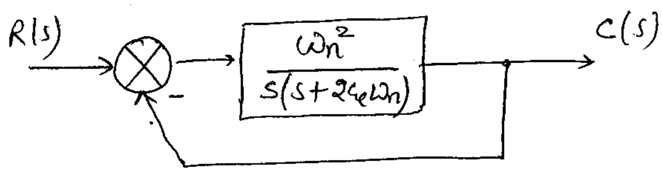
Therefore; $C(s) = \frac{R(s)}{1+sT} = \frac{1}{1+sT} = \frac{1}{T}\left(\frac{1}{s+\frac{1}{T}}\right)$

Taking inverse laplace of above function;

$$\boxed{C(t) = \frac{1}{T} e^{-t/T} u(t)}$$

This gives the impulse response of first order system.

TIME RESPONSE OF SECOND ORDER SYSTEM :⇒



For second order system ; $G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$, $H(s) = 1$

where ω_n = natural freq. of oscillations

ζ = (zeta) damping factor

Therefore ; $$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

a) Unit step response of second order system :⇒

For unit step response $r(t) = u(t)$ or $R(s) = \frac{1}{s}$

Thus ; $$C(s) = \left[\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right] \cdot R(s)$$

or $$C(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

Taking partial fraction, we get

$$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{A}{s} + \frac{Bs+C}{\omega_n^2+2\zeta\omega_n s+s^2} \quad -(1)$$

or $\omega_n^2 = As^2 + 2\zeta\omega_n A\cdot s + A\omega_n^2 + Bs^2 + Cs$

Compare s^2 terms; $0 = A + B$

" s terms; $0 = 2\zeta\omega_n A + C$

" s^0 terms; $\omega_n^2 = A\omega_n^2$ or $A = 1$

$\therefore B = -1$ & $C = -2\zeta\omega_n$

Substitute the value of A, B and C in eqⁿ. (1),

$$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{1}{s} - \frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2-\zeta^2\omega_n^2+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}$$

$$= \frac{1}{s} - \frac{s+\zeta\omega_n+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

Let $\omega_d^2 = \omega_n^2(1-\zeta^2)$ (damped ~~natural~~ freq. of osscillation)

or $$C(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta}{\sqrt{1-\zeta^2}}\cdot\frac{\omega_d}{(s+\omega_n\zeta)^2+\omega_d^2}$$

Taking inverse Laplace, we get

$$c(t) = 1 - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\cdot e^{-\zeta\omega_n t}$$

or $c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}\cos\omega_d t + \zeta\sin\omega_d t\right]$

let $\zeta = \cos\phi$ then $\sqrt{1-\zeta^2} = \sin\phi$

or $c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t\right]$

or $\boxed{c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)}$

This is the unit step response of second order system.

The error signal for the system is

$$e(t) = r(t) - c(t)$$
$$= 1 - \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)\right]$$

or $e(t) = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)$

Steady state error $e_{ss} = \lim_{t\to\infty} e(t)$

or $\boxed{e_{ss} = 0}$

<u>System classification based on damping factor</u>

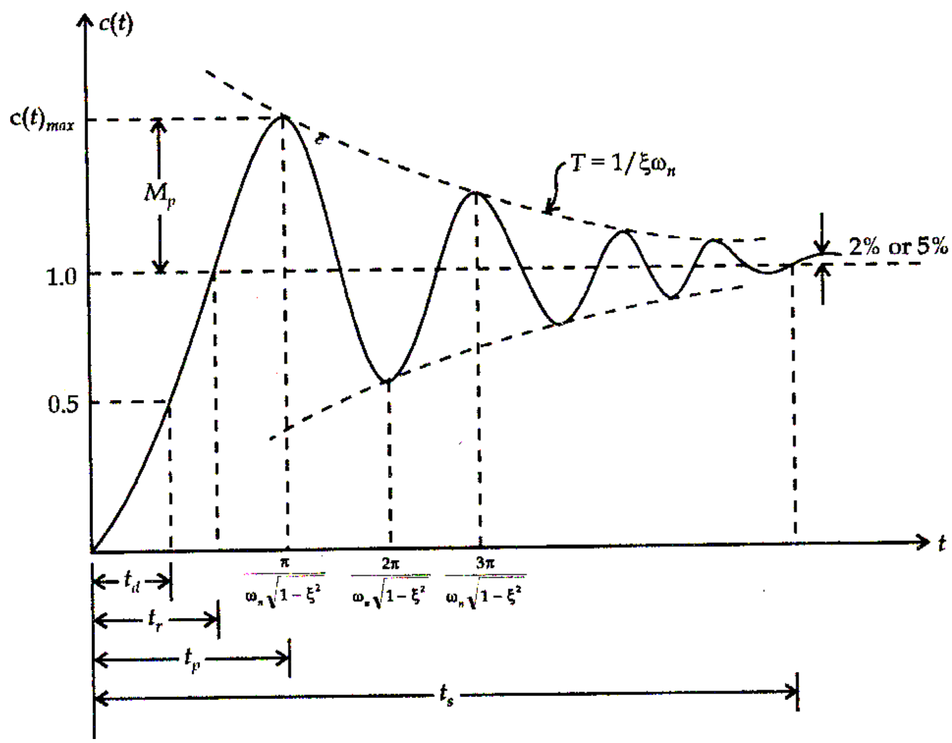
damping factor	System Type
$\zeta = 0$	undamped
$0 < \zeta < 1$	underdamped
$\zeta = 1$	critically damped
$\zeta > 1$	overdamped.

The above response is derived for underdamped system by default.

TIME RESPONSE SPECIFICATIONS OF SECOND ORDER SYSTEM ⇒

Consider a second order system with unit step input with all initial conditions are zero. The time response of the system is

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$



The following are the common transient response specification;

1) Delay Time (t_d) :- It is the time required for the response to reach 50% of its final value in first time.

2) Rise Time (t_r) :- It is the time required for the response to rise from 10% to 90% of its final value for ~~under~~ over damped system and 0 to 100% for under damped systems.

3) Peak Time (t_p) :- It is the time required for the response to reach the first peak of the response.

4) Maximum Overshoot (M_p) :- It is the normalized difference between the peak of the time response and steady output.

5) Settling time (t_s) :- It is the time required for the response to reach and stay within the specified range (2% to 5%) of its final value.

6) Steady state error (e_{ss}) :- It is the difference between the actual output and desired output as 't' $\to \infty$.

i.e. $e_{ss} = \lim_{t \to \infty} \left[r(t) - C(t) \right]$

Expression For Rise Time (t_r) :-

We know that, the response of second order system is

$$C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

For rise time $c(t) = 1$,

$$\therefore \quad 1 = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

or $\quad e^{-\zeta\omega_n t}\sin(\omega_d t+\phi)=0$

$\because \; e^{-\zeta\omega_n t} \neq 0$

$\therefore \; \sin(\omega_d t+\phi) = 0 = \sin n\pi$

for $n=1$, (first time in response)

$t = t_r$

$\therefore \; \sin(\omega_d t_r+\phi) = \sin\pi$

or $\quad \omega_d t_r + \phi = \pi$

or $$\boxed{t_r = \frac{\pi-\phi}{\omega_d} = \frac{\pi-\phi}{\omega_n\sqrt{1-\zeta^2}}}$$

where $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

Expression For Peak Time (t_p) :-

We know that, response of second order system is

$$c(t) = 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi)$$

For peak time, (maximum value) $\frac{d\,c(t)}{dt}=0$

or $$0-\left[\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left\{\cos(\omega_d t+\phi)\cdot\omega_d\right\} + \sin(\omega_d t+\phi)\cdot\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cdot(-\zeta\omega_n)\right]=0$$

or $$\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\omega_n\sqrt{1-\zeta^2}\cdot\cos(\omega_d t+\phi)\right] = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi)\cdot\zeta\omega_n$$

or $\quad \sin(\omega_d t+\phi)\cdot\zeta - \cos(\omega_d t+\phi)\cdot\sqrt{1-\zeta^2} = 0$

As we know, $\zeta = \cos\phi$ and $\sqrt{1-\zeta^2} = \sin\phi$.

$\therefore \; \sin(\omega_d t+\phi)\cdot\cos\phi - \cos(\omega_d t+\phi)\cdot\sin\phi = 0$

or $$\sin\left[(\omega_d t + \cancel{\phi}) - \cancel{\phi}\right] = 0 = \sin n\pi$$

for $t = t_p$, $n = 1$

$$\therefore \quad \omega_d t_p = \pi$$

or $$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

⇒ Expression for Maximum Overshoot ($\% M_p$) :-

The response of second order system is ;

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

For maximum overshoot, we calculate $c(t)$ at t_p

i.e. $$c(t_p) = 1 - \frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}} \sin(\omega_d t_p + \phi)$$

since $$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\therefore \quad c(t_p) = 1 - \frac{e^{-\pi\zeta/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}} \sin(\pi + \phi)$$

or $$c(t_p) = 1 + e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Now $$M_p = c(t_p) - 1 = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

or $$\boxed{\% M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100}$$

⇒ Settling Time :-

$$\boxed{t_s = \frac{4}{\zeta\omega_n}}$$ for underdamped system.

Numerical problems

Q-1) When a second order system is subjected to an unit step input, the values of $\zeta = 0.5$ and $\omega_n = 6$ rad/sec. Determine the rise time, peak time, settling time and peak overshoot.

Given $\zeta = 0.5$, $\omega_n = 6.0$ rad/sec

1) Rise time $t_r = \dfrac{\pi - \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{3.14 - 1.047}{5.19}$

or $t_r = 0.403$ sec

2) Peak time $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{3.14}{6\sqrt{1-(0.5)^2}} = 0.605$ sec.

3) settling time $t_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{(0.5)(6)} = 1.33$ sec

4) Max. Overshoot $M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 = 16.3\%$

Q-2) The open loop transfer function of a servo system with unity feedback is given by $G(s) = \dfrac{10}{(s+2)(s+5)}$

Determine the damping ratio, undamped frequency of oscillation. What percentage overshoot of the response to a unit step input.

Given, $H(s) = 1$, $G(s) = \dfrac{10}{(s+2)(s+5)}$

$\therefore$ characteristic equation: $1 + G(s)H(s) = 0$

or $1 + \dfrac{10}{(s+2)(s+5)} \cdot 1 = 0$

or $s^2 + 7s + 20 = 0$

Compare this equation with standard CE i.e.

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Thus, $\omega_n^2 = 20$ or $\omega_n = \underline{4.47 \text{ rad/sec}}$

$2\zeta\omega_n = 7$ or $\zeta = \dfrac{7}{2(4.47)} = \underline{0.7826}$.

and $\% M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$

$$= e^{\frac{-(3.14)(0.7826)}{\sqrt{1-(0.7826)^2}}} \times 100 = \underline{1.92}\ \%$$

ERROR ANALYSIS :- TYPE OF SYSTEM

Generalised open loop transfer function is,

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)(1+sT_c)\ldots}{s^m(1+sT_1)(1+sT_2)(1+sT_3)\ldots}$$

where $s = -\frac{1}{T_a}, -\frac{1}{T_b}, -\frac{1}{T_c} \ldots$ are zeros

and $s = -\frac{1}{T_1}, -\frac{1}{T_2}, -\frac{1}{T_3} \ldots$ are poles.

Also, here s^m in denominator, where 'm' shows the number of poles at the origin. Thus, it is also called Type-'m' system. Therefore,

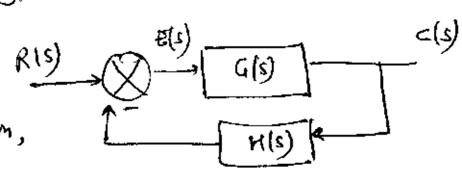
$$\boxed{\text{Type of system} = \text{No. of poles lies at the origin}}$$

for e.g. if $m = 1$, type '1' system
if $m = 2$, type '2' system
if $m = 0$, type-0 system.

STEADY STATE ERROR :-

It is defined as the difference between the input and output of the system during steady state. For accuracy the steady state error should be minimum.

Consider a closed loop system,



Here, $C(s) = E(s) \cdot G(s)$

Now, $E(s) = R(s) - C(s)H(s)$

or $E(s) = R(s) - E(s) \cdot G(s) \cdot H(s)$

or $E(s)\left[1+G(s)H(s)\right] = R(s)$

or $$\boxed{E(s) = \frac{R(s)}{1+G(s)\,H(s)}}$$

The steady state error of the system is given by,

$$\boxed{e_{ss} = \lim_{s\to 0} s\,.\,E(s) = \lim_{s\to 0} s\,.\,\frac{R(s)}{1+G(s)H(s)}}$$

⇒ STATIC ERROR COEFFICIENTS :⇒

STEATY STATE ERROR FOR DIFFERENT TYPES OF SYSTEM

1) Static Position Error Coefficients ⇒ (K_p)

We know that, steady state error

$$e_{ss} = \lim_{s\to 0} s\,.\,\frac{R(s)}{1+G(s)\,H(s)}$$

For unit step input $R(s) = \frac{1}{s}$, thus

$$e_{ss} = \lim_{s\to 0} s\,.\,\frac{1/s}{1+G(s)H(s)} = \frac{1}{1+\lim_{s\to 0} G(s)\,H(s)}$$

or $$e_{ss} = \frac{1}{1+K_p}$$

where $\boxed{K_p = \lim_{s\to 0} G(s)\,H(s)}$ is called Position error coefficient.

For Type - 0 system; $G(s)\,H(s) = \frac{K(1+sT_a)(1+sT_b)\,\text{--}}{(1+sT_1)(1+sT_2)\,\text{--}}$

Thus, $$e_{ss} = \frac{1}{1+\lim_{s\to 0} \frac{K(1+sT_a)(1+sT_b)}{(1+sT_1)(1+sT_2)}}$$

or $\boxed{e_{ss} = \frac{1}{1+K}}$ This is steady state error for Type - 0 system subjected to unit step input.

For Type-1 system ; $G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)\ldots}$

Thus, $e_{ss} = \frac{1}{1+\lim\limits_{s\to 0}\frac{K(1+sT_a)(1+sT_b)}{s(1+sT_1)(1+sT_2)}} = \frac{1}{1+\infty}$

or $\boxed{e_{ss} = 0}$

This is steady state error for type-1 system subjected to unit step input.

for Type-2 System :- $G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)}{s^2(1+sT_1)(1+sT_2)}$

Thus, $e_{ss} = \frac{1}{1+\lim\limits_{s\to 0}\frac{K(1+sT_a)(1+sT_b)}{s^2(1+sT_1)(1+sT_2)}} = \frac{1}{1+\infty}$

or $\boxed{e_{ss} = 0}$

This is the steady state error for Type-2 system subjected to unit step input.

2) STATIC VELOCITY ERROR COEFFICIENTS :⇒

We know that, steady state error

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{R(s)}{1+G(s)H(s)}$$

For unit ramp input $R(s) = \frac{1}{s^2}$

$$\therefore\ e_{ss} = \lim_{s\to 0}\frac{s\cdot\frac{1}{s^2}}{1+G(s)H(s)} = \lim_{s\to 0}\frac{1}{s+s\cdot G(s)H(s)}$$

Or $e_{ss} = \frac{1}{\lim\limits_{s\to 0} s\cdot G(s)H(s)} = \frac{1}{K_v}$

where $\boxed{K_v = \lim\limits_{s\to 0} s\cdot G(s)H(s)}$ is called velocity error coefficients

For Type-0 system ; $G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)}{(1+sT_1)(1+sT_2)}$

Thus, $e_{ss} = \frac{1}{\lim_{s\to 0} s \cdot \frac{K(1+sT_a)(1+sT_b)}{(1+sT_1)(1+sT_2)}} = \frac{1}{0}$

or $\boxed{e_{ss} = \infty}$

This is the steady state error for Type-0 systems subjected to unit ramp input.

For Type 1 systems : $G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)}{s(1+sT_1)(1+sT_2)}$

Thus ; $e_{ss} = \frac{1}{\lim_{s\to 0} s \cdot \frac{K(1+sT_a)(1+sT_b)}{s(1+sT_1)(1+sT_2)}} = \frac{1}{K}$

or $\boxed{e_{ss} = \frac{1}{K}}$

This is the steady state error for Type-1 systems subjected to unit ramp input.

For Type-2 systems :- $G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)}{s^2(1+sT_1)(1+sT_2)}$

Thus, $e_{ss} = \frac{1}{\lim_{s\to 0} s \cdot \frac{K(1+sT_a)(1+sT_b)}{s^2(1+sT_1)(1+sT_2)}}$

or $e_{ss} = \frac{1}{\lim_{s\to 0} \frac{K(1+sT_a)(1+sT_b)}{s(1+sT_1)(1+sT_2)}} = \frac{1}{\frac{K}{0}}$

or $\boxed{e_{ss} = \frac{1}{\infty} = 0}$

This is the steady state error for Type-2 systems subjected to unit ramp input.

3) Static acceleration error coefficients :⇒

We know that, steady state error

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

For unit parabolic input $R(s) = \frac{1}{s^3}$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s^3}}{1+G(s)H(s)} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s) H(s)}$$

$$\text{or } e_{ss} = \frac{1}{\lim_{s \to 0} s^2 \cdot G(s) H(s)} = \frac{1}{K_a}$$

where $\boxed{K_a = \lim_{s \to 0} s^2 G(s) H(s)}$ is called acceleration error coefficients

For Type-0 system ; $G(s) H(s) = \frac{K(1+sT_a)(1+sT_b)}{(1+sT_1)(1+sT_2)}$

Thus, $e_{ss} = \frac{1}{\lim_{s \to 0} s^2 \cdot \frac{K(1+sT_a)(1+sT_b)}{(1+sT_1)(1+sT_2)}} = \frac{1}{0}$

or $\boxed{e_{ss} = \infty}$

This is the steady state error for Type-0 system subjected to unit parabolic input.

For Type-1 system ; $G(s) H(s) = \frac{K(1+sT_a)(1+sT_b)}{s(1+sT_1)(1+sT_2)}$

Thus, $e_{ss} = \frac{1}{\lim_{s \to 0} s^2 \cdot \frac{K(1+sT_a)(1+sT_b)}{s(1+sT_1)(1+sT_2)}} = \frac{1}{0}$

or $\boxed{e_{ss} = \infty}$

This is the steady state error for Type-1 system subjected to unit parabolic input.

For Type-2 systems ; $G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)}{s^2(1+sT_1)(1+sT_2)}$

Thus, $e_{ss} = \lim_{s \to 0} s^2 \cdot \frac{K(1+sT_a)(1+sT_b)}{s^2(1+sT_1)(1+sT_2)} = \frac{1}{K}$

or $\boxed{e_{ss} = \frac{1}{K}}$

This is the steady state error for Type-2 system subjected to unit parabolic input.

Q-6. The open loop transfer function of unity feedback system is given by

$$G(s) = \frac{50}{(1+0.1s)(s+10)}.$$

Determine the static error coefficients K_p, K_v and K_a.

Soln:- i) $K_p = \lim_{s \to 0} G(s)H(s)$

$$= \lim_{s \to 0} \frac{50}{(1+0.1s)(s+10)} = \underline{\underline{5}}$$

ii) $K_v = \lim_{s \to 0} s \cdot G(s)H(s) = \lim_{s \to 0} s \cdot \frac{50}{(1+0.1s)(s+10)} = \underline{\underline{0}}$

iii) $K_a = \lim_{s \to 0} s^2 G(s)H(s) = \lim_{s \to 0} s^2 \cdot \frac{50}{(s+10)(1+0.1s)} = \underline{\underline{0}}$