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B.Tech. Examination, 2017

(Third Semester)

(All Branches)

Paper - II

MATHEMATICS - III

Time Allowed : Three Hours

Maximum Marks : 100

Q. 1. (a) Define analytic function. Show that the function

$f(z) = u + iv$ where :

$$f(z)\begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} & ; \quad z \neq 0 \\ 0 & ; \quad z = 0 \end{cases}$$

satisfy the Cauchy-Reimann equations at $z = 0$. Is the function analytic at $z = 0$.

(b) Prove that an analytic function with constant modulus is constant.

Q. 2. (a) State and prove Taylor's theorem.

(b) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5+4\cos\theta} d\theta$

Q. 3. (a) (i) Prove that $\mu_4 = \mu' - 4\mu_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$.

(ii) $V_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$.

(b) By the method of Least square, find the straight line that best fits the following data :

x :	1	2	3	4	5
y :	14	27	40	55	68

Q. 4. (a) Find the coefficient of correlation between the values of x and y :

Q. 5. (a) Find the rate of convergence of Newton-Raphson's method.

(b) Using Regula-Falsi method to obtain the smallest positive root of $x^4 - 3x - 8 = 0$. The root lies between 1.5 and 2.

Q. 6. (a) Using appropriate method of interpolation find the population for the year 1936.

Year	1901	1911	1921	1931	1941	1951
Population (Thousand)	12	15	20	27	39	52

(b) Solve the following system of linear equation by Gauss-Seidel method :

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

Q. 7. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson $\frac{1}{3}$rd rule. Hence obtain the approximate value of π by taking interval 9 equal parts.

(b) Use Eular's modified method to compute y for x = .05. Given that $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 0$, $y_0 = 1$ correct upto three decimal places.

Q. 8. (a) Using the complex variable techniques evaluate the integral :

$$\int_0^\infty \frac{1}{x^4+16}\,dx$$

(b) Find the first four terms of the Taylor's series expansion of the complex variable function

$$f(z)=\frac{z+1}{(z-3)\,(z-4)} \text{ about } z = 2.$$

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B.Tech. Examination, 2016
(Third Semester)
(All Branches)
Paper - II

MATHEMATICS - III

Time Allowed : Three Hours

Maximum Marks : 100

Q. 1. (a) If $f(z) = u + iv$ is an analytic function of z and

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - 2\cos hy}$$

Prove that :

$$f(z) = \frac{1}{2}\left[1 - \cot\frac{z}{2}\right] \text{ when } f\left(\frac{\pi}{2}\right) = 0$$

(b) (i) Use Cauchy integral formula to evaluate :

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where C is the circle $|z| = 3$

(ii) Evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{(z-1)(z-2)}$ where C is the circle $|z| = 4$.

Q. 2. (a) State and prove Laurent's theorem.

(b) Apply calculus of residues to prove that :

$$\int_0^{\pi} \frac{\cos 2\theta \, d\theta}{1 - 2p\cos\theta + p^2} = \frac{\pi p^2}{1-p^2} \quad (0 < p < 1)$$

Q. 3. (a) Prove that :

$$V_4 = \mu_4 + 4\mu_3 \bar{x} + 6\mu_2 \bar{x}^2 + \bar{x}^4.$$

$y = a + bx$ (b) By the method of least squares, find the straight line that best fits the following data :

x :	1	2	3	4	5
y :	14	27	40	55	68

Q. 4. (a) If $z = ax + by$ and r is the correlation co-efficient between x and y. Show that :

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_x\sigma_y$$

(b) If θ is the acute angle between the two regression lines in the case of two variables x and y, show that :

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2},$$ where r, σ_x, σ_y have their usual meanings. Explain the significance of the formula when $r = 0$ and $r = \pm 1$

Q. 5. (a) Explain any four :

(i) Moment and Kurtosis

(ii) Statistical quality control method

(iii) Analytic and singular point

(iv) Newton Raphson method

(v) Residue and C.R equation in polar form

(b) Find the real root of the equation $xe^x = \cos x$ is the interval (0, 1) using Regula Falsi method correct to four decimal places.

Q. 6. (a) From the table, estimate the number of students who obtained marks between 40 and 45.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

(b) Solve the following system of equations using Gauss-Seidal iterative method :

$2x + 10y + z = 51,\ 10x + y + 2z = 44,\ x + 2y +$

Q. 7. (a) Find the cubic Lagrange's interpolating polynomial from the following data :

x	0	1	2	5
f(x)	2	3	12	147

60

(b) Evaluate $\int_0^1 \frac{dx}{1+x}$ using :

(i) Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$

(ii) Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$

Hence compute an approximate value of π in each case.

Q. 8. (a) Given the initial value problem :

$y' = 1 + y^2$, $y(0) = 0$

Find y(0.6) by Runge-Kutta fourth order method

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Hence compute an approximate value of π in each case.

Q. 8. (a) Given the initial value problem :

$y' = 1 + y^2, \ y(0) = 0$

Find y(0.6) by Runge-Kutta fourth order method taking h = 0.2.

(b) Write difference between :

(i) Binomial and Poisson distribution

(ii) Taylor's and Laurent theorem

(iii) Time series and forecasting

(iv) Euler's and Ricard's method

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B. Tech. Examination, 2015

(Third Semester)

(All Branches)

Paper-II

MATHEMATICS-III

Time Allowed : Three Hours

Maximum Marks : 100

Q. 1. (a) Show that the function defined by $f(z) = \sqrt{|xy|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfied there. C–R

(b) Evaluate $\int_C \frac{z^2+1}{z^2-1} dz$ where C is circle,

(i) $|z| = 3/2$ (ii) $|z-1| = 4$.

 P.T.O.

(b) Evalue : **10**

$$\int_c \frac{z}{z^2+1}\,dz \text{ where}$$

(i) C is $\left|z+\frac{1}{z}\right| = 2$

(ii) C is $|z+i| = 1$

Q. 2. (a) State and proof Laurent theorem. **10**

(b) Using the complex variable techniques, evaluate the real integral. **10**

$$\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta}\,d\theta$$

Q. 3. (a) Find Karl Pearson's coefficient of skewness for the following data. **10**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of Student	10	12	18	25	16	14	8

$x' = x - \bar{x}$
$y' = y - \bar{y}$

(3)

$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}$

x	1	3	5	7	8	10
y	8	12	15	17	18	20

$X' = x - \bar{x}$
$Y' = y - \bar{y}$

(b) The Regression lines of y on x and x on y are respectively $y = ax + b$, $x = cy + d$,

show that $\sqrt{\frac{\sigma_y}{\sigma_x}} = \sqrt{a/c}$

$\bar{x} = \frac{bc+d}{1-ac}, \bar{y} = \frac{ad+b}{1-ac}$

Q. 5. (a) Explain any four :

(i) Skewness and control charts.

(ii) Analysis of variance (one way)

(iii) Analytic and singular point

P.T.O.

(iv) Picards method.

(v) Residue and C-R Equation.

(b) Find a real root of the Equation $x = e^{-x}$ using the Newton-Raphson method.

Q. 6. (a) The population of a town was as given. Estimate the population for the year 1925.

Years (x) :	1891	1901	1911	1921	1931
Population (y) :	46	66	81	93	101

(b) Solve the system by Crout's method :

$$x + y + z = 3$$

$$2x - y + 3z = 16$$

$$3x + y - z = -3$$

Q. 7. (a) Using the divided difference formula find $f'(3)$ from the following table :

x	1	2	4	8	10
f(x)	0	1	5	21	27

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using :

(i) Simpson's $\frac{1}{3}$ rule.

(ii) Simpson's $\frac{3}{8}$ rule.

(iii) Trapezoidal rule.

Q. 8. (a) Find the value of $y(1.1)$ using Runge-Kutta method of fourth order, given that $\frac{dy}{dx} = y^2 + xy$. $y(1) = 1.0$. Take $h = 0.05$.

(b) Write difference between :

(i) Poisson and Normal distribution

(ii) Taylor's and Laurent theorem.

(iii) Correlation and Regression.

(iv) Chi-square test and t-test.