5007

B.Tech. Examination, 2017

(First Semester)

(All Branches)

MATHEMATICS - I

Paper - IV

Time Allowed : Three Hours

Maximum Marks : 100

Note : Attempt any five questions.

Q. 1. (a) Evaluate $D^n (\tan^{-1} x)$ at $x = 0$ when n is of the form 2P, 4P + 1, 4P + 3 where P is any integer and D stands for $\frac{d}{dx}$.

(b) Trace :

(i) $r = a \cos 3\theta$

(ii) $y^2 = x^2 - x^4$

Q. 2. (a) Find maximum and minimum value of $\sin x \sin y \sin (x + y)$.

(b) Find the dimension of rectangular box of maximum capacity whose surface area is given when :

(i) box is open at top

(ii) box is closed.

Q. 3. (a) If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r \sin \theta \cos \phi$, $v = r \sin \theta \sin \phi$, $w = r \cos \theta$, calculate $J\left(\frac{x, \; y, \; z}{r, \; \theta, \; \phi}\right)$.

(b) Find the Taylor's series expansion of $f(x, y) = x^3 + xy^2$ about $(2, 1)$.

Q. 4. (a) Find the rank of matrix.

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

(b) Find A^{-1} and verify Cayley Hamilton theorem,

where :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q. 5 (a) Prove a square matrix A of order n has n linearly independent eigen vector, then a matrix P can be found such that $P^{-1}AP$ is a diagonal matrix.

(b) Express the matrix :

$$A = \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$$

as a sum of Hermitian and skew Hermitian matrix.

Q. 6. (a) Evaluate :

$$\iint (x^2 + y^2)\, dx\, dy$$

through the area enclosed by the curve $y = 4x$, $x + y = 3$, $y = 0$ & $y = 2$.

(b) Evaluate $\iint_R (x+y)^2\, dx\, dy$ where R with verties (1, 0), (3, 1), (2, 2), (0, 1) using the

transformation $u = x + y$ & $v = x - 2y$.

Q. 7. (a) Show that (any two) :

(i) $$\int_0^a \frac{dx}{\sqrt[n]{a^n - x^n}} = \frac{\pi}{n}\operatorname{cosec}\left(\frac{\pi}{n}\right)$$

$n > 0$

(ii) $$\int_0^{\pi/2} \tan^P\theta \, d\theta = \frac{\pi}{4}\sec\frac{P\pi}{2}$$

(iii) $$\int_0^1 \frac{dx}{(1 - x^n)^{1/n}} = \frac{\pi}{n\sin\frac{\pi}{n}}$$

(b) Find the mass of an octant of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ the density at any point being $\rho = kxyz$.

Q. 8. (a) Verify Green theorem in the plane for

$$\oint \left(3x^2 - 8y^2\right)dx + (4y - 6xy)\,dy$$

where C is boundary of region bounded by $x \geq 0, y \leq 0, 2x - 3y = 6$.

(6)

(b) Find the scalar potential function f for :

$$\vec{A} = y^2\hat{i} + 2xy\hat{j} = z^2\hat{k}$$

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B.Tech. Examination, 2015

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MATHEMATICS - I

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Q. 1. (a) If $Y = \sin(m \sin^{-1} x)$ Prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ and hence find y_n at $x = 0$.

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$. Prove that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{x + y + z}$$

Q. 2. (a) (i) Trace the curve $x^3 + y^3 = 3axy$.

(ii) $a^2y^2 = x^3 (2a - x)$.

(b) ObtainTaylor's expansion of $\tan^{-1} (y/x)$ about (1, 1) upto and including the II-degreeterm. Hence compute $f(1.1, 0.9)$.

Q. 3. (a) Prove that $JJ' = 1$.

(b) Prove that the function :

$u = 3x + 2y - z$, $v = x - 2y + z$,

$w = x (x + 2y - z)$ are not independent and find a relation between them.

Q. 4. (a) Discuss the extreme values (Maxima & Minima) of the function $x^3 + y^3 - 3axy$.

(b) Find the maxima and minima of $u = x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$. Interpret the result geometrically.

Q. 5. (a) Reduce the Matrix A to its normal form when :

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

and hence find the Rank.

(b) Find that for what values of λ, μ the equations

$x + y + z = 6$, $x + 2y + 3z = 10$,

$x + 2y + \lambda z = \mu$.

(i) No solution

(ii) Unique solution

(iii) Infinite solution

Q. 6. (a) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ hence

Compute A^{-1}.

Q. 7. (a) Change the order of integration in the double integral $\int_0^{2a} \int_{\sqrt{2x-x^2}}^{\sqrt{2ax}} V\, dx\, dy$

where V is a function of x and y.

(b) To prove that $B(m, n) = \frac{\lceil m \lceil n}{\lceil (m+n)}$, $(m, n > 0)$.

Q. 8. (a) (i) If $V = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$, Find div V and curl V.

(ii) div $(\phi A) = \phi$ div V + (grad ϕ). A.

(iii) Find the constant a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = h$ at the point $(1, -1, 2)$.

(b) State the stoke theorem. Verify this theorem for $\vec{F}(x, y, z) = xz\hat{i} - y\hat{j} + x^2y\hat{k}$ where the surface S in the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $2x + y + 2z = 8$ which is not included on xy – plane.