5257

B.Tech. Examination, 2013

(First Semester)

(All Branches)

MATHEMATICS - I

Paper - IV

Time : Three Hours] **[Maximum Marks : 100**

Note :– Attempt **any five.**

1. (a) If $y^{1/m} + y^{-1/m} = 2x$ prove that

$$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$$

(b) If z be a homogenous function of degree n show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x dy} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

2. (a) Trace the curve

(i) $y = x + \frac{1}{x}$

(ii) $y^2 = x^2 - x^4$

[P. T. O.

(b) Expand x^y in power of $(x-1)$ and $(y-1)$ upto the third degree terms.

3. (a) If $u = x + 2y + z$,

$$v = x - 2y + 3z$$

$$w = 2xy - xz + 4yz - 2z^2$$

Show that they are functionally related and find the relation between them.

(b) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4m and is surmounted by hemispherical ends. If the radius is increased by 0.01m and the length by 0.05 m find the percentage change in volume of the balloon.

4. (a) Prove that if the perimeter of a triangle is constant, its area is maximum when the triangle is equilateral.

(b) Find the dimension of rectangular box of maximum capacity whose surface area is given when :

(i) box is open at top.

(ii) box is closed.

5. (a) Factorise the matrix

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

into the form LU, where L is Lower triangular and U is triangular matrix.

(b) Define Rank of a matrix. Find the rank of matrix.

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$$

6. (a) For what value of K the equation :

$$x+y+z = 1$$
$$2x+y+4z = K$$
$$4x+y+10z = K^2$$

has a solution.

(b) Find the value of K such that the following equation have unique solution :

$$\lambda x + 2y - 2z - 1 = 0$$

$$4x + 2\lambda y - z - 2 = 0$$

$$6x + 6y + \lambda z - 3 = 0$$

7. (a) State and prove Cayley - Hamilton Theorem.

(b) Change the order of integration in the double integral

$$\int_{0}^{2a} \int_{\sqrt{2ax - x^2}}^{\sqrt{2ax}} v\,d \times dy$$

8. (a) Prove that

$$\Gamma m \, \Gamma m + 1/2 = \frac{\sqrt{\pi}\, \Gamma 2m}{2^{2m-1}}$$

(b) State :

(i) Gauss divergence Theorem

(ii) Stoke Theorem

(iii) Green Theorem

●