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B.Tech. Examination, 2017

(Third Semester)

(C.S. Branch)

Paper - VI

(Discrete Mathematical Structures)

Time Allowed : Three Hours

Maximum Marks : 100

Note : Attempt any five questions.

Q. 1. (a) (i) Prove that for any two sets A and B :

$$(A \cap B)' = A' \cup B'$$

(ii) Prove that a countable union of sets is countable.

(b) Prove that the function $f : R \to R$ defined as $f(x) = 2x + 3, \ \forall \ x \in R$ is both one to one and onto.

Q. 2. (a) Prove by using induction that the sum of cubes of 3 consecutive integers is divisible by 9.

(b) (i) Prove that for any integer n "if 3n + 2 is even then n is even" by method of contraposition.

(ii) Show without using a truth table that :

$$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$$

Q. 3. (a) (i) Show that if a, b are arbitrary elements of a group G, then $(ab)^2 = a^2b^2$ if G is abelian.

(ii) Find the product of two permutations and show that it is not commutative :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

(b) State and prove Lagrange's theorem.

Q. 4. (a) (i) Use K-map to find minimal sum of Boolean expression :

$$f(x, y, z) = \Sigma(1, 2, 4, 5, 6, 11, 12, 13, 14, 15)$$

(ii) Show that the relation of "parallel to" is not partial order relation on the set of lines.

(b) (i) Consider the lattice D_{30}

(1) Draw the Hasse diagram of D_{30}.

(2) Is D_{30} complemented ?

(3) Is D_{30} distributive ?

(ii) Find the number of sub-algebra of the Boolean algebra D_{110}.

Q. 5. (a) (i) Show that if 20 people are selected, then one may choose a subset of 3 so that all were born on same day of the week.

(ii) Determine whether $\{p \wedge (p \to q)\} \to q$ is tautology, contradiction or contingency.

(b) (i) Prove that $\sqrt{5}$ is an irrational number.

(ii) Find the closed form of generating function $a_n = 2^n$.

Q. 6. (a) (i) Using truth table show that :

$$(p \leftrightarrow q) \equiv (p \vee q) \rightarrow (p \wedge q)$$

(ii) Check the validity of following argument :

"If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore there was no ball game".

(b) Prove that $x^2 + 5x + 11$ is $0(x^3)$ but x^3 is not big $0\,(x^2 + 5x + 11)$.

Q. 7. (a) (i) Define bipartite graph and planar graph with examples.

(ii) Using generating function, solve the recurrence relation :

$$a_{n+2} - 2a_{n+1} + a_n = 2_n, n \geq 0, a_0 = 2, a_1 = 1$$

(b) Write short notes on any two :

(i) Hamiltonian and Eulerian graph with examples.

(ii) Reflexive, symmetric and transitive relations.

(iii) Union, intersection and sum of two graphs.

(iv) Polya's counting theorem.

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B.Tech. Examination, 2014

(Third Semester)

(C.S. Branch)

Paper - VI

(Discrete Mathematical Structures)

Time Allowed : Three Hours

Maximum Marks : 100

Note : Attempt any five from the following :

Q. 1. (a) Show that the function $f(x) = k$, where k is a constant, is primitive recursive. Also state and prove pigeon hole principle. **10**

(b) Show that a relation R, defined on the set of real numbers as $(a, b)\ R\ (c, d)$ iff $a^2 + b^2 = c^2 + d^2$. Show that R is an equivalence relation. **10**

Q. 2. (a) Make a truth table for the following : **10**

(i) $(p \vee q) \wedge r$

(ii) $(p \vee \sim q) \Rightarrow r$

(iii) $(p \downarrow q) \wedge (p \downarrow r)$

(b) Is $((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q$ a tautology? **10**

Q. 3. (a) Solve $a_n - 3a_{n-1} = 2$, $n \geq 2$ with $a_0 = 1$ **10**

P.T.O.

(b) Determine the generating function of the following numeric function : **10**

(i) $a_n = 2^n$, if n is even
$= -2^n$, if n is odd.

Q. 4. (a) Differentiate between semigroup and subgroup with example. **10**

(b) What do you mean by group isomorphism. Explain with the help of example. **10**

Q. 5. (a) Define Eulerian Graph and prove that a non-empty connected graph G is Eulerian iff its vertices are all of even degree. **10**

(b) Prove that the chromatic number of a tree is always 2 and chromatic polynomial is $\lambda(\lambda - 1)^{n-1}$. **10**

Q. 6. (a) Use Karnaugh map to simplify the following expression : **10**

(i) $X = A'B'CD + A'B'CD' + AB'C'D' + AB'CD'$

(ii) $X = A'B'C'D' + AB'C'D' + A'B'CD' + AB'CD'$

(b) Prove Demorgan's Law. **10**

Q. 7. Write short note on any two : **20**

(a) Hamiltonian Graph.

(b) Walk, path.

(c) Venn Diagram.

(d) Reflexive, Symmetric and Transitive Relation.

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