Assignment 1

The folder `Course Materials/Assignments/Assignment 1/` on webCafé contains data on the prices of listed Google options and on the LIBOR (continuously compounded) rates on June 22, 2007. Google has a well-established policy of not paying dividends on its stock.

1. Plot the Google volatility surface on June 22, 2007.
   Note: Some of the steps we followed in constructing the volatility surface for SPX options were based on the fact that listed SPX options are European. Listed stock options are American. You should adjust the procedure as appropriate in view of this difference. Indicate what adjustments you made (if any).

2. Calibrate the Black, CEV and SABR models to the Google option data. Report the calibrated parameter values and the RMSE for each calibration and produce graphs showing how the model IV's for each of the three models compare to the market IV's.

3. Use each of the models you have calibrated in part 2 to value the Google-Linked Cliquet Notes described in the term sheet in Figure 1. Since the Notes have a guaranteed minimum Redemption Amount of $1,100 per Note at the Redemption Date, focus on determining the amount by which the value of the Notes exceeds the value of a 5-year ZCB with face value equal to $1,100. Report the estimated value of this amount, as well as the number of simulated paths, the size of the time step and the standard error of your estimate (try to keep the standard error below $0.50). Explain how you applied each model to price the Notes (e.g., how you dealt with the non-flat term structure of interest rates).

4. Consider the Heston model with the following parameters: $\sqrt{v_0} = 0.2577$, $\kappa = 0.2656$, $\bar{v} = .1851$, $\sigma_v = 0.2992$, $\rho = -0.4042$. Compute the RMSE for this calibration of the Heston model and produce a graph showing how the model IV's compare to the market IV's.

5. Use the calibration in part 4 to value the Google-linked Cliquet Notes. Report the estimated amount by which the value of the Notes exceeds the value of a 5-year ZCB with face value equal to $1,100, as well as the number of simulated paths, the size of the time step and the standard error of your estimate (try to keep the standard error below $0.50). Explain how you applied the Heston model to price the Notes.

<div style="border: 1px solid black; padding: 1em;">

Five Year Minimum Coupon Cliquet Notes
Linked to Google Common Stock

Security Type	Structured Notes
Description	The Notes are redeemable by cash payment of the Notional Amount plus the annual Cliquets on the Reference Index, subject to a Global Floor
Reference Index	The official close price of Google common stock
Issuer	XXX Bank
Issue Size	10,000 Notes
Notional Amount	USD 1,000 per Note
Issue Price	TBD
Issue Date	June 22, 2007
Maturity Date	June 22, 2012
Redemption Date	Maturity Date
Interest	None
Redemption	The Notes will be redeemed by cash payment of the Redemption Amount on the Redemption Date
Redemption Amount	$\text{Notional} \times \left(1 + \max \left[\text{Global Floor}, \sum_{i=1}^{5} \text{Cliquet}_i \right] \right)$
Cliquets	$\text{Cliquet}_i = \max \left[0, \min \left[\text{Cliquet Cap}, \dfrac{S_i - S_{i-1}}{S_{i-1}} \right] \right]$ where $S_i = \text{Reference Index on Issue Date} + i$ years
Global Floor	10%
Cliquet Cap	10%

</div>

Figure 1: Term Sheet for Google-Linked Cliquet Notes