Systems and Control Engineering

SC649: Embedded Controls and Robotics

Assignment 3

Team
Pranav Gupta (22B2179)
Rohan Mekala (22B2106)
Sahil Sudhakar (210010055)

Instructor
Prof. Leena Vachhani

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Contents

Work Distribution

1. **Pranav Gupta** (`22B2179`): understanding frenet frame and python implementation
2. **Rohan Mekala** (`22B2106`): understanding the model and control law
3. **Sahil Sudhakar** (`210010055`): automation of running test cases and data visualisation

1 Choice of Amplitude for Reference Trajectory

1.1 Observations

For different amplitudes:

- $A = 5$:

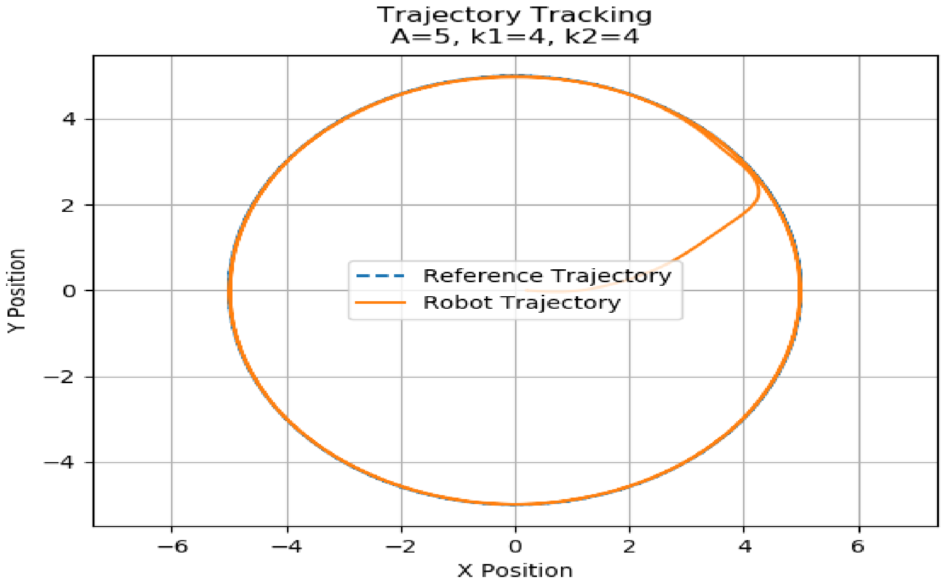


Figure 1: Trajectory Tracking for $A = 5$ (Click to play)

- $A = 8$:

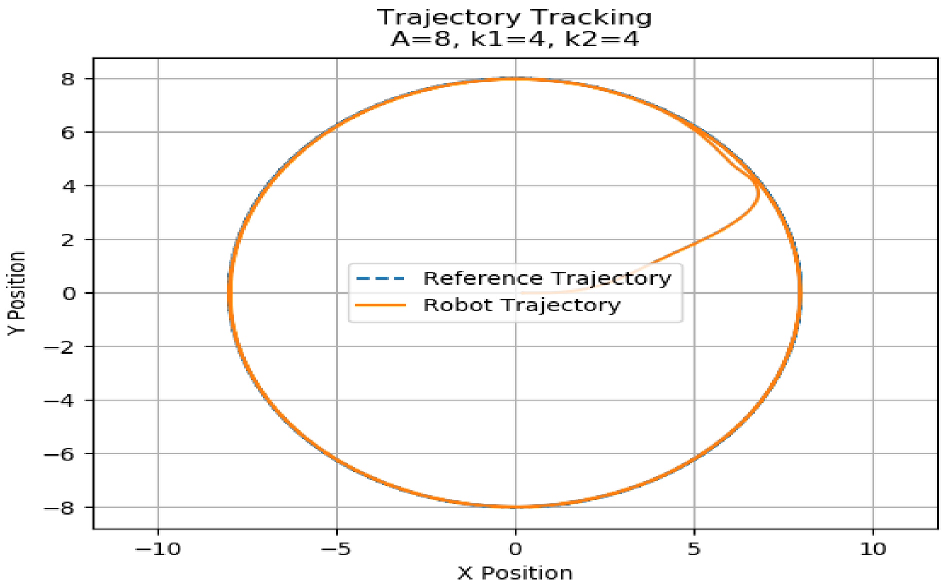


Figure 2: Trajectory Tracking for $A = 8$ (Click to play)

- $A = 12$:

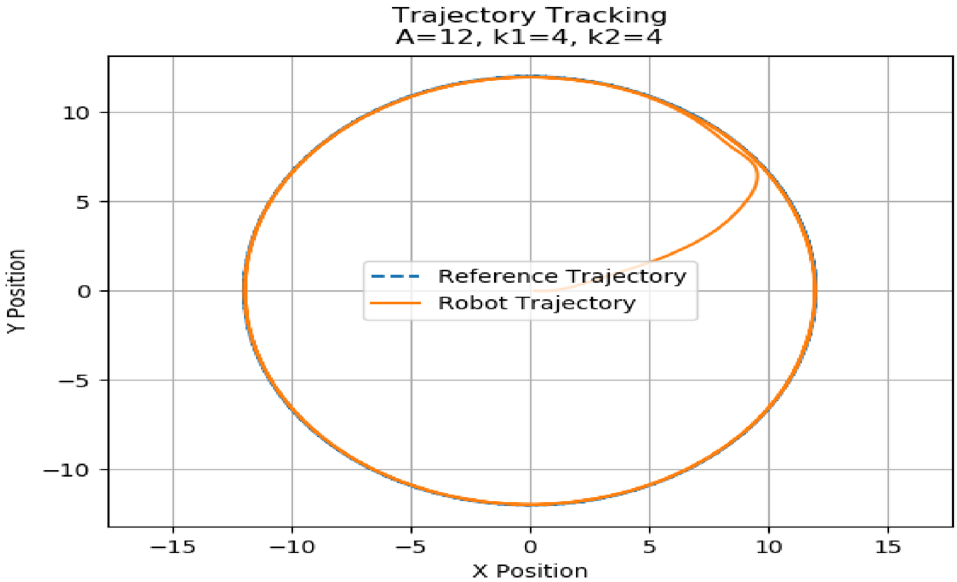


Figure 3: Trajectory Tracking for $A = 12$ (Click to play)

link: drive.google.com/drive/folders/1gWPNkXLT5S5GkosSYY6G4AvSyILF0KUd?usp=sharing to folder containing experimental results (.csv, .png, .gif, .mp4)

1.2 Analysis

The controller is generally successful in tracking the desired trajectory relatively closely, its performance naturally varying with the proportional gains. We ran a tootal of 75 runs covering all combinations of $A \in \{5, 8, 12\}$ and $k_1, k_2 \in \{1, 2, 4, 8, 16\}$. From these, we draw the following insights:

1. Increasing A primarily increases the settling time, with the transient and steady state responses scaling accordingly to maintain their characteristics.

2. Transient response is smoother for cases where $k_1 \sim k_2$ and squigglier otherwise.

Steady State

1. The steady state trajectory of the controller when $k_1 = k_2$ is approximately uniform circular motion.

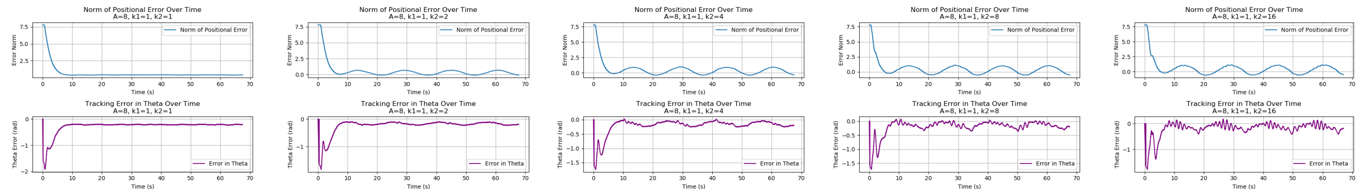
2. Whenever $k_2 > k_1$, the emergent steady state trajectory is distended along the 1st and 3rd quadrants, the eccentricity increasing with k_2/k_1.

3. Similarly, $k_1 > k_2$ results in steady state trajectories distended along the 2nd and 4th quadrants, the eccentricity increasing with k_1/k_2.

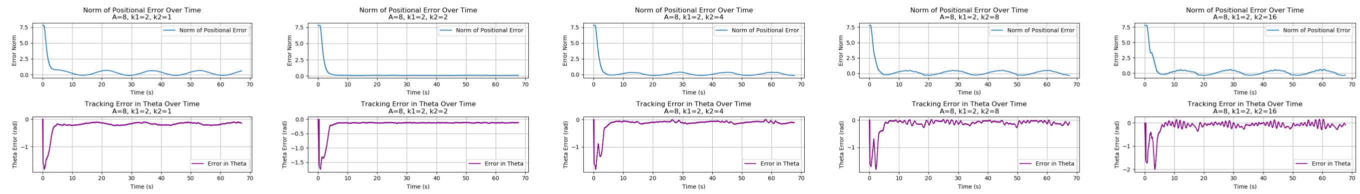
4. In cases of slight imbalance, the steady state trajectory remains within the desired circle. However in cases of high imbalance, the steady state trajectory may even overshoot the reference trajectory.

2 Effect of Control Gains (K_1, K_2) on Tracking Performance

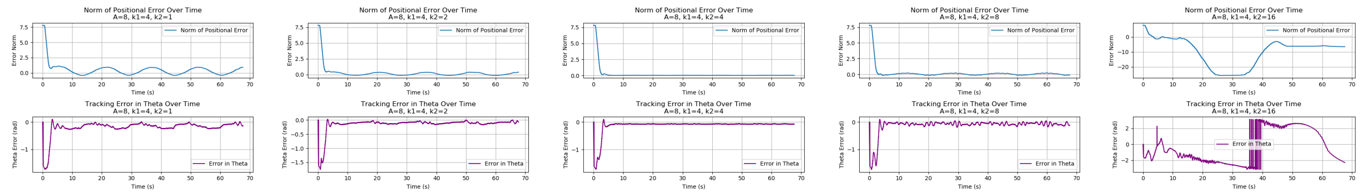
2.1 Observations



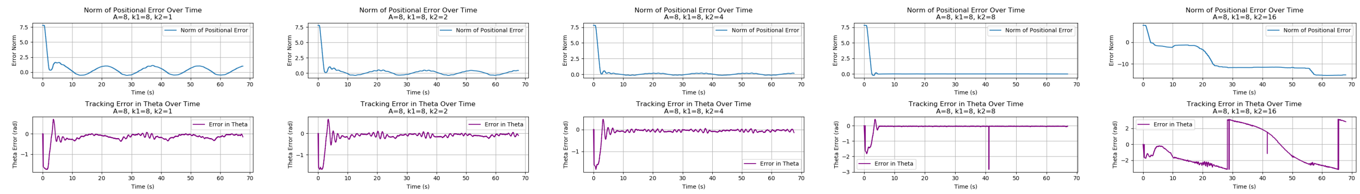
(a) $K_1 = 1, K_2 = 1$ (b) $K_1 = 1, K_2 = 2$ (c) $K_1 = 1, K_2 = 4$ (d) $K_1 = 1, K_2 = 8$ (e) $K_1 = 1, K_2 = 16$



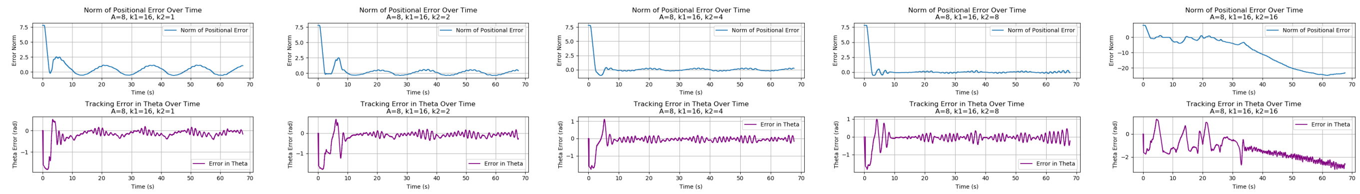
(f) $K_1 = 2, K_2 = 1$ (g) $K_1 = 2, K_2 = 2$ (h) $K_1 = 2, K_2 = 4$ (i) $K_1 = 2, K_2 = 8$ (j) $K_1 = 2, K_2 = 16$



(k) $K_1 = 4, K_2 = 1$ (l) $K_1 = 4, K_2 = 2$ (m) $K_1 = 4, K_2 = 4$ (n) $K_1 = 4, K_2 = 8$ (o) $K_1 = 4, K_2 = 16$



(p) $K_1 = 8, K_2 = 1$ (q) $K_1 = 8, K_2 = 2$ (r) $K_1 = 8, K_2 = 4$ (s) $K_1 = 8, K_2 = 8$ (t) $K_1 = 8, K_2 = 16$



(u) $K_1 = 16, K_2 = 1$ (v) $K_1 = 16, K_2 = 2$ (w) $K_1 = 16, K_2 = 4$ (x) $K_1 = 16, K_2 = 8$ (y) $K_1 = 16, K_2 = 16$

Figure 4: Trajectory Tracking for Different Combinations of K_1 and K_2 (Amplitude $A = 8$)

2.2 Analysis

As visible in the grid, a significant oscillating steady state error in the positional and angular tracking can be observed as we deviate from a $k_1 : k_2$ ratio of 1. Furthermore, higher the imbalance in the $k_1 : k_2$ ratio from 1, the more the error magnitude.

1. **Low Gains (K_1 and K_2 values of 1 to 4)**

- **Behavior**: Slow response with visible delays in correcting errors.
- **Positional Error**: Gradual reduction with longer settling times, indicating under-damped behavior.
- **Theta Error**: Slow convergence with minimal oscillations.

2. Moderate Gains (K_1 and K_2 values of 4 to 8)

- **Behavior**: Balanced response with quicker corrections.
- **Positional Error**: Faster reduction, minor oscillations indicating good stability.
- **Theta Error**: Effective and smooth correction.

3. High Gains (K_1 and K_2 values of 8 to 16)

- **Behavior**: Aggressive response with over-corrections.
- **Positional Error**: Quick reduction with significant oscillations, indicating over-damped behavior.
- **Theta Error**: Rapid fluctuations with potential instability.

3 Bijectivity of Transformation Between Control Inputs

1. **Control Inputs to Angular and Linear Velocities**:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -l_1 \sin\theta \\ \sin\theta & l_1 \cos\theta \end{pmatrix} \begin{pmatrix} 1 & -l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

The criteria for bijectivity of this transformation is

$$\begin{vmatrix} \cos\theta & -l_1 \sin\theta \\ \sin\theta & l_1 \cos\theta \end{vmatrix} \begin{vmatrix} 1 & -l_2 \\ 0 & 1 \end{vmatrix} \neq 0 \iff l_1 \neq 0 \iff P \text{ doesn't lie on steering wheel axle}$$

which is also stated in the book in **49.3.1** $\rightarrow$ *Car*

2. **Conversion to Steering Angle**: Assuming rear-wheel drive and wheelbase length L of the MIT Racecar, the conversion from angular velocity ω to the steering angle ζ is given by:

$$\zeta = \arctan\left(\frac{u_2 L}{u_1}\right)$$

This relationship is valid only if $u_1 \neq 0$. It is easy to notice that ζ is not unique to a pair of (u_1, u_2) but rather only to their ratio. Hence, this relation is clearly never bijective because it results in the same steering angle for all values of (u_1, u_2) at some fixed ratio to each other.