Assignment 2

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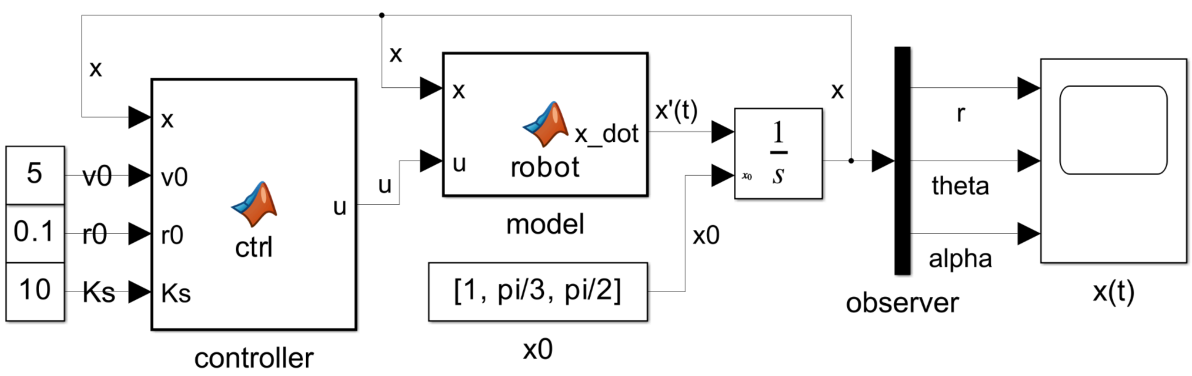
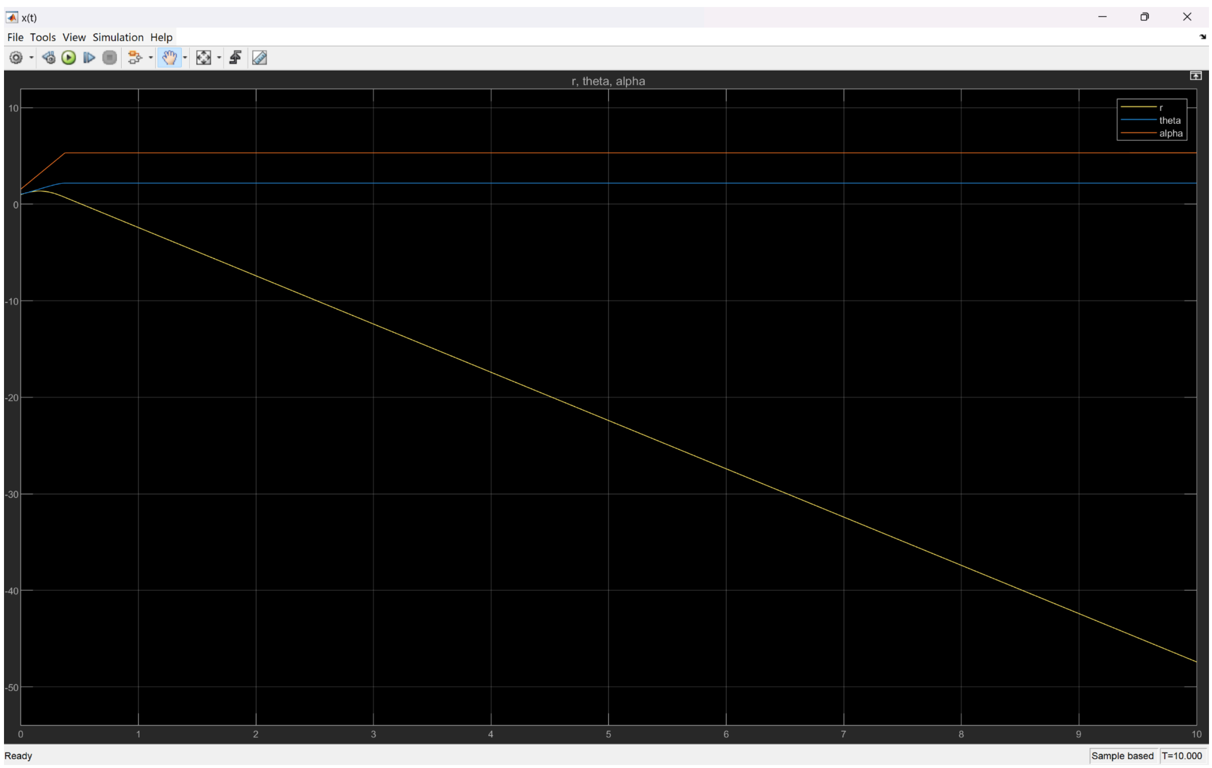
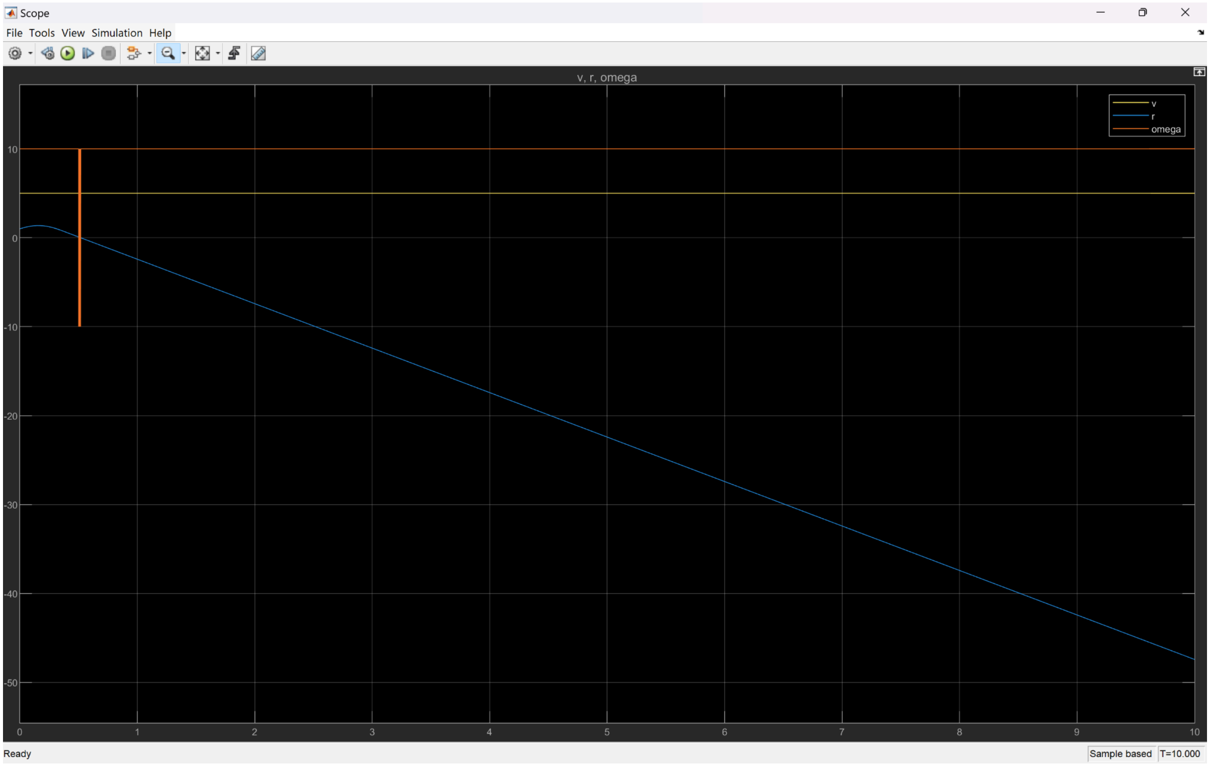
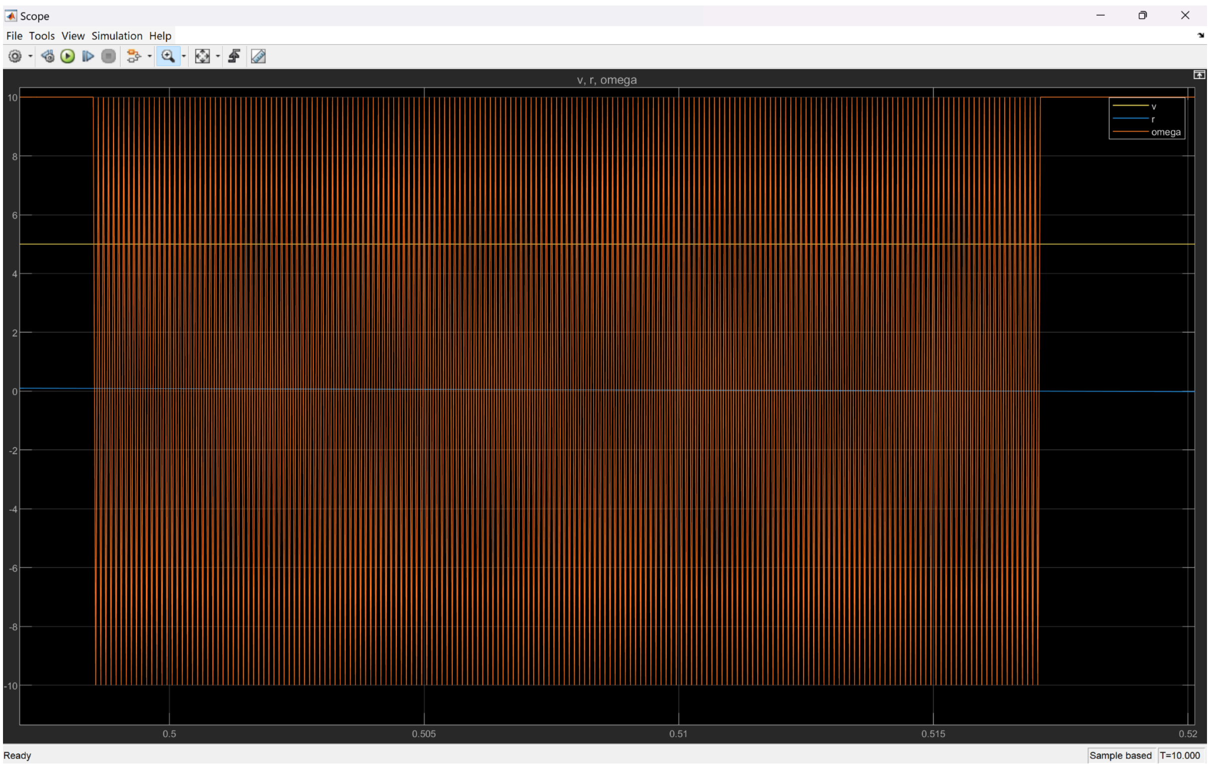


This is the block diagram setup that we have implemented in Simulink.
Note: In the assignment, it is mentioned that v is constant, however, with that assumption, the system response kept diverging for all possible initial conditions and cases. Here is the response plot for one random case:-



Note: All the simulations in this assignment have been run with a fixed step size of 5e-05 using ode4 solver with a stop time of 10 seconds.
We can clearly observe from this sample simulation that R diverges drastically implying the robot will never settle at the home position. This is the inherent issue with keeping v as constant which causes ω to fluctuate with a very high frequency between +Ks and -Ks at alternate simulation timesteps when the robot first comes very close to home position (R~0) which causes the forestated numerical instability.



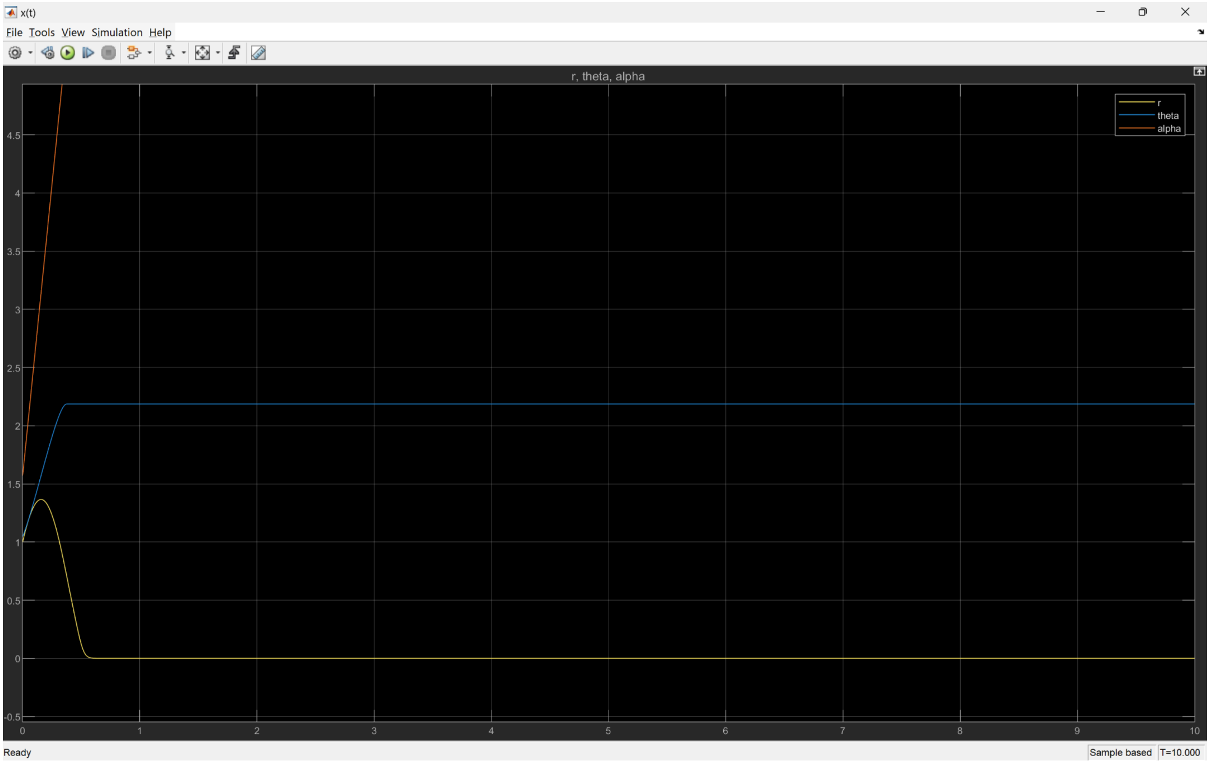


High frequency ω fluctuations at R=0

Owing to this issue, we decided to tweak the control law to control v to vary monotonically with r to ensure that the robot slows down as it approaches the home position and comes to rest at the home position. The primary goal was to pick a decaying function (with r as the independent variable) for v. We decided to proceed with the exponential decay function ($1-e^{-r/r0}$) as that was giving a very short settling time. Here is the final control law that we implemented:-

$v = v_0 * (1-\exp(-r/r_0))$
$\omega = -K_s * \mathrm{sgn}(\alpha - \theta - \pi)$

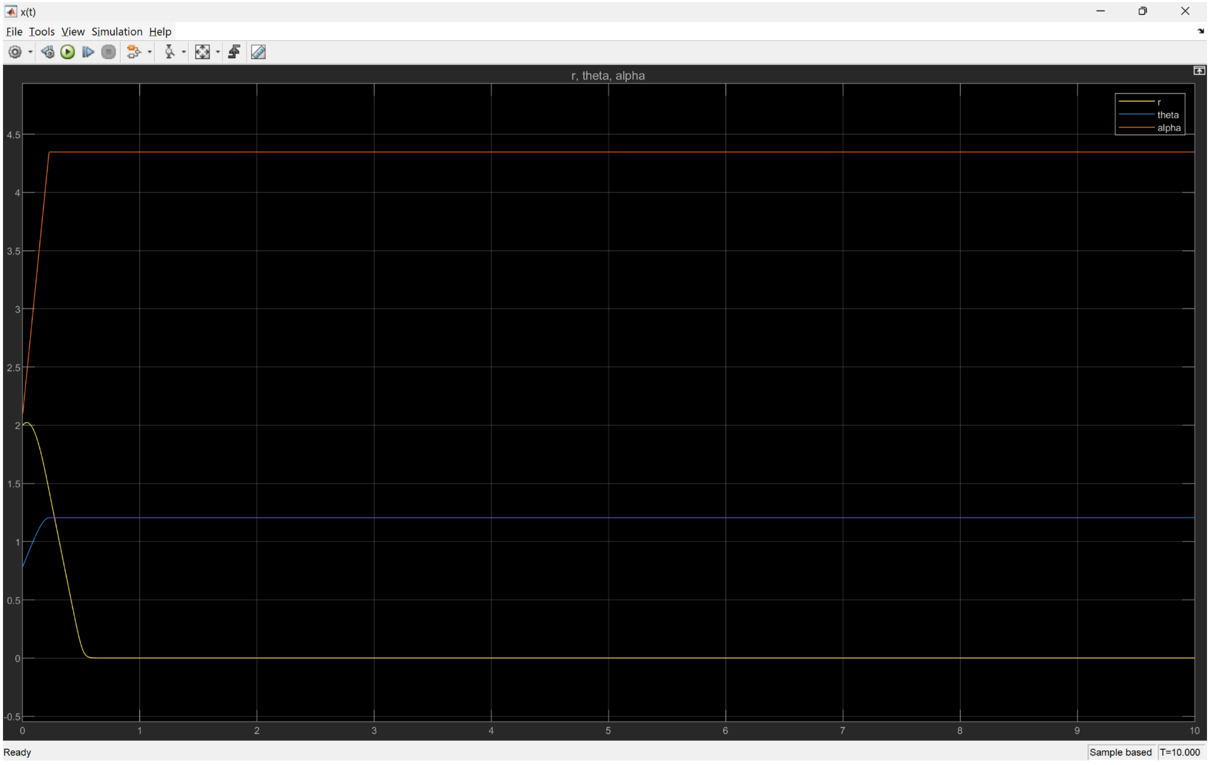
This is how we reached the final block diagram setup as depicted in the first figure.



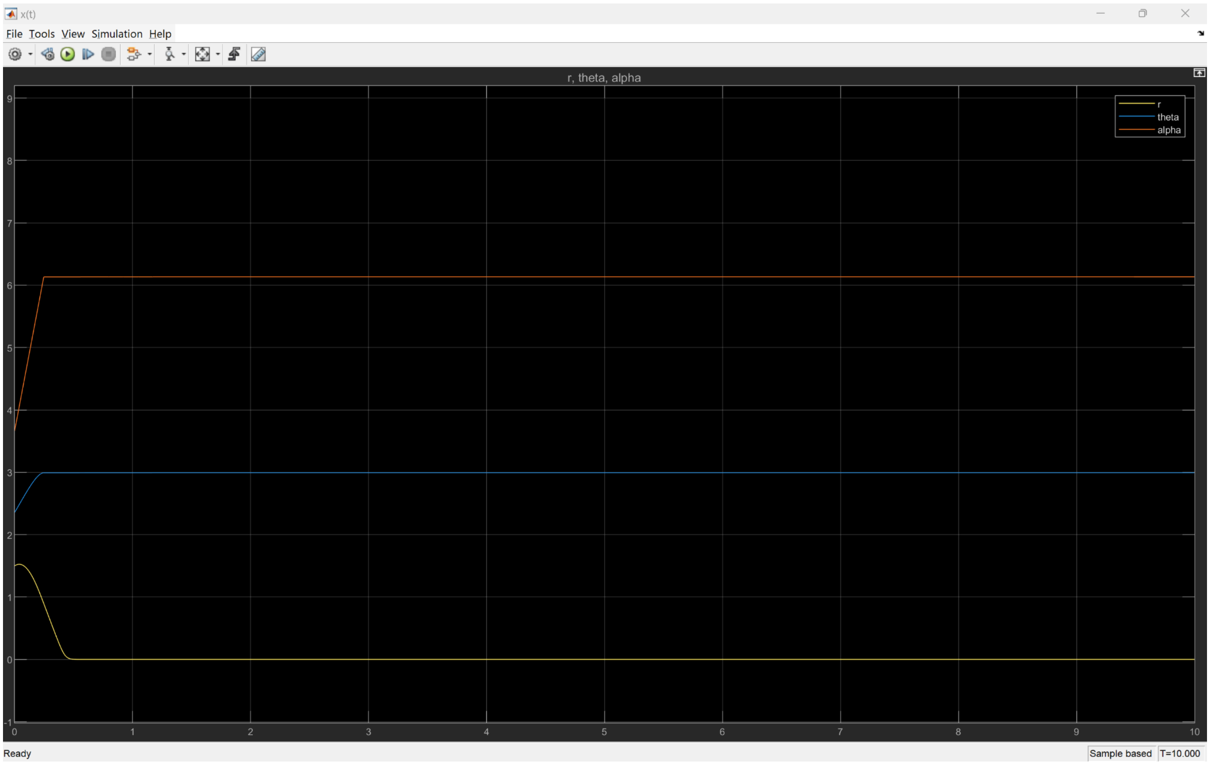
Therefore, with the help of this tweaked control law, we have fixed the issue of divergence and we can observe that the system response converges with a very short settling time and the robot is able to settle at the home position. This response was for initial conditions {R, θ, α} = {1,π/3, π/2}.

Let's check the system response for different initial positions of the robot in each quadrant:-

1st quadrant:-
{R, θ, α} = {2,π/4, 2π/3}

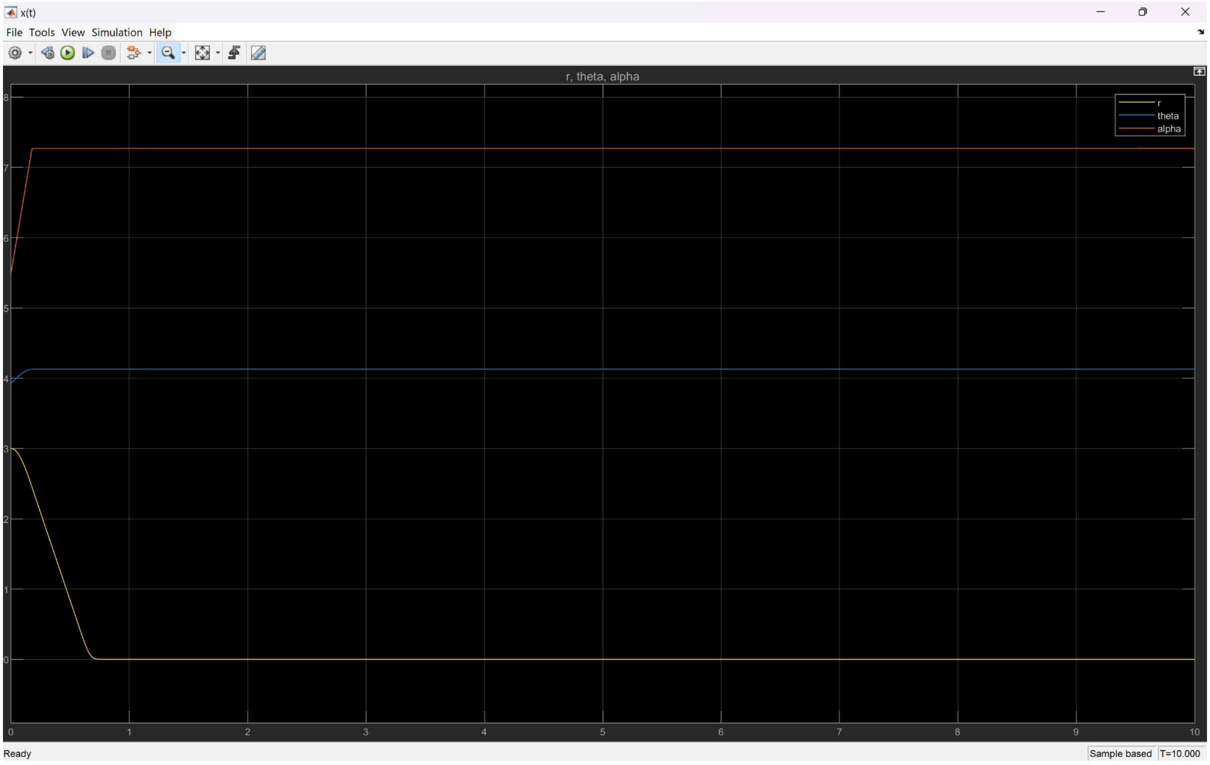


2nd quadrant:-
{R, θ, α} = {1.5,3π/4, 7π/6}



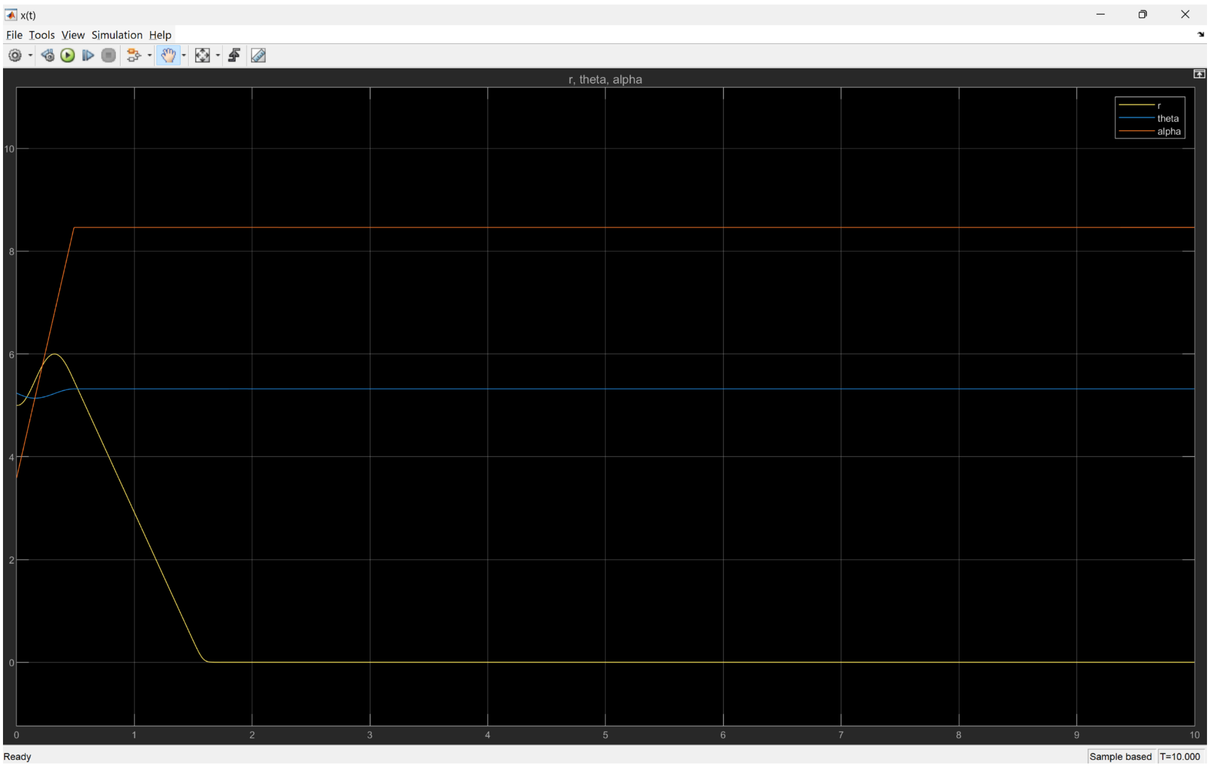
3rd quadrant:-

{R, θ, α} = {3,5π/4, 7π/4}



4th quadrant:-

{R, θ, α} = {5,5π/3, 8π/7}

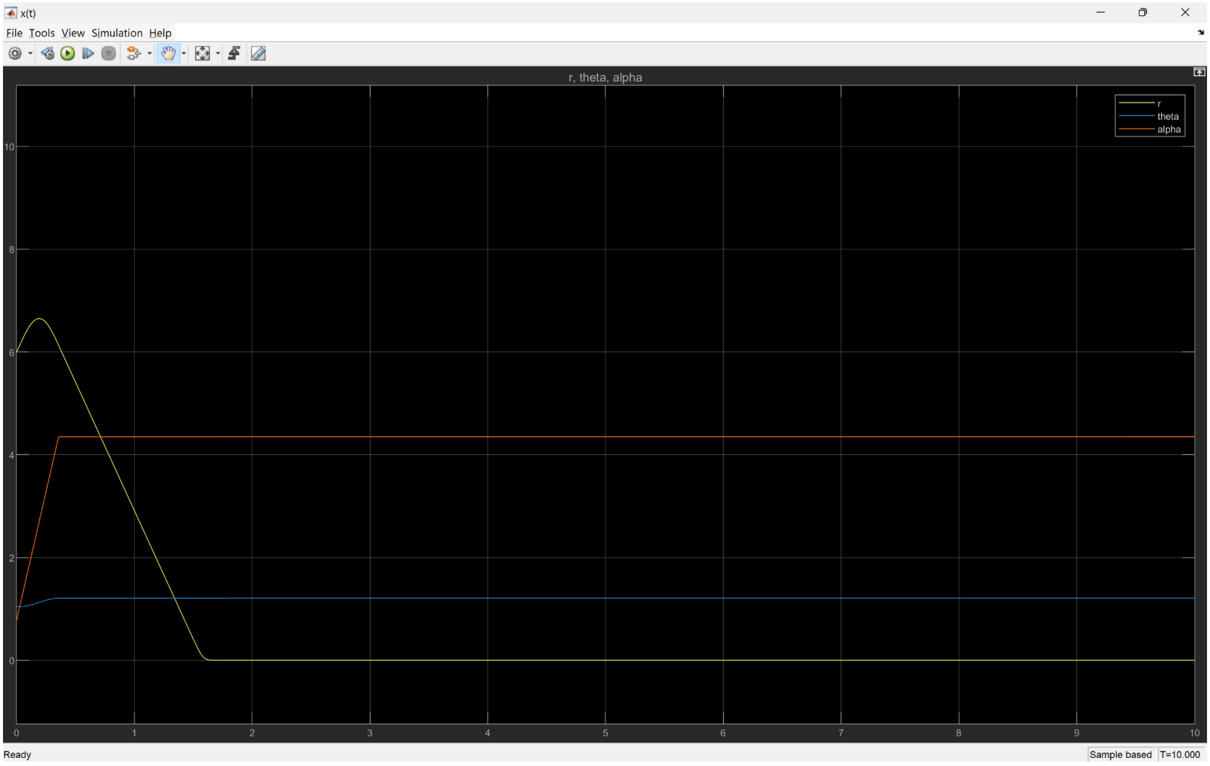


Let’s check the system response for different control gains:-

As per our tweaked control law, we have 3 control gains: v_0, r_0 and K_s. Let's vary each of them and check the system response.

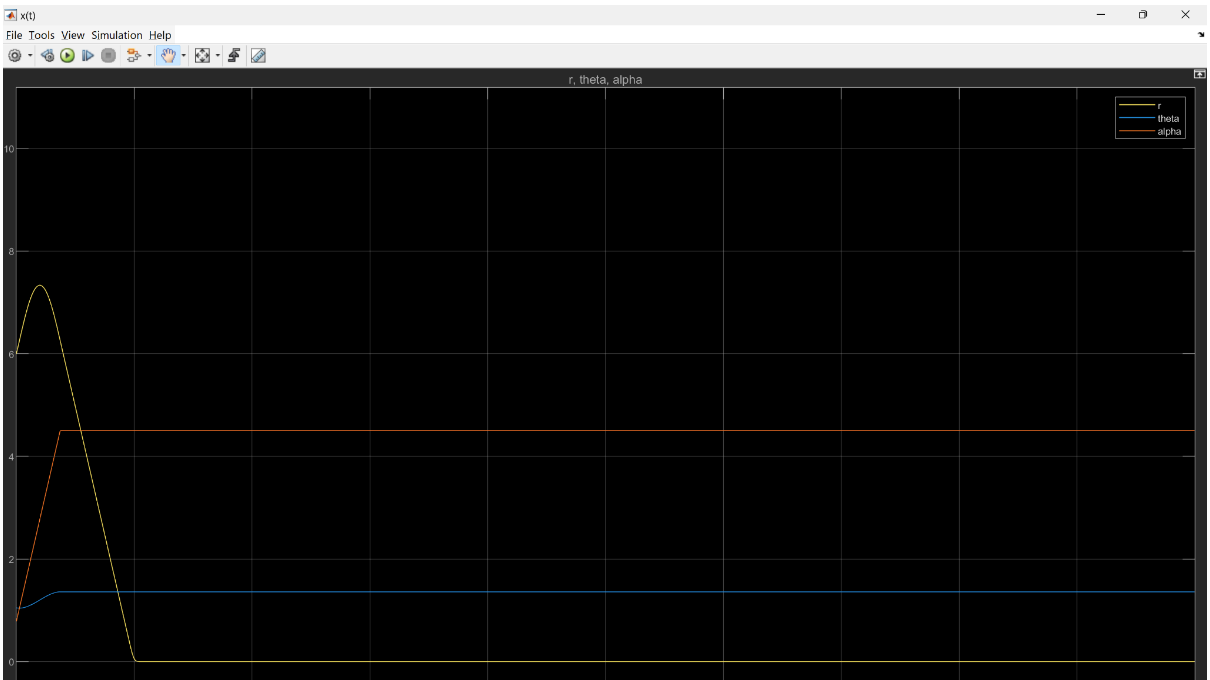
$\{R, \theta, \alpha\} = \{6, \pi/3, \pi/4\}$

a) Varying v_0:-
   $r_0 = 0.1$
   $K_s = 10$

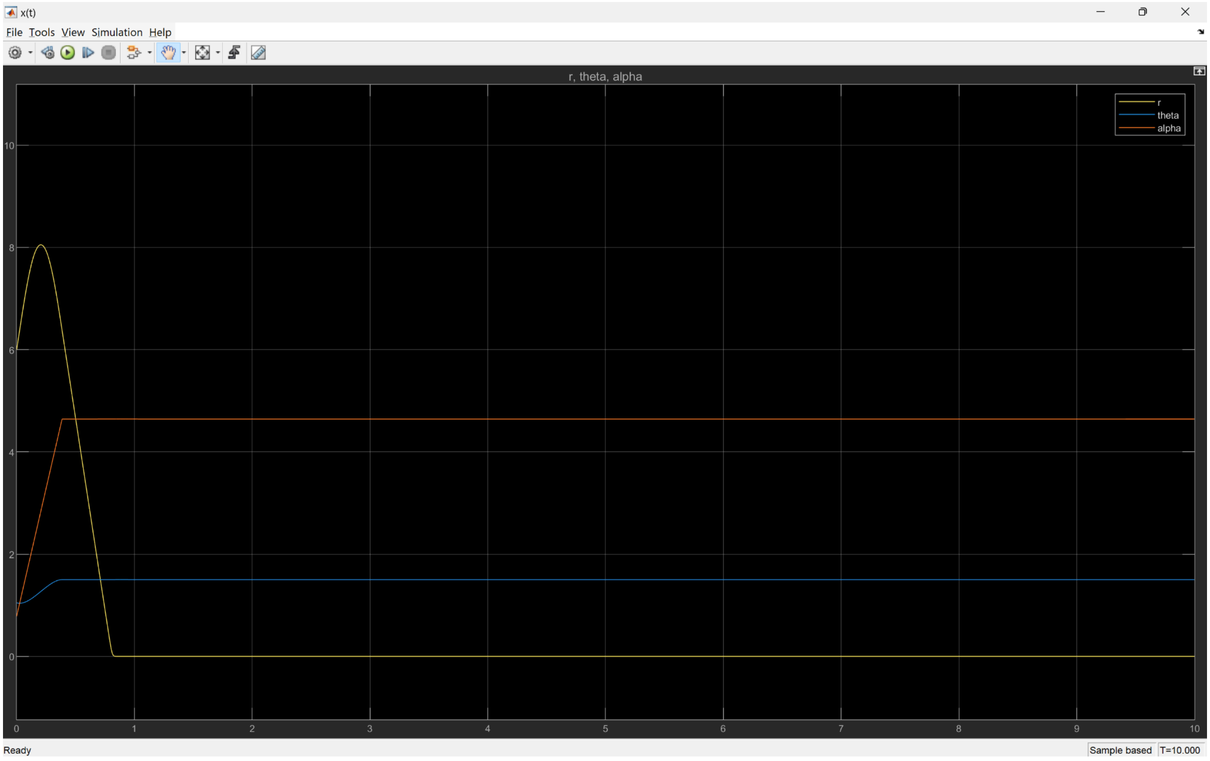


1) $v_0 = 5$

2) $v_0 = 10$



3) $v_0 = 15$



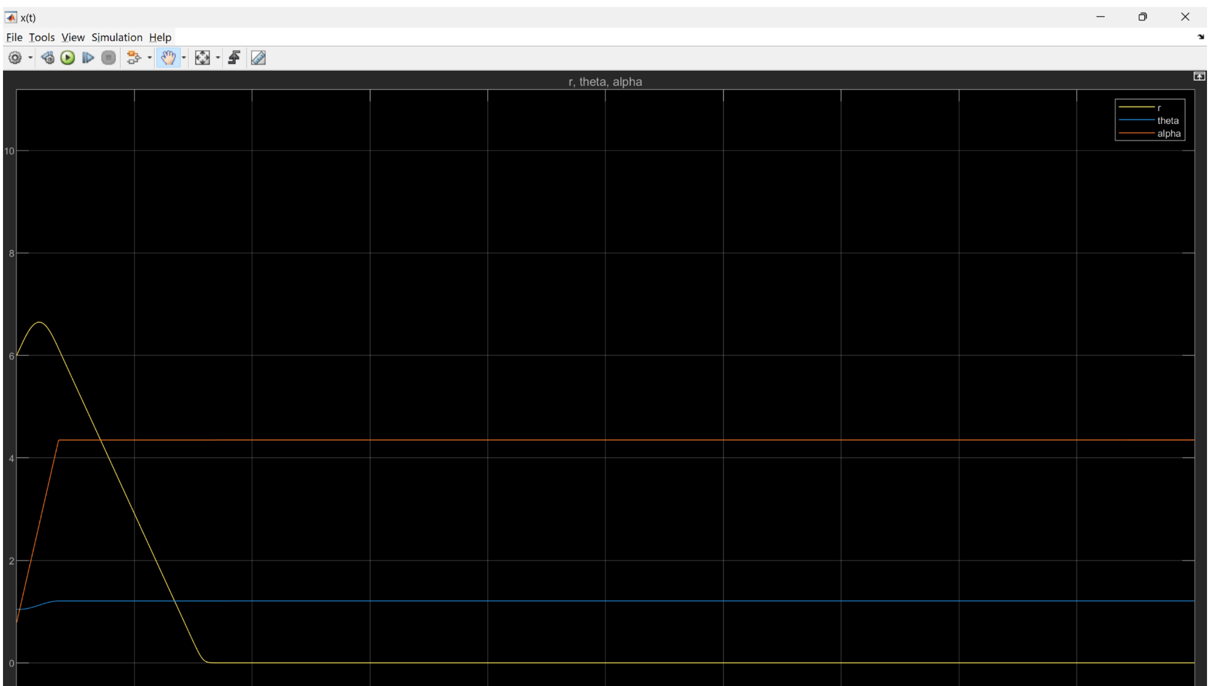
On increasing v_0, we can observe that the setting time decreases but at the cost of increasing the maximum peak overshoot. Moreover, we can observe an increase in the change of θ on increasing v_0. This is compliant with the physical system as increasing v_0 essentially increases the velocity which should decrease the time required to reach the home position while also causing increases in change of its orientation (θ) and inertia (which can be linked to the maximum peak overshoot).

b) Varying r_0:-

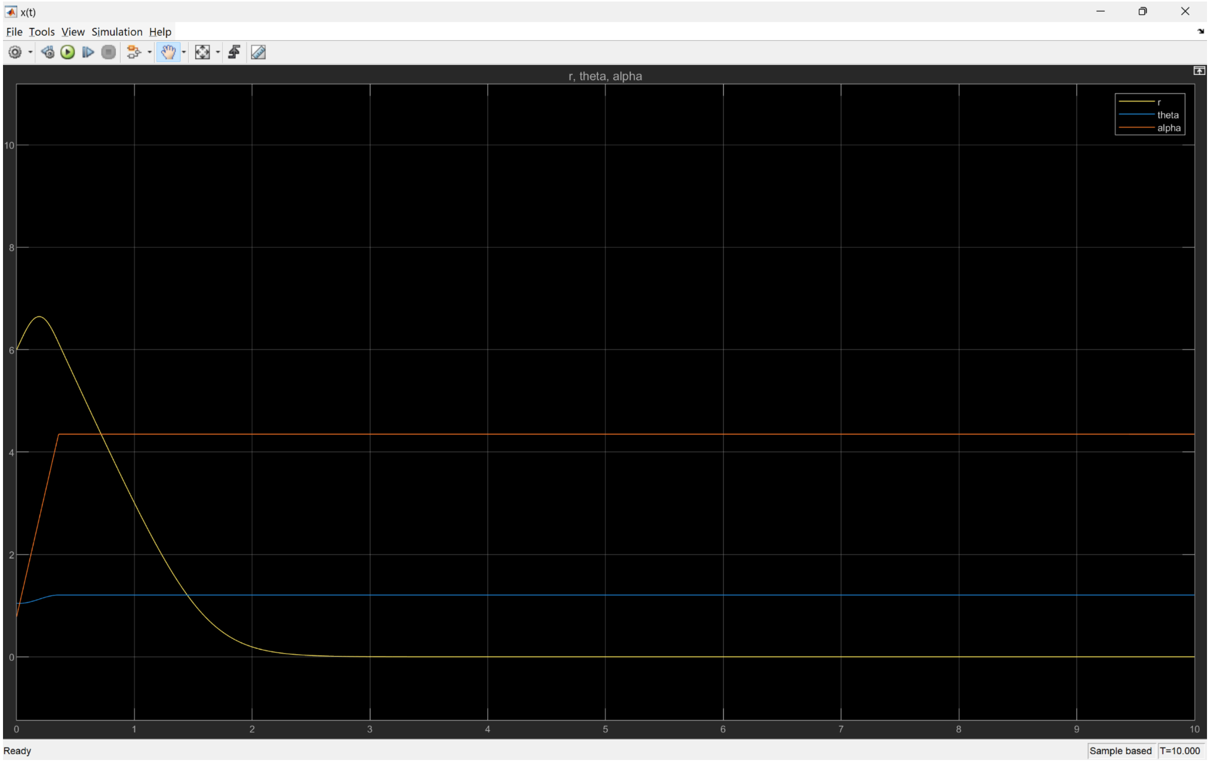
$v_0 = 5$

$K_s = 10$

1) $r_0 = 0.1$



2) $r_0 = 1.2$


x(t)
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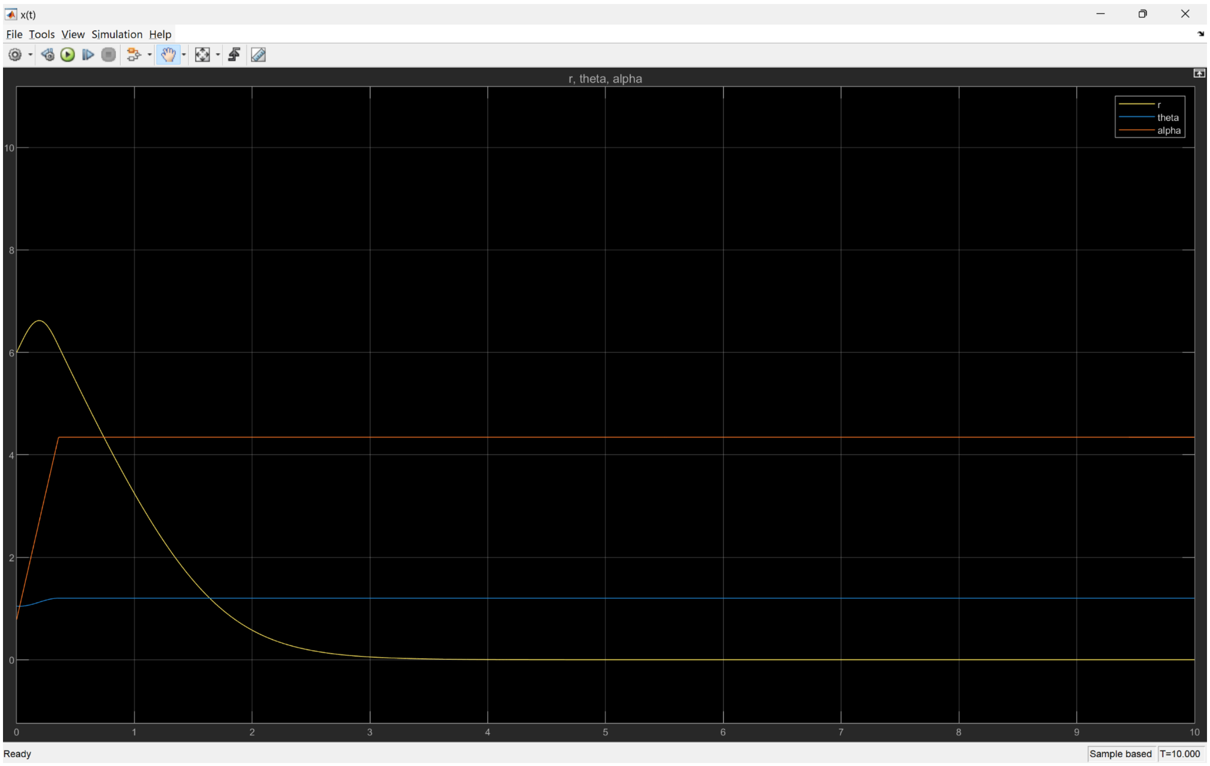
r, theta, alpha
r
theta
alpha
Ready
Sample based
T=10.000

3) $r_0 = 2$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r
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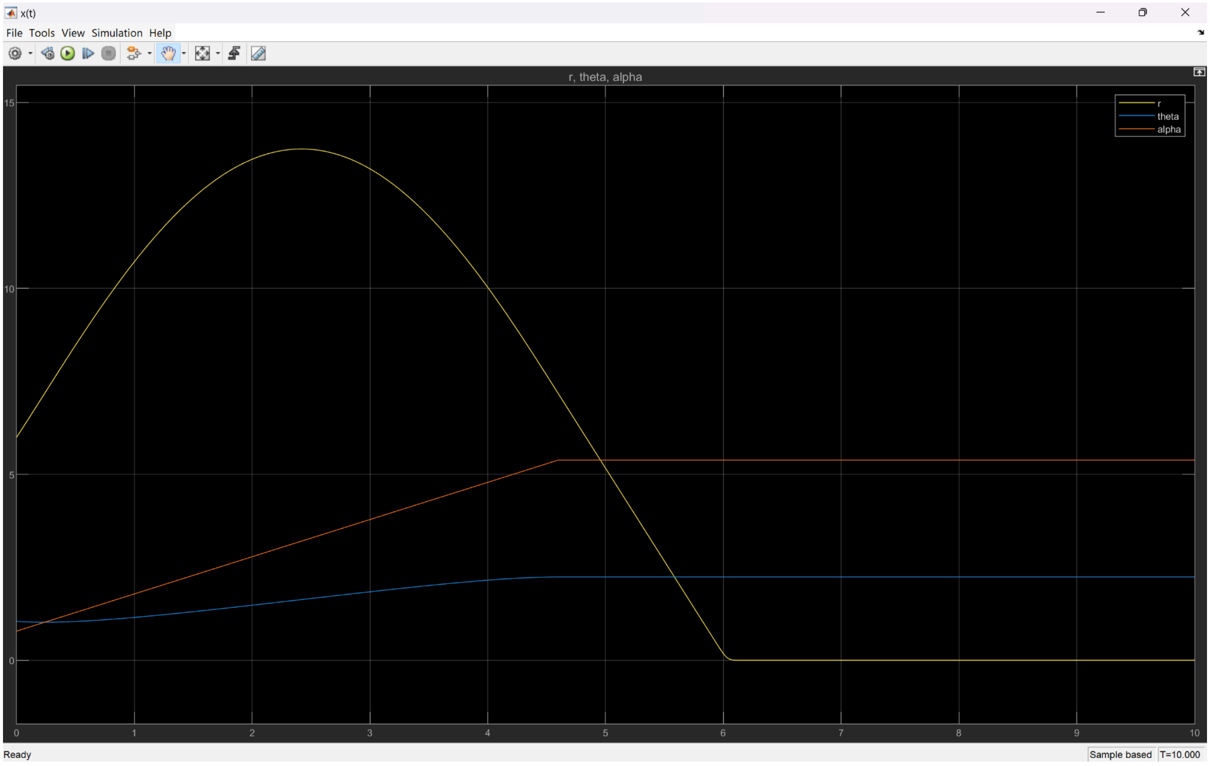
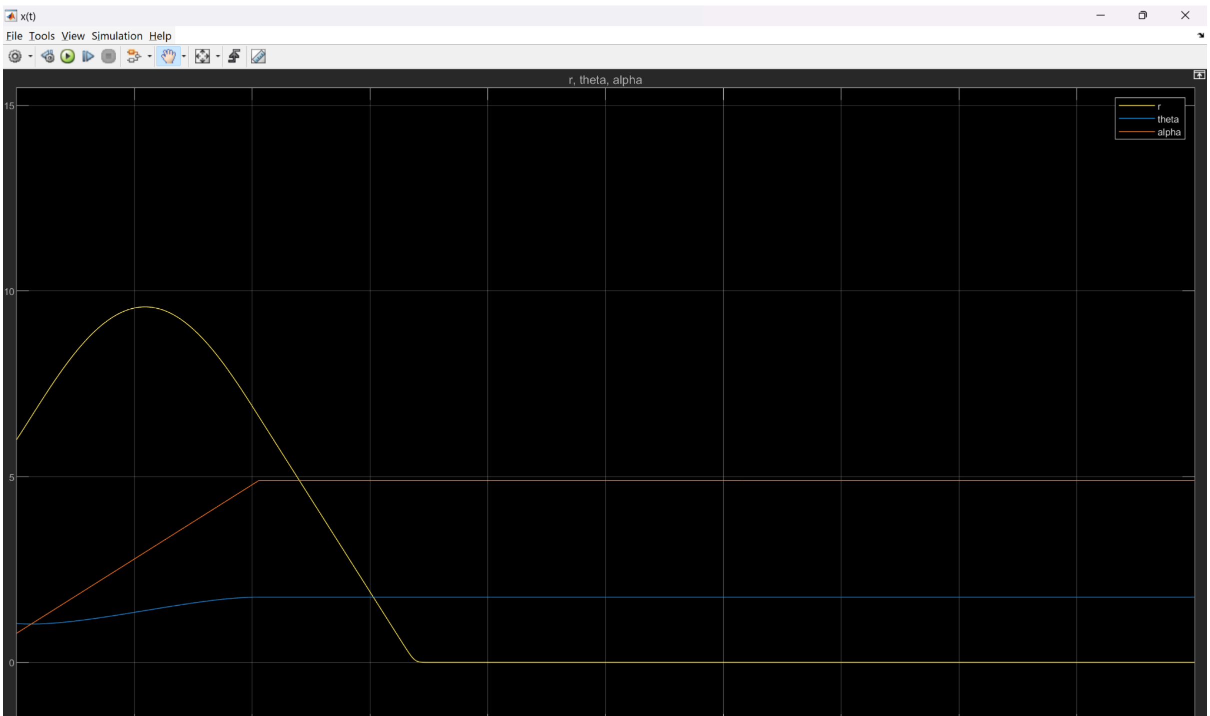
We can observe that on increasing r_0, the settling time will increase as the exponential decay decreases. Therefore, we should choose a small value of r_0 for our control law to reduce the settling time as much as possible.

c) Varying K_s:-
$v_0 = 5$
$r_0 = 0.1$

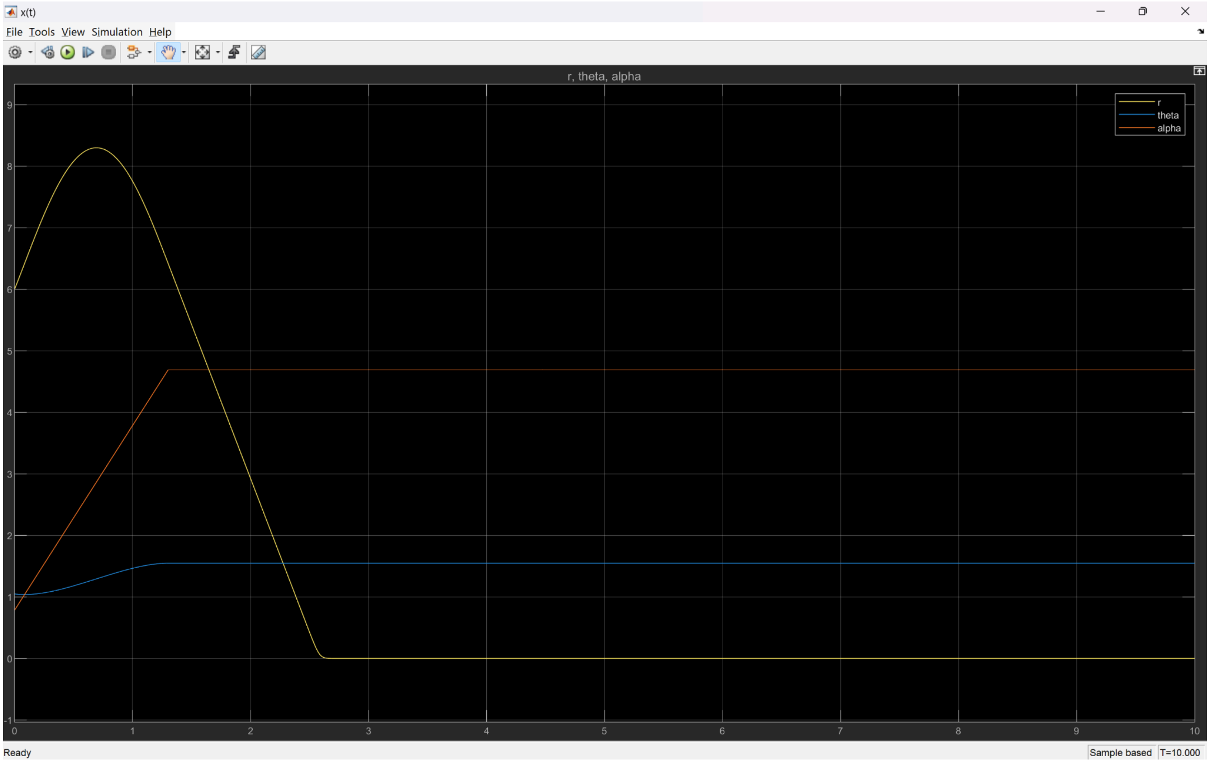
1) $K_s = 1$



2) $K_s = 2$



3) $K_s = 3$


x(t)
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r, theta, alpha
r
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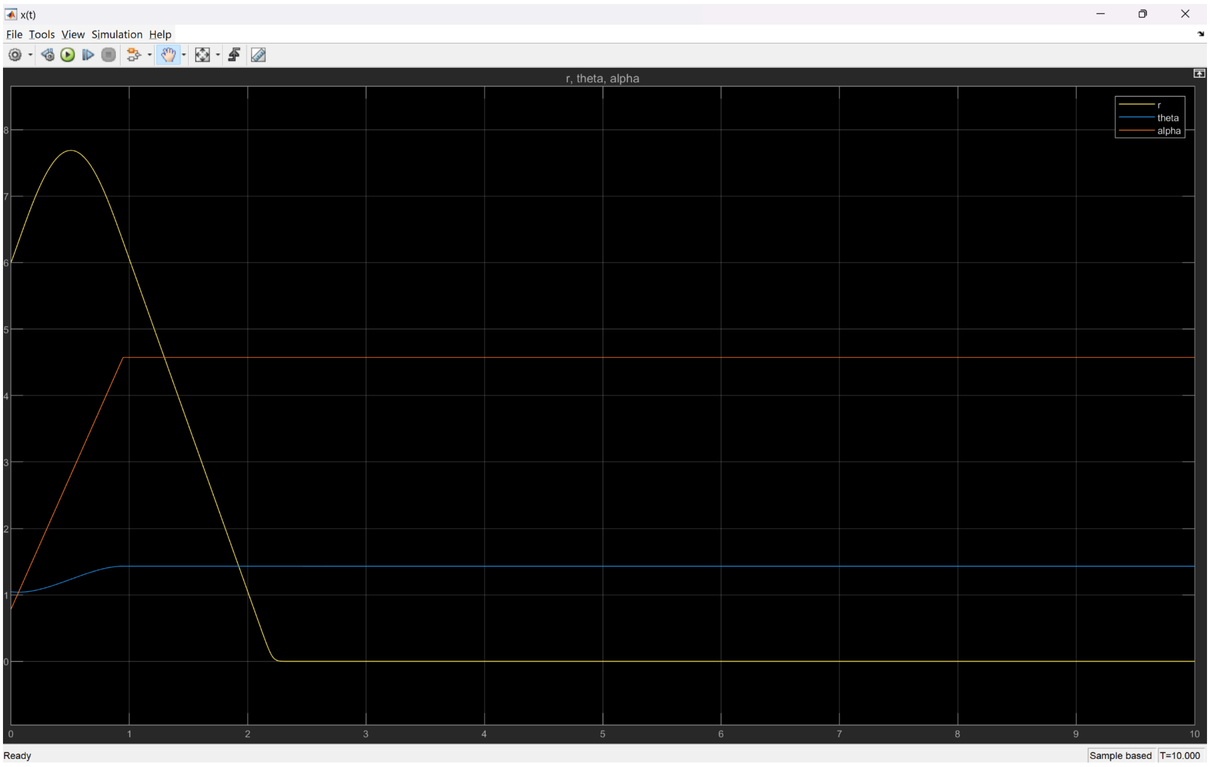
Sample based
T=10.000

4) $K_s = 4$


x(t)
File Tools View Simulation Help
r, theta, alpha
r
theta
alpha
Ready
Sample based
T=10.000

Increasing K_s decreases the settling time slightly, but the more important effect that it has is decreasing the maximum peak overshoot. Physically, what this means is that the robot is more responsive to control with a high latency to counter any existing inertia and direct the robot towards the home position. We can observe that as K_s increases, the states (R, θ, α) reach their steady state values faster which further highlights this point.

Stability of the controller:-

As observed from the above sections, the exponential decay control law used to control v to vary with r has ensured that inspite of changing the initial position or the control gains, the system response converges with a short settling time as compared to the constant v control strategy and therefore this controller is stable for all conditions.