

Theory Of Computation – Final

Time: 2 hours

Name:

Languages:

1. [5 points] Which of the following are languages over the alphabet $\{a, b\}$ (select all that apply)
 - a) $\{a\}$
 - b) $\{b, a\}$
 - c) ab
 - d) the empty set
 - e) $\{a, aa, aaa, aaaa\}$

Regular and Non Regular Languages:

1. [10 points] Draw the state diagram of the DFA that recognizes the language over.
 $\Sigma = \{0, 1\} : A = \{w \in \{0, 1\}^* : w \text{ does not contain the string } 1010 \text{ as a substring}\}$

2. [10 points] Convert the below regular expression to NFA.

$aUbU(aUb)^*(aUbb)$

3.[10 points] Prove that the language $L = \{ 0^n \mid n \text{ is a power of } 2 \}$ is not regular.

CFL and NonCFL :

1. [10 points] Design a CFG or PDA for $L = \{a^{3k+1}b^{3k-1} \mid k \geq 1\}$

2. [5 points] Consider the following grammar G:

$$S \rightarrow 1S1 \mid T$$

$$T \rightarrow 1X1 \mid X$$

$$X \rightarrow 0X0 \mid 1$$

Is G ambiguous?

3. [10 points] $L = \{x \in \{a, b\}^* : |x| \text{ is even and the first half of } x \text{ has one more "a" than does the second half}\}$. Show that L is non CFL.

Turing Machines:

1. [10 points] Construct a state transition diagram of a Turing Machine that decides the language $L = \{w \in \{a,b\}^*, |w|_a = 1 + |w|_b\}$ where $|w|_a$ denotes the number of occurrences of a in w and $|w|_b$ denotes the number of occurrences of b in w.

Decidability & Undecidability:

1.[10 points] Show that $L = \{ \langle M, w, t \rangle : M \text{ is a TM and } M \text{ runs more than } t \text{ steps on input } w \}$ is decidable.

2. [10 points] Show that the language
 $\text{SUB} = \{\langle M_1, M_2 \rangle : M_1, M_2 \text{ are TMs and } L(M_1) \subseteq L(M_2) \text{ is undecidable. You can use the fact that } A_{\text{TM}} \text{ is undecidable.}$