Math2310 - Fall '22

Syllabus - Lecture 16

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Review

1 Optimization on the boundary

- continuous functions on closed and bdd domain attain max and min and are bounded

- <u>exmpl</u> the gradient of the function $(x, y) \mapsto \left\| \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) \right\|$

- <u>exmpl</u> the gradient of the function $(x, y) \mapsto \dfrac{1}{1 + \left\| \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) \right\|^2}$

- why extremal points in the interior are critical points

- the direction of the gradient and rate of change

2 Optimization on the boundary - parametric aproach

- parameterizing the boundary of a domain and optimizing in lower dimensions.

- <u>exmpl</u> $f(x, y) = x\, e^y$ on $\left\{ \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) : x \geq 1, y \geq \frac{1}{2}, xy \geq 2 \right\}$

Topics

1 Optimization on the boundary - Lagrange multipliers

- directions of gradients and boundary tangent directions

- <u>defn</u> constraints

- optimization under constraints

- the method of Lagrange multipliers

  - motivation

  - method of Lagrange multipliers

  - the role of the multiplier λ

  - <u>exmpl</u> $f(x, y) = x\, e^y$ on $\left\{ \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) : x \geq 1, y \geq \frac{1}{2}, xy \leq 2 \right\}$

  - <u>exmpl</u> $f(x, y) = x + y$ on $\left\{ \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) : y < -e^x \right\}$

  - shortcomings: once candidate is found, no "second derivative test" is available

2 Integrals over multivariable domains

- Examples and motivation.

References

Textbook

- [Ste] Chap 14.7 (complete) - Maximum and minimum values

- [Ste] Chap 14.8 (complete) - Lagrange multipliers (skip 2 constraints).

- [Ste] Chap 15.1 (complete) - Double integrals over rectangles

Videos

- [Lagrange multipliers, using tangency to solve constrained optimization - YouTube](#)

- [Lagrange Multipliers | Geometric Meaning & Full Example - YouTube](#)

- [Lagrange Multipliers - YouTube](#)