Math2310 - Fall '22

Syllabus - Lecture 04

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Review

- <u>prop</u> Generalized Pythagoras theorem - Cosine law.

- [5] Using the Cosine law to prove the identity

$$a_1 b_1 + a_2 b_2 = \|\vec{a}\| \|\vec{b}\| \cos(\theta)$$

  where θ is the angle formed by $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$.

- Equation of a plane using the normal vector [6]

- Equation of a line in $\mathbb{R}^2$ using a normal vector

  - rotation of a vector by $90°$ counterclockwise

Topics

1 The cross product [1,2,3]

- Algebraic expression and properties.

- Geometric properties

  - magnitude

  - direction

- Differences between $\mathbb{R}^3$ and $\mathbb{R}^2$

- Chirality/handedness.

- Signed area

- Inequalities of the cross product

  - when it vanishes

  - when it is maximal

- Cross product as area of parallelogram: $\mathbb{R}^2$ and $\mathbb{R}^3$

- The volume of the parallelogram and the triple product formula

- Using the cross product to find a normal to a parameterized plane [4]

2 Projections

- unit vectors and normalization

- `defn` projections

- using dot products to compute projections onto:

    - lines [7,8]

    - planes [9]

References

Videos

1. [Cross products | Chapter 10, Essence of linear algebra - YouTube](#)

2. [Cross Product of Two Vectors Explained! - YouTube](#)

3. [The Vector Cross Product - YouTube](#)

4. [Normal to a Plane using the Vector Product of Two Vectors- Worked Example - YouTube](#)

5. [Proof of Law of Cosines using Dot Product - YouTube](#)

6. [Normal vector from plane equation | Vectors and spaces | Linear Algebra | Khan Academy - YouTube](#)

7. [Projection Vector Proof - YouTube](#)

8. [Orthogonal Projections - Scalar and Vector Projections - YouTube](#)

9. [Projection of a vector onto a plane - YouTube](#)

Textbook

- [Ste] Chap 12.3 The Dot Product pp847 - 854

- [Ste] Chap 12.5 Equations of Lines and Planes pp864 - 870

- [Ste] Chap 12.4 The Cross product pp855-864

Additional material