

# Supplementary File to “Pixel-level Non-local Image Smoothing with Objective Evaluation”

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In this supplementary file, we provide:

- detailed Haar transformation and inverse Haar transformation;
- more comparisons of different image smoothing methods on the datasets of NKS, [1], [6], [9].

## I. DETAILED HAAR TRANSFORMATION AND INVERSE HAAR TRANSFORMATION

We perform standard Haar transformation and inverse Haar transformation with no modification. Moreover we set  $q = 4$ ,  $m = 16$  in all experiments, so the similar pixels matrix  $\mathbf{S} \in \mathbb{R}^{4 \times 16}$  could be represented by columns as  $\mathbf{S} = [\mathbf{s}_1^4, \dots, \mathbf{s}_{16}^4] \in \mathbb{R}^{4 \times 16}$ . The Haar transformation includes horizontal and vertical transformation. We first apply the horizontal transformation. Specifically, we multiply the similar pixels matrix  $\mathbf{S} \in \mathbb{R}^{4 \times 16}$  and the horizontal transformation matrix  $\mathbf{H}_r \in \mathbb{R}^{16 \times 16}$ :

$$\begin{aligned} \mathbf{t}_i^4 &= \frac{1}{\sqrt{16}} \left( \sum_{j=1}^8 \mathbf{s}_j^4 + (-1)^{i-1} \sum_{j=9}^{16} \mathbf{s}_j^4 \right), \text{ when } i = 1, 2; \\ \mathbf{t}_i^4 &= \frac{1}{\sqrt{8}} \left( \sum_{j=8(i-3)+1}^{8(i-3)+4} \mathbf{s}_j^4 - \sum_{j=8(i-3)+5}^{8(i-2)} \mathbf{s}_j^4 \right), \text{ when } i = 3, 4; \\ \mathbf{t}_i^4 &= \frac{1}{\sqrt{4}} \left( \sum_{j=4(i-5)+1}^{4(i-5)+2} \mathbf{s}_j^4 - \sum_{j=4(i-5)+3}^{4(i-5)+4} \mathbf{s}_j^4 \right), \text{ when } i = 5, \dots, 8; \\ \mathbf{t}_i^4 &= \frac{1}{\sqrt{2}} (\mathbf{s}_{2(i-9)+1}^4 - \mathbf{s}_{2(i-9)+2}^4), \text{ when } i = 9, \dots, 16. \end{aligned} \quad (1)$$

We stack the all these column vectors to form  $\mathbf{T} = [\mathbf{t}_1^4, \dots, \mathbf{t}_{16}^4] \in \mathbb{R}^{4 \times 16}$ . We then represent  $\mathbf{T}$  by rows as  $\mathbf{T}^4 = [\mathbf{t}^1^\top, \dots, \mathbf{t}^4^\top]^\top \in \mathbb{R}^{4 \times 16}$ , and perform vertical Haar transformation. Specifically, we multiply the matrix  $\mathbf{T}^4 \in \mathbb{R}^{4 \times 16}$  and the vertical transformation matrix  $\mathbf{H}_l \in \mathbb{R}^{4 \times 4}$ :

$$\begin{aligned} \hat{\mathbf{t}}^1 &= \frac{1}{\sqrt{4}} \sum_{i=1}^4 \mathbf{t}^i, \quad \hat{\mathbf{t}}^2 = \frac{1}{\sqrt{4}} \left( \sum_{i=1}^2 \mathbf{t}^i - \sum_{i=3}^4 \mathbf{t}^i \right), \\ \hat{\mathbf{t}}^3 &= \frac{1}{\sqrt{2}} (\mathbf{t}^1 - \mathbf{t}^2), \quad \hat{\mathbf{t}}^4 = \frac{1}{\sqrt{2}} (\mathbf{t}^3 - \mathbf{t}^4). \end{aligned} \quad (2)$$

After the thresholding step, we could get the thresholded representation matrix  $\tilde{\mathbf{T}} \in \mathbb{R}^{4 \times 16}$ . We next perform inverse vertical Haar transformation and inverse horizontal Haar transformation. We first apply the inverse vertical transformation. Specifically, we multiply the inverse vertical transformation matrix  $\mathbf{H}_{il} \in \mathbb{R}^{4 \times 4}$  and the thresholded representation matrix  $\tilde{\mathbf{T}}^4 \in \mathbb{R}^{4 \times 16}$ :

$$\begin{aligned} \tilde{\mathbf{t}}^1 &= \frac{1}{\sqrt{4}} (\hat{\mathbf{t}}^1 + \hat{\mathbf{t}}^2) + \frac{1}{\sqrt{2}} \hat{\mathbf{t}}^3, \\ \tilde{\mathbf{t}}^2 &= \frac{1}{\sqrt{4}} (\hat{\mathbf{t}}^1 + \hat{\mathbf{t}}^2) - \frac{1}{\sqrt{2}} \hat{\mathbf{t}}^3, \\ \tilde{\mathbf{t}}^3 &= \frac{1}{\sqrt{4}} (\hat{\mathbf{t}}^1 - \hat{\mathbf{t}}^2) + \frac{1}{\sqrt{2}} \hat{\mathbf{t}}^4, \\ \tilde{\mathbf{t}}^4 &= \frac{1}{\sqrt{4}} (\hat{\mathbf{t}}^1 - \hat{\mathbf{t}}^2) - \frac{1}{\sqrt{2}} \hat{\mathbf{t}}^4. \end{aligned} \quad (3)$$

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We stack all these row vectors to form  $\tilde{\mathbf{T}}^4 = [(\tilde{\mathbf{t}}^4)^T, \dots, (\tilde{\mathbf{t}}^4)^T]^T \in \mathbb{R}^{4 \times 16}$ . We then represent  $\tilde{\mathbf{T}}$  by columns as  $\tilde{\mathbf{T}} = [\tilde{\mathbf{t}}_1^4, \dots, \tilde{\mathbf{t}}_{16}^4] \in \mathbb{R}^{4 \times 16}$ , and perform inverse horizontal Haar transformation. Specifically, we multiply the matrix  $\tilde{\mathbf{T}} \in \mathbb{R}^{4 \times 16}$  and the inverse horizontal transformation matrix  $\mathbf{H}_{ir} \in \mathbb{R}^{16 \times 16}$ :

$$\begin{aligned}
\tilde{s}_1^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_5^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_9^4, \\
\tilde{s}_2^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_5^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_9^4, \\
\tilde{s}_3^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_5^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{10}^4, \\
\tilde{s}_4^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_5^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{10}^4, \\
\tilde{s}_5^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_6^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{11}^4, \\
\tilde{s}_6^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_6^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{11}^4, \\
\tilde{s}_7^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_6^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{12}^4, \\
\tilde{s}_8^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 + \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_3^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_6^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{12}^4, \\
\tilde{s}_9^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_7^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{13}^4, \\
\tilde{s}_{10}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_7^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{13}^4, \\
\tilde{s}_{11}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_7^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{14}^4, \\
\tilde{s}_{12}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) + \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_7^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{14}^4, \\
\tilde{s}_{13}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_8^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{15}^4, \\
\tilde{s}_{14}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 + \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_8^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{15}^4, \\
\tilde{s}_{15}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_8^4 + \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{16}^4, \\
\tilde{s}_{16}^4 &= \frac{1}{\sqrt{16}}(\tilde{\mathbf{t}}_1^4 - \tilde{\mathbf{t}}_2^4) - \frac{1}{\sqrt{8}}\tilde{\mathbf{t}}_4^4 - \frac{1}{\sqrt{4}}\tilde{\mathbf{t}}_8^4 - \frac{1}{\sqrt{2}}\tilde{\mathbf{t}}_{16}^4.
\end{aligned} \tag{17}$$

We stack all these column vectors together and form the smoothed similar pixel matrix  $\tilde{\mathbf{S}} = [\tilde{s}_1^4, \dots, \tilde{s}_{16}^4] \in \mathbb{R}^{4 \times 16}$ .

## II. MORE COMPARISONS OF DIFFERENT IMAGE SMOOTHING METHODS

Here, we conduct more comparisons of different image smoothing methods on the datasets of NKS, [1], [6], [9]. In Figures 1-5, we compare PSNR, SSIM [4], FSIM [7], and visual quality of different methods on image smoothing on the dataset of NKS. In Figures 6-26, we compare the visual quality of different methods on image smoothing on the datasets of [1], [6], [9]. The comparison results demonstrate that the PNLS method achieves better visual quality than the other image smoothing methods.

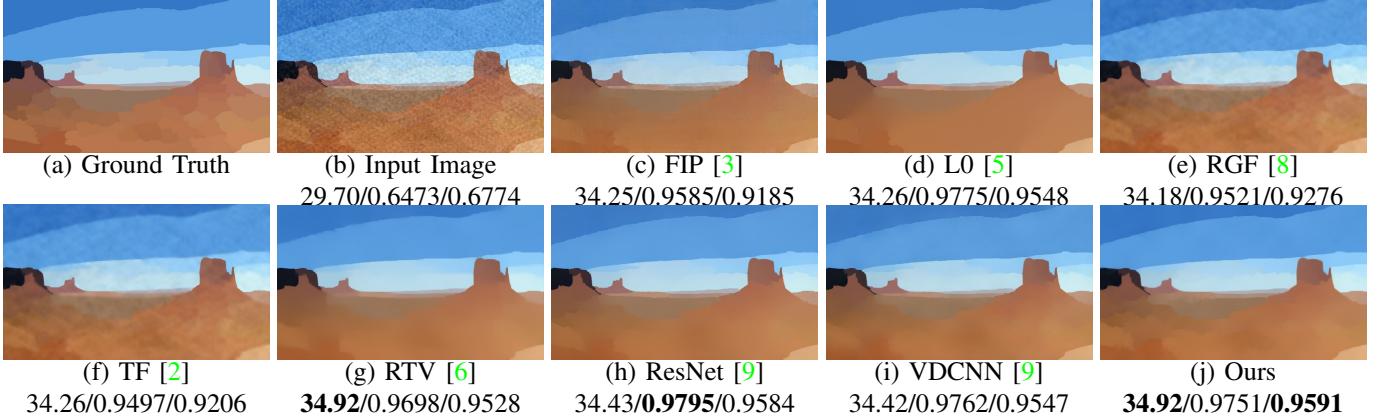


Fig. 1. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S03\_T07” from our NKS dataset. The best results are highlighted in **bold**.

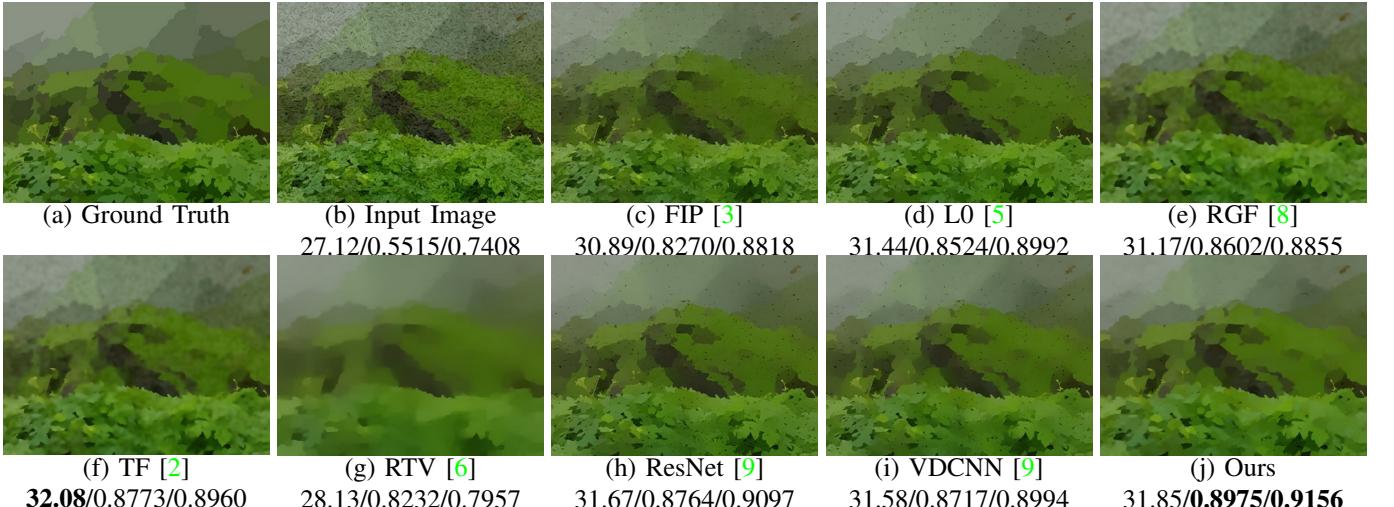


Fig. 2. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S07\_T02” from our NKS dataset. The best results are highlighted in **bold**.

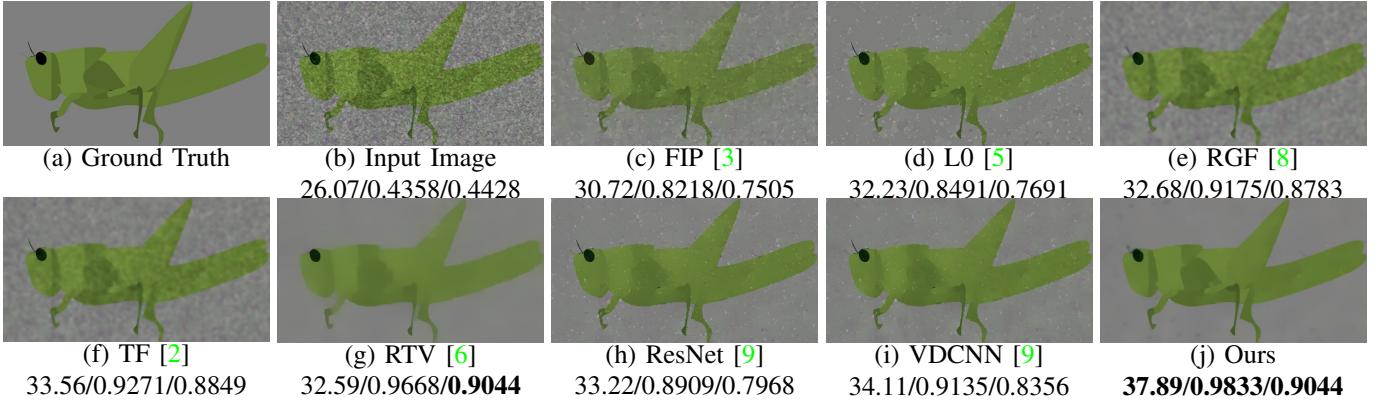


Fig. 3. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S09\_T09” from our NKS dataset. The best results are highlighted in **bold**.

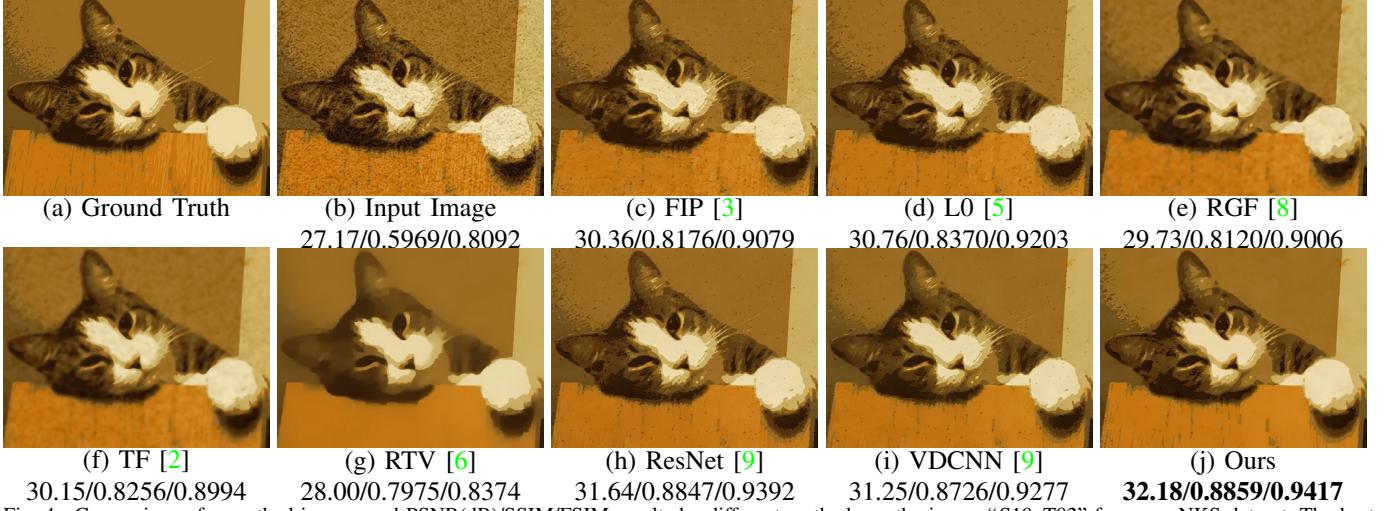


Fig. 4. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S10\_T02” from our NKS dataset. The best results are highlighted in **bold**.

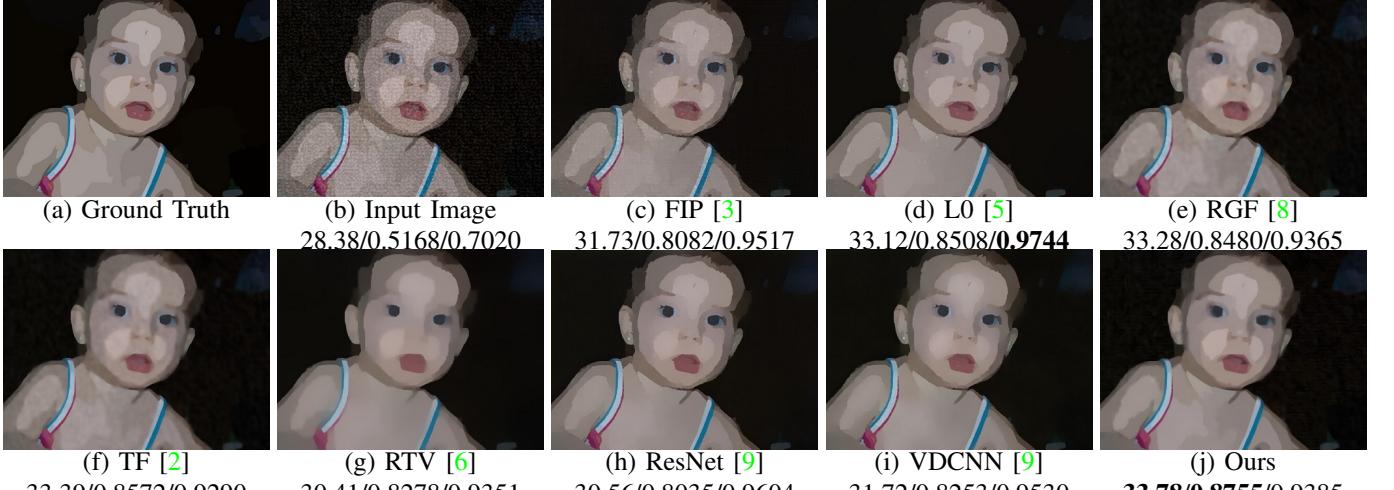


Fig. 5. Comparison of smoothed images and PSNR(dB)/SSIM/FSIM results by different methods on the image “S14\_T06” from our NKS dataset. The best results are highlighted in **bold**.

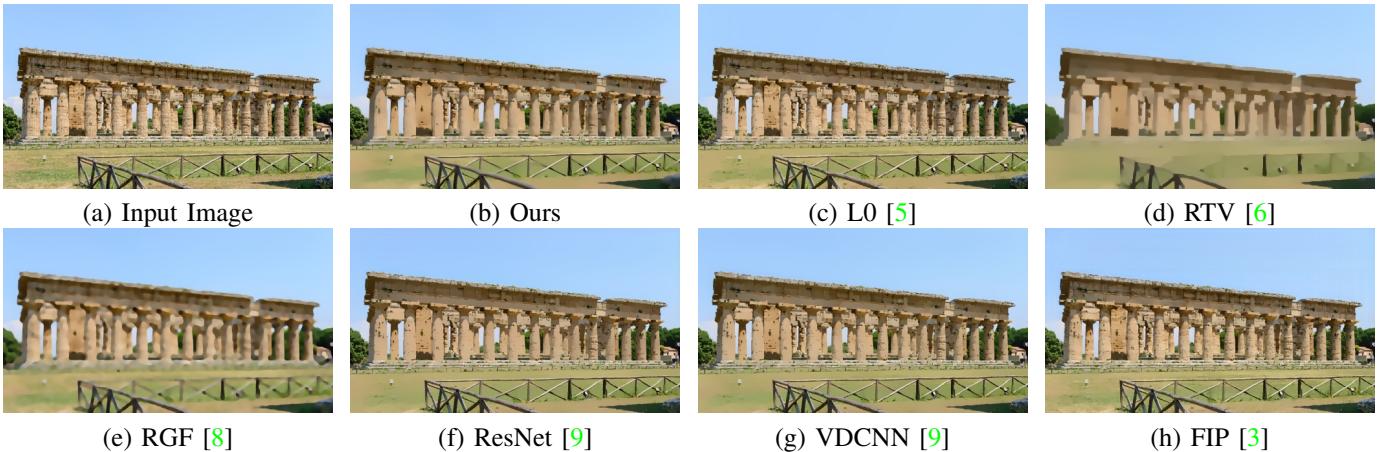


Fig. 6. Comparison of smoothed images by different methods on the image “0073” from the DIV2K dataset [1].

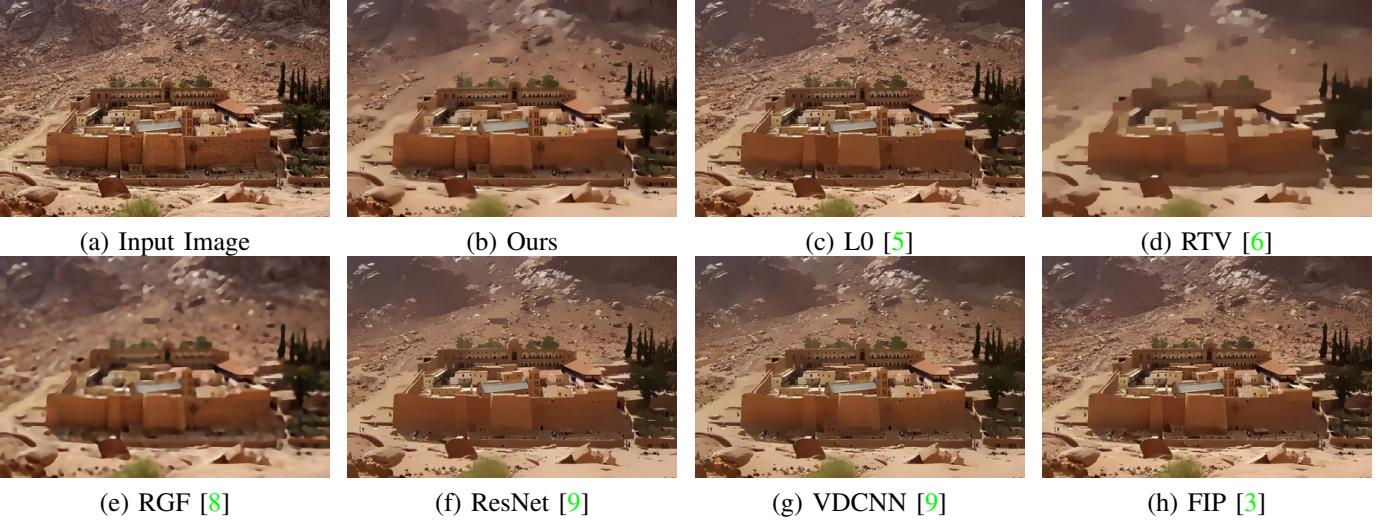


Fig. 7. Comparison of smoothed images by different methods on the image “0102” from the DIV2K dataset [1].

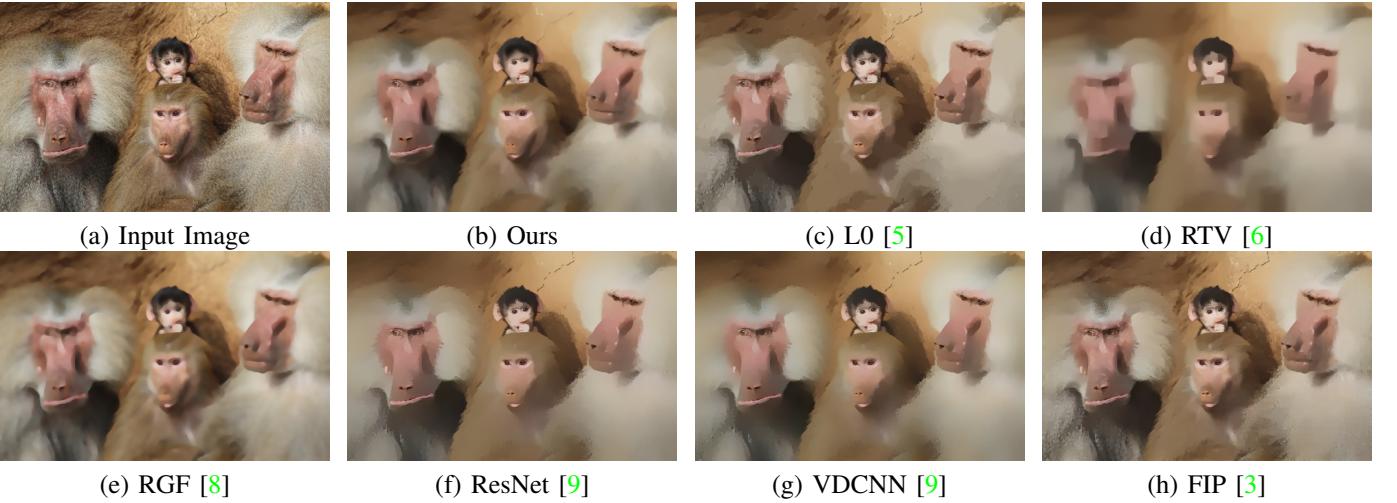


Fig. 8. Comparison of smoothed images by different methods on the image “0105” from the DIV2K dataset [1].

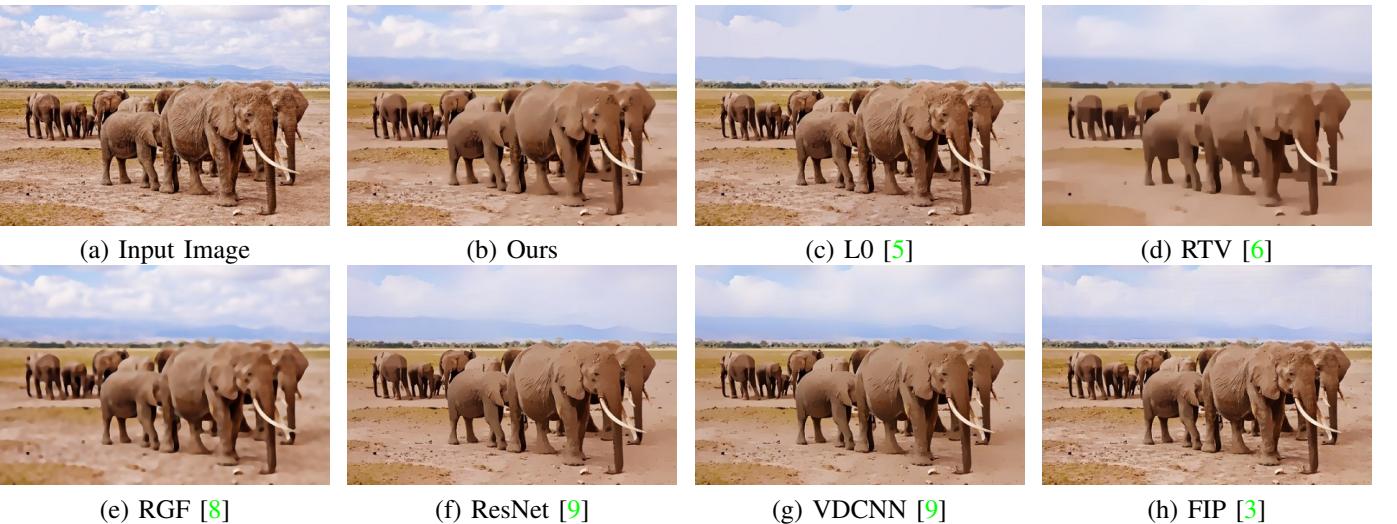


Fig. 9. Comparison of smoothed images by different methods on the image “0117” from the DIV2K dataset [1].

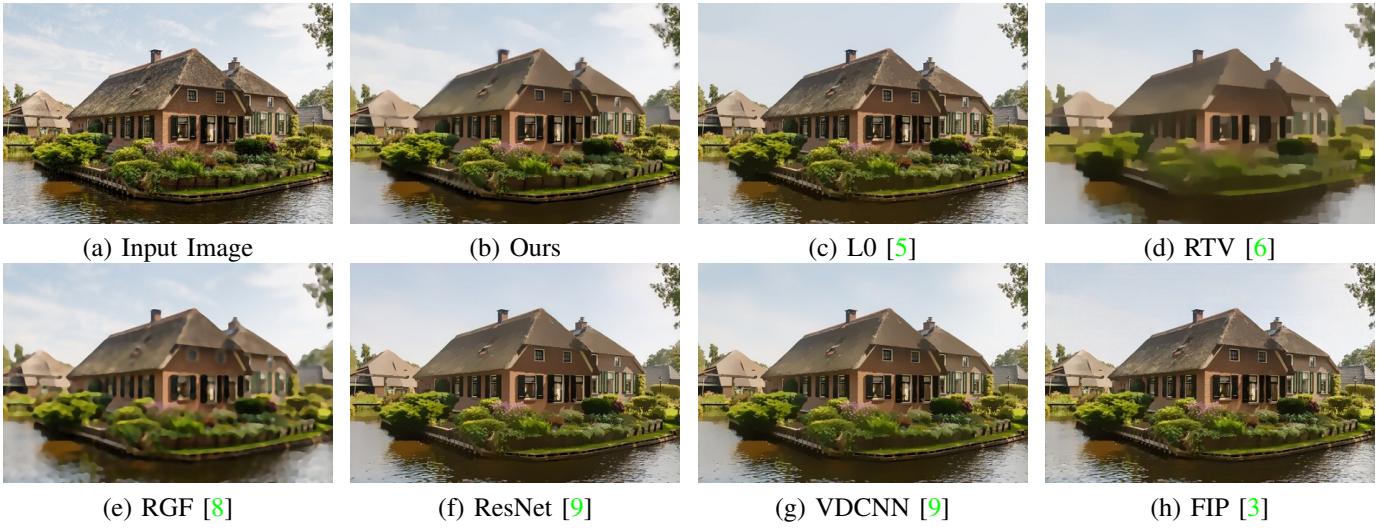


Fig. 10. Comparison of smoothed images by different methods on the image “0146” from the DIV2K dataset [1].

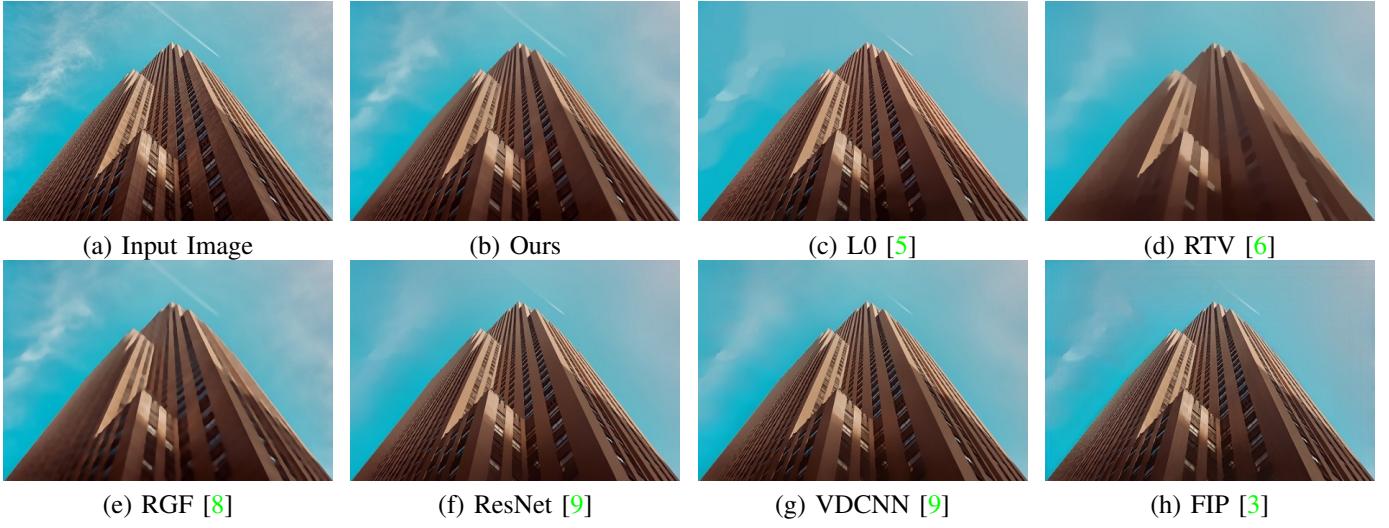


Fig. 11. Comparison of smoothed images by different methods on the image “0154” from the DIV2K dataset [1].

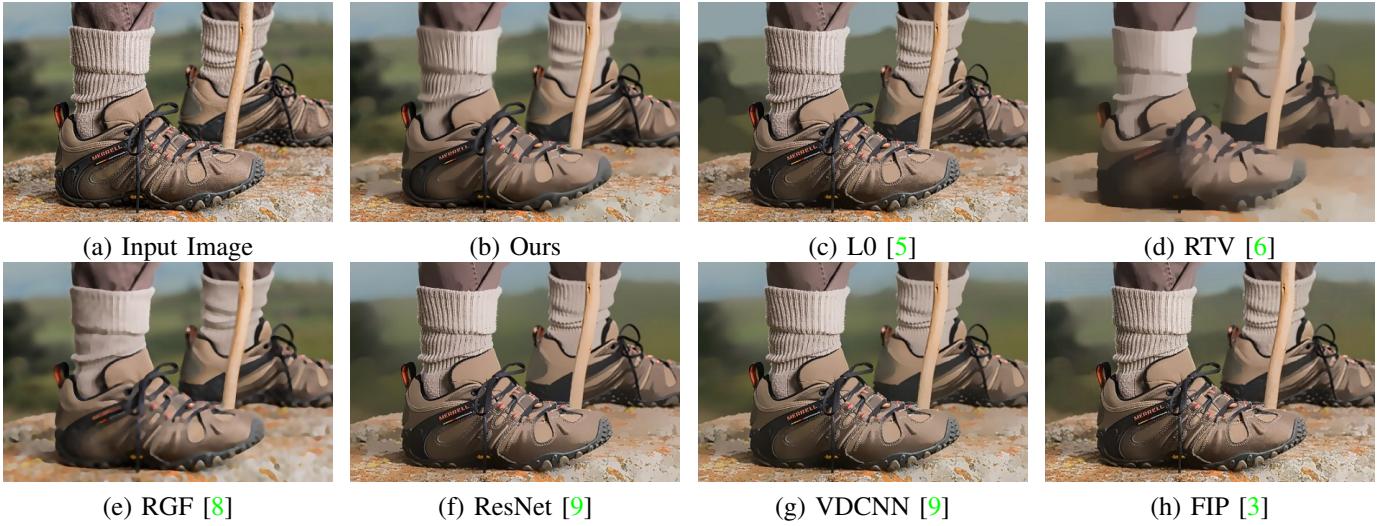


Fig. 12. Comparison of smoothed images by different methods on the image “0166” from the DIV2K dataset [1].

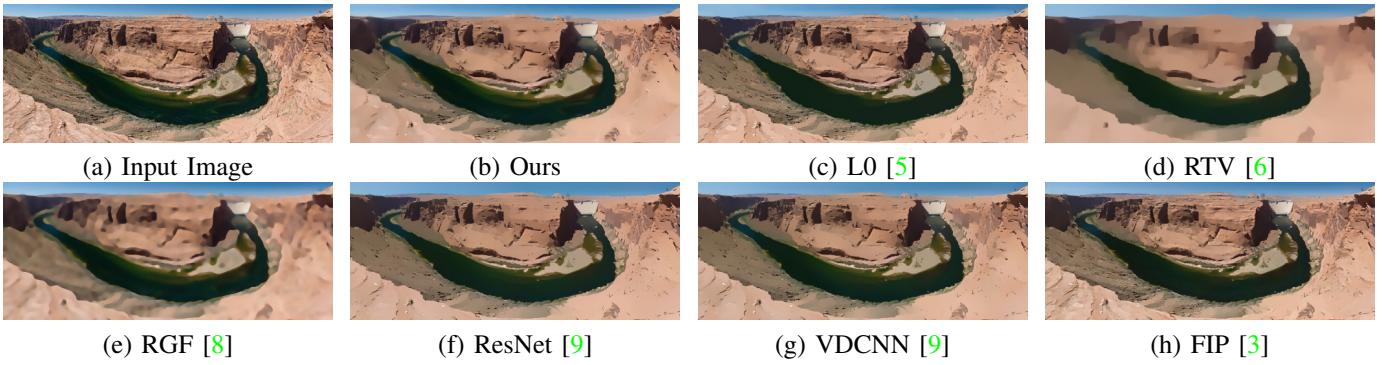


Fig. 13. Comparison of smoothed images by different methods on the image “0205” from the DIV2K dataset [1].

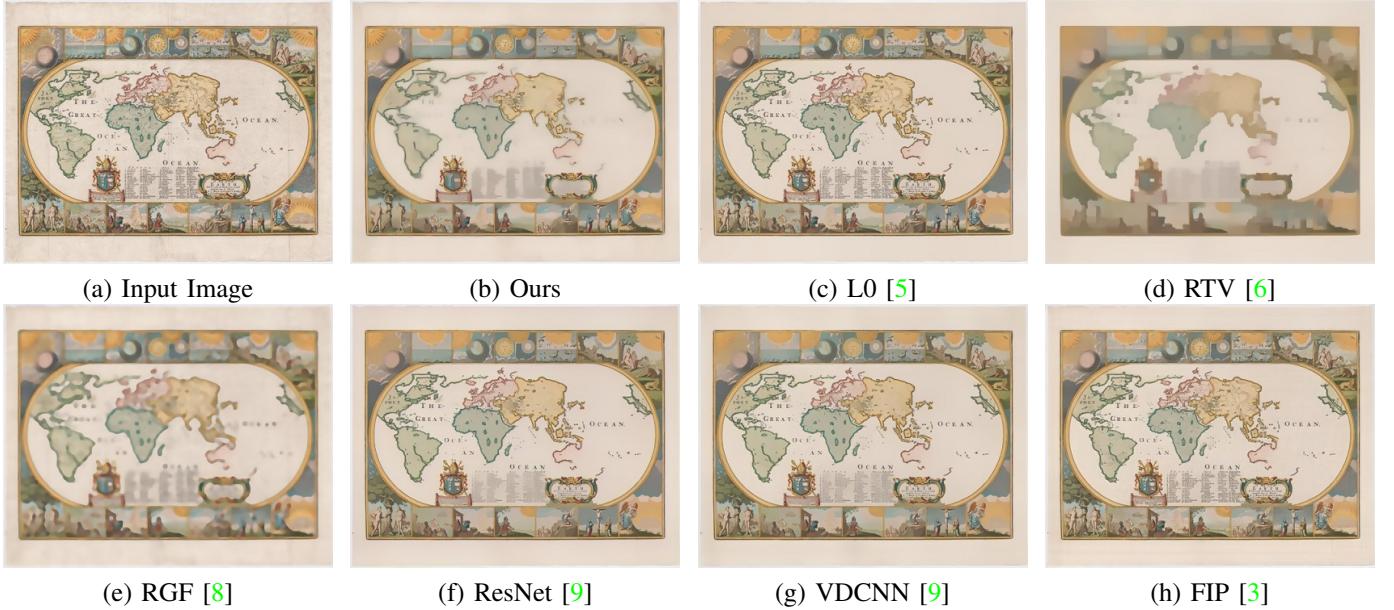


Fig. 14. Comparison of smoothed images by different methods on the image “0404” from the DIV2K dataset [1].

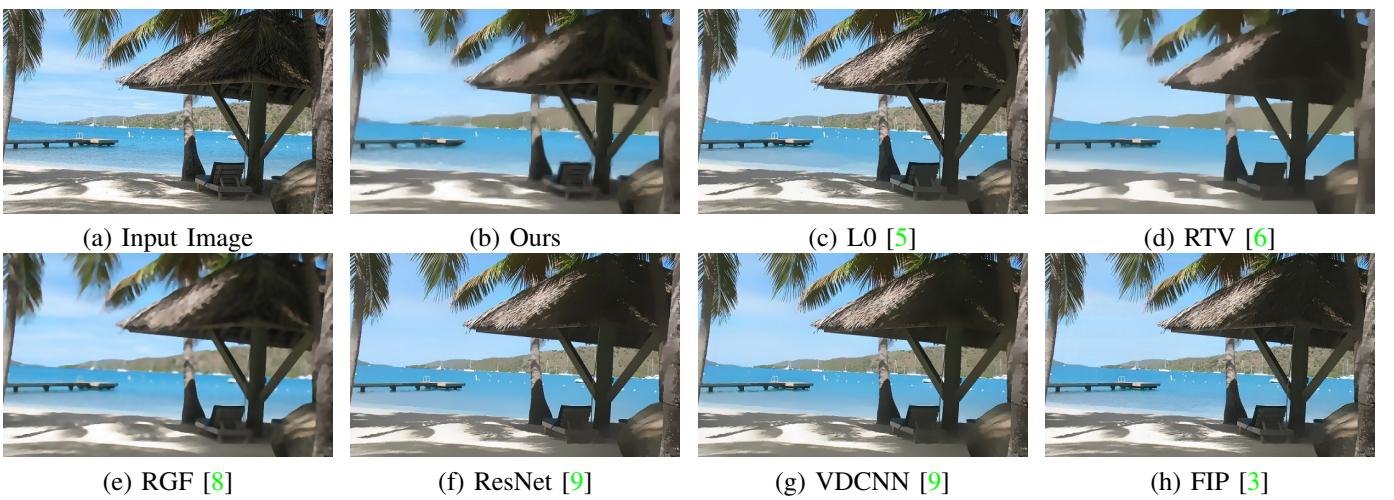


Fig. 15. Comparison of smoothed images by different methods on the image “0094” from the dataset in [9].

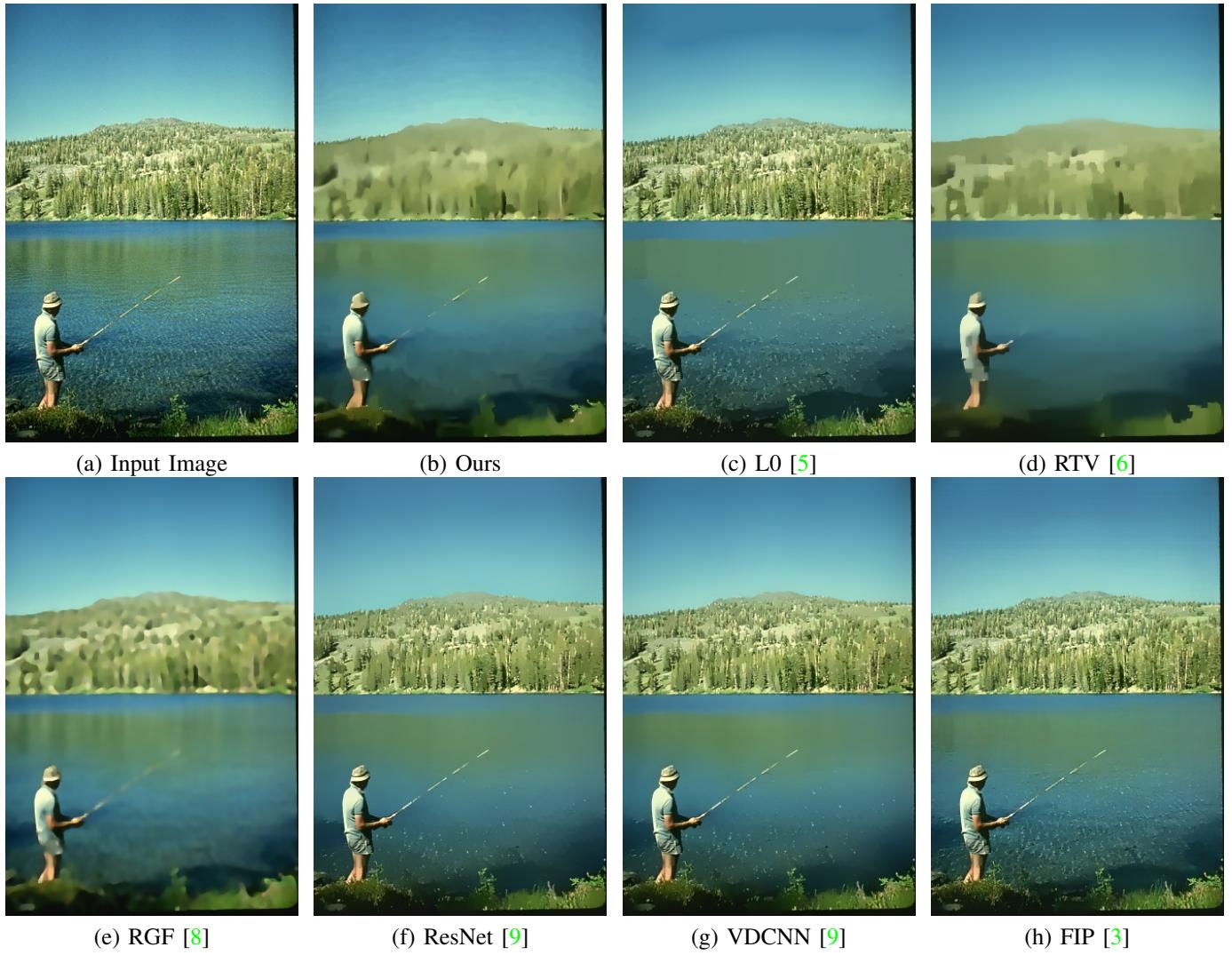


Fig. 16. Comparison of smoothed images by different methods on the image “0115” from the dataset in [9]

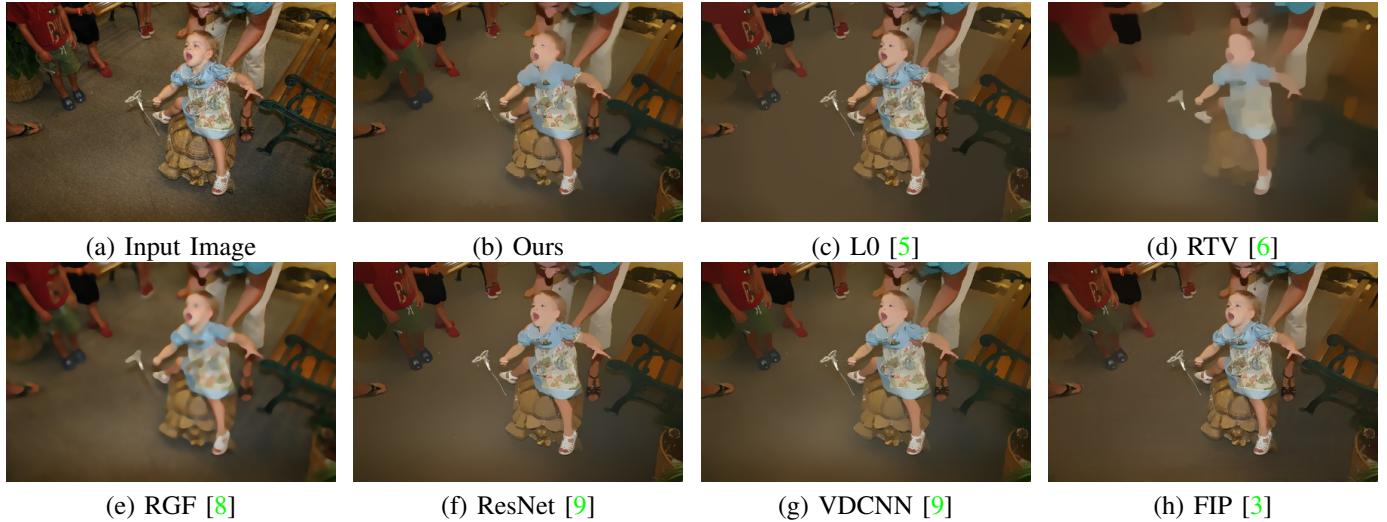


Fig. 17. Comparison of smoothed images by different methods on the image “0169” from the dataset in [9]

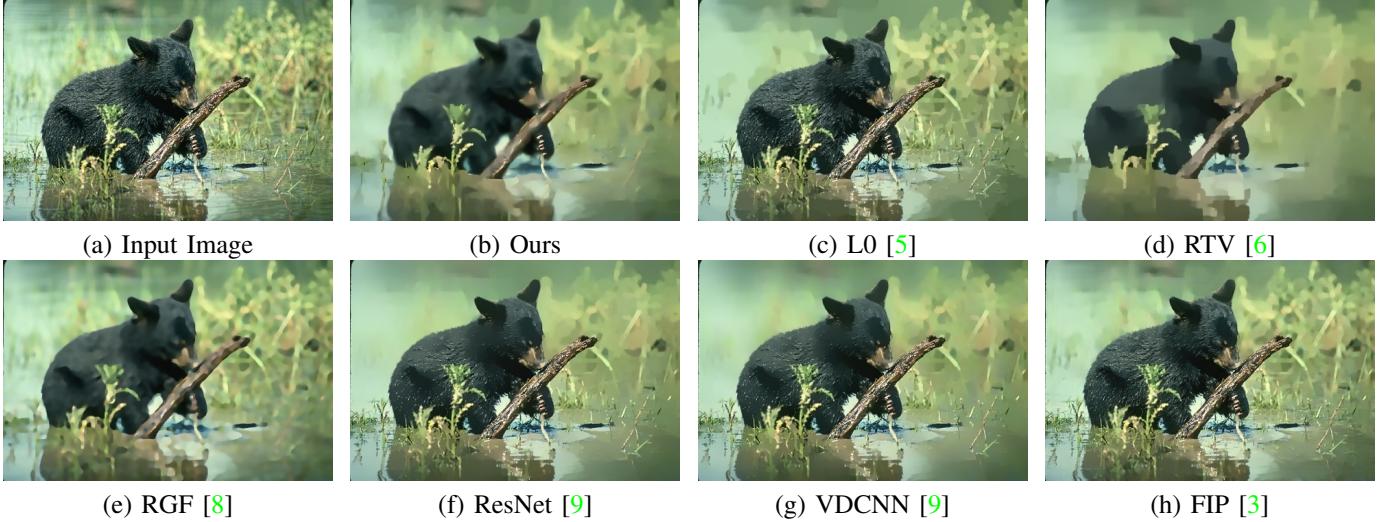


Fig. 18. Comparison of smoothed images by different methods on the image “0314” from the dataset in [9].

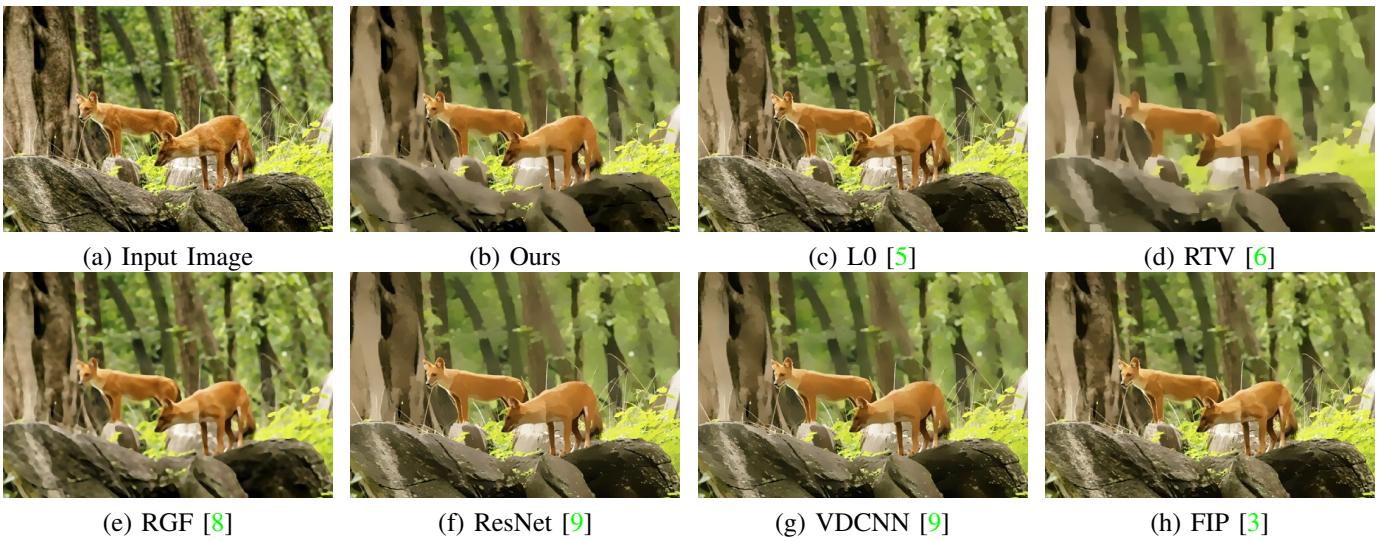


Fig. 19. Comparison of smoothed images by different methods on the image “0334” from the dataset in [9].

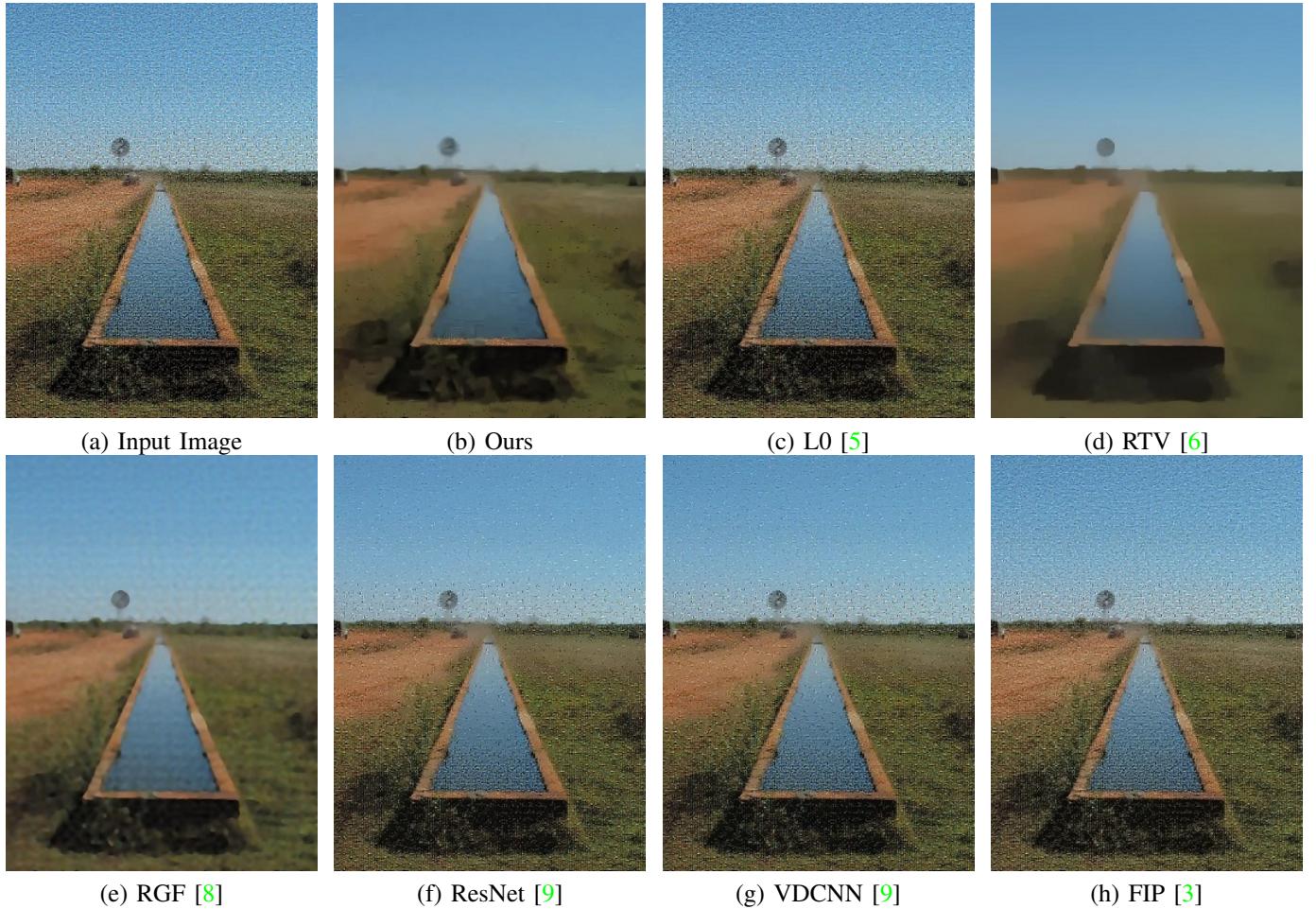


Fig. 20. Comparison of smoothed images by different methods on the image “02\_23” from the dataset in [6]



Fig. 21. Comparison of smoothed images by different methods on the image “03\_11” from the dataset in [6]

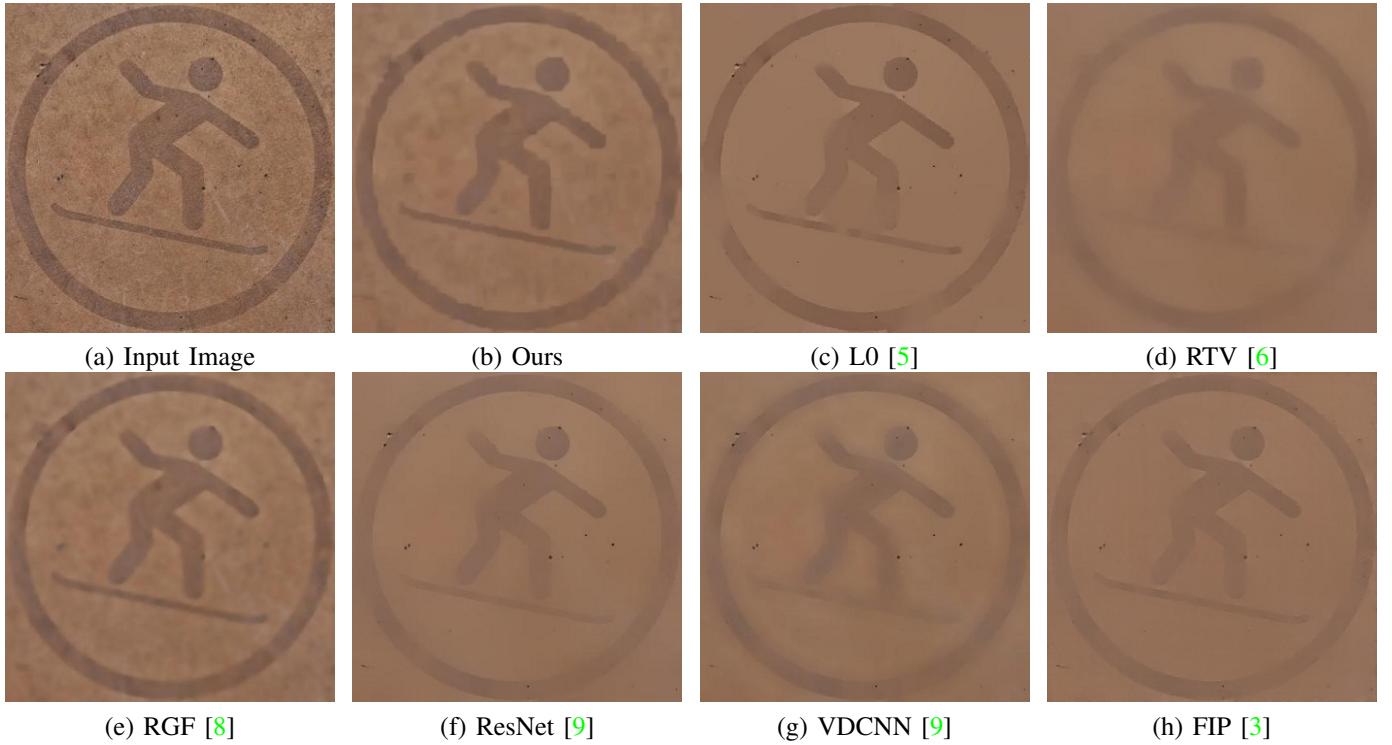


Fig. 22. Comparison of smoothed images by different methods on the image “11\_07” from the dataset in [6]

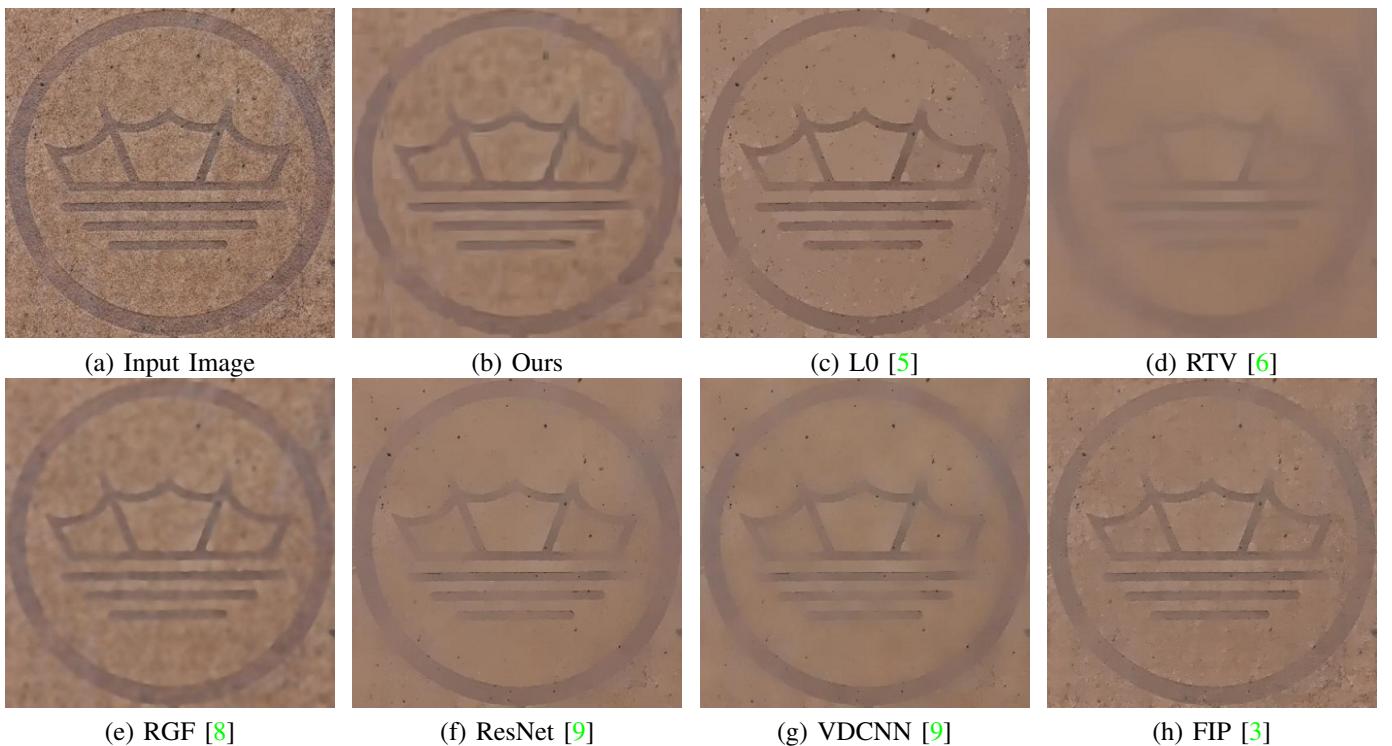


Fig. 23. Comparison of smoothed images by different methods on the image “11\_08” from the dataset in [6]

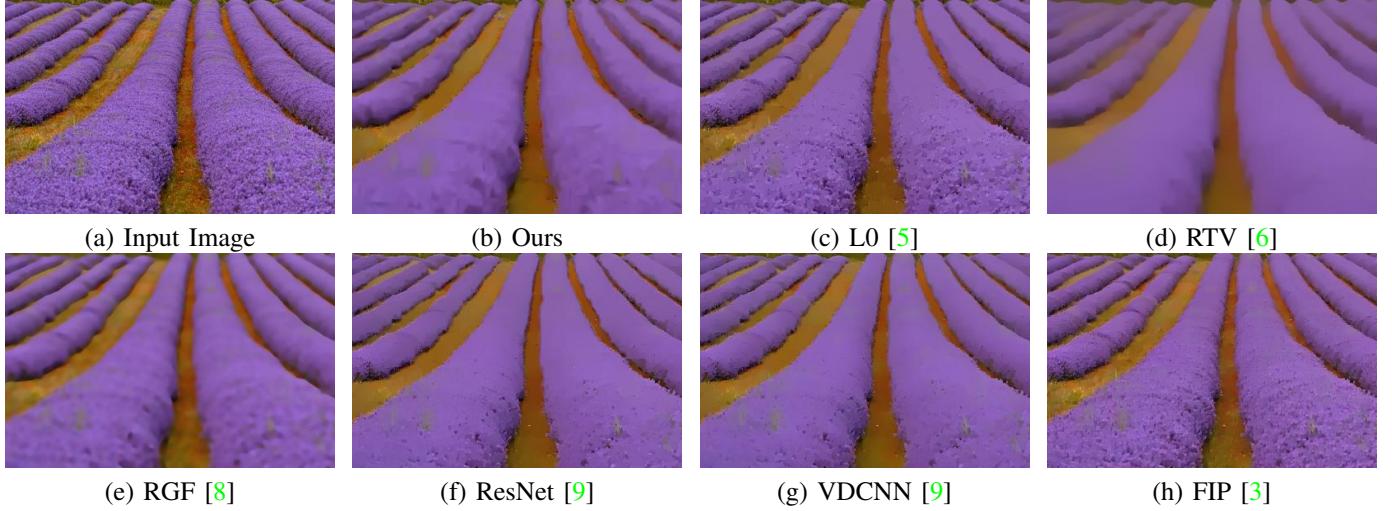


Fig. 24. Comparison of smoothed images by different methods on the image “11\_17” from the dataset in [6]

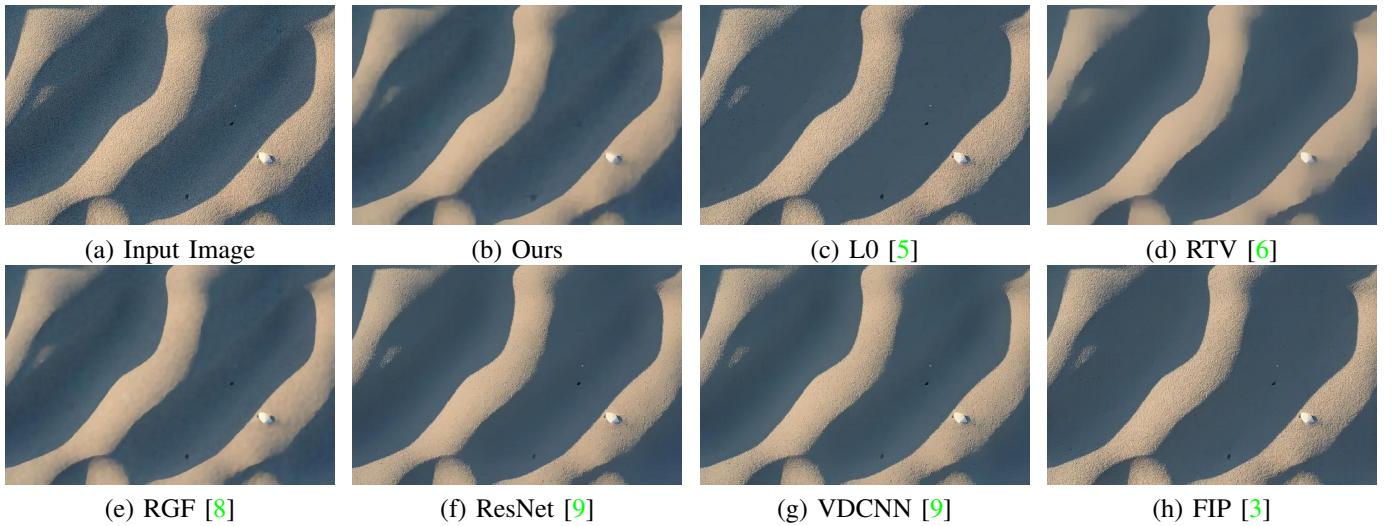


Fig. 25. Comparison of smoothed images by different methods on the image “11\_26” from the dataset in [6]

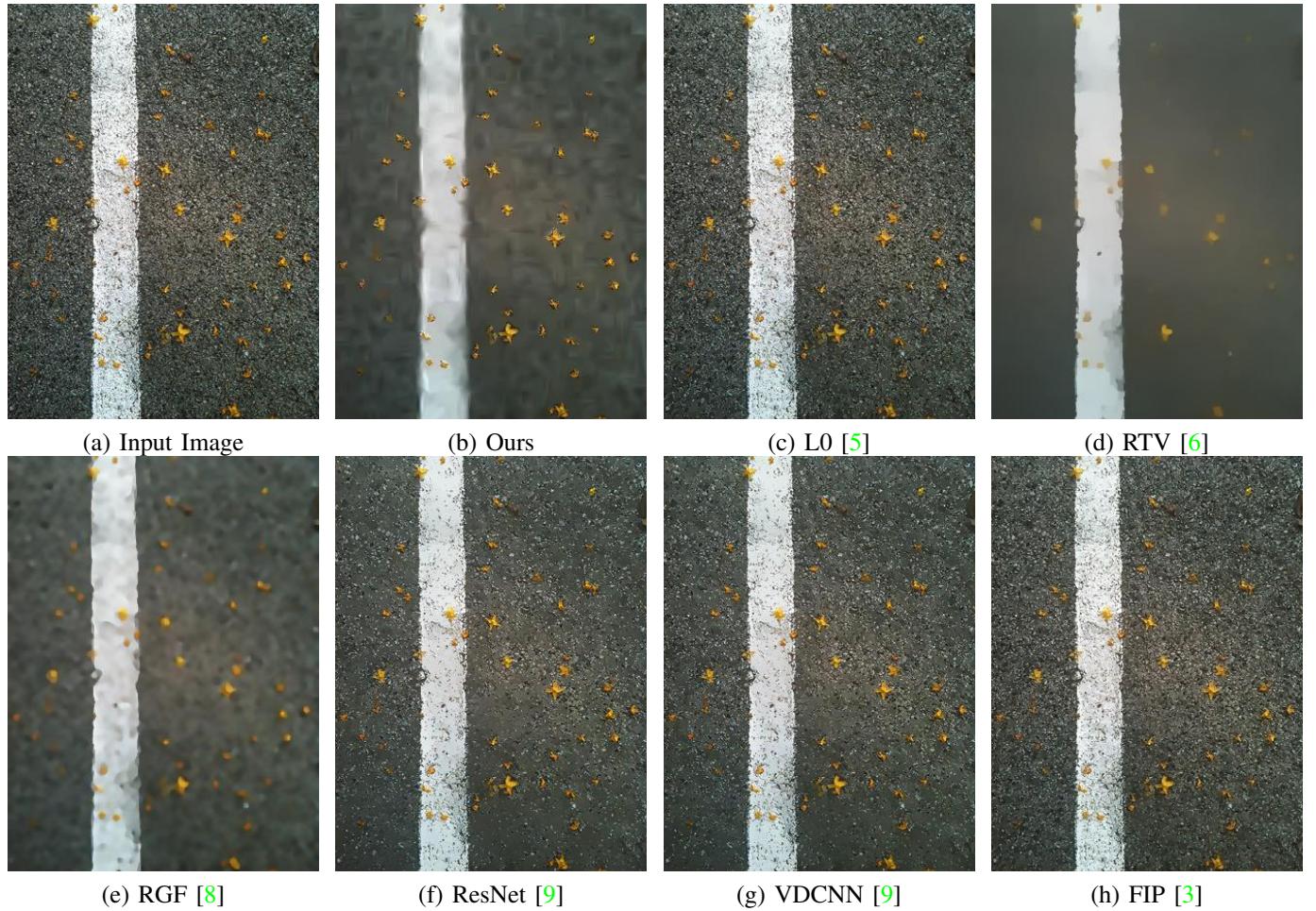


Fig. 26. Comparison of smoothed images by different methods on the image “12\_53” from the dataset in [6]

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