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EXPERIMENT 5. OPTIMUM FILTERING: FIR WIENER FILTER IMPLEMENTATION FOR NOISE REMOVAL

PART 1

LABORATORY REPORT

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Write **'rng default;'** in the beginning of your code so that the random number generation settings will be the same for all of us. **Do not forget** to **upload all MATLAB codes** to **ODTUClass.**

Task

1) **In this task, you will implement the filtering of a signal in noise. Set the parameters as follows:**
   N=1000, P is the sum of the last digit of student ID of lab group members'. If it is equal to 0,1,2,3 or 4, set P=5. Otherwise continue. a = 0.8, noise standard deviation= 1, P_{step}=0. Write R_x, r_{dx} and h. Note that these are matrices as described in equation (13) of the experiment manual. Therefore, R_x will have size P*P, r_{dx} and h will have sizes P*1.

   Write your P value. Plot input, output and desired signals on the same figure and attach it below. Also plot error signal in another figure. Write the MSE and LSE values you obtained.

   P=8

   MSE is theoretical, LSE is the practical error. As the number of samples increases, LSE approaches MSE under ideal conditions. As seen from Figure 1, MSE is obtained as 0.5781 and LSE is obtained as 0.6150, which was expected as practical error should be close to theoretical error. Figures 2,3, and 4 show input, output, desired and error signals.

```
Rx =

    3.9183    2.4044    1.9195    1.4927    1.0890    0.9995    0.8491    0.6911
    2.4044    3.9183    2.4044    1.9195    1.4927    1.0890    0.9995    0.8491
    1.9195    2.4044    3.9183    2.4044    1.9195    1.4927    1.0890    0.9995
    1.4927    1.9195    2.4044    3.9183    2.4044    1.9195    1.4927    1.0890
    1.0890    1.4927    1.9195    2.4044    3.9183    2.4044    1.9195    1.4927
    0.9995    1.0890    1.4927    1.9195    2.4044    3.9183    2.4044    1.9195
    0.8491    0.9995    1.0890    1.4927    1.9195    2.4044    3.9183    2.4044
    0.6911    0.8491    0.9995    1.0890    1.4927    1.9195    2.4044    3.9183


rdx =

    2.9490
    2.3478
    1.9178
    1.5039
    1.1357
    0.9255
    0.7165
    0.5422


h =

    0.6026
    0.1747
    0.0725
    0.0218
    0.0226
    0.0080
   -0.0167
   -0.0326


MSE =

0.578050630974656707165307805264 41


LSE =

    0.6150
```

Figure 1: Computed values for given input parameters with P=8

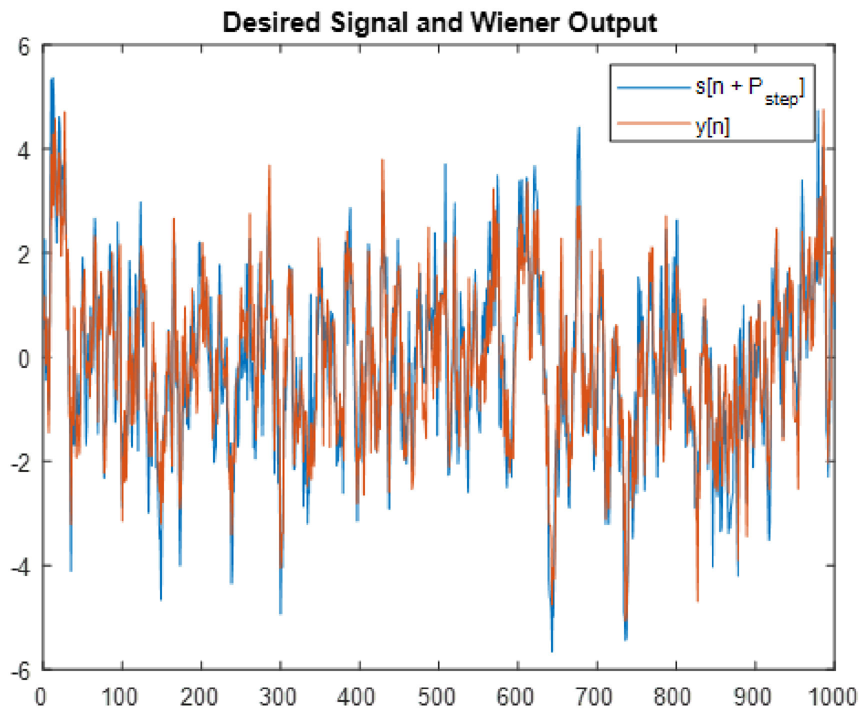


Figure 2: Wiener output and desired signal on same plot

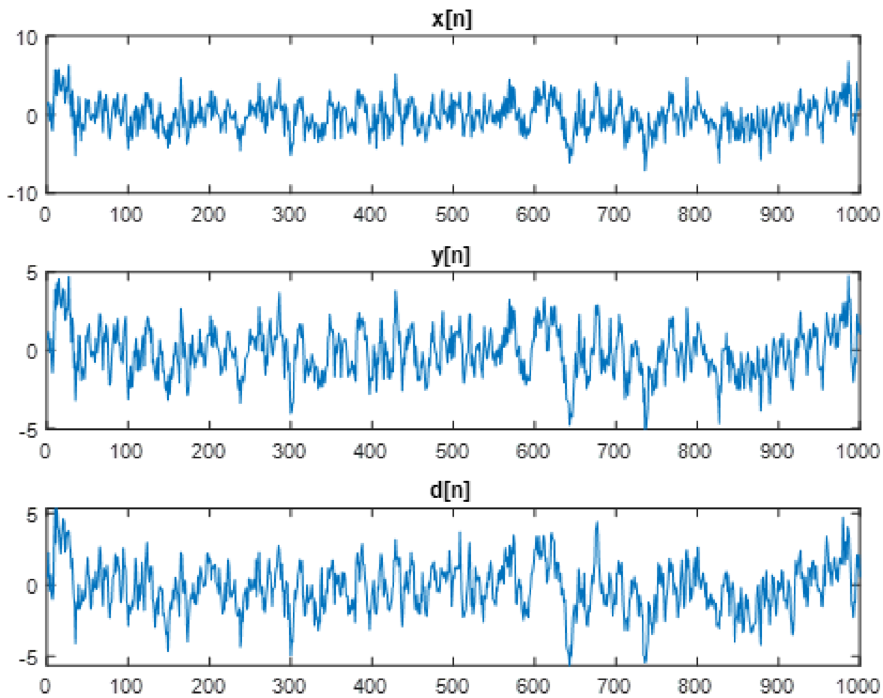


Figure 3: Input, output and desired signals respectively for Wiener filter

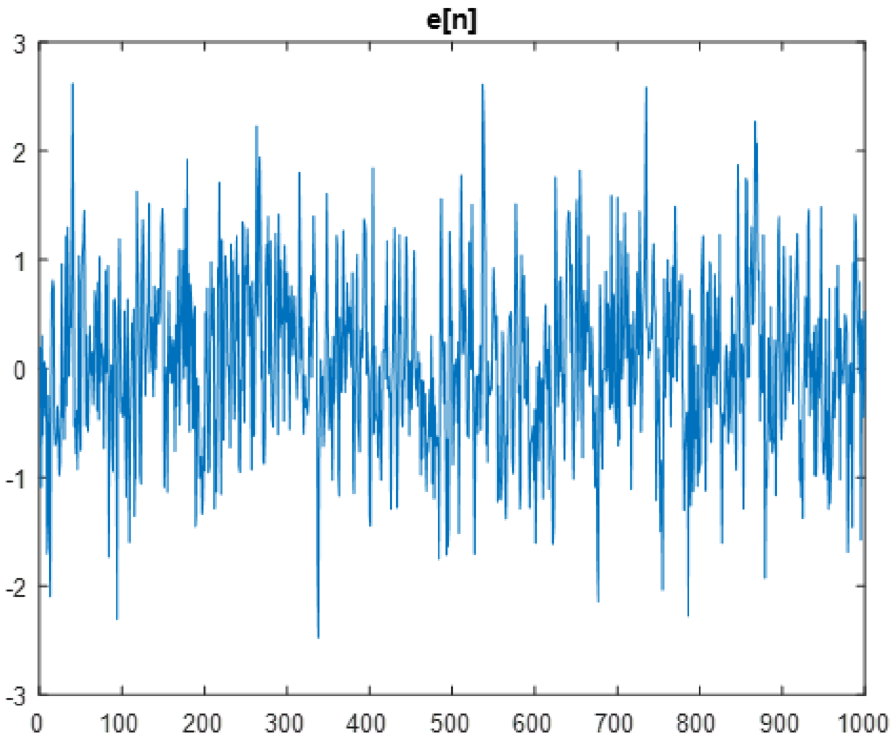


Figure 4: Error signal computed from the output of the filter and the desired signal

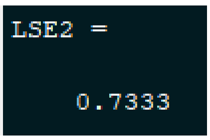
2) Compute LSE with Parks-McClellan Filter (use firpm command):

Desired Response of the filter:

Normalized Frequency (0-1 corresponds to 0-fs/2)	*Desired Amplitude*
0	1
0.4	1
0.8	0
1	0

Filter Order: 30

LSE is found as 0.7333:



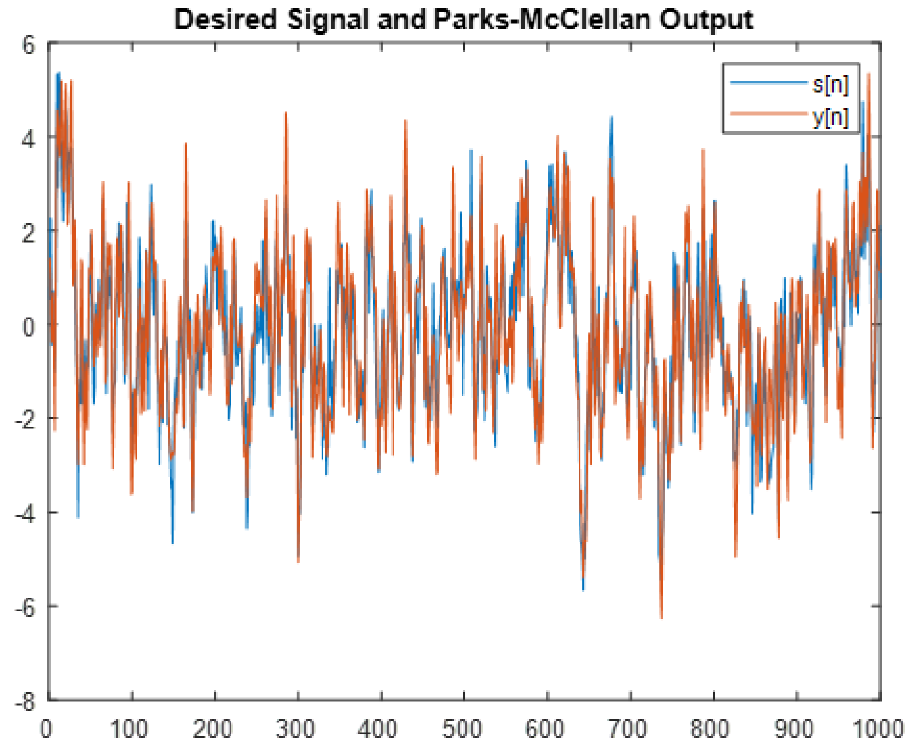


Figure 5: Parks-McClellan output and desired signal on same plot

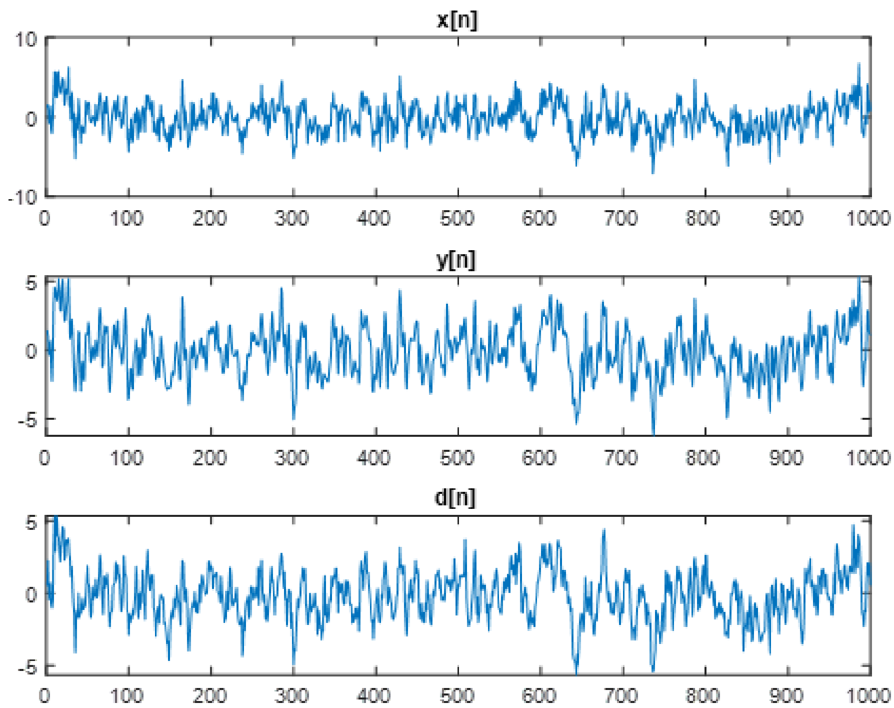


Figure 6: Input, output and desired signals respectively for Parks-McClellan filter

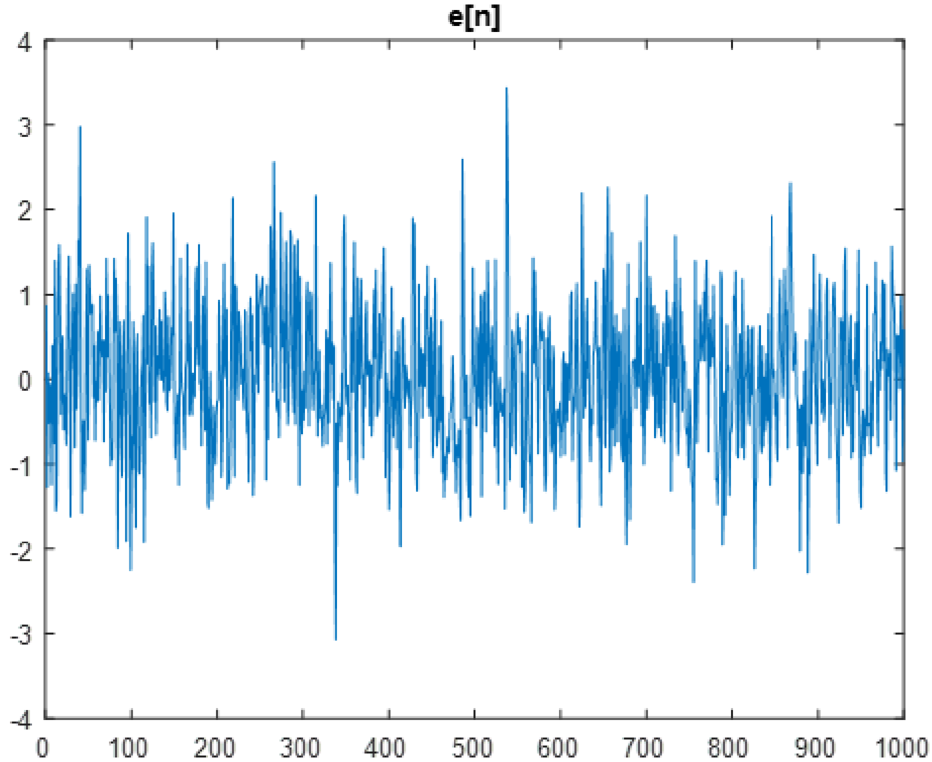


Figure 7: Error signal computed from the output of the filter and the desired signal

3) Compare the results of 1) and 2). Also comment on the results.

LSE is calculated as 0.6150 with a Wiener filter with an order of 8. LSE is calculated as 0.7333 with a Parks-McClellan filter with an order of 30. In optimum filtering, as in Wiener filter, statistical characteristics of the input signal are used to extract the useful information. Whereas, Parks-McClellan filter does not use prior knowledge about the signal, which is, thus, a deterministic filter. As seen from LSE results, having prior knowledge decreases the error between filter output and the desired signal. Not only that, but also the order of the filter has significantly decreased. A Wiener filter of order 8 gave more accurate results than a Parks-McClellan filter of order 30.

4) In this task you will perform prediction of a signal in noise. Perform the predictions for P_{step}=1,2,3,4,5. Write MSE and LSE values obtained for each case (please use a table). Also add the plot of input, desired and predicted signals on the same figure, and error signal on another figure for a specific P_{step}. Choose P_{step} as follows:
   - 0<= sum of the last digit of student ID of lab group members'<=3 P_{step}=1
   - 4<= sum of the last digit of student ID of lab group members'<=7 P_{step}=2
   - 8<= sum of the last digit of student ID of lab group members'<=11 P_{step}=3
   - 12<= sum of the last digit of student ID of lab group members'<=15 P_{step}=4
   - 16<= sum of the last digit of student ID of lab group members'<=18 P_{step}=5

For instance, if your student ID sum up to 10, you should calculate MSE and LSE values for ALL different prediction steps, but you only need to add a plot for P_{step}=3 case. Comment on your results. What happens as you increase P_{step}?

As P_{step} increases error values are increasing both theoretically and practically. This is because correlations between predicted elements and the elements that we are predicting from decreases. If we write the theoretical correlations between d[n] and x[n], which is $r_{dx}[m] = 0.8^{|m+P_{step}|}\frac{1}{1-0.8^2}$, we see that correlations decrease as clearly seen from Table 1. Therefore, we will expect more erroneous results. Also, when we investigate Figure 8, the peak value of y[n] decreases due to decrease in correlation between d[n] and x[n]. Filter coefficients are calculated from $h = R_x^{-1} r_{dx}$. Since Rx does not change, decrease in rdx also decreases the filter coefficients. Thus, output of the filter decreases as a result and we see a decreased peak value at the output if we increase P_{step}.

Table 1: MSE and LSE values for different values of P_{step}

P_{step}	1	2	3	4	5
MSE	1.3700	1.8768	2.2011	2.4087	2.5146
LSE	1.4689	1.9916	2.3818	2.6307	2.7551

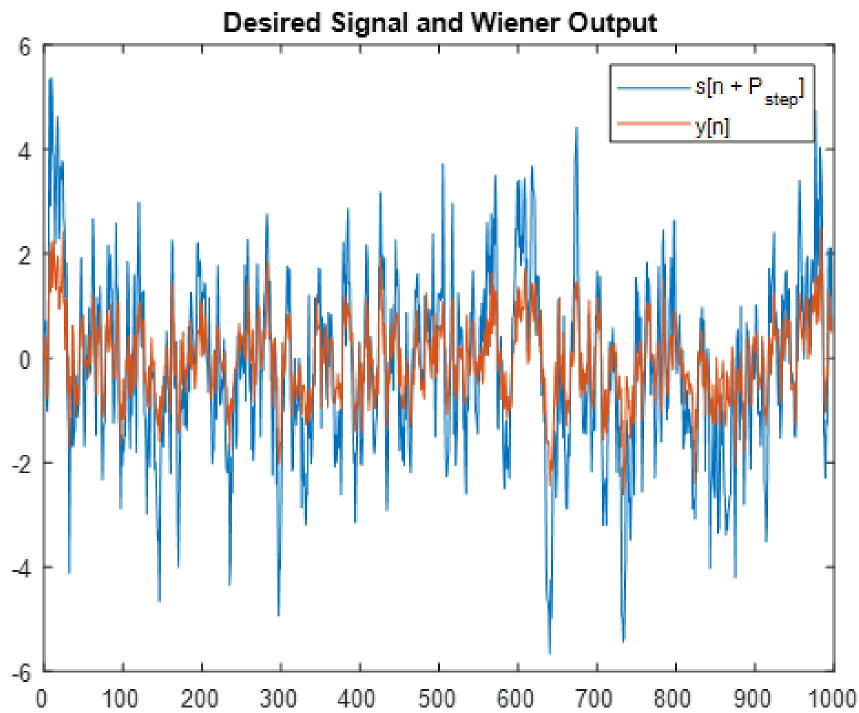


Figure 8: Wiener filter output and desired signal for P_{step}=3

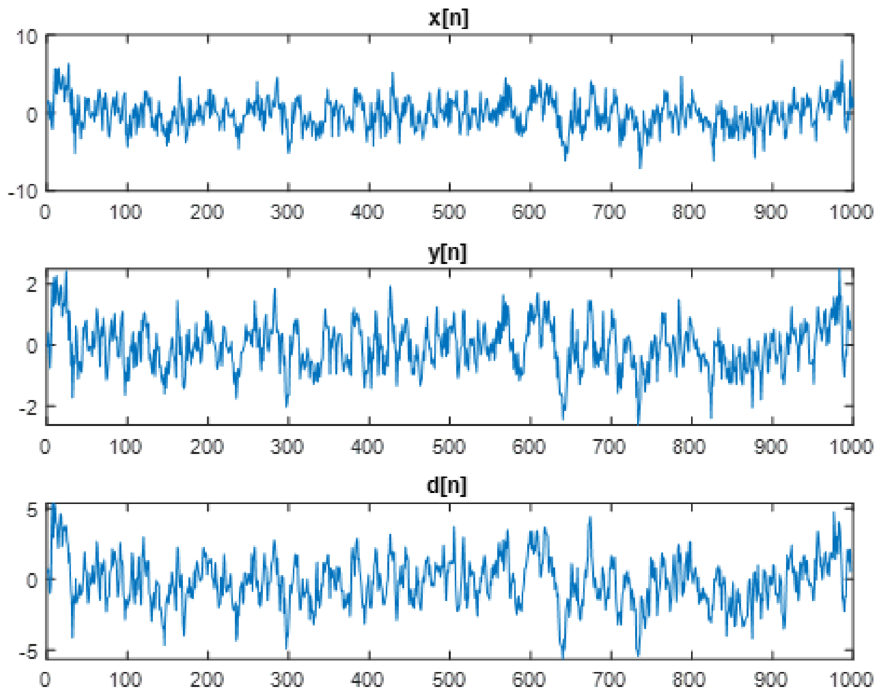


Figure 9: Input, output and desired signals respectively with a Wiener filter and P_{step}=3

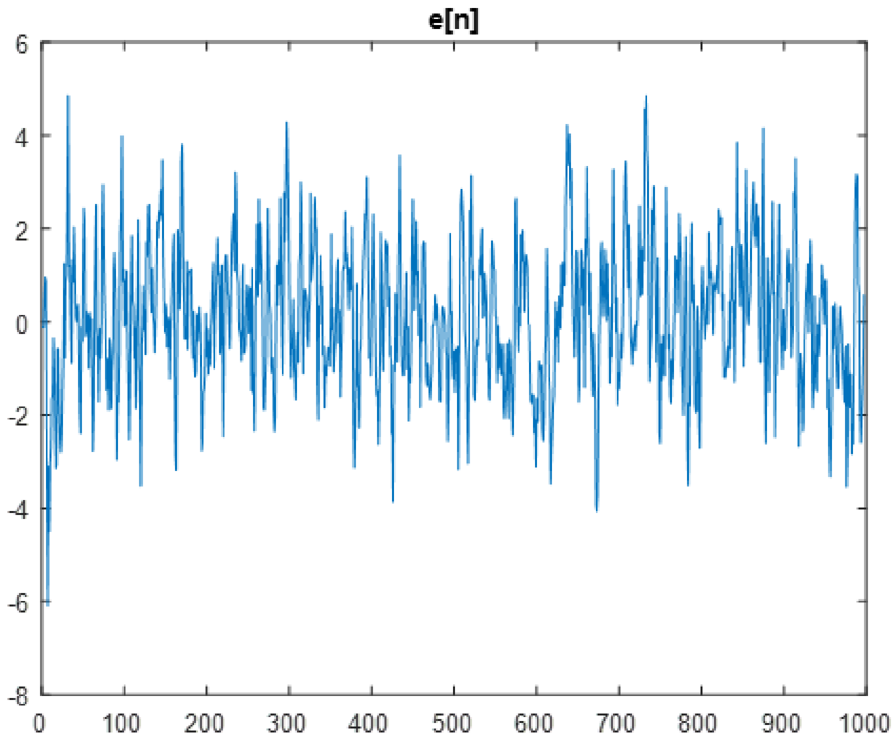


Figure 10: Error signal for P_{step}=3