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EXPERIMENT 6. SYSTEM IDENTIFICATION WITH ADAPTIVE PROCESSING, DESIGN AND IMPLEMENTATION OF LMS FILTER

PART 1

LABORATORY REPORT

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Simulation Parameters

K = 10000	- number of samples
N = 5	- length of FIR filter (unknown system)
h[n] = [1 -0.8 0.6 -0.4 0.2]	- unknown system impulse response
σ_v	- noise standard deviation of v
M	- adaptive filter length
μ	- step size of LMS algorithm

6.5.1. System Identification in MATLAB

a)

- Take M=5, μ =0.001, σ_v=0.
- Plot $e^2[n]$ versus n . Attach this plot. **Determine** the n value where convergence occurred. **Write** this value.

The n value where convergence occurred approximately at 2500 for a threshold value of μ/10.

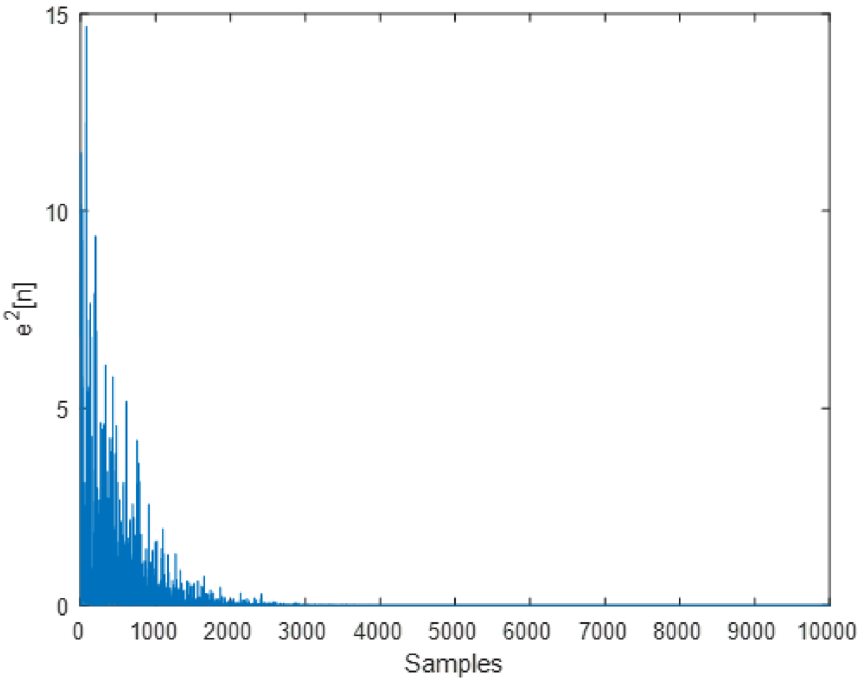

Figure 1: Convergence of the experiment where M=5, μ =0.001, σ_v=0

b) Change μ =0.1. Plot $e^2[n]$ versus n . Attach this plot. **Determine** the n value where convergence occurred. **Write** this value.

The n value where convergence occurred at 25.

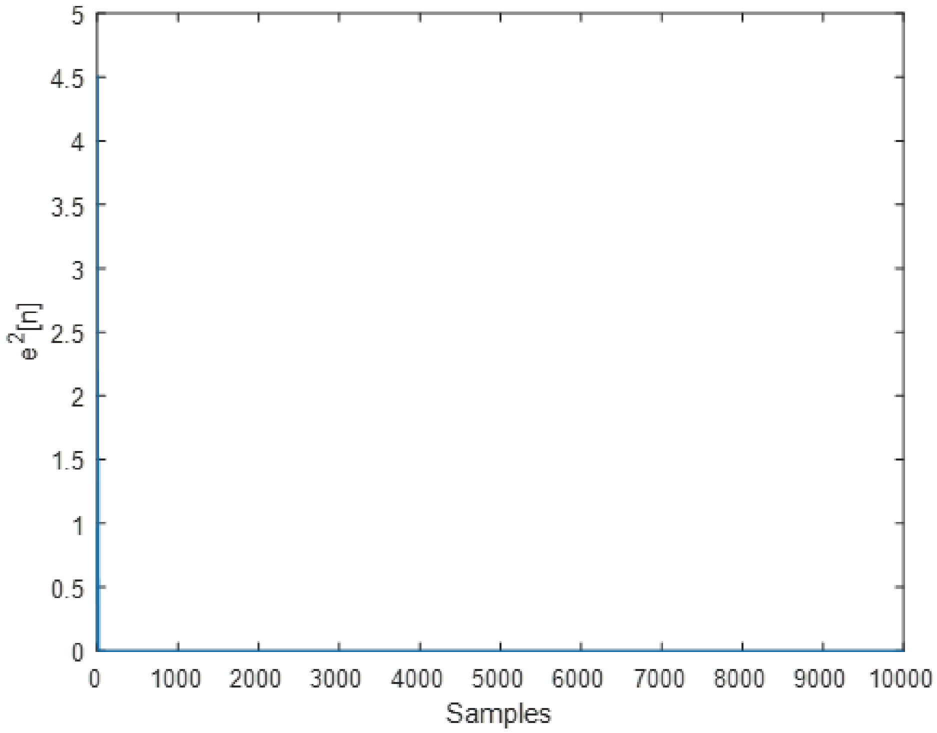

Figure 2: Convergence of the experiment where M=5, μ =0.1, σ_v=0

c) **Comment** on the effect of step size on the **convergence speed** based on your plots and observations.

Convergence speed increases when we increase the μ parameter. We obtain the gradient of the error and by moving in the opposite direction to the gradient we eventually reach the minimum point. This is called gradient descent. When our step size is small it takes more time to reach the minimum point. Hence, the convergence speed increases when we increase the μ parameter. In our experiment it took approximately 10 times more iterations to reach the minimum error as can be seen in Figure 1 and Figure 2, which was expected.

d) Write the coefficients of the adaptive filter after convergence. **Comment** on it.

Table 1: Adaptive Filter Coefficients for the experiment where M=5, σ_v=0

Adaptive Filter Coefficients	**1.0**	**-0.8**	**0.6**	**-0.4**	**0.2**
Unknown System Coefficients	**1.0**	**-0.8**	**0.6**	**-0.4**	**0.2**

As there is no noise component in our signal, the impulse response of the filter should be equal to the unknown system impulse response ideally. Because there is nothing to filter out. In our experiment we obtained the same coefficients with the unknown system coefficients as expected as seen from Table 1.

e) Change noise standard deviation from σ_v=0 to σ_v=0.2. Take μ =0.001. Plot $e^2[n]$ versus n . Attach this plot. **Compare** it with the noiseless case, **part a)**. **Comment** on it. **What are the adaptive filter coefficients?**

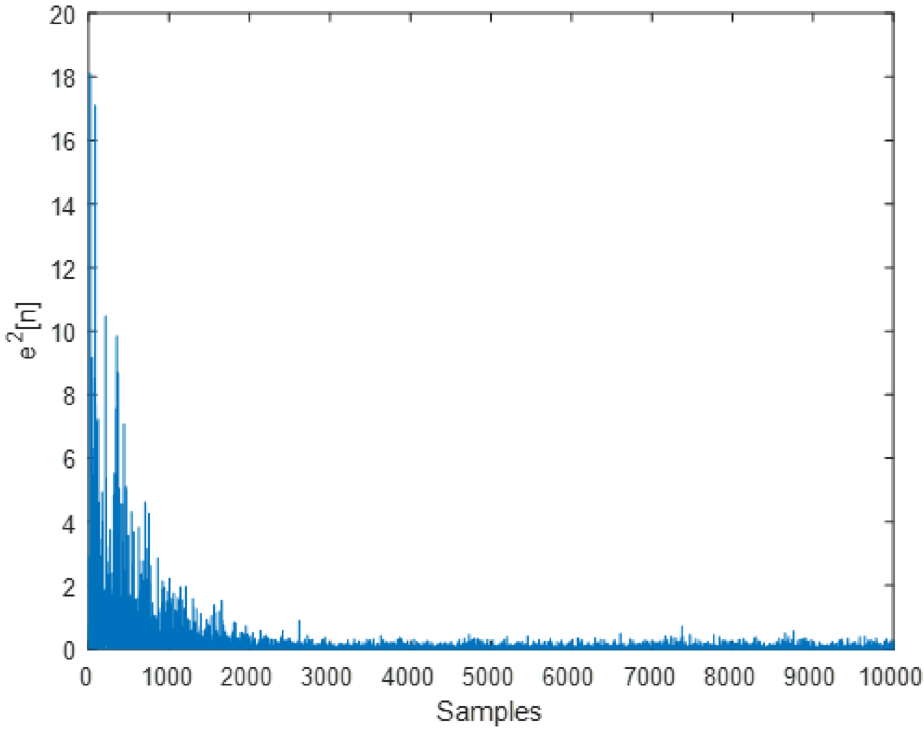

Figure 3: Convergence of the experiment where M=5, μ =0.001, σ_v=0.2

Table 2: Adaptive Filter Coefficients for the experiment where M=5, μ =0.001, σ_v=0.2

Adaptive Filter Coefficients	**0.9926**	**-0.8009**	**0.5953**	**-0.4039**	**0.2027**

In this case, we have AWGN in our signal. As our filter has a finite number of coefficients it is impossible to filter all of the noise. We expect to converge to a minimum point using gradient descent with our adaptive filter. However, it is not possible to reach the minimum point with gradient descent; the results will oscillate around it after some point. In our experiment we obtained filter coefficients that are similar to the unknown filter coefficients but with a small error.

f) Keep the noise standard deviation at σ_v=0.2. Take μ =0.1. Plot $e^2[n]$ versus n. **Attach** this plot. **Compare** it with the noiseless case, **part b)**. **Comment** on it. **What are the adaptive filter coefficients?**

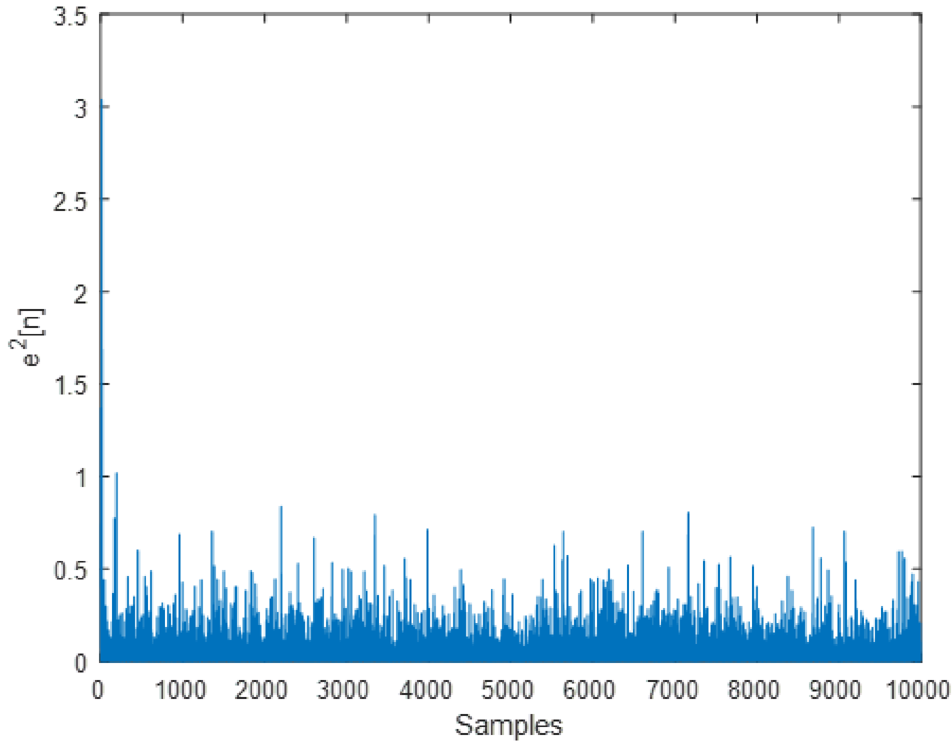

Figure 4: Convergence of the experiment where M=5, μ =0.1, σ_v=0.2

Table 3: Adaptive Filter Coefficients for the experiment where M=5, μ =0.1, σ_v=0.2

Adaptive Filter Coefficients	**1.0645**	**-0.7741**	**0.6525**	**-0.3636**	**0.2299**

Similar to Part e, this time we have AWGN in our signal and everything else is the same as in Part b. We expect to reach a minimum error which is greater than 0 because we cannot filter the AWGN completely. Our result oscillated after some point and after convergence we obtained an LSE value of 0.0592.

g) Write the approximate final error power after convergence for **part e)** and **part f)**. Comment on the effect of step size on the **misadjustment**.

For Part e, LSE value is 0.0408 and for Part f LSE value is 0.0592. In gradient descent, when step size is large it is harder to reach the bottom of the error curve because it is very likely to oscillate around the minimum point with a large step size. Hence, a larger step means faster convergence but it comes with a decrease in precision. In our experiment, we obtained higher error when we increased the step size, which agrees with the expected result. Thus, when the step size increases, misadjustment also increases.

h)

- Take M=4, μ =0.001, σ_v=0.
- Plot $e^2[n]$ versus n. **Attach** this plot. **Compare** it with the previous noiseless case where M=5, **part a)**. **Write** the coefficients of the adaptive filter after convergence. **Comment** on it.

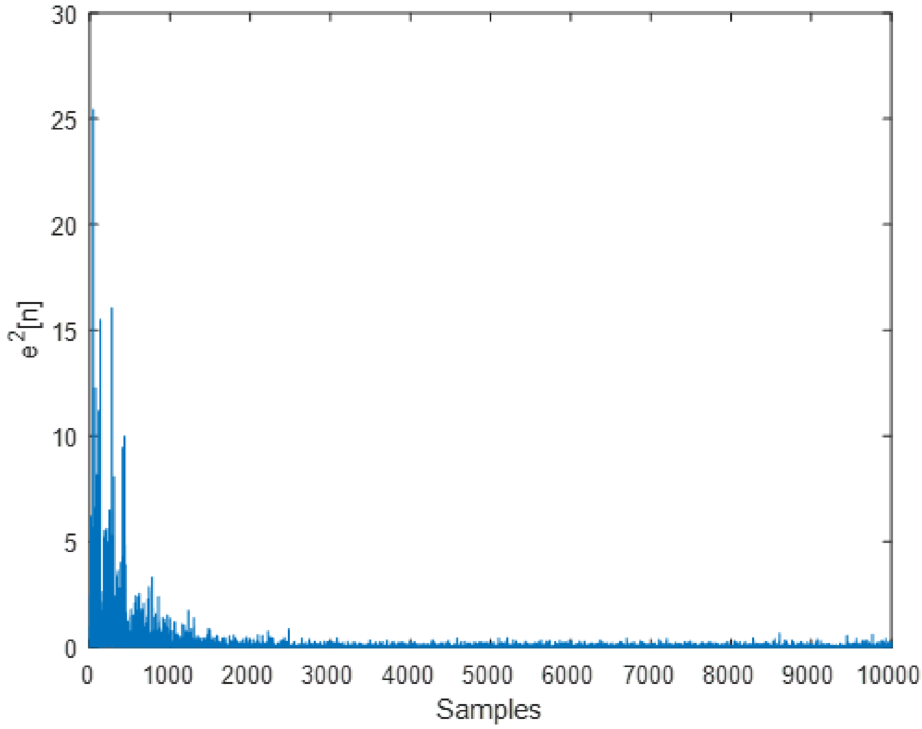

Figure 5: Convergence of the experiment where M=4, μ =0.001, σ_v=0

Table 4: Adaptive Filter Coefficients for the experiment where M=4, μ =0.001, σ_v=0

Adaptive Filter Coefficients	**0.9971**	**-0.8074**	**0.6022**	**-0.4041**

When we decrease the number of filter coefficients, we try to simulate the frequency response of a 5 point filter with a 4 point filter. First, 4 coefficients are not the same as the unknown filter coefficients because our representation capacity is not enough for reaching the solution. We see the effect of higher order terms, in this case it is 5th, in the first 4 coefficients.

i)

- Take M=10, μ =0.001, σ_v=0.
- Plot $e^2[n]$ versus n. **Attach** this plot. **Compare** it with the previous noiseless case where M=5, **part a). Write** the coefficients of the adaptive filter after convergence. **Comment** on it.

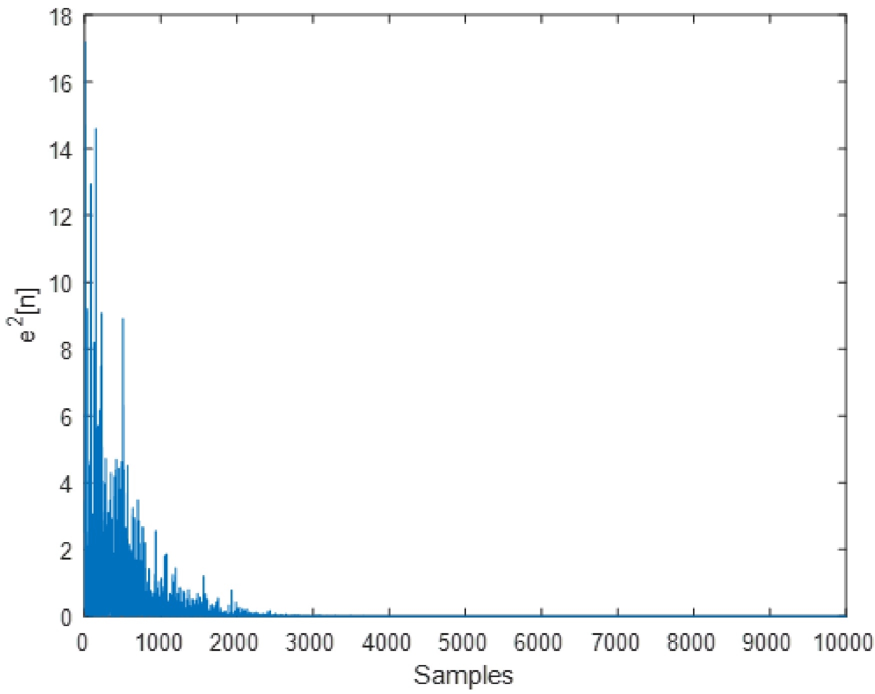

Figure 6: Convergence of the experiment where M=10, μ =0.001, σ_v=0

Table 5: Adaptive Filter Coefficients for the experiment where M=10, μ =0.001, σ_v=0

AFC	**1**	**-0.8**	**0.6**	**-0.4**	**0.2**	**0**	**0**	**0**	**0**	**0**

We set the filter length equal to 10. Our target filter is a 4th order filter and now we are using more than necessary coefficients to replicate the response of the filter. In our experiment, we see that filter coefficients at k=0 to k=N-1 are the same with the unknown filter, and coefficients with k=N to k=M-1 came out as 0. Hence it is not necessary to use a higher order filter.

6.5.2. Noise Cancellation in MATLAB

j)

- Take M=5, μ =0.001, σ_v=1.
- H is the same as system identification.
- Let x[n] be a 200 Hz sin wave, which is sampled with 10 kHz.
- Plot e[n] and x[n] on the same graph. **Attach** this plot.
- Plot $(e[n]-x[n])^2$ versus n. **Attach** this plot. **Determine** the n value where convergence occurred. **Write** this value.

The n value where convergence occurred approximately at 3000.

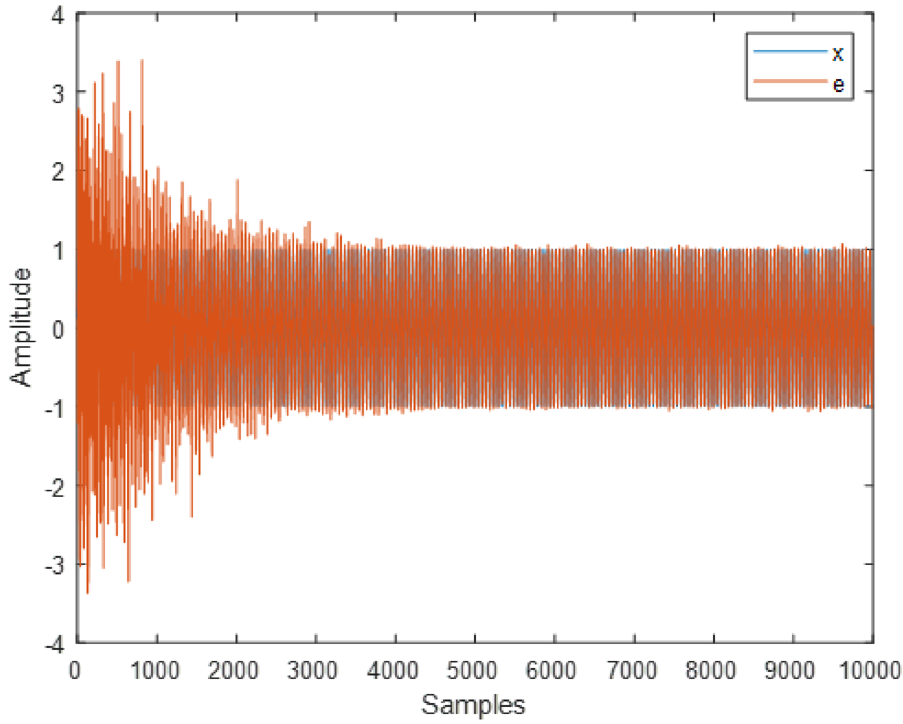

Figure 7: Results of noise cancellation experiment where M=5, μ =0.001, σ_v=1

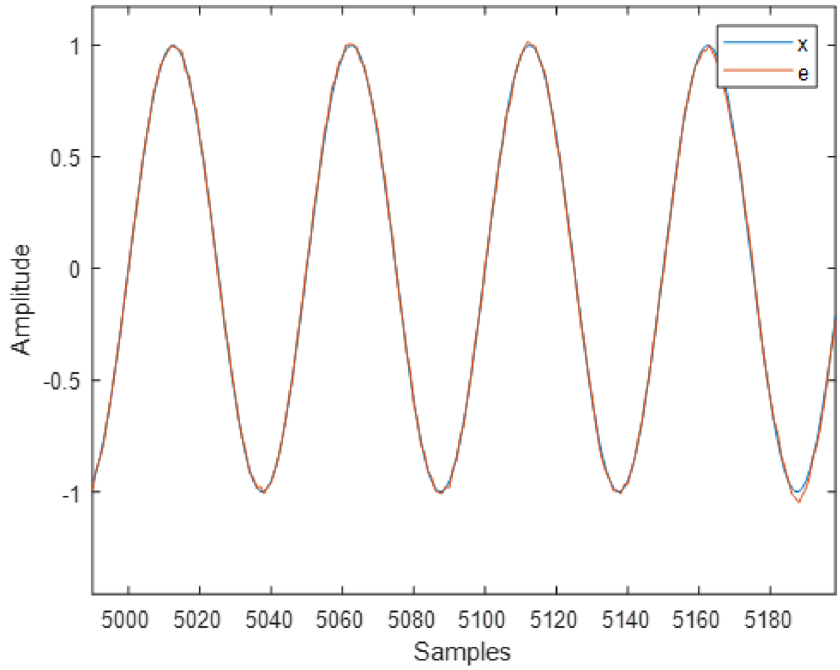

Figure 8: Zoomed in results of the experiment where M=5, μ =0.001, σ_v=1

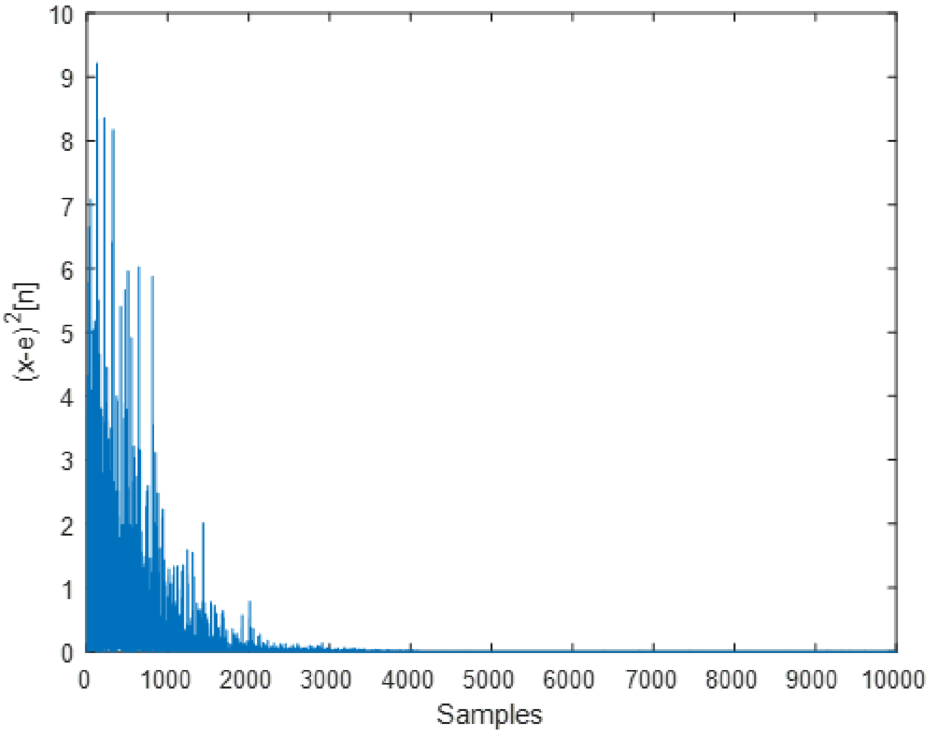

Figure 9: Convergence of the experiment where M=5, μ =0.001, σ_v=1

k) Change μ =0.01. Plot e[n] and x[n] on the same graph . Plot $(e[n]-x[n])^2$ versus n. **Attach** these plots. **Determine** the n value where convergence occurred. **Write** this value.

The n value where convergence occurred approximately at 250.

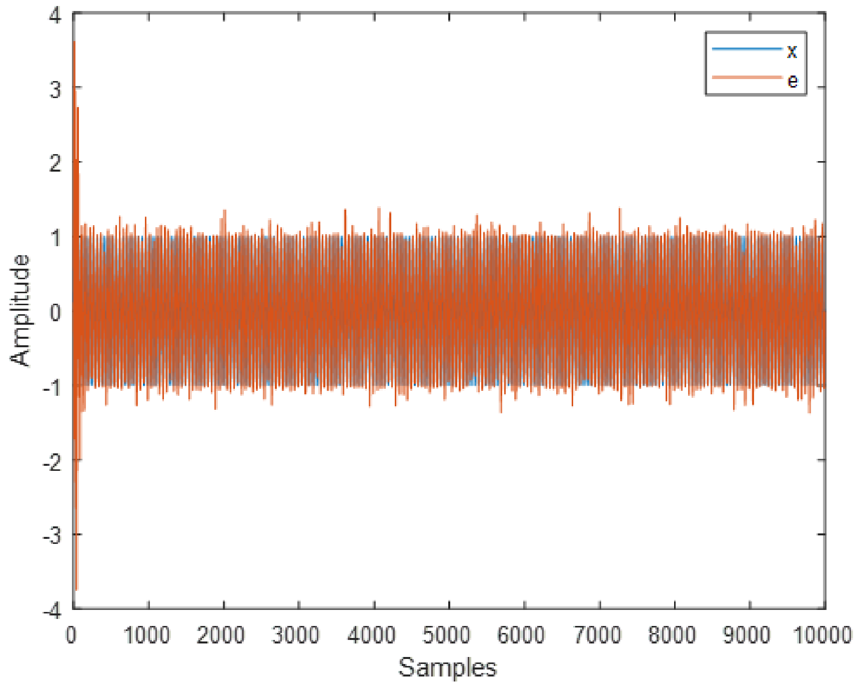

Figure 10: Results of noise cancellation experiment where M=5, μ =0.01, σ_v=1

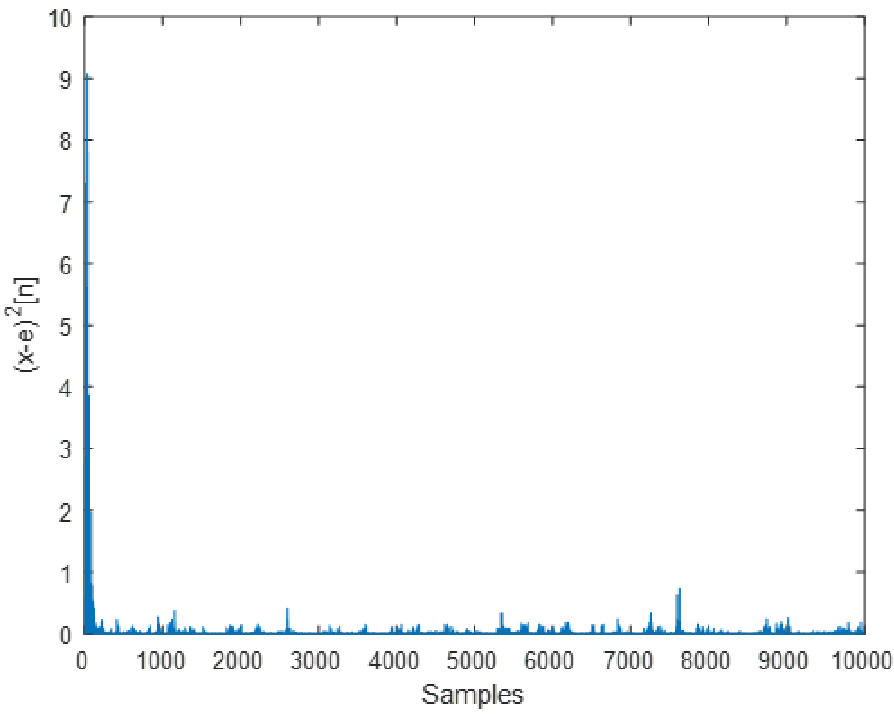

Figure 11: Convergence of the experiment where M=5, μ =0.01, σ_v=1

l) Write the coefficients of the adaptive filter after convergence. **Comment** on it.

We expect adaptive filter coefficients to converge to the channel filter coefficients. Because input is noise and we define our cost function as the difference between channel filter output added to signal and adaptive filter output. As our adaptive filter does not get any information about the x[n] at its input we expect y[n] to converge to s[n] and hence, the error approaches to x[n]. However, because of the existence of the noise, we cannot converge completely as seen from Table 6 and Table 7.

Table 6: Adaptive Filter Coefficients for the experiment where M=5, μ =0.001, σ_v=1 (Part j)

Adaptive Filter Coefficients	**0.9983**	**-0.7981**	**0.6057**	**-0.3907**	**0.2130**

Table 7: Adaptive Filter Coefficients for the experiment where M=5, μ =0.001, σ_v=1 (Part k)

Adaptive Filter Coefficients	**0.9986**	**-0.8040**	**0.6052**	**-0.4012**	**0.2044**

m) Change noise standard deviation from σ_v=1 to σ_v=2. Take μ =0.001. Plot $(e[n]-x[n])^2$ versus n . **Attach** this plot. Compare it with **part j)**. **Comment** on it.

$$h_k[n+1] = h_k[n] + \mu e[n]x[n-k], \quad k = 0,1,\ldots,M-1 \qquad \textbf{(1)}$$

As we try to estimate filter coefficients, we multiply step size with the error and x[n-k], which is v[n-k] for noise cancellation as seen from Equation 1. There are two effects playing a role in the rate of convergence, one is the error and the other is the input. As the error is decreasing constantly, the main effect for convergence rate is the input of the adaptive filter, which is the noise and its power.

When we increase the noise standard deviation, the noise power increases. The increase of the power increases convergence rate according to Equation 1, because, to increase the power of noise we just simply multiply the noise with standard deviation, which in this case is 2, so we end up with an equivalent system of doubled step size and unit variance. Thus, the convergence rate is increased, and we expect the convergence after the 1500th sample approximately as expected and seen from Figure 12.

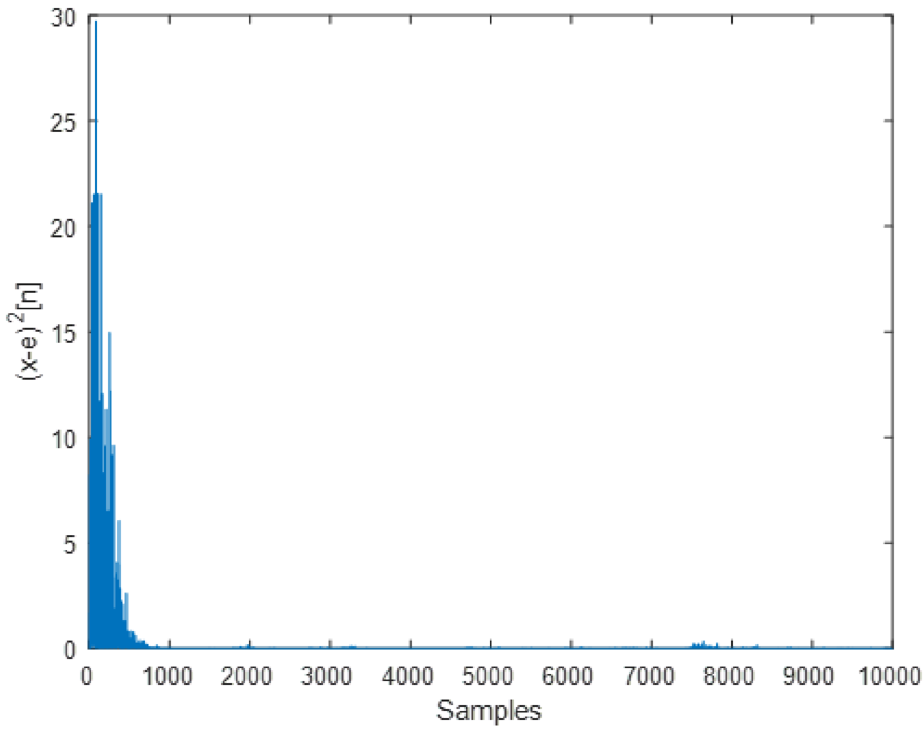

Figure 12: Convergence of the experiment where M=5, μ =0.01, σ_v=2

Table 8: Adaptive Filter Coefficients for the experiment where M=5, μ =0.01, σ_v=2

Adaptive Filter Coefficients	**1.0076**	**-0.7914**	**0.6119**	**-0.3890**	**0.2133**

n)

- Take M=4, μ =0.001, σ_v=1.
- Plot $(e[n]-x[n])^2$ versus n . **Attach** this plot. Compare it with **part j)**. **Write** the coefficients of the adaptive filter after convergence. **Comment** on it.

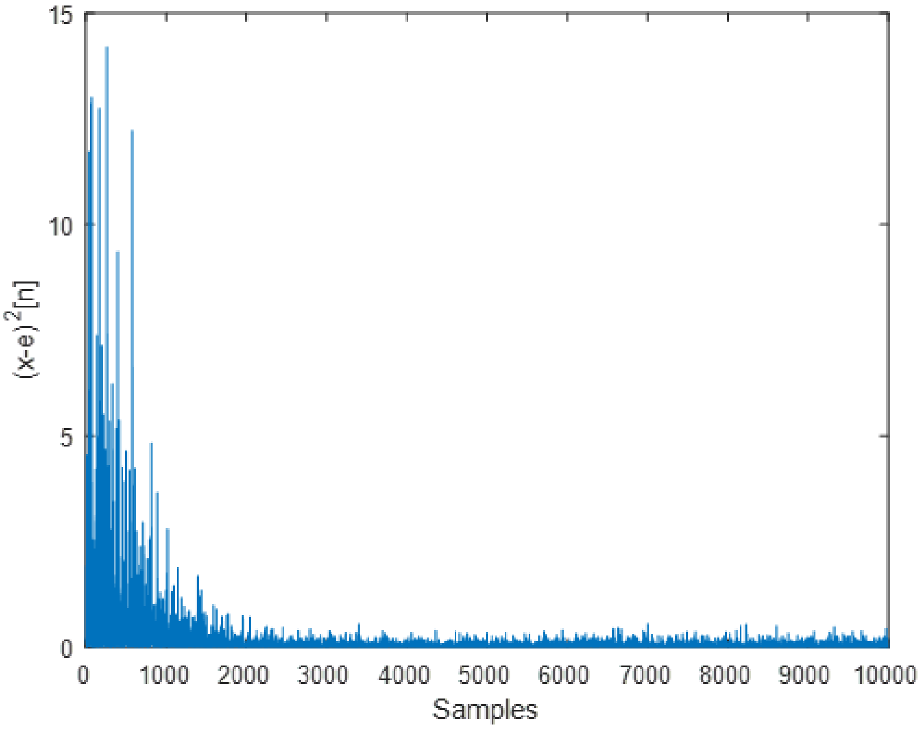

Figure 13: Convergence of the experiment where M=4, μ =0.001, σ_v=1

Table 9: Adaptive Filter Coefficients for the experiment where M=4, μ =0.001, σ_v=1

Adaptive Filter Coefficients	**1.0198**	**-0.7791**	**0.6177**	**-0.3686**

When we decrease the number of filter coefficients, we try to simulate the frequency response of a 5 point filter with a 4 point filter. First, 4 coefficients are not the same as the channel filter coefficients because our representation capacity is not enough for reaching the solution. We see the effect of higher order terms, in this case it is 5th, in the first 4 coefficients.

o)

- Take M=10, μ =0.001, σ_v=1.
- Plot $(e[n]-x[n])^2$ versus n. **Attach** this plot. Compare it with **part j). Write** the coefficients of the adaptive filter after convergence. **Comment** on it.

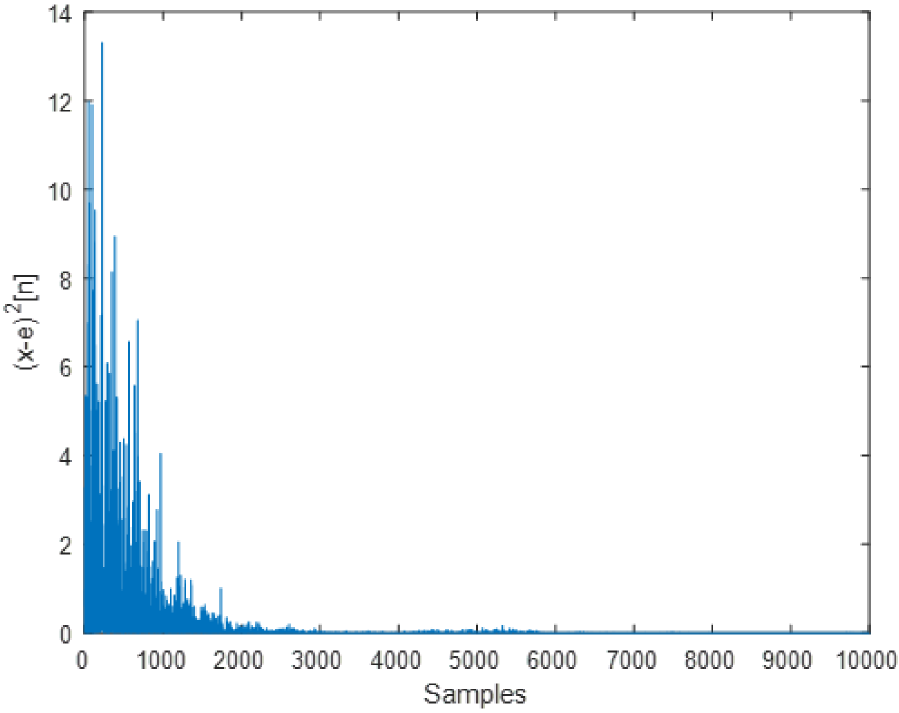

Figure 14: Convergence of the experiment where M=10, μ =0.001, σ_v=1

Table 10: Adaptive Filter Coefficients for the experiment where M=10, μ =0.001, σ_v=1

AFC	**1.0162**	**-0.7866**	**0.6100**	**-0.3937**	**0.2027**	**-0.0018**	**-0.0062**	**-0.0098**	**-0.0139**	**-0.0176**

We set the filter length equal to 10. Our target filter is a 4th order filter and now we are using more than necessary coefficients to replicate the response of the filter. In our experiment, we see that filter coefficients at k=0 to k=N-1 are the same with the channel filter, and coefficients with k=N to k=M-1 came out as approximately 0. Hence it is not necessary to use a higher order filter.