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EXPERIMENT 4. DECIMATION, INTERPOLATION AND PHASE-LOCKED LOOP

PART 1
LABORATORY REPORT

Student 1: Abdullah Canbolat, 2304244
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Let M, i.e., decimation factor, be equal to the last digit of your student ID. You can use the student ID of either of the group members. If M is zero or one then take M+2.
2167054

Let K be the 2nd last digit of your student ID, e.g., if your student ID is 1234567, then K is equal to 6. .If K is zero or one then take K+2.
2167054

Note that for M and K, you should use the same student ID.

Let L, i.e., interpolation factor, be equal to the last digit of the remaining student ID. That is, if you used your student ID to decide M, then you should use your partner's student ID to decide L or vice versa. If L is zero or one take L+2. If L=M, then take L+1.
2232395

Let N be the 2nd last digit of the student ID, which is used to determine L. If N is zero or one take N+2.
2232395

To get full credit, please include ALL plots, comments, etc., as indicated, with APPROPRIATE TIME and FREQUENCY axis. Otherwise, you will LOSE some points.

IMPORTANT NOTE:** **In the report, some parts are parametrized with respect to M, K, L, and N. Because of that reason, there might be some incompatible tasks such as plot a sine with frequency 100 Hz, in 0-20 ms (in this case, it will contain only two periods) and plot its interpolated and decimated versions (which may not contain even one waveform). In those situations, please consult me, so that we can arrange better parametrization. If there is no problem, you can proceed with your tasks.

For the remaining parts take M, K, N and L as defined at the beginning of the report, and **write** the corresponding values for these parameters. If you do not **write** corresponding values, you will lose some points.

M=4, K=5, L=5, N=9

a) Write a code in MATLAB to implement **decimation** operation. Given the input signal, and decimation factor, M, the code obtains the output. The **input** and **output** signals are **plotted** both in **time** and **frequency**.

MATLAB code for function "decimation" is added to MATLAB .zip files.

a)1) Generate a **100*K Hz (for instance if K is 4, 100*4=400 Hz) sine sequence** in **0-20*N ms time range (for instance if N is 5, 20*5=100 ms)** with **sampling rate 4000*K samples/sec**. **Plot** it and output in **time** using the **same time axis**, i.e., time difference between consecutive sequence elements is **1/(4000*K)**. Also please **plot** both of the waveforms on the **same figure**. **Do not forget** to indicate which waveform belongs to which signal. You can use **legend** command in MATLAB for this purpose. **Comment** on your results.

In decimation operation, we just apply a low-pass filter with cut-off frequency π/M and then take (Mn)th element, which is done by resample command, it is expected to see the time range is decreased at the output, which is observed on Figure 1. Therefore, the duration of the signal decreased by 4. Ideally, amplitudes of input and output should be the same, however in our results, we saw that amplitude of the output is less than the amplitude of input, this is because the filter used is not an ideal filter.

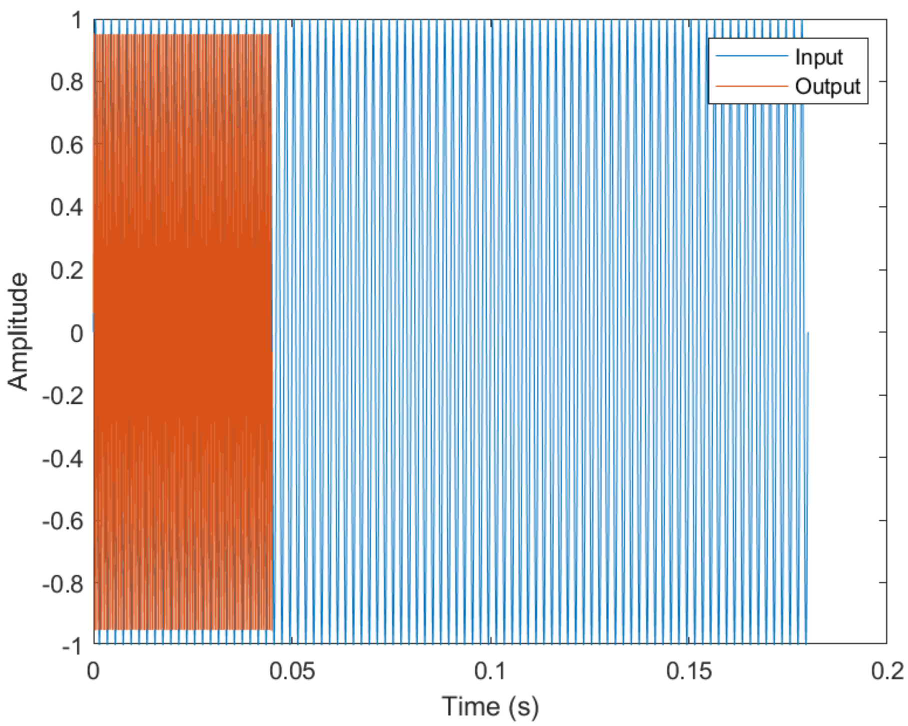

Figure 1: Sine wave with frequency 500 Hz and its decimated output by 4

a)2) Plot the input and output in part a)1) in frequency. Please use fftshift command in your plots so that center frequency corresponds to 0 Hz in your plot. Also please **plot** both of the frequency contents on the **same figure**. **Do not forget** to indicate which frequency content belongs to which signal. **Comment** on your results.

As explained in Part a)1) decimation operation increases the frequency of the waveform by discarding the elements between M*(n-1) and M*nth elements, also, total time of the waveform decreases if the sample rate is kept constant, therefore, it will generate more periodic components for the same time. Furthermore, the energy of the signal decreases as we sample some of the elements in the signal, with the rate of decimation. Therefore, we see the plot in Figure 2, which shows that output of the signal has M = 4 times frequency and the Fourier coefficient is decreased by M.

We can also explain this occurrence by Parseval's relation:
Since the time range is decreased at the output, we see increased frequency at the output. Power of the frequencies are decreased. We can understand the decrease in power by Parseval's Theorem. Our time signal lost its samples and its energy is ¼ of this previous value. By Parseval's Theorem, power of frequencies should also decrease with the same ratio, which is what we observe in Figure 2.

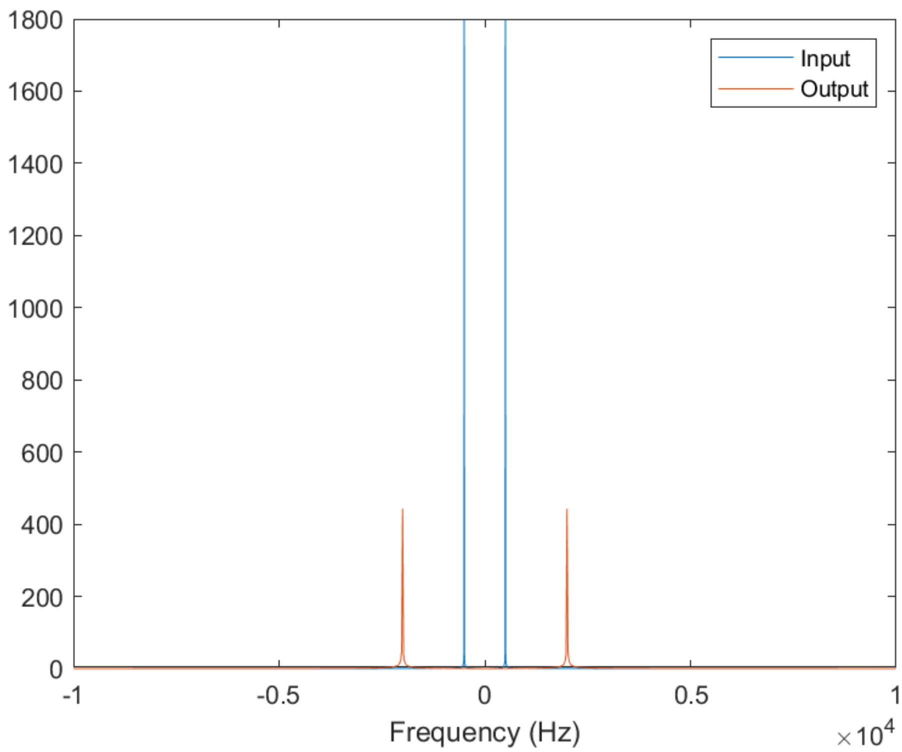

Figure 2: DFT waveform of signals in Figure 1

a)3) Generate a **100*K Hz triangle sequence** in **0-20*N ms time range** with **sampling rate 4000*K samples/sec**. **Plot** it and output in **time** using the **same time axis**, i.e., time difference between consecutive sequence elements is **1/(4000*K)**. Also please **plot** both of the waveforms on the **same figure**. **Do not forget** to indicate which waveform belongs to which signal. **Comment** on your results.

Duration of the signal decreased to ¼ times its previous value and the fundamental frequency is increased to 4 times its previous value. Which is expected considering Heisenberg's Uncertainty Principle. Also, from Figure 3, we can see that the power of the signal is decreased, because the duration is decreased with the same samples. If we have a closer look we will observe the change in shape of waveform

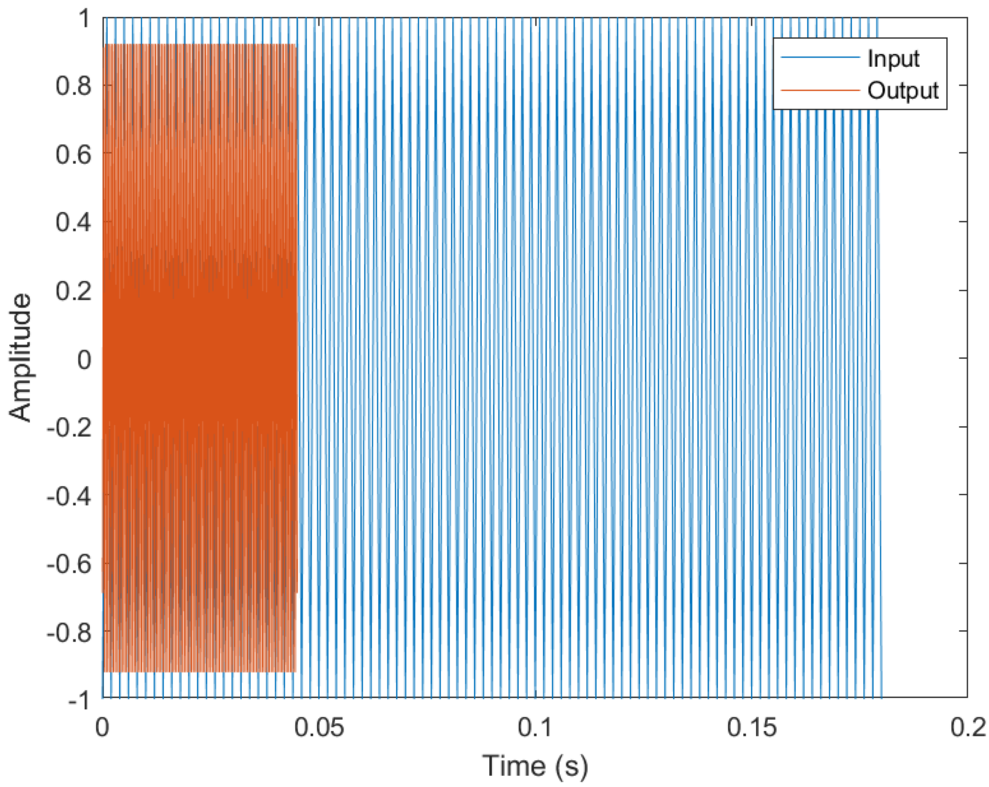

Figure 3: Triangle wave with frequency 500 Hz and its decimated output by 4

a)4) Plot the input and output in part a)3) in frequency. Please use fftshift command in your plots so that center frequency corresponds to 0 Hz in your plot. Also please **plot** both of the frequency contents on the **same figure**. **Do not forget** to indicate which frequency content belongs to which signal. **Comment** on your results.

As we can see from Figure 4, the power of the first and second harmonics are decreased. On the other hand, higher order harmonics are diminished because of the low pass filtering. In this operation we filter out the frequency samples higher than 10000/4 because they might cause aliasing after expanding the frequency domain. Frequency of all harmonics smaller than 2.5 kHz are increased by M = 4. All other comments are also valid for triangular wave case.

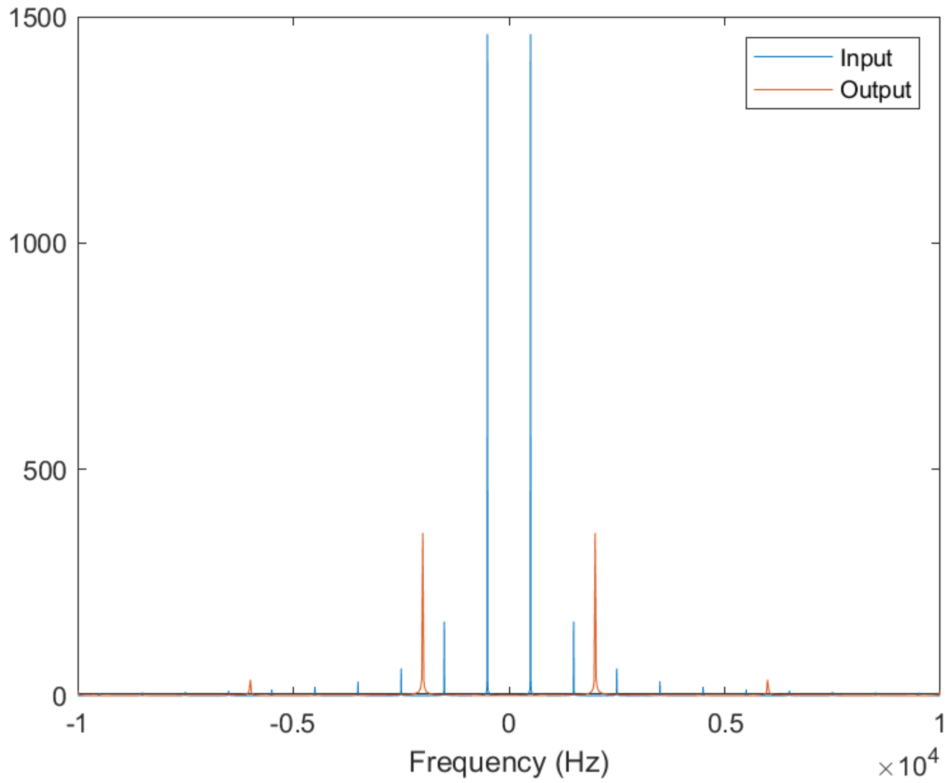

Figure 4: DFT waveform of signals in Figure 3

b) Write a code in MATLAB to implement **interpolation** operation. Given the input signal, and interpolation factor, L, the code obtains the output. The **input** and **output** signals are **plotted** both in **time** and **frequency**.

MATLAB code for function "interpolation" is added to MATLAB .zip files.

b)1) Take the **sine sequence in part a)1) as input**. **Plot** it and output in **time** using the **same time axis**, i.e., time difference between consecutive sequence elements is **1/(4000*K)**. Also please **plot** both of the waveforms on the **same figure**. **Do not forget** to indicate which waveform belongs to which signal. **Comment** on your results.

When we look at Figure 5, we see that the duration of the signal is increased by L = 5. It is expected since we add L-1 zeros between consecutive terms in the original signal. Then, we smooth the signal by applying a low-pass filter with a cutoff frequency of π/L in the discrete domain. That is why it looks similar to the original curve and duration is increased when sampling rate is kept constant.

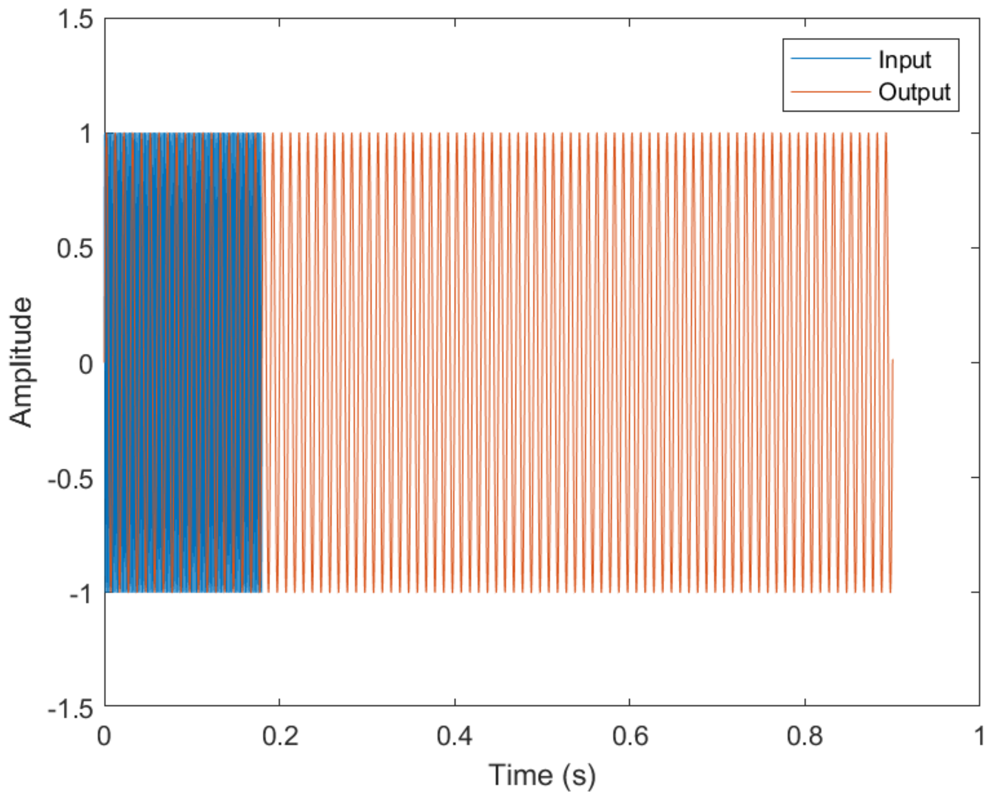

Figure 5: Sine wave with frequency 500 Hz and its interpolated output by 5

b)2) Plot the output in part b)1) in frequency. Please use fftshift command in your plots so that center frequency corresponds to 0 Hz in your plot. Also please **plot** both of the frequency contents on the **same figure**. **Do not forget** to indicate which frequency content belongs to which signal. **Comment** on your results.

When we look at Figure 6, we see that frequency of the output signal is decreased. Also, zero padded signal has the same power as the original signal, by using a filter with gain L, we restore the amplitude of the signal in the time domain. Therefore, we see a Fourier coefficient with amplitude increased by L and frequency decreased by L.

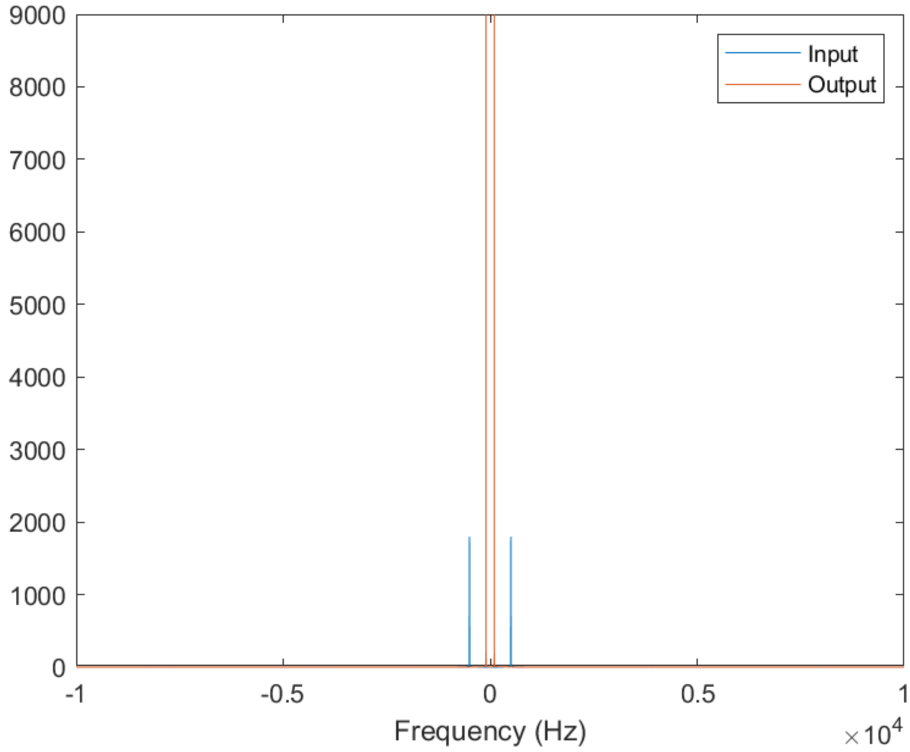

Figure 6: DFT waveform of signals in Figure 5

b)3) Take the **triangle sequence in part a)3) as input**. **Plot** it and output in **time** using the **same time axis**, i.e., time difference between consecutive sequence elements is **1/(4000*K)**. Also please **plot** both of the waveforms on the **same figure**. **Do not forget** to indicate which waveform belongs to which signal. **Comment** on your results.

We get the similar results for triangle wave as in Part b)1), in other words, duration of the signal is increased by L and frequency is decreased by L.

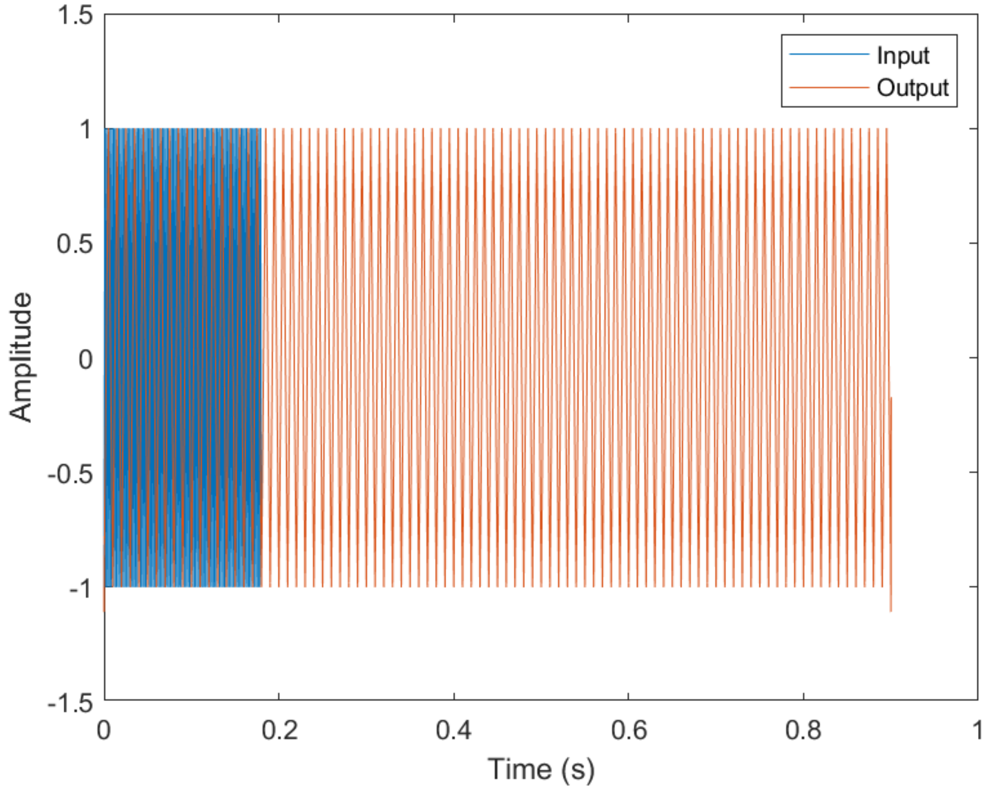

Figure 7: Triangle wave with frequency 500 Hz and its interpolated output by 5

b)4) Plot the input and output in part b)3) in frequency. Please use fftshift command in your plots so that center frequency corresponds to 0 Hz in your plot. Also please **plot** both of the frequency contents on the **same figure**. **Do not forget** to indicate which frequency content belongs to which signal. **Comment** on your results.

A similar comment to Parts a)4) and b)2) can be made for the frequency spectrum.However, for this time, we interpolated the signal, therefore, we see the same number of harmonics at the output, but it is confined in the range [-π/L, π/L] because of the low-pass filter. Also, frequency of harmonics are decreased by L, and their fourier coefficients increased by L.

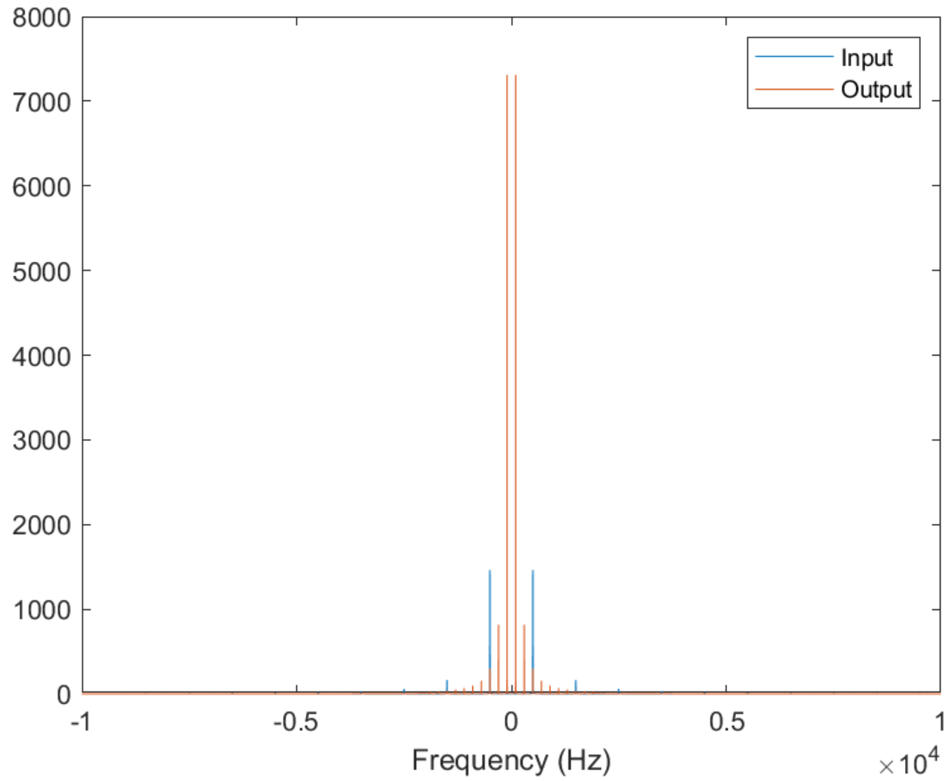

Figure 8: DFT waveform of signals in Figure 7

c) Write a code for sampling rate change with a **rational factor**. **Plot** the **input** and **output** both in **time** and **frequency**.

MATLAB code for function "sampling rate change with a rational factor" is added to MATLAB .zip files.

Increase the sampling rate by L/M in the following tasks.

IMPORTANT NOTE 2:** **L/M may be an integer. For example, if L is equal to 6 and M is equal to 3, L/M is equal to 2, which corresponds to upsampling with 2. In such as case, try (L+1)/M, which corresponds to 7/3 or try (L-1)/M, which corresponds to 5/3, so that you can get a rational factor. If your M is greater than L use M/L. Also indicate the value of the value in your report.

M=4, L=5, L/M=5/4

c)1) Take the **sine sequence in part a)1) as input**. **Plot** it and output in **time** using the **same time axis**, i.e., time difference between consecutive sequence elements is **1/(4000*K)**. Also please **plot** both of the waveforms on the **same figure**. **Do not forget** to indicate which waveform belongs to which signal. **Comment** on your results.

Duration of the signal increased by 5/4 because we first interpolated it with 5 then we decimated the resulting signal by 4. Therefore, we have an increase in duration by L/M, and decrease in frequency by L/M, as seen in Figures 9, 10.

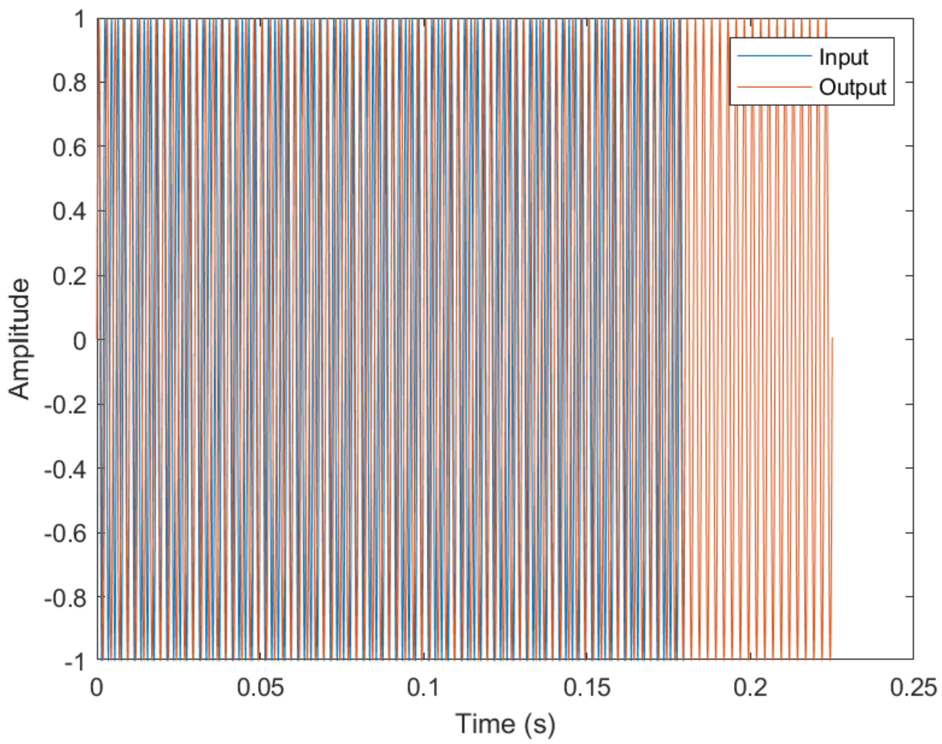

Figure 9: Sine wave with frequency 500 Hz and its resampled output with rational rate 4/5

c)2) Plot the output in part c)1) in frequency. **Please use fftshift command in your plots so that center frequency corresponds to 0 Hz in your plot.** Also please **plot** both of the frequency contents on the **same figure**. **Do not forget** to indicate which frequency content belongs to which signal. **Comment** on your results.

We see that the frequencies are shifted by a 1/rational factor i.e., M/L = 4/5, after this operation. Amplitude of DFT at frequencies are affected proportional to the ratio L/M.

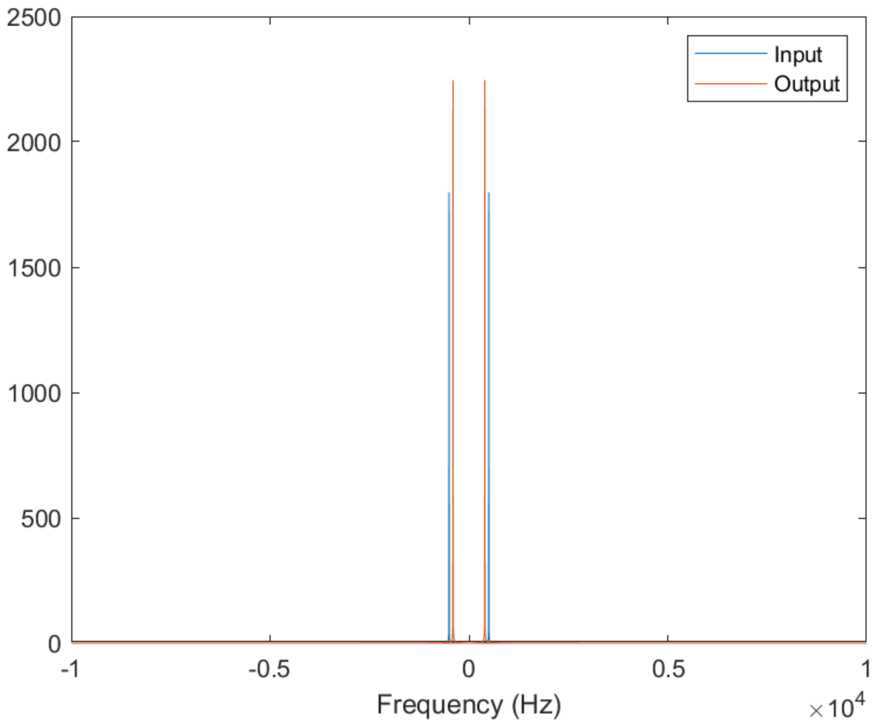

Figure 10: DFT waveform of signals in Figure 9

c)3) Take the **triangle sequence in part a)3) as input**. **Plot** it and output in **time** using the **same time axis**, i.e., time difference between consecutive sequence elements is **1/(4000*K)**. Also please **plot** both of the waveforms on the **same figure**. **Do not forget** to indicate which waveform belongs to which signal. **Comment** on your results.

Duration of the triangular wave is multiplied with L/M=5/4, similar to the sinusoidal. This operation is done by upsampling by 5, filtering, and downsampling by 4. In time domain the change in duration can be observed.

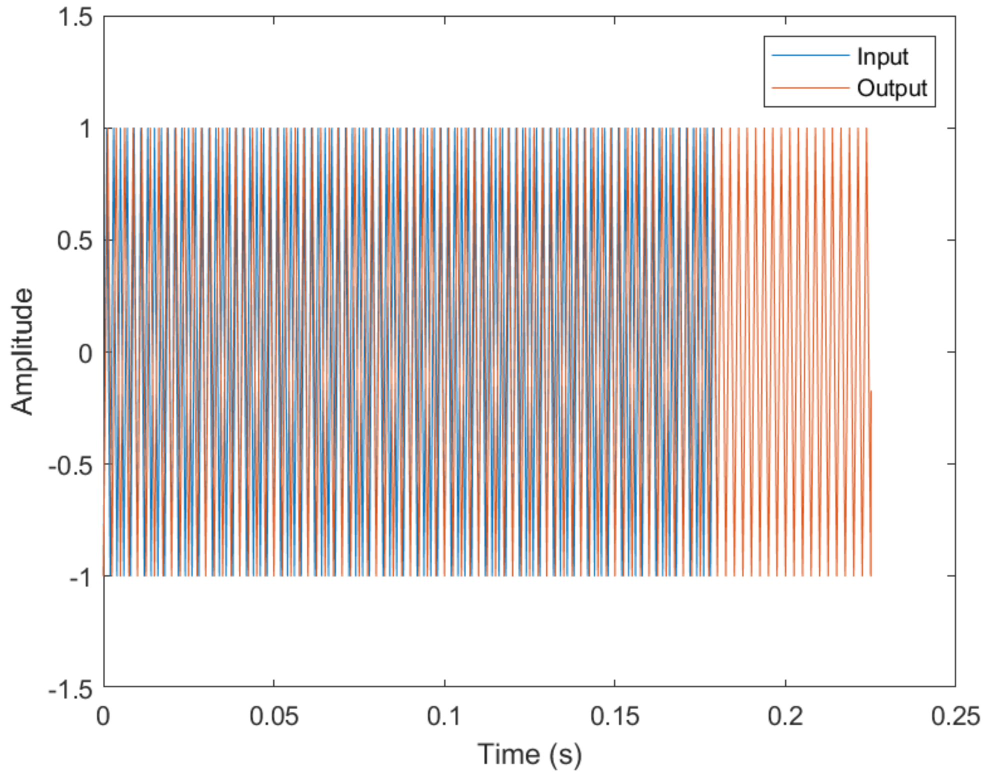

Figure 11: Triangle wave with frequency 500 Hz and its resampled output with a rate of 4/5

c)4) **Plot the input and output in part c)3) in frequency**. **Please use fftshift command in your plots so that center frequency corresponds to 0 Hz in your plot.** Also please **plot** both of the frequency contents on the **same figure**. **Do not forget** to indicate which frequency content belongs to which signal. **Comment** on your results.

All other comments in Parts b)4) and a)4) are applicable here, but, cut-off frequency is chosen to be π/5, which is the minimum of both of the cut-off frequencies of the low-pass filter used in Parts a, and b. Thus, no harmonics observed at the output after 2000 Hz, if we look at Figure 12. Also, since the previous comments are applicable, we get first increase by L and decrease by M in Fourier coefficients of input signal. So, we get an overall L/M increase. The argument is similar for frequency of the harmonics.

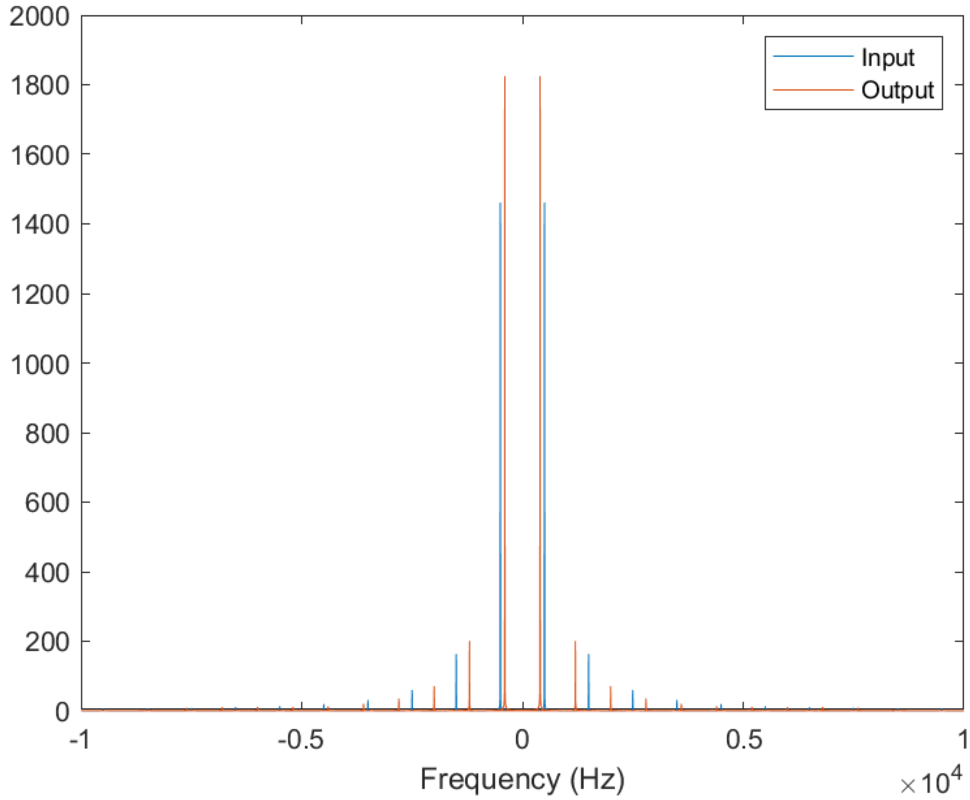

Figure 12: DFT waveform of signals in Figure 11

d) Implement a **digital PLL** structure. In this case, assume that there is a **sinusoid** where **you would lock onto its frequency and phase.** You should be able to **change the frequency and phase of this sinusoid.** The second sinusoid is an **internally produced** one. You can match its frequency with the first sinusoid. You can create **internally produced sinusoid by using sin or cos functions of MATLAB**. For this purpose, you need to **detect the frequency of the first sinusoid by using FFT magnitude.** After the frequency of the second sinusoid is brought to the lock range of the PLL, PLL works on lock onto the fine frequency and phase.

MATLAB code for the function "PLL" is added to MATLAB .zip files.

IMPORTANT NOTE 3: ***If L+M is less than or equal to 5, i.e., 1-2-3-4-5, take L+M=6. Also indicate this situation and the assigned value in your report. If L+M is greater than or equal to 6, you may proceed with your report.***

M=4, L=5, L+M=9

IMPORTANT NOTE 4: ***If (L+M)/2 is greater than or equal to 6 take it 5. Also indicate this situation and assigned value in your report. If (L+M)/2 is less than or equal to 5, you may proceed with your report.***

M=4, L=5, L+M=9, *(L+M)/2*=9/2

d)1) Let the **incoming** sine wave frequency be (L+M) *100 Hz. Also let it phase to be random. Note that you should add the random phase in **RADIANS (see MATLAB help for sin).** Take the **sampling rate as 4000*(L+M) samples/sec**. Create a **sine sequence with appropriate time range, so that you can observe and have AT LEAST 15 WAVEFORMS (periods) of the input signal. Detect the frequency of this input sinusoid** by using **FFT magnitude and let the internally produced sinusoid have this frequency**. **Plot** the **input and locked signal on the same figure** with **different colors**.

Hint: Once you decide on the frequency, you should lock onto the phase.

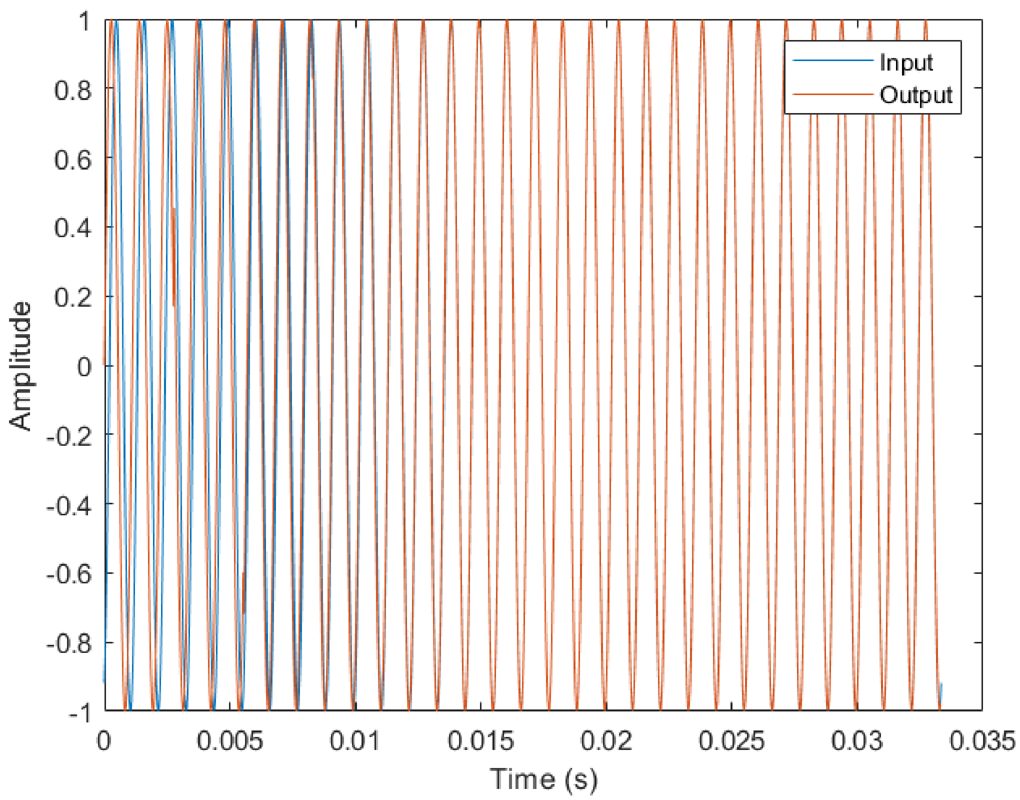

Figure 13: Sine wave with frequency 900 Hz and the output signal that is locked onto it

d)2) Repeat d)1) with the **frequency of sine as 50*(L+M)/2 Hz**. **Observe** whether locking occurs or not. **Comment** on your results.

Once e[n] is low-pass filtered to remove the twice frequency component, $sin(2\pi 2f_0 t + \Theta)$ **, g[n]=sin(Θ)=Θ. However, for the frequency (L+M)*25 Hz, the frequency is low, thus, the low-pass can not eliminate the first part of the equation, so it can not lock to the phase, which is observed in Figure 15.**

$$e[n] = \frac{1}{2}(sin(2\pi 2f_0 t + \Theta) + sin(\Theta)) \quad \textbf{(1)}$$

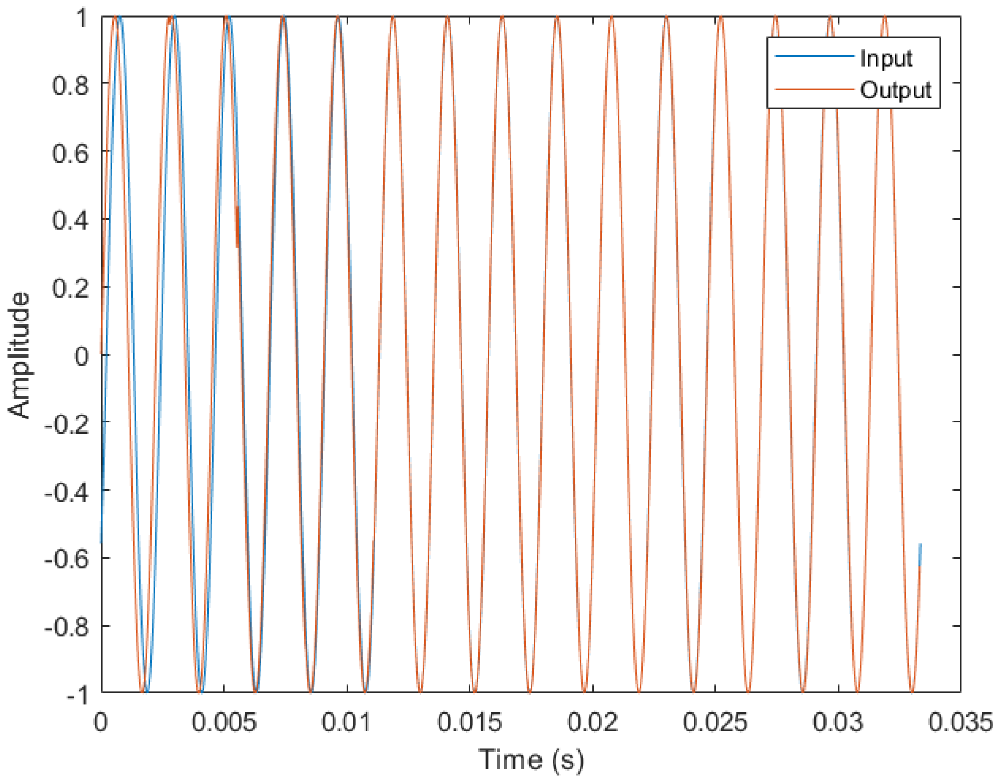

Figure 14: Sine wave with frequency 450 Hz and the output signal that is locked onto it

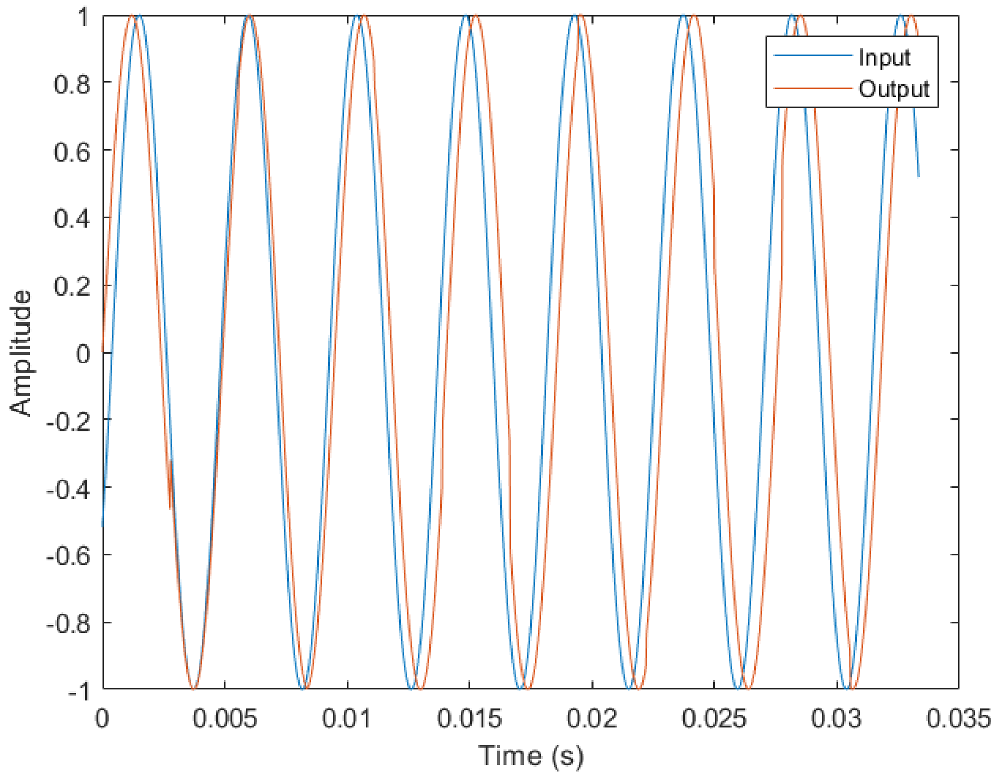

Figure 15: Sine wave with frequency 225 Hz and the output signal that is locked onto it