Consequences and Limits of Nonlocal Strategies

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Definition

A hypothetical cooperative game which is played by 2 or more players against a referee, where the goal of players is to jointly win the game.

Catch? The ONLY communication allowed is between the players and the referee, where each player receives a random question from a known probability distribution and responds with an answer to the referee. Finally, referee collects the answers and decides if players won or not.

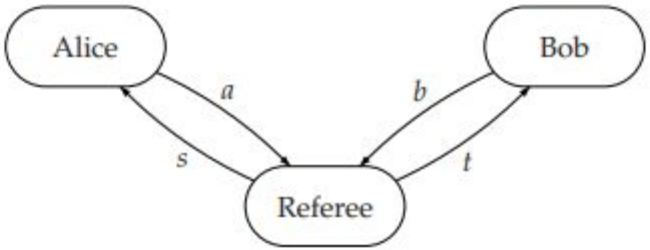


Figure 1: The communication structure of a nonlocal game.

Importance in Study

- NL games are a natural abstractions of multi-prover interactive proof systems, that is, systems consisting of a single round of interaction with two provers
- Provide an intuitive setting in which the quantum-physical notion of ***non-locality*** can be understood
- In quantum information theory, a *Bell inequality* is analogous to an upper bound on the probability with which Alice and Bob can win a non local game using a classical strategy
- A violation of a Bell inequality is equivalent to a situation in which a quantum strategy wins a particular non local game with a higher probability than is possible for any classical strategy

Mathematical Formulation

A non local game *G* is defined as $G = G(V, \pi)$ where π is the probability distribution on the question set *S* x *T*, and *V* is a predicate on *S* x *T* x *A* x *B*. A pair of question (*s, t*) from (*S* x *T*) is chosen from the distribution by the referee, and pair of answers (*a*, *b*) from (*A* x *B*) is sent by Alice and Bob.

The game is won by players Alice and Bob if the predicate *V* evaluates to **`true`**, else it is lost. The predicate *V* is often denoted by (*a, b* | *s, t*) rather than (*s, t, a, b*) to highlight the conditional dependence of replies (*a, b*) on given questions (*s, t*).

Classical value of an NL Game

- The maximum probability with which Alice and Bob can win a game G
- Denoted as $\omega_c(G)$
- Always obtained by some deterministic strategy, given that any probabilistic strategy can be expressed as a convex combination of deterministic strategies. Thus,

$$\omega_c(G(V,\pi)) = \max_{a,b} \sum_{s,t} \pi(s,t) V(a(s), b(t) \mid s, t),$$

  where the maximum is over all functions $a : S \to A$ and $b : T \to B$

- A deterministic strategy is a restricted type of classical strategy where a and b are simply functions of s and t, respectively.

Introduction to Quantum Strategies

- The most important part of the quantum strategies for any non local game is the presence of an entangled shared state($|\psi\rangle$) by 2 players in the beginning.
- The strategy is that on receiving some set of input (s, t); Alice performs measurement corresponding to her input, *s*; & gets output *a*. Bob gets output b in a similar way.
- Mathematically, $|\psi\rangle \in A \otimes B$, where *A* represents Alice's space and *B* represents Bob's.
- We also need two collections of positive semi-definite matrices $\{X_s^a : s \in S, a \in A\}; \{Y_t^b : t \in T, b \in B\}$ satisfying $\sum_{a \in A} X_s^a = 1$ and $\sum_{b \in B} Y_t^b = 1 \, \forall s \in S; t \in T$

Introduction to Quantum Strategies (contd.)

- Here, this collection $\{X_s^a : a \in A\}$ describes the measurement performed by Alice whenever she receives the question s, and likewise for Bob.
- The probability that Alice answers with ‘a’ and Bob with ‘b’ when input is {s, t} is given by the expression - $<\psi|X_s^a \otimes Y_t^b|\psi>$.
- Therefore, the probability of winning the game using quantum strategy is given by -
$$\sum_{(s,t)\in S\times T} \pi(s,t) \sum_{(a,b)\in A\times B} <\psi|X_s^a \otimes Y_t^b|\psi> V(a,b|s,t)$$
where $\pi_{(s,t)}$ denotes the probability of occurrence of input (s, t).
- The quantum value of a game, denoted by w_q(G) is the supremum of winning probabilities of over all quantum strategies of Alice and Bob.
- We measure the outcomes using **Observables**, that are simply Hermitian matrices with real eigenvalues.
- A perfect quantum strategy is one that wins with probability *p* = 1.

Examples of Non Local Games

1. CHSH Game

CHSH game [2] is an example of a Non Local game where all *S, T, A, B* are the same set; {0, 1}. The input given to players is again $s \in S; t \in T$ and both players output 'a' and 'b' respectively. The game is won if $a \oplus b = s \wedge t$. It can easily be shown that it is impossible to win the game for all (s, t). Let's say Alice outputs x_0 for s = 0 and x_1 for s = 1; and Bob outputs y_0 for t = 0 and y_1 for t = 1. Now, we need to satisfy 4 equations, i.e for all pairs of (s, t) :-

- (0, 0) - $x_0 \oplus y_0 = 0$
- (0, 1) - $x_0 \oplus y_1 = 0$
- (1, 0) - $x_1 \oplus y_0 = 0$
- (1, 1) - $x_1 \oplus y_1 = 1$

XORing all 4 equations, we get 0 = 1; which is obviously wrong, hence it is impossible to win the game with probability 1; however it can be seen that we can win the game with probability ¾ with the simple classical strategy of outputting 0 on each input.

- First, we define the quantum entangled state shared by Alice and Bob, which is the bell state; - $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$
- Then we define the quantum states - $\phi_1(\theta) = -sin(\theta)|0\rangle + cos(\theta)|1\rangle$ & $\phi_0(\theta) = cos(\theta)|0\rangle + sin(\theta)|1\rangle$ and using them, we define the following states, which would be later used to calculate probabilities -

$$X_0^a = |\phi_a(0)\rangle\langle\phi_a(0)| \qquad X_1^a = |\phi_a(\pi/4)\rangle\langle\phi_a(\pi/4)|$$

$$Y_0^b = |\phi_b(\pi/8)\rangle\langle\phi_b(\pi/8)| \qquad Y_1^b = |\phi_b(-\pi/8)\rangle\langle\phi_b(-\pi/8)|$$

- It can also be shown that for a *d* dimension maximally entangled state $|\psi\rangle$, $\langle\psi|\overline{A} \otimes B|\psi\rangle = (1/d)(Tr(A^*B))$, where $\overline{A}$ stands for conjugate of A; while A^*stands for conjugate transpose of A. Thus, what we have is that the probability of Alice and Bob answering (s, t) with (a, b) is $(1/2)(Tr(X_s^a Y_t^b))$.
- It can be verified that for each case the correct answer is given with probability $cos^2(\pi/8)$ = 0.85 > 0.75 & the probability for wrong answer is similarly $sin^2(\pi/8)$ Hence, it is shown that quantum strategy outperforms classical strategy.

2. Odd Cycle Game

- Formally, this game can be defined as:

Let $n \geq 3$ be an odd integer, let $S = T = \mathbb{Z}_n$, and let $A = B = \{0, 1\}$. Take π to be the uniform distribution over the set $\{(s,t) \in \mathbb{Z}_n \times \mathbb{Z}_n : s = t \text{ or } s + 1 \equiv t (\mod n)\}$ and let V be defined as

$$V(a, b|s, t) = \begin{cases} 1 & \text{if } a \oplus b = [s + 1 \equiv t \mod n] \\ 0 & \text{if } otherwise \end{cases}$$

- The game can be imagined as Alice and Bob trying to convince the referee that an odd cycle of length n is 2-colorable. The referee sends a vertex (either same or adjacent) to both of them. Both of them send a color back to the referee. If the vertices were same, then colors should agree otherwise they should be different.
- The value of this game is $1 - 1/2n$ any deterministic strategy should fail for at least one of the possible pairs (s,t) as an odd cycle cannot be 2-colored.
- The best strategy is that both Alice and Bob let *a = s mod 2* and *b = t mod 2*, which fails when *s = 0* and *t = n-1* (both are even).

- A quantum strategy can obtain a success probability quadratically closer to 1
- The following strategy wins the game with probability: $\cos^2(\pi/4n) \geq 1 - (\pi/4n)^2$
- The shared state is again $\psi = (|00> + |11>)/\sqrt{2}$
- Define $X_s^a = |\phi_a(\alpha_s)\rangle\langle\phi_a(\alpha_s)|$ and $Y_t^b = |\phi_b(\beta_t)\rangle\langle\phi_b(\beta_t)|$, where $|\phi_0(\theta)\rangle$ and $|\phi_1(\theta)\rangle$ are as defined for the CHSH game and,

$$\alpha_s = \left(\frac{\pi}{2} - \frac{\pi}{2n}\right)s + \frac{\pi}{4n},$$
$$\beta_t = \left(\frac{\pi}{2} - \frac{\pi}{2n}\right)t,$$

- Given questions (s,t), the probability that Alice and Bob answer the same bit is $cos^2(\alpha_s - \beta_t)$ and that they answer differently is $sin^2(\alpha_s - \beta_t)$
- When s = t, then they need to answer the same bit, which they do with probability $cos^2(\pi/4n)$ (just putting *s* and *t* equal)
- When $s + 1 \equiv t \mod n$, $\alpha_s - \beta_t = \pi/2 - \pi/4n$ and they need to answer differently. Thus the probability is $sin^2(\pi/2 - \pi/4n) = cos^2(\pi/4n)$

3. Magic Square Games

- Based on the fact that there does not exist a 3 × 3 binary matrix with the property that each row has even parity and each column has odd parity
  - Parity : #1's in a row/column is odd, then parity = 1 else 0
- Alice is asked to fill in the values in either a row or a column of the matrix, while Bob is asked to fill in a single entry of the matrix, that is randomly chosen among the three entries given to Alice
- Mathematically, $S = \mathbb{Z}_6$ denote six possible questions to Alice, and $T = \mathbb{Z}_9$ denote the nine possible questions to Bob, and hence probability distribution

$$\pi = \{(s, t) : \text{entry } \in S \times T : t \text{ is in triplet } s\}$$

- The game is **won** if parity conditions are met by Alice's answers ($A = \{0, 1\}^3$), and Bob's answer ($B = \{0, 1\}$) is consistent with Alice's answers
- Classically, $\omega_c(G)$ = 17/18 for the magic square game.
- Remarkably, a **perfect quantum strategy** exists for this game!

- The strategy utilises notion of observables with the 3 x 3 matrix $\begin{pmatrix} \sigma_x \otimes \sigma_y & \sigma_y \otimes \sigma_x & \sigma_z \otimes \sigma_z \\ \sigma_y \otimes \sigma_z & \sigma_z \otimes \sigma_y & \sigma_x \otimes \sigma_x \\ \sigma_z \otimes \sigma_x & \sigma_x \otimes \sigma_z & \sigma_y \otimes \sigma_y \end{pmatrix}$
- Alice and Bob share copy of entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$

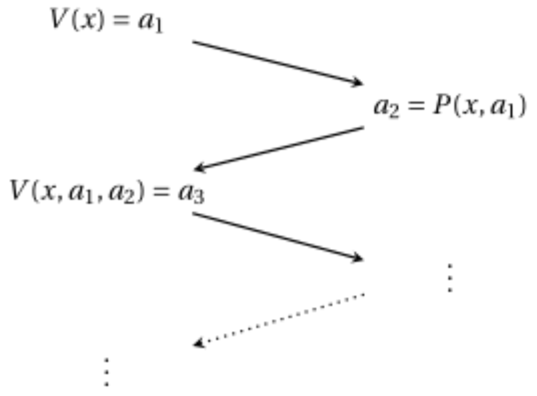
- To answer any question, Alice and Bob simply measure their shared state wrt corresponding observables in the matrix
- Why this works? 3 key observations:
  - $|\psi\rangle$ is a -1 eigenvector of each of the operators $\sigma_i \otimes \sigma_i \; ; \; i \in \{x, y, z\}$, therefore
  $$\langle\psi|\sigma_i \otimes \sigma_i|\psi\rangle = -1$$
  Hence Alice and Bob's answer are always in agreement (product of measurement = (-1)(-1) = 1)
  - Pauli matrices anti-commute in pair i.e. $\sigma_x\sigma_y = -\sigma_y\sigma_x, \quad \sigma_x\sigma_z = -\sigma_z\sigma_x, \quad \sigma_y\sigma_z = -\sigma_z\sigma_y.$
  Therefore, Alice can simultaneously measure the three observables within whichever row or column she was asked, i.e., output is independent of order of measurements
  - The product of the observables in each row is equal to 1 $\otimes$ 1, whereas it -1 $\otimes$ 1 in each column, thus satisfying the parity condition for Alice.

Reference: https://people.vcu.edu/~sgharibian/courses/CMSC491/notes/Lecture%205%20-%20Non-local%20games.pdf

Connections with Multi Prover Interactive Systems

Multi Prover Interactive Systems

- An abstract machine that models computation as the exchange of messages between two parties: a **prover** and a **verifier** to ascertain whether a given string belongs to a language or not.
  - The prover possesses unlimited computational resources but cannot be trusted, while the verifier has bounded computation power but is assumed to be always honest.
- It becomes natural to consider prover strategies that entail sharing entangled quantum information prior to the execution of the proof system
- *What happens when the provers can employ quantum strategies?* No changes are made to the verifier, who remains classical, and all communication between the verifier and the provers remains classical



General representation of an interactive proof protocol.

One-round two-prover interactive proof system

- The interaction is restricted to two stages: a query stage where the verifier sends information to the provers, and a response stage where the provers send information to the verifier
- One can associate a nonlocal game G_x to each string *x* with the property that $\forall$ yes-inputs *x*, $w_q(G_x)$ is close to 1, else $w_q(G_x)$ is close to 0

Paper presents two examples of natural two-prover one-round proof systems that are classically sound, but become unsound when the provers use quantum strategies: *Graph Coloring* and *3-SAT* proof systems

1. Graph Coloring proof system

- The idea of odd cycle game can be generalized to any graph *G* and integer number of colors *k*.
- The verifier sends vertices (either same or adjacent) to each prover (2 here), and requires that the colors be the same whenever each prover gets the same vertex and different whenever the provers get adjacent vertices.
- If *G* is *k*-colorable, then the provers can answer based on a valid coloring.
- If not, there must be some inconsistency for some (*s,t*) and so the value of game cannot be 1.
- The verifier can amplify the difference between the 2 cases: colorable or not
- This proof system breaks down in case of entangled provers. There exists [4] a sequence of graphs G_n such that:
  - For any n, there is a perfect quantum strategy for the Graph Coloring proof system with graph G_n and k = n colors.
  - For sufficiently large n, G_n is not k-colorable.

2. 3-SAT Proof System

- We consider a non local game G_f involving Alice and Bob, where Alice is provided with a clause and Bob is given a variable from the clause. Alice needs to provide a valid assignment for the clause, satisfying the clause, while Bob needs to provide an assignment for the variable satisfying Alice's choice.
- In Mathematical terms, let the variables be x_0, x_1,...x_{n-1} and the clauses be c_0, c_1,...c_{m-1}. Then, $S = \mathbb{Z}_m, \ T = \mathbb{Z}_n, \ A = \{0,1\}^3, \ B = \{0,1\}$
- The predicate *V(a, b | s, t)* takes value 1 iff Alice's assignment for the variables in c_s satisfies c_s & is consistent with assignment x_t = b. If f is satisfiable then, $w_c(G_f) = 1$
- If *f* is unsatisfiable, then $w_c(G_f) \leq 1 - 1/3m$ as out of all 3m possibilities, atleast 1 would violate the clause. However a simple counterexample can be derived based on Magic Square game, where f is unsatisfiable, but there is a perfect quantum strategy for the above two prover proof system.

2. 3-SAT Proof System (contd.)

- We consider 9 boolean variables for this Magic Square game, x_{00}, x_{01}, x_{02}, x_{10}, x_{11}, x_{12}, x_{20}, x_{21} *and* x_{22}. They represent a 3 x 3 boolean matrix.
- There are 6 parity conditions in the magic square game, each row has even parity & each column has odd parity & each condition can be expressed with 4 clauses, $(\overline{x_{00}} \vee \overline{x_{01}} \vee \overline{x_{02}}) \wedge (\overline{x_{00}} \vee x_{01} \vee x_{02}) \wedge (x_{00} \vee \overline{x_{01}} \vee x_{02}) \wedge (x_{00} \vee x_{01} \vee \overline{x_{02}})$
- Thus we would have 24 such clauses. This clause is satisfied only when $x_{00} \oplus x_{01} \oplus x_{02} = 0$. Thus, as we know, this formula is unsatisfiable, but since we have perfect strategy for Magic Square game, the quantum strategy defeats this 3 SAT game with certainty.

Binary games and XOR games

Brief Overview

- Binary games are games in which the sets A and B are {0, 1}.
- The paper shows some basic results regarding measurements and values for these games, and proceeds to talk about XOR games.
- XOR games are a restriction of binary games in which the value of the predicate ***V*** depends only on $a \oplus b$, and not 'a' and 'b' independently.
- CHSH and Odd cycle games are examples of these games. In the further slides we show the upper bounds of quantum values of these games.
- We then state upper bounds on the amount of entanglement required for Alice and Bob to play XOR games optimally and nearly optimally.

Optimality of projective measurements for binary games

- The paper shows that for a binary game G, if we let $|\psi\rangle \in \mathcal{A} \otimes \mathcal{B}$ be any fixed state shared between Alice and Bob, then among the set of all strategies for G, there is an optimal strategy for which all of Alice's measurements $\{X_s^0, X_s^1\}$ and Bob's measurements $\{Y_t^0, Y_t^1\}$ are projective measurements on $\mathcal{A}$ and $\mathcal{B}$, respectively.
- Matrices may always be taken to be projection matrices, even when restricted to the support of the vector $|\psi\rangle$
- Consequently, general measurements can be simulated by projective measurements.

Perfect Strategies for Binary Games

- The paper shows that for any binary game *G*, if there exists a perfect quantum strategy, then the game must have a perfect classical strategy as well (i.e. $\omega_c(G) = 1$)
- The result can be proved by assuming that a perfect quantum strategy for a game is given and then constructing a perfect classical strategy.
- The classical strategy should be such that the answers given by it for any pair of questions (*s, t*) should be given by the quantum strategy with some non zero probability.

Bounds on values of XOR games

- Tsirelson's Correspondence [3] -

  Let S and T be finite, nonempty sets, and let $\{c_{s,t} : (s, t) \in S \times T\}$ be a collection of real numbers in the range $[-1, 1]$. Then the following are equivalent:

  1. There exists a positive integer *n*, complex Hilbert spaces A and B with finite dimension n, a unit vector $|\psi\rangle \in A \otimes B$, a collection $\{A_s : s \in S\}$ of ±1 observables on A, and a collection $\{B_t : t \in T\}$ of ±1 observables on B, such that $\langle\psi|A_s \otimes B_t|\psi\rangle = c_{s,t} \forall (s,t) \in S \times T$
  2. There exists a positive integer m and two collections $\{|u_s\rangle : s \in S\}$ and $\{|v_t\rangle : t \in T\}$ of unit vectors in R^m such that $\langle u_s|v_t\rangle = c_{s,t} \forall (s,t) \in S \times T$
- Moreover, if the first item holds for a fixed choice of *n*, then the second holds for *m* = $2n^2$; and if the second item holds for a fixed choice of *m*, then the first holds for $n = 2^{\lceil m/2 \rceil}$

Bounds on values of XOR games (contd.)

- Let's define trivial random strategy for Alice and Bob as one where they ignore their inputs and answer uniformly generated random bits. If $\tau(G)$ denotes the success probability of game (G, π) when Alice and Bob are restricted to this trivial strategy, then $\tau(G) = (1/2)\sum_{c\in 0,1}\sum_{s,t}\pi(s,t)V(c|s,t)$
- Advantage over trivial Strategy -
  - Let G(V, π) be an XOR game and let m = min(|S|, |T|). Then,
    $w_q(G) - \tau(G) = (1/2)max_{\{|u_s\rangle,|v_t\rangle\}}\sum_{s,t}\pi(s,t)(V(0|s,t) - V(1|s,t))\langle u_s|v_t\rangle$
- Upper bound for XOR games with weak classical strategies -
  - We consider the regime where the success probability of the best classical strategy is not much better than τ(G). We use Grothendieck's constant K_G for this.
  - Let G be a XOR game, then $w_q(G) - \tau(G) \leq K_G(w_c(G) - \tau(G))$

Bounds on values of XOR games (contd.)

- Upper Bound for XOR games with strong quantum strategies -
  - Now, we consider games like Odd Cycle game ($w_c(G) = 1 - 1/2n$) where classical strategies perform well as compared to trivial strategies.
  - Let G be an XOR game with classical value ωc(G). Then $\omega_q(G) \leq g(\omega_c(G))$, where g is as defined above, i.e.,

$$\omega_q(G) \leq \begin{cases} \gamma_1 \omega_c(G) & \textit{if } \omega_c(G) \leq \gamma_2 \\ \sin^2\left(\frac{\pi}{2}\omega_c(G)\right) & \textit{if } \omega_c(G) > \gamma_2, \end{cases}$$

where $\gamma_1 = 1.1382$ & $\gamma_2 = 0.74202$

Bounds on entanglement for XOR games

- For any XOR game G with m = min(|S|, |T|), there exists an optimal strategy for Alice and Bob in which they share a maximally-entangled state on $\lceil m/2 \rceil$ qubits
- The number of qubits is exponential in the size of their inputs
- But a sub optimal strategy can be obtained using polynomial number of shared qubits. This follows from the Johnson-Lindenstrauss lemma [5] :

For $\varepsilon \in (0, 1)$ and n a positive integer, let K be a positive integer such that

$$K \geq 4 \, (\varepsilon^2/2 - \varepsilon^3/3)^{-1} \log n$$

Then for any set V of n points in R^d , there is a mapping $f : R^d \to R^K$ such that for all $|u\rangle, |v\rangle$,

$$(1 - \varepsilon)\| \, |u\rangle - |v\rangle \, \|^2 \leq \| \, f(|u\rangle - f(|v\rangle \, \|^2 \leq (1 + \varepsilon)\| \, |u\rangle - |v\rangle \, \|^2$$

Bounds on entanglement for XOR games (contd.)

- Using the lemma, we can arrive at the theorem,

Let $G = G(V, \pi)$ be an XOR game with quantum value $\omega_q(G)$. Let $0 < \epsilon < 1/10$, and suppose K is an even integer such that

$$K \geq 4(\epsilon^2/2 - \epsilon^3/3)^{-1} \log(|S| + |T| + 1)$$

Then, if Alice and Bob share a maximally entangled state on $K/2$ qubits, they can win with probability greater than $\omega_q(G) - \epsilon$

- Oded Regev has described to us an improved form of this theorem where K has no dependence on |S| and |T|.

Connections with multi prover interactive proof systems

- We need to study upper bounds on quantum values of games as they are related to the soundness property of multi prover interactive proof systems.
- For example in case of Odd Cycle Game (simple proof system for the two-colorability of odd cycles), the correct response for verifier is to reject.
- This is valid for a classical system, but if the quantum value of the game were to be 1, then it would not be a valid quantum proof system.
- The upper bound on the quantum value proves that it is a valid quantum proof system, and with a polynomial number of repetitions, the probability of verifier incorrectly accepting the game can be made 0.
- Similarly, applying upper bounds on the number of entangled qubits help in analysing the complexity class of such systems

Connections with multi prover interactive proof systems (contd.)

For $0 \le s < c < 1$, let $^{\oplus}MIP_{c,s}[2]$ denote the class of all languages L recognized by classical two-prover interactive proof systems of the following form:

- They operate in one round, each prover sends a single bit in response to the verifier's question, and the verifier's decision is a function of the parity of those two bits.
- If $x \notin L$ then, whatever strategy Alice and Bob follow, the Prover's acceptance probability is at most s (the soundness probability).
- If $x \in L$ then there exists a strategy for Alice and Bob for which the Prover's acceptance probability is at least c (the completeness probability).

For $0 \le s < c < 1$, let $^{\oplus}MIP^*_{c,s}[2]$ denote the class corresponding to the previous definition, with all communication classical, but where the provers may share previous quantum entanglement.

- For all $\varepsilon \in (0, 1/16)$, if $s = 11/16 + \varepsilon$ and $c = 12/16$ then $^{\oplus}MIP_{c,s}[2] = NEXP$ [6]
- For all s and c such that $0 \le s < c \le 1$, $^{\oplus}MIP^*_{c,s}[2] \subseteq EXP$.

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Thank You!

We are open to questions and suggestions, if any.