Autoregressive Model

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Autoregressive Model

Durbin Watson Test for the auto correlation:

H_o: $\rho = 0$ H_a: $\rho > 0$

Use the lmtest library for the Durbin Watson test.

```
Blaisdell <- read.csv("/cloud/project/Blaisdell.csv")
library(lmtest)
```

```
## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
dwtest(Company.Sales~Industry.Sales,data=Blaisdell)
```

```
##
##  Durbin-Watson test
##
## data:  Company.Sales ~ Industry.Sales
## DW = 0.73473, p-value = 0.0001748
## alternative hypothesis: true autocorrelation is greater than 0
```

```
#library(nlme)
#glmod <- gls(Company.Sales~Industry.Sales,correlation=corAR1(form=~Quarter),data=Blaisdell)
#summary(glmod)
#intervals(glmod,which="var-cov")
```

#Cocharane-Orcutt Procedure

```
library(Hmisc)
```

```
## Loading required package: lattice

## Loading required package: survival

## Loading required package: Formula

## Loading required package: ggplot2

##
## Attaching package: 'Hmisc'
```

```
## The following objects are masked from 'package:base':
##
##     format.pval, units
```

```r
#lets do it manually
f<-lm(Company.Sales~Industry.Sales,data=Blaisdell)
et<-f$residuals
et1<-Lag(et, shift = 1)

d1<-sum(na.omit(et1*et))
d2<-sum(na.omit(et1)^2)
rho<-d1/d2

Ytnew=Blaisdell$Company.Sales - rho*Lag(Blaisdell$Company.Sales, shift = 1)
Xtnew=Blaisdell$Industry.Sales - rho*Lag(Blaisdell$Industry.Sales, shift = 1)

f1<-lm(Ytnew~Xtnew)
summary(f1)
```

```
##
## Call:
## lm(formula = Ytnew ~ Xtnew)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.097039 -0.056815  0.009902  0.034553  0.125048
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.394111   0.167230  -2.357   0.0307 *
## Xtnew        0.173758   0.002957  58.767   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06715 on 17 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.9951, Adjusted R-squared:  0.9948
## F-statistic:  3454 on 1 and 17 DF,  p-value: < 2.2e-16
```

```r
dwtest(Ytnew~Xtnew)
```

```
##
##  Durbin-Watson test
##
## data:  Ytnew ~ Xtnew
## DW = 1.6502, p-value = 0.1517
## alternative hypothesis: true autocorrelation is greater than 0
```

```r
#building function uses Spearman's rho autocorrelation
library(orcutt)
#coch<- cochrane.orcutt(f)
#summary(coch)
#it did not converge, changing the convergence criteria to (4th decimal)
coch1<-cochrane.orcutt(f,convergence = 4, max.iter=100)
summary(coch1)
```

```
## Call:
## lm(formula = Company.Sales ~ Industry.Sales, data = Blaisdell)
##
##                  Estimate Std. Error t value  Pr(>|t|)
## (Intercept)    1.6685563  1.4040722   1.188     0.251
## Industry.Sales 0.1606538  0.0067976  23.634 1.924e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0649 on 17 degrees of freedom
## Multiple R-squared:  0.9705 ,  Adjusted R-squared:  0.9687
## F-statistic: 558.6 on 1 and 17 DF,  p-value: < 1.924e-14
##
## Durbin-Watson statistic
## (original):    0.73473 , p-value: 1.748e-04
## (transformed): 1.72256 , p-value: 2.774e-01
```

Hildreth-Lu

```
library(HoRM)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
prg1<-function(x,y,rh){
n<-length(rh)
out<-matrix(0,nrow=n,ncol=2)
out[,1]<-rh
for (i in 1:n){
d<-anova(hildreth.lu(y=y,x=x,rho=rh[i]))
out[i,2]<-d$"Sum Sq"[2]
}
out
}
rh<-seq(0.1,1,by=0.01)
hl<-prg1(Blaisdell$Company.Sales,x=Blaisdell$Industry.Sales,rh)
which.min(hl[,2])
```

```
## [1] 87
```

```
hl[87,]
```

```
## [1] 0.96000000 0.07167118
```

First Difference

```
rho=1
Ytnew=Blaisdell$Company.Sales - rho*Lag(Blaisdell$Company.Sales, shift = 1)
Xtnew=Blaisdell$Industry.Sales - rho*Lag(Blaisdell$Industry.Sales, shift = 1)

mean(Blaisdell$Company.Sales)-mean(Blaisdell$Industry.Sales)*0.168488
```

```
## [1] -0.304041
```

```r
f1<-lm(Ytnew~Xtnew -1)
summary(f1)
```

```
##
## Call:
## lm(formula = Ytnew ~ Xtnew - 1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.08958 -0.03231  0.02412  0.05344  0.15139
##
## Coefficients:
##       Estimate Std. Error t value Pr(>|t|)
## Xtnew 0.168488   0.005096   33.06   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06939 on 18 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.9838, Adjusted R-squared:  0.9829
## F-statistic:  1093 on 1 and 18 DF,  p-value: < 2.2e-16
```

```r
dwtest(f1)
```

```
##
##  Durbin-Watson test
##
## data:  f1
## DW = 1.7389, p-value = 0.3279
## alternative hypothesis: true autocorrelation is greater than 0
```

prediction for the Cochrane-Orcutt model

```r
f<-lm(Company.Sales~Industry.Sales,data=Blaisdell)
et<-f$residuals
et1<-Lag(et, shift = 1)

d1<-sum(na.omit(et1*et))
d2<-sum(na.omit(et1)^2)
rho<-d1/d2

Ytnew=Blaisdell$Company.Sales - rho*Lag(Blaisdell$Company.Sales, shift = 1)
Xtnew=Blaisdell$Industry.Sales - rho*Lag(Blaisdell$Industry.Sales, shift = 1)

f1<-lm(Ytnew~Xtnew)
MSE<-summary(f1)$sigma^2

#transforming the coeficents back to original form
b0 <- summary(f1)[[4]][1,1]/(1-rho); print(b0)
```

```
## [1] -1.068524
```

```r
s.b0 <- summary(f1)[[4]][1,2]/(1-rho)
b1 <- summary(f1)[[4]][2,1]; print(b1)
```

```
## [1] 0.1737583
```

```r
s.b1 <- summary(f1)[[4]][2,2]
correct.y.hats <- b0 + b1*Blaisdell$Company.Sales
MSE<-summary(f1)$sigma^2


X.prime<-Xtnew
X.bar.prime <- mean(X.prime[-1])

X.n.plus.1 <- 175.3
X.n <- rev(Blaisdell$Industry.Sales)[1]
X.n.plus.1.prime <- X.n.plus.1 - rho*X.n

# Point forecast:

Y.hat.n.plus.1 <- b0 + b1*X.n.plus.1
Y.n <- rev(Blaisdell$Industry.Sales)[1]
e.n <- Y.n - (b0 + b1*X.n)
Y.hat.FORECAST.n.plus.1 <- Y.hat.n.plus.1 + rho*e.n

print(paste("forecasted response at time n+1 is:", round(Y.hat.FORECAST.n.plus.1,4) ))
```

```
## [1] "forecasted response at time n+1 is: 119.6062"
```

```r
# Prediction interval:

alpha <- 0.05
n<-length(Blaisdell$Company.Sales)
s.pred <- sqrt(MSE*(1 + (1/n) + (X.n.plus.1.prime -X.bar.prime)^2/(sum((X.prime[-1]-X.bar.prime)^2))))
s.pred
```

```
## [1] 0.0756265
```

```r
pred.L <- Y.hat.FORECAST.n.plus.1 - qt(1-alpha/2,df=n-3)*s.pred
pred.U <- Y.hat.FORECAST.n.plus.1 + qt(1-alpha/2,df=n-3)*s.pred


print(paste(100*(1-alpha) ,"percent PI for response at time n+1 is:", round(pred.L,4), ",", round(pred.U
```

```
## [1] "95 percent PI for response at time n+1 is: 119.4466 , 119.7657"
```