NAME GURDEEP SINGH

ROLL NO 2314502446

PROGRAM MASTER OF BUSINESS ADMINISTRATION

SEMESTER 2nd SEMESTER

COURSE NAME OPERATIONS RESEARCH

COURSE CODE DMBA 205

1. What is Operations Research? Explain the Methodology used to solveOperationsResearch Problems in brief.

Ans 1.

Operations Research

Operations Research (OR) is a discipline that utilizes mathematical models, statistical analysis, and optimization techniques to aid in decision-making and problem-solving. It involves applying scientific methods to complex problems in order to help organizations make better decisions and improve their operations. OR is used in various industries and sectors, including manufacturing, logistics, finance, healthcare, and telecommunications, among others.

Methodology Used to Solve Operations Research Problems

Problem Formulation The first step in solving an Operations Research problem is to clearly define the problem at hand. This includes identifying the objectives, constraints, and variables involved. Formulating the problem in a mathematical or logical framework is crucial for the subsequent analysis.

Model Development Once the problem is formulated, the next step is to develop a mathematical model that represents the real-world situation. This model could be deterministic, where all inputs and outputs are known, or stochastic, where there is uncertainty involved. The model helps in understanding the problem better and provides a basis for analysis.

Data Collection and Analysis Data plays a crucial role in Operations Research. It is important to collect relevant data that will be used in the model. This data is then analyzed to identify patterns, trends, and relationships that can help in decision-making. Statistical analysis is often used to analyze the data and draw meaningful conclusions.

Model Solution After developing the model and analyzing the data, the next step is to find a solution to the problem. This involves using various mathematical techniques such as linear programming, integer programming, network analysis, simulation, and queuing theory. These techniques help in optimizing the solution and finding the best course of action.

Solution Validation Once a solution is obtained, it is important to validate it to ensure that it is feasible and optimal. This involves checking if the solution satisfies all the constraints and achieves the desired objectives. Validation is crucial as it ensures that the solution is practical and can be implemented in the real world.

Implementation and Monitoring The final step in solving an Operations Research problem is to implement the solution in the real world and monitor its performance. This involves tracking key metrics and making adjustments as necessary to ensure the effectiveness of the solution. Implementation and monitoring are crucial for ensuring that the solution delivers the desired results and provides value to the organization.

In conclusion, Operations Research is a valuable tool that can help organizations make better decisions and improve their operations. By following a systematic approach and using advanced analytical methods, OR enables organizations to solve complex problems, optimize their processes, and achieve their objectives more effectively.

2. Solve the following linear programming problem using its Dual form:

Minimize Z = 3x1 + 4x2

Subject to: 4x1 + x2 ≥ 30

-x1 - x2 ≤ -18

x1 +3x2 ≥ 28

where x1, x2 ≥ 0

Ans 2.

Problem is

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Min *Z* | = |  | 3 | *x*1 | + | 4 | *x*2 | |
| subject to |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | |  | 4 | *x*1 | + |  | *x*2 | ≥ | 30 | | - |  | *x*1 | - |  | *x*2 | ≤ | -18 | | Here *b*2 = -18 < 0, so multiply this constraint by -1 to make *b*2 > 0. | | | | | | | | |  |  | *x*1 | + |  | *x*2 | ≥ | 18 | |  |  | *x*1 | + | 3 | *x*2 | ≥ | 28 | |
| and *x*1,*x*2≥0; |

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate  
  
1. As the constraint-1 is of type '≥' we should subtract surplus variable *S*1 and add artificial variable *A*1  
  
2. As the constraint-2 is of type '≥' we should subtract surplus variable *S*2 and add artificial variable *A*2  
  
3. As the constraint-3 is of type '≥' we should subtract surplus variable *S*3 and add artificial variable *A*3  
  
After introducing surplus,artificial variables

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Min *Z* | = |  | 3 | *x*1 | + | 4 | *x*2 | + | 0 | *S*1 | + | 0 | *S*2 | + | 0 | *S*3 | + | *M* | *A*1 | + | *M* | *A*2 | + | *M* | *A*3 | |
| subject to |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  | 4 | *x*1 | + |  | *x*2 | - |  | *S*1 |  |  |  |  |  |  | + |  | *A*1 |  |  |  |  |  |  | = | 30 | |  |  | *x*1 | + |  | *x*2 |  |  |  | - |  | *S*2 |  |  |  |  |  |  | + |  | *A*2 |  |  |  | = | 18 | |  |  | *x*1 | + | 3 | *x*2 |  |  |  |  |  |  | - |  | *S*3 |  |  |  |  |  |  | + |  | *A*3 | = | 28 | |
| and *x*1,*x*2,*S*1,*S*2,*S*3,*A*1,*A*2,*A*3≥0 |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration-1 |  | *Cj* | 3 | 4 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| *B* | *CB* | *XB* | *x*1 | *x*2 | *S*1 | *S*2 | *S*3 | *A*1 | *A*2 | *A*3 | MinRatio *XBx*1 |
| *A*1 | *M* | 30 | (4) | 1 | -1 | 0 | 0 | 1 | 0 | 0 | 304=7.5→ |
| *A*2 | *M* | 18 | 1 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 181=18 |
| *A*3 | *M* | 28 | 1 | 3 | 0 | 0 | -1 | 0 | 0 | 1 | 281=28 |
| *Z*=76*M* |  | *Zj* | 6*M* | 5*M* | -*M* | -*M* | -*M* | *M* | *M* | *M* |  |
|  |  | *Zj*-*Cj* | 6*M*-3↑ | 5*M*-4 | -*M* | -*M* | -*M* | 0 | 0 | 0 |  |

Positive maximum *Zj*-*Cj* is 6*M*-3 and its column index is 1. So, the entering variable is *x*1.  
  
Minimum ratio is 7.5 and its row index is 1. So, the leaving basis variable is *A*1.  
  
∴ The pivot element is 4.  
  
Entering =*x*1, Departing =*A*1, Key Element =4  
  
*R*1(new)=*R*1(old) ÷4  
  
*R*2(new)=*R*2(old) - *R*1(new)  
  
*R*3(new)=*R*3(old) - *R*1(new)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration-2 |  | *Cj* | 3 | 4 | 0 | 0 | 0 | *M* | *M* |  |
| *B* | *CB* | *XB* | *x*1 | *x*2 | *S*1 | *S*2 | *S*3 | *A*2 | *A*3 | MinRatio *XBx*2 |
| *x*1 | 3 | 7.5 | 1 | 0.25 | -0.25 | 0 | 0 | 0 | 0 | 7.50.25=30 |
| *A*2 | *M* | 10.5 | 0 | 0.75 | 0.25 | -1 | 0 | 1 | 0 | 10.50.75=14 |
| *A*3 | *M* | 20.5 | 0 | (2.75) | 0.25 | 0 | -1 | 0 | 1 | 20.52.75=7.4545→ |
| *Z*=31*M*+22.5 |  | *Zj* | 3 | 3.5*M*+0.75 | 0.5*M*-0.75 | -*M* | -*M* | *M* | *M* |  |
|  |  | *Zj*-*Cj* | 0 | 3.5*M*-3.25↑ | 0.5*M*-0.75 | -*M* | -*M* | 0 | 0 |  |

Positive maximum *Zj*-*Cj* is 3.5*M*-3.25 and its column index is 2. So, the entering variable is *x*2.  
  
Minimum ratio is 7.4545 and its row index is 3. So, the leaving basis variable is *A*3.  
  
∴ The pivot element is 2.75.  
  
Entering =*x*2, Departing =*A*3, Key Element =2.75  
  
*R*3(new)=*R*3(old) ÷2.75  
  
*R*1(new)=*R*1(old) - 0.25*R*3(new)  
  
*R*2(new)=*R*2(old) - 0.75*R*3(new)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration-3 |  | *Cj* | 3 | 4 | 0 | 0 | 0 | *M* |  |
| *B* | *CB* | *XB* | *x*1 | *x*2 | *S*1 | *S*2 | *S*3 | *A*2 | MinRatio *XBS*3 |
| *x*1 | 3 | 5.6364 | 1 | 0 | -0.2727 | 0 | 0.0909 | 0 | 5.63640.0909=62 |
| *A*2 | *M* | 4.9091 | 0 | 0 | 0.1818 | -1 | (0.2727) | 1 | 4.90910.2727=18→ |
| *x*2 | 4 | 7.4545 | 0 | 1 | 0.0909 | 0 | -0.3636 | 0 | --- |
| *Z*=4.9091*M*+46.7273 |  | *Zj* | 3 | 4 | 0.1818*M*-0.4545 | -*M* | 0.2727*M*-1.1818 | *M* |  |
|  |  | *Zj*-*Cj* | 0 | 0 | 0.1818*M*-0.4545 | -*M* | 0.2727*M*-1.1818↑ | 0 |  |

Positive maximum *Zj*-*Cj* is 0.2727*M*-1.1818 and its column index is 5. So, the entering variable is *S*3.  
  
Minimum ratio is 18 and its row index is 2. So, the leaving basis variable is *A*2.  
  
∴ The pivot element is 0.2727.  
  
Entering =*S*3, Departing =*A*2, Key Element =0.2727  
  
*R*2(new)=*R*2(old) ÷0.2727  
  
*R*1(new)=*R*1(old) - 0.0909*R*2(new)  
  
*R*3(new)=*R*3(old) + 0.3636*R*2(new)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration-4 |  | *Cj* | 3 | 4 | 0 | 0 | 0 |  |
| *B* | *CB* | *XB* | *x*1 | *x*2 | *S*1 | *S*2 | *S*3 | MinRatio *XBS*1 |
| *x*1 | 3 | 4 | 1 | 0 | -0.3333 | 0.3333 | 0 | --- |
| *S*3 | 0 | 18 | 0 | 0 | (0.6667) | -3.6667 | 1 | 180.6667=27→ |
| *x*2 | 4 | 14 | 0 | 1 | 0.3333 | -1.3333 | 0 | 140.3333=42 |
| *Z*=68 |  | *Zj* | 3 | 4 | 0.3333 | -4.3333 | 0 |  |
|  |  | *Zj*-*Cj* | 0 | 0 | 0.3333↑ | -4.3333 | 0 |  |

Positive maximum *Zj*-*Cj* is 0.3333 and its column index is 3. So, the entering variable is *S*1.  
  
Minimum ratio is 27 and its row index is 2. So, the leaving basis variable is *S*3.  
  
 ∴ The pivot element is 0.6667.  
  
Entering =*S*1, Departing =*S*3, Key Element =0.6667  
  
*R*2(new)=*R*2(old) ÷0.6667  
  
*R*1(new)=*R*1(old) + 0.3333*R*2(new)  
  
*R*3(new)=*R*3(old) - 0.3333*R*2(new)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration-5 |  | *Cj* | 3 | 4 | 0 | 0 | 0 |  |
| *B* | *CB* | *XB* | *x*1 | *x*2 | *S*1 | *S*2 | *S*3 | MinRatio |
| *x*1 | 3 | 13 | 1 | 0 | 0 | -1.5 | 0.5 |  |
| *S*1 | 0 | 27 | 0 | 0 | 1 | -5.5 | 1.5 |  |
| *x*2 | 4 | 5 | 0 | 1 | 0 | 0.5 | -0.5 |  |
| *Z*=59 |  | *Zj* | 3 | 4 | 0 | -2.5 | -0.5 |  |
|  |  | *Zj*-*Cj* | 0 | 0 | 0 | -2.5 | -0.5 |  |

Since all *Zj*-*Cj*≤0

Hence, optimal solution is arrived with value of variables as :  
*x*1=13,*x*2=5  
  
Min *Z*=59

3. A firm marketing a product has four salesman S1, S2, S3 and S4. There are three customers to whom a sale of each unit to be made. The chance of making a sale to a customer depend on the salesman customer support. The data depicts the probability with which each of the salesman can sell to each of the customers.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Salesman | | | |
| Customer | S1 | S2 | S3 | S4 |
| C1 | 0.7 | 0.4 | 0.5 | 0.8 |
| C2 | 0.5 | 0.8 | 0.6 | 0.7 |
| C3 | 0.3 | 0.9 | 0.6 | 0.2 |

If only one salesman is to be assigned to each of the customers, what combination of salesman and customers shall be optimal. Give further that the profit obtained by selling one unit of C1 is Rs. 500, whereas it is respectively Rs 450 and Rs. 540 for sale to C2 and C3. What is the expected profit?

Ans 3a.

Multiply each customers profit value with the probability of each salesman and customer

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Customer | Salesmen | | | |
|  | S1 | S2 | S3 | S4 |
| C1 | 0.7\*500 | 0.4\*500 | 0.5\*500 | 0.8\*500 |
| C2 | 0.5\*450 | 0.8\*450 | 0.6\*450 | 0.7\*450 |
| C3 | 0.3\*540 | 0.9\*540 | 0.6\*540 | 0.2\*540 |

We have to assign on salesmen to one customer

Since there are four salesman and three customers we have to add one customer (dummy cell) So the matrix (table) will be as follow:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Customer | Salesmen | | | |
|  | S1 | S2 | S3 | S4 |
| C1 | 350 | 200 | 250 | 400 |
| C2 | 225 | 360 | 270 | 315 |
| C3 | 162 | 486 | 324 | 108 |
| C4 | 0 | 0 | 0 | 0 |

Row Deduction

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Customer | Salesmen | | | |
|  | S1 | S2 | S3 | S4 |
| C1 | 150 | 0 | 50 | 200 |
| C2 | 0 | 135 | 45 | 90 |
| C3 | 54 | 378 | 216 | 0 |
| C4 | 0 | 0 | 0 | 0 |

Optimal Assignment

C1 to S2 =200

C2 to S1 =225

C3 to S4 =108

C4 to S3 =000

Total Expected Profit is Rs. 533

Assignment Set – 2

1. What is Monte Carlo simulation? Explain Monte Carlo Simulation Procedure in brief.

Ans 1.

Monte Carlo Simulation: A Powerful Tool for Decision Making

Monte Carlo simulation is a computational technique used to assess the impact of risk and uncertainty in decision-making processes. Named after the famous Monte Carlo Casino in Monaco, known for its games of chance, this method involves using random sampling and probability distributions to model different possible outcomes in a problem. It provides a range of possible outcomes and the probabilities they will occur for any choice of action, helping decision-makers to quantify the risk and make more informed decisions.

Monte Carlo Simulation Procedure

The Monte Carlo simulation procedure involves several key steps:

Problem Formulation: The first step is to clearly define the problem and the variables involved. This includes identifying the decision to be made, the uncertain factors affecting the decision, and the range of possible outcomes.

Model Development: Next, a mathematical model is developed to represent the problem. This model includes the relationships between the variables and the probability distributions that describe the uncertainty in the system.

Random Sampling: Monte Carlo simulation relies on random sampling to generate possible outcomes. Random numbers are generated based on the specified probability distributions for each variable.

Simulation Runs: The simulation is run multiple times, with each run representing a different possible outcome. The number of runs is typically large enough to ensure a reliable estimate of the outcomes.

Analysis of Results: Once the simulation runs are completed, the results are analyzed to determine the range of possible outcomes and their probabilities. This analysis provides insights into the risk and uncertainty associated with the decision.

Decision Making: Finally, based on the results of the simulation, a decision is made. The decision-maker can use the information provided by the simulation to assess the risks and benefits of different options and make an informed choice.

Example of Monte Carlo Simulation

For example, consider a project manager who needs to estimate the completion time for a construction project. There are several uncertain factors that could affect the project's timeline, such as weather conditions, availability of resources, and unforeseen delays. By using Monte Carlo simulation, the project manager can model these factors using probability distributions and generate multiple possible scenarios for the project's completion time.

Based on the simulation results, the project manager can assess the likelihood of completing the project on time and budget and identify potential risks. This information can help in making decisions such as resource allocation, scheduling, and risk management strategies.

Monte Carlo simulation is a powerful tool for decision-making in complex and uncertain situations. It provides a systematic way to assess risks and uncertainties and helps decision-makers make more informed choices. By simulating different scenarios and analyzing the results, Monte Carlo simulation enables organizations to optimize their decision-making processes and improve their outcomes.

Top of Form

2. Asmallprojectiscomposedofsevenactivities,whosetimeestimatesarelistedin the table below:

|  |  |  |  |
| --- | --- | --- | --- |
| Activity (i – j) | Estimated Duration (Weeks) | | |
| Optimistic | Most Likely | Pessimistic |
| 1 – 2 | 1 | 1 | 7 |
| 1 – 3 | 1 | 4 | 7 |
| 1 – 4 | 2 | 2 | 8 |
| 2 – 5 | 1 | 1 | 1 |
| 3 – 5 | 2 | 5 | 14 |
| 4 – 6 | 2 | 5 | 8 |
| 5 – 6 | 3 | 6 | 15 |

Draw the network diagram of activities in the project.

Find the expected duration and variance for each activity. What is the expected project length?

Calculate the variance and standard deviation of the project length. What is the probability that the project will be completed atleast 4 weeks earlier than expected time.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Z | 0.67 | 1.00 | 1.33 | 2.00 |
| Prob. | 0.2514 | 0.1587 | 0.0918 | 0.0228 |

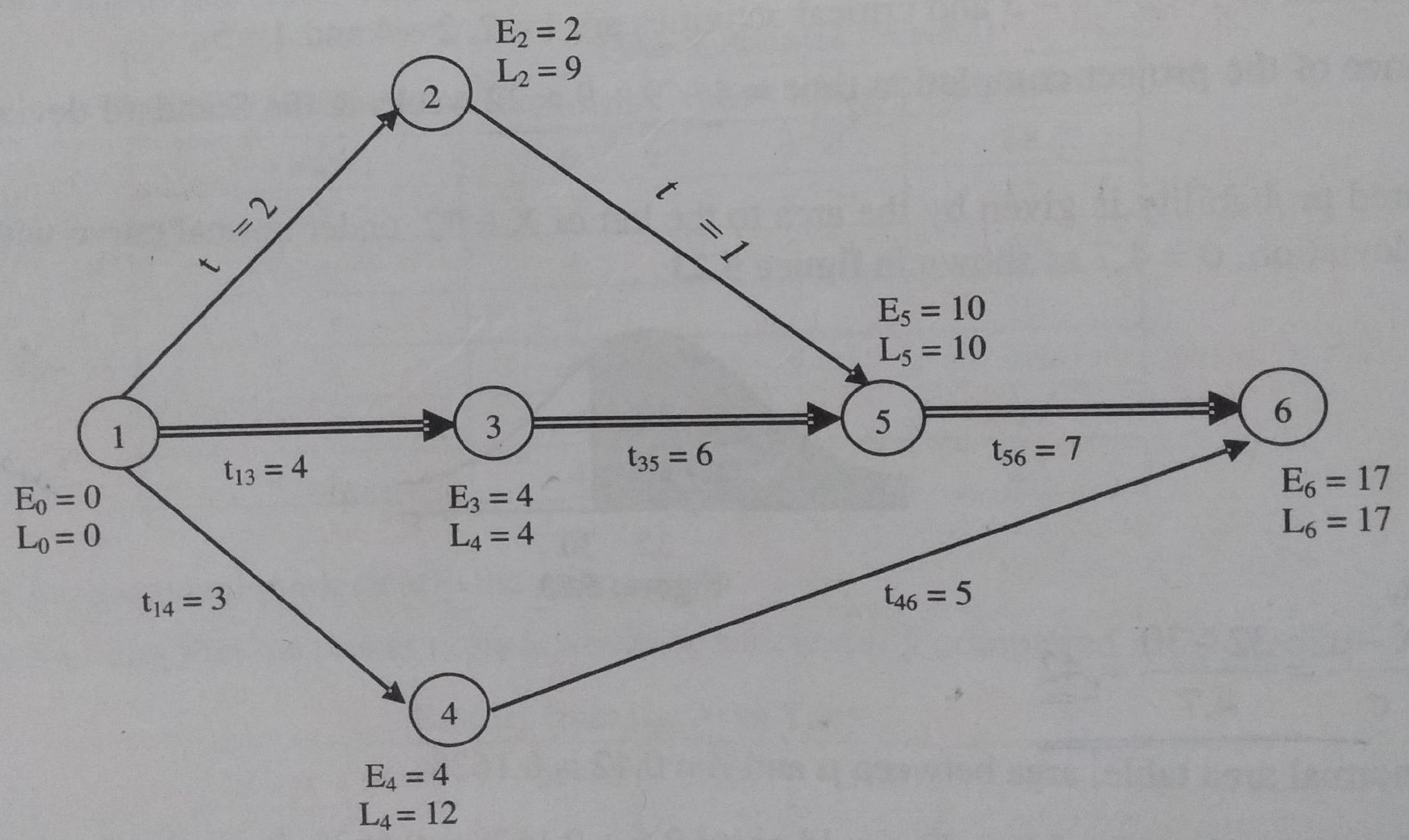
Ans 2.

Making use of time estimates and , the calculations for expected time and variance for activities are shown in table below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Activity  Sequence | Time Duration (Weeks) | |  |  |  |
|  |  |  |
|  | 1 | 1 | 7 | 2 | 1 |
|  | 1 | 4 | 7 | 4 | 1 |
|  | 2 | 2 | 8 | 3 | 1 |
|  | 1 | 1 | 1 | 1 | 0 |
|  | 2 | 5 | 14 | 6 | 4 |
|  | 2 | 5 | 8 | 5 | 1 |
|  | 3 | 6 | 15 | 7 | 4 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 2 | 2 | 8 | 3 | 0 |
|  | 1 | 1 | 1 | 1 | 4 |
|  | 2 | 5 | 14 | 6 | 1 |
|  | 2 | 5 | 8 | 5 | 4 |
|  | 3 | 6 | 15 | 7 |  |

1. The network diagram can be drawn as follows:



Network Diagram

The evpecned duration and variance of each activity is given in table above. We now calculate the expected

|  |  |  |
| --- | --- | --- |
| Variwes Paths | Length of the Path | |
|  |  | weeks |
|  |  | weeks |
|  |  | weeks |

Among these putis, the critical path is weeks

2. Wuriance of the project length is the sum of the variances of the activities on the critical path. Hence, Variance,

deys (trom table above). Therefore, Standard deviation of the project weeks

Phobubiliry that the project will be completed atleast 4 weeks earlier than expected time is given by;

3. There is a game between the two players A and B where A is maximizing player and B is minimizing player. Player A wins B’s coin if the two coins total are equal to odd number and losses his coin if total to two coins is even. It is game of 1, 2, 5, 10 and 50 rupees coins. Determine the payoff matrix, the optimal strategies for each player and the value of the game to A.

Ans 3.

To solve this, let's first construct the payoff matrix:

Let's represent the coins as 1, 2, 5, 10, and 50 rupees, respectively. Player A is the maximizing player and Player B is the minimizing player. The possible totals can be odd (win for A) or even (loss for A). The matrix will have rows for Player A's choices and columns for Player B's choices, and the entries will represent the payoff to A.

Here's the payoff matrix:

| 1 2 5 10 50

-------------------------

1 | 0 -1 1 -1 1

2 | -1 0 -1 1 -1

5 | 1 -1 0 1 -1

10 | -1 1 -1 0 1

50 | 1 -1 1 -1 0

In this matrix:

* A positive number represents a win for A (A gains B's coin).
* A negative number represents a loss for A (A loses their coin to B).
* Zero represents no change in coins.

To find the optimal strategies for each player, we can use the concept of mixed strategies. Let 𝑝p be the probability that A chooses 1 rupee, 𝑞q be the probability that A chooses 2 rupees, 𝑟r be the probability that A chooses 5 rupees, 𝑠s be the probability that A chooses 10 rupees, and 𝑡t be the probability that A chooses 50 rupees.

Let's assume B's optimal strategy is to randomize their choices such that all coins are equally likely to be chosen. This assumption simplifies the calculation, and it's often the case in such games where one player is maximizing and the other is minimizing.

The expected payoff for A when choosing 1 rupee is:

𝐸1=𝑝(−1)+𝑞(1)+𝑟(−1)+𝑠(1)+𝑡(1)=𝑞+𝑠+𝑡−𝑝−𝑟

Similarly, the expected payoffs for A when choosing 2, 5, 10, and 50 rupees are:

𝐸2=𝑝(1)+𝑞(−1)+𝑟(1)+𝑠(−1)+𝑡(−1)=𝑝+𝑟−𝑞−𝑠−𝑡

𝐸5=𝑝(−1)+𝑞(1)+𝑟(−1)+𝑠(1)+𝑡(−1)=𝑞+𝑠−𝑝+𝑟−𝑡

𝐸10=𝑝(1)+𝑞(−1)+𝑟(1)+𝑠(−1)+𝑡(1)=𝑝+𝑟−𝑞−𝑠+𝑡

𝐸50=𝑝(−1)+𝑞(1)+𝑟(−1)+𝑠(1)+𝑡(0)=𝑞+𝑠−𝑝+𝑟

To find the optimal strategy for A, we need to find the values of p, q, r, s, and t that maximize A's expected payoff while considering the constraints p+q+r+s+t=1 and 𝑝,𝑞,𝑟,𝑠,𝑡≥0

The optimal strategy for B is to choose all coins with equal probability, i.e., 𝑞=𝑟=𝑠=𝑡=0.2

By solving the above system of equations, we can find the optimal strategies for A and the value of the game to A.

Let's solve for the optimal strategies for A and the value of the game to A.

1. Optimal Strategy for A:

We want to maximize E1​, E2​, E5​, E10​, and E50​ subject to the constraint 𝑝+𝑞+𝑟+𝑠+𝑡=1 and 𝑝,𝑞,𝑟,𝑠,𝑡≥0

Let's set up the equations:

* + - 𝐸1=𝑞+𝑠+𝑡−𝑝−𝑟
    - 𝐸2=𝑝+𝑟−𝑞−𝑠−𝑡
    - 𝐸5=𝑞+𝑠−𝑝+𝑟−𝑡
    - 𝐸10=𝑝+𝑟−𝑞−𝑠+𝑡
    - 𝐸50=𝑞+𝑠−𝑝+𝑟
    - 𝑝+𝑞+𝑟+𝑠+𝑡=1

1. Solving the System of Equations:
   * Adding all equations, we get: 2𝑝+2𝑞+2𝑟+2𝑠+2𝑡=2
   * Dividing by 2, we get: 𝑝+𝑞+𝑟+𝑠+𝑡=1 which is our constraint.
   * Therefore, the equations are consistent with the constraint.
2. Optimal Strategies:
   * The equations are symmetrical, so any permutation of the solutions is also valid.
   * One possible optimal strategy for A is p=0.2, q=0.2, r=0.2, s=0.2, 𝑡=0.2t=0.2.
3. Value of the Game to A:

Using the optimal strategy, the expected payoff to A is

𝐸total=0.2×𝐸1+0.2×𝐸2+0.2×𝐸5+0.2×𝐸10+0.2×𝐸50

Calculating Etotal​, we get the value of the game to A.

Let's calculate 𝐸totalEtotal​ to find the value of the game to A.

Substituting the optimal strategy p=0.2, q=0.2, r=0.2, s=0.2, t=0.2 into the expressions for E1​, E2​, E5​, E10​, and E50​:

𝐸1=0.2(−1)+0.2(1)+0.2(−1)+0.2(1)+0.2(1)=0

𝐸2=0.2(1)+0.2(−1)+0.2(1)+0.2(−1)+0.2(−1)=−0.2

𝐸5=0.2(−1)+0.2(1)+0.2(−1)+0.2(1)+0.2(−1)=−0.2

𝐸10=0.2(1)+0.2(−1)+0.2(1)+0.2(−1)+0.2(1)=0.2

𝐸50=0.2(−1)+0.2(1)+0.2(−1)+0.2(1)+0.2(0)=0

Now, calculating the total expected payoff to A:

𝐸total=0.2×𝐸1+0.2×𝐸2+0.2×𝐸5+0.2×𝐸10+0.2×𝐸50

𝐸total=0×0.2+(−0.2)×0.2+(−0.2)×0.2+0.2×0.2+0×0.2

𝐸total=0−0.04−0.04+0.04+0

𝐸total=−0.08

So, the value of the game to Player A is -0.08, which means, on average, Player A loses 0.08 rupees per game against an optimal strategy from Player B.