OHIO UNIVERSITY MATHEMATICS DEPARTMENT

Control Theory & Adaptive Cruise Control

Abdalah El-Barrad, Colton Smith, & Dylan Denner; Advised by Gurgen Hayrapetyan

ABSTRACT

Using a PID (Proportion, Integral, Derivative) closed loop tracking control, we implemented a cruise control system to dynamically preserve a specified distance from a lead car by adjusting the throttle.

INTRODUCTION

Control Theory

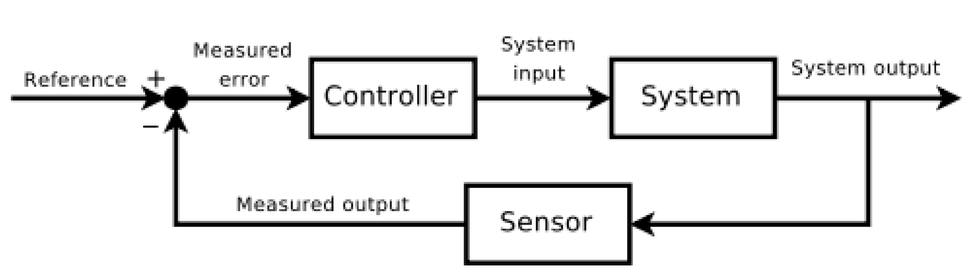
Control theory is a discipline that deals with dynamics (typically nonlinear) to control a system using a controller. Such problems include robotics, investment strategies, temperature control, and aircraft motion.

A controller starts with modeling a system. After running the initial values on the system, the controller will continuously adjust the controls to attempt to minimize the error. A visualization of this is shown below.

The Problem

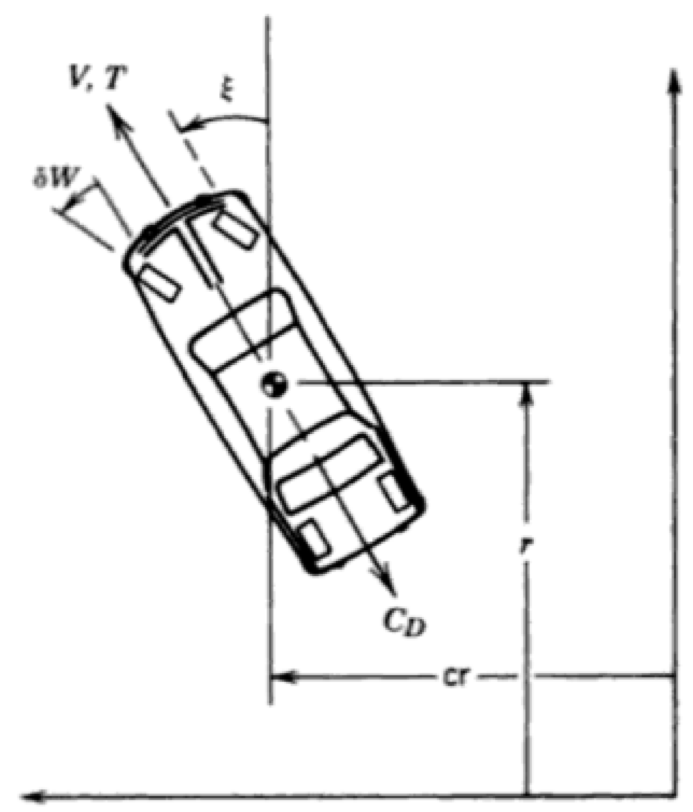
The problem we tried to solve was creating a cruise control system that also tracks and maintains a specified distance away from the car directly in front of it. The car is assumed to have a sensor to track the distance between it and the lead car. This would be used in a self navigating car.

Negative Feedback Control System



Wikipedia. 15 October 2008. "Control Theory"

Problem figure



Stengel, Robert F. 1986. "Optimal Control and State Estimation"

THE MODEL

Dynamical System/Process

$$\dot{v} = \frac{\delta T \cos^2(w_0) - D(v)}{m_0}$$

$$\dot{\xi} = \frac{v \tan(w_0)}{l}$$

$$\dot{x} = v \sin(\xi)$$

$$\dot{y} = v \cos \xi$$

Linearization

With the dynamical system above, we linearize the system about our initial conditions to simplify it. We do this by finding the Jacobian of our output and control which in this case is thrust.

$$\dot{x} = Fx + Gu$$

$$\dot{x} = \begin{bmatrix} \frac{D'(0)}{m_0} & 0 & 0 & 0 \\ \frac{\tan(w_0)}{l} & 0 & 0 & 0 \\ \sin(\xi_0) & 0 & 0 & 0 \\ \cos(\xi_0) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ \xi \\ x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\cos^2(w_0)}{m_0} \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta T$$

Output Function

To get the output function from the linearized one, we use the following formula to get the our outcome:

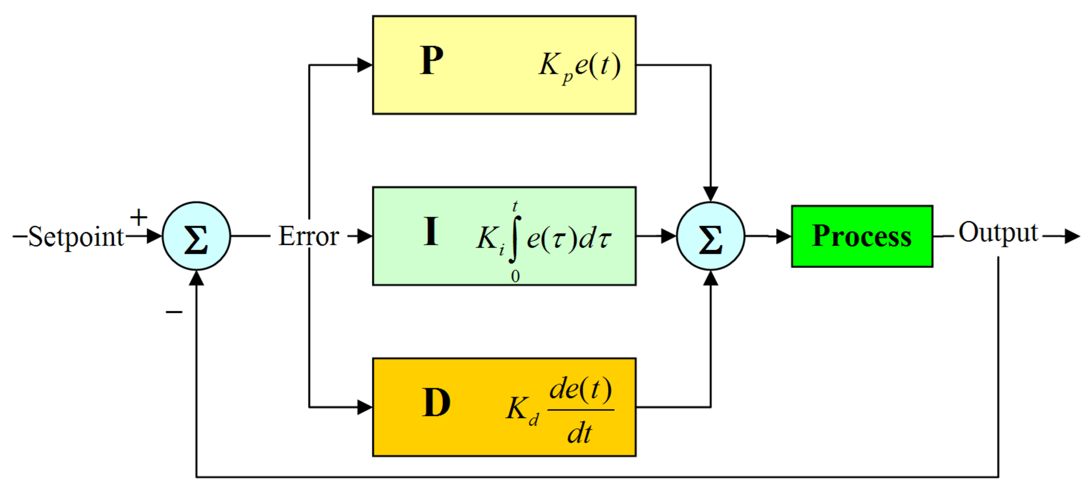
$$Y(s) = H(sI - F)^{-1} GU(s)$$

(where H is a binary function that determines the output)

$$Y(s) = \frac{-\cos^2(w_0)\sin(\xi_0)}{m_0 s^2 + D'(0)s}$$

PID Control

Our model uses a Proportion Integral Derivative (PID) closed loop tracking control. PID controls are very common in control theory. The goal of the PID control is to adjust the error with respect to the current, past, and future errors.



With the output function, we can find adequate K values that work for the system.

$$T_0(s) = \frac{N(s)}{D(s)}$$

$$C(s) = K_p + K_i \frac{1}{s} + K_d s$$

$$T_c(s) = \frac{C(s)N(s)}{D(s) + C(s)N(s)}$$

Finalization

Assuming that all the poles of the transfer function are -1, we were able to get the following K values:

$$K_p = \frac{-3m_0}{\cos^2(w_0)\sin(\xi_0)}$$

$$K_i = \frac{-m_0}{\cos^2(w_0)\sin(\xi_0)}$$

$$K_d = \frac{D'(0) - 3m_0}{\cos^2(w_0)\sin(\xi_0)}$$

Solving that, we were able to test our controller in C++ with a simulator created by Dr. Hayrapetyan.

Possible Improvements

- Sensor measurements – Using probabilistic methods to obtain best estimates of front car
- Linearizion – Using multiple initial conditions to make the output functions more accurate.
- MIMO – Making the control system to multiple input/multiple output problem instead
- Optimal control – Use optimal control to save on something such as gas or driving distance

CONCLUSIONS

Using high level mathematics, we are able to model, optimize, and control real world problems. Control theory is a combination optimization, differential equations, dynamical systems, linear algebra, calculus of variations, probability, and other more theoretical areas of mathematics that can be used across disciplines to solve many problems.

BIBLIOGRAPHY

1 Stengel Robert F. *Optimal Control and State Estimation*. New York: Dover. 1994. Print

2 Hayrapetyan, Gurgen. 2017. *Class Resources. http://gurgentus.name/work/class-resrouces/*

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