In[]:=
```
ClearAll[f, f1, f2, g, s, x, x1, x2, W1, W2, w11, w12, w21, w22]
```

# Градиент простой функции

In[]:=
```
activation[x_] := Exp[x];
f[z_, w_] := activation[z.w]; (* экспонента от скалярного произведения x и w *)
x = {x1, x2}; W1 = {w11, w12};
TableForm[{
  "f(x,W1)=" f[x, W1],
  "gradient=" MatrixForm[Grad[f[x, W1], W1]]}]
```

Out[]//TableForm=

$$f(x,W1) = e^{w11\,x1+w12\,x2}$$

$$\text{gradient} = \begin{pmatrix} e^{w11\,x1+w12\,x2}\,x1 \\ e^{w11\,x1+w12\,x2}\,x2 \end{pmatrix}$$

## Усложним функцию активации

```
activation[x_] := Exp[-x^2];
f[z_, w_] := activation[z.w];
(* экспонента от минус квадарата скалярного произведения x и w *)
x = {x1, x2}; W1 = {w11, w12};
{f[x, W1], MatrixForm[Grad[f[x, W1], W1]]}
```

Out[]=

$$\left\{e^{-(w11\,x1+w12\,x2)^2}, \begin{pmatrix} -2\,e^{-(w11\,x1+w12\,x2)^2}\,x1\,(w11\,x1+w12\,x2) \\ -2\,e^{-(w11\,x1+w12\,x2)^2}\,x2\,(w11\,x1+w12\,x2) \end{pmatrix}\right\}$$

In[]:=
```
activation[x_] := Exp[-x^2];
f[z_, w_] := activation[z.w];
(* экспонента от минус квадарата скалярного произведения x и w *)
x = {x1, x2}; W3 = {{w11, w12}, {w21, w22}};
MatrixForm[{Grad[f[x, W3], W3[[1]]], Grad[f[x, W3], W3[[2]]]}]
```

Out[]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -2\,e^{-(w11\,x1+w21\,x2)^2}\,x1\,(w11\,x1+w21\,x2) \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -2\,e^{-(w12\,x1+w22\,x2)^2}\,x1\,(w12\,x1+w22\,x2) \end{pmatrix} \\ \begin{pmatrix} -2\,e^{-(w11\,x1+w21\,x2)^2}\,x2\,(w11\,x1+w21\,x2) \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -2\,e^{-(w12\,x1+w22\,x2)^2}\,x2\,(w12\,x1+w22\,x2) \end{pmatrix} \end{pmatrix}$$

# Градиент сложной функции

In[•]:=
```
f2[x_, w1_, w2_] := f[f[x, w1], w2];
W2 = {w21, w22};
MatrixForm[Grad[f2[x, W1, W2], Join[W1, W2]]]
```

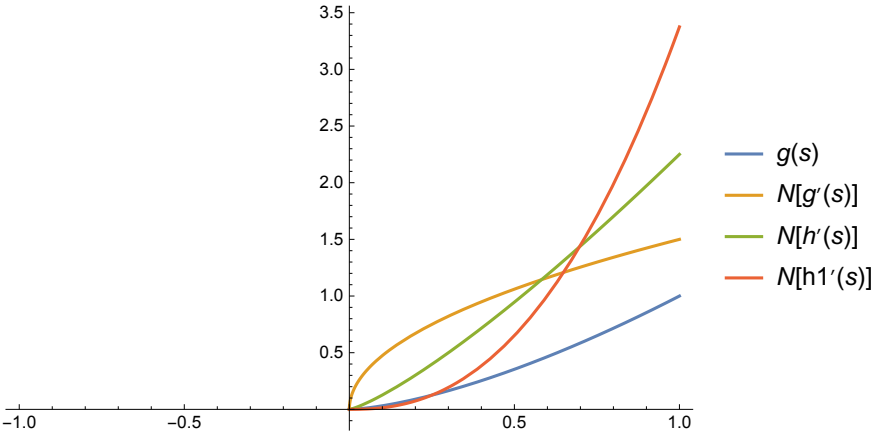
Out[•]//MatrixForm=

$$\begin{pmatrix} -2\, e^{-\left(e^{-(w11\,x1+w12\,x2)^2}.\{w21,w22\}\right)^2}\, e^{-(w11\,x1+w12\,x2)^2}.\{w21,\,w22\}\left(e^{-(w11\,x1+w12\,x2)^2}.\{\{0,0,1,0\},\{0,0,0,1\}\right. \\ -2\, e^{-\left(e^{-(w11\,x1+w12\,x2)^2}.\{w21,w22\}\right)^2}\, e^{-(w11\,x1+w12\,x2)^2}.\{w21,\,w22\}\left(e^{-(w11\,x1+w12\,x2)^2}.\{\{0,0,1,0\},\{0,0,0,1\}\right. \\ -2\, e^{-\left(e^{-(w11\,x1+w12\,x2)^2}.\{w21,w22\}\right)^2}\, e^{-(w11\,x1+w12\,x2)^2}.\{w21,\,w22\}\, e^{-(w11\,x1+w12\,x2)^2}. \\ -2\, e^{-\left(e^{-(w11\,x1+w12\,x2)^2}.\{w21,w22\}\right)^2}\, e^{-(w11\,x1+w12\,x2)^2}.\{w21,\,w22\}\, e^{-(w11\,x1+w12\,x2)^2}. \end{pmatrix}$$

# Графики производных

In[•]:=
```
g[s_] := s^1.5;
(*g[s_]:=Exp[-s^2/4];*)
(*g[s_]:=Tanh[s];*)
h[s_] := g[g[s]];
h1[s_] := g[h[s]];
Plot[{g[s], N[g'[s]], N[h'[s]], N[h1'[s]]},
 {s, -1, 1}, PlotLegends → "Expressions", PlotRange → Full]
```

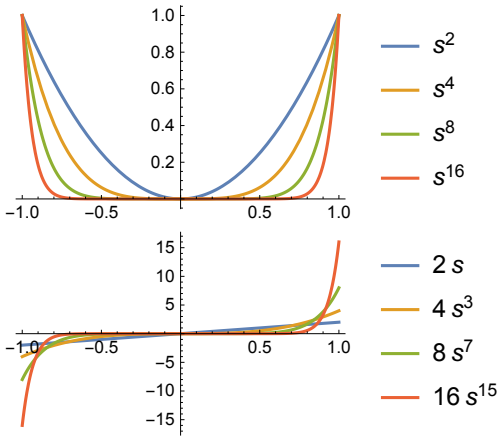
Out[•]=



# Проблема “взрыва” градиента степенных функций

```
In[•]:= g[s_] := s^2;
        t = RecurrenceTable[{a[n + 1] == g[a[n]], a[1] == g[s]}, a, {n, 1, 4}];
        dt = D[t, s];
        TableForm[{Plot[t, {s, -1, 1}, PlotLegends → "Expressions", PlotRange → Full],
          Plot[dt, {s, -1, 1}, PlotLegends → "Expressions", PlotRange → Full]}]
```

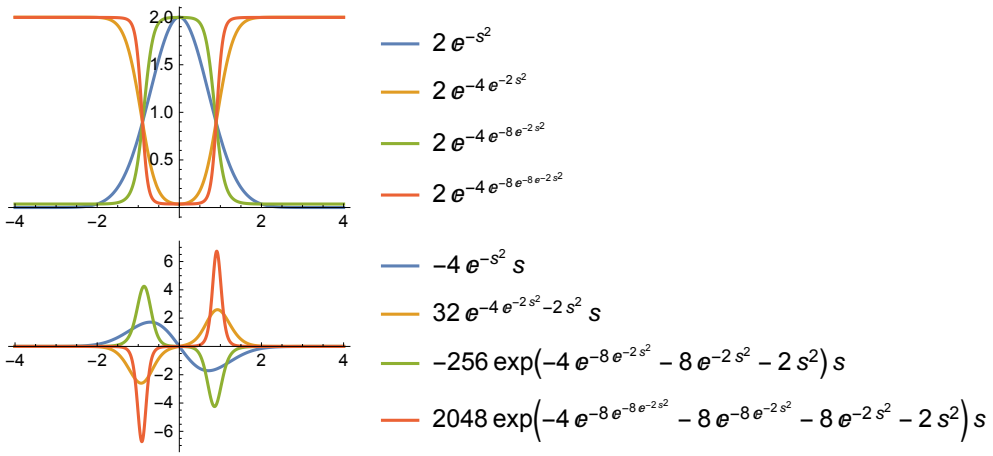
Out[•]//TableForm=




# Проблема “угасания” градиента негативно-степенных функций

```
In[•]:= g[s_] := 2 Exp[-s^2];
        t = RecurrenceTable[{a[n + 1] == g[a[n]], a[1] == g[s]}, a, {n, 1, 4}];
        dt = D[t, s];
        TableForm[{Plot[t, {s, -4, 4}, PlotLegends → "Expressions", PlotRange → Full],
          Plot[dt, {s, -4, 4}, PlotLegends → "Expressions", PlotRange → Full]}]
```

Out[•]//TableForm=




# Решение проблем “взрыва”/”угасания”- специальные

# функции

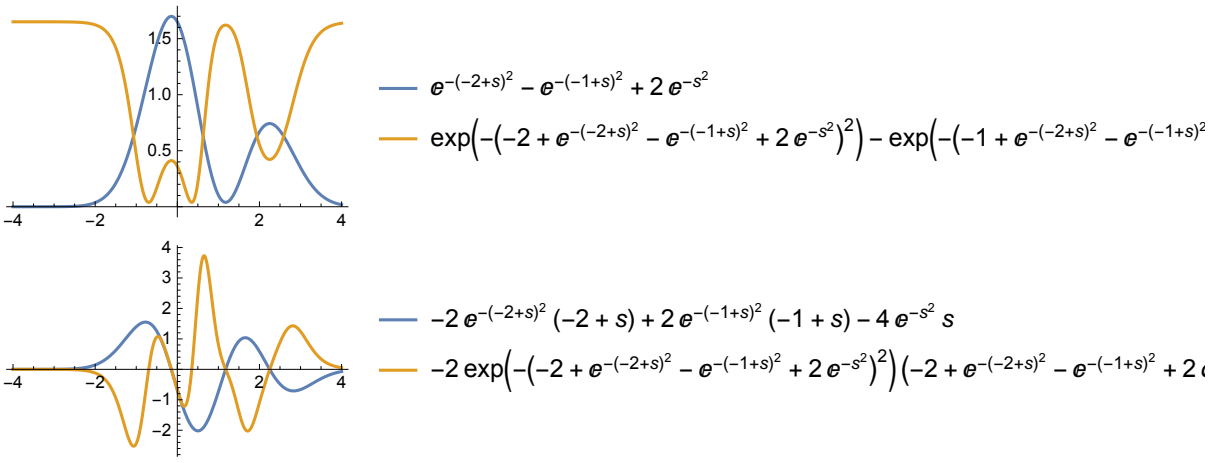
```
g[s_] := Tanh[s];
t = RecurrenceTable[{a[n + 1] == g[a[n]], a[1] == g[s]}, a, {n, 1, 4}];
dt = D[t, s];
TableForm[{Plot[t, {s, -2, 2}, PlotLegends → "Expressions", PlotRange → Full],
  Plot[dt, {s, -2, 2}, PlotLegends → "Expressions", PlotRange → Full]}]
```

Out[•]//TableForm=


# Проблема локальных минимумов

In[•]:=
```
g[s_] := 2 Exp[-s^2] - Exp[-(s - 1)^2] + Exp[-(s - 2)^2];
t = RecurrenceTable[{a[n + 1] == g[a[n]], a[1] == g[s]}, a, {n, 1, 2}];
dt = D[t, s];
TableForm[{Plot[t, {s, -4, 4}, PlotLegends → "Expressions", PlotRange → Full],
  Plot[dt, {s, -4, 4}, PlotLegends → "Expressions", PlotRange → Full]}]
```

Out[•]//TableForm=




In[•]:=
```
N[13470 / 50]
```

Out[•]=
```
269.4
```