Relations

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# Operations on Relations

- $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, etc. set theoretical

  $(R_1, R_2 \subseteq A \times B)$

## Composition of relations


$R \subseteq A \times B$, $S \subseteq B \times C$

$S \circ R = \{(a,c) \mid \exists b \in B \ \ aRb \wedge bSc\}$

a misleading notation $\rightarrow$ $R \circ S$

e.g., $R = S$ : a is a parent of b

$R \circ S$ ; a is a grandparent of b

$R^0 = id$, $R^1 = R$, $R^{k+1} = R^k \circ R$

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n-ary relations $R \subseteq A_1 \times A_2 \times \dots \times A_n$

e.g., $(n, q, r)$ : "r is remainder of n divided by q"

ternary relⁿ → degree is 3

e.g., Relational Databases

Organize data in tables (relations)

$R_1$

| Stid | Name | Surname | Dept | GPA | semester |
|---|---|---|---|---|---|
| 12345 | Ahmet | Can | CS | 3.43 | 4 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

$R_2$

| Stid | TakeCourse |
|---|---|
| 12345 | 5710223 |
| ⋮ | ⋮ |

Operations

□ Relational Algebra

- projection
- selection
- join

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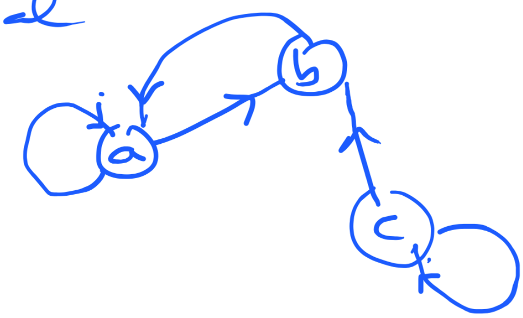
# Representing Relations

e.g., $R_1 = \{(a,a), (a,b), (c,b), (b,a), (c,c)\}$
on $A = \{a, b, c\}$

- bit matrix

$$M_{R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

- graphical



a loop

$(a, b)$

## Operations on Relations

$R, S$ : relations $\quad R \cup S \quad M_{R \cup S} = M_R \vee M_S$

logical or

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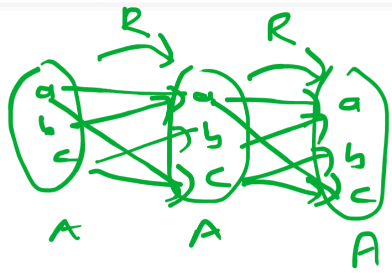



$M_{R\cap S} = M_R \wedge M_S$

logical AND

$M_{R\circ S} = M_S \odot M_R$ → boolean product

awkward notation

e.g., $A=\{a,b,c\}$

$R=\{(a,a),(a,c),(b,a),(c,b),(c,c)\}$

$$M_R = \begin{bmatrix}1&0&1\\1&0&0\\0&1&1\end{bmatrix}$$

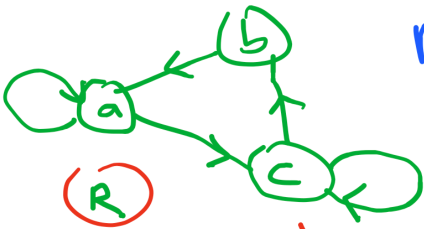
$$M_{R^2}=M_{R\circ R} = \begin{bmatrix}1&0&1\\1&0&0\\0&1&1\end{bmatrix}\odot\begin{bmatrix}1&0&1\\1&0&0\\0&1&1\end{bmatrix}=\begin{bmatrix}1&1&1\\1&0&1\\1&1&1\end{bmatrix}$$

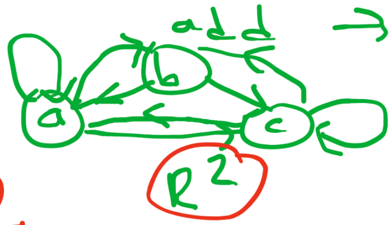
$\parallel$

$M_R^2$

mult → logical and

add → logical or




apply rule of trans. one step

# Closure of Relations

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P: property of rel^n's such as refl., symm., ...

R: relation on A

A closure of R wrt P is the smallest rel^n S with property P containing R

① Reflexive closure of R $= R \cup \Delta$

$\Delta = \{(a,a) \mid a \in A\}$

$$\underbrace{\begin{bmatrix} * & & T_1 \\ & * & \\ T_2 & & \ddots \; * \end{bmatrix}}_{M_R} \longrightarrow \underbrace{\begin{bmatrix} 1 & & T_1 \\ & 1 & \\ T_2 & & \ddots \; 1 \end{bmatrix}}_{M_S = M_R \vee M_\Delta}$$

② Symmetric closure of R $= R \cup R^{-1}$

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$

③ Transitive closure of R $= R^*$

$$R^* = \bigcup_{i=1}^{\infty} R^i = R^1 \cup R^2 \cup R^3 \cup \cdots$$

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$$R^* = \bigcup_{i=1}^{n} R^i = R^1 \cup R^2 \cup R^3 \cup \dots \cup R^n$$

$n = |A|$

$\exists k < n$ s.t. $R^k = R^{k+1}$

stop at k+1 !

e.g.,

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

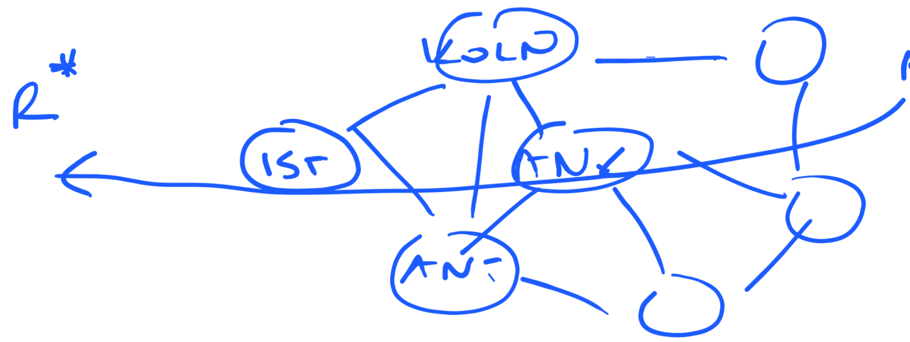
$$\boxed{M_{R^k} = M_{R^{k-1}} \odot M_R}$$

$M_{R^*} = ?$

$$M_{R^2} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad M_{R^3} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$M_R^3 = M_R^2$

$$M_{R^*} = M_R \vee M_R^2 \vee M_R^3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \checkmark$$




R

| Source | Dest. |
|---|---|
| IST | KOLN |
| IST | ANT |
| ⋮ | |