CENG 371 - Scientific Computing
Fall 2023
Homework 4

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Answer 1

All the RMM algorithm files are uploaded.

Answer 2

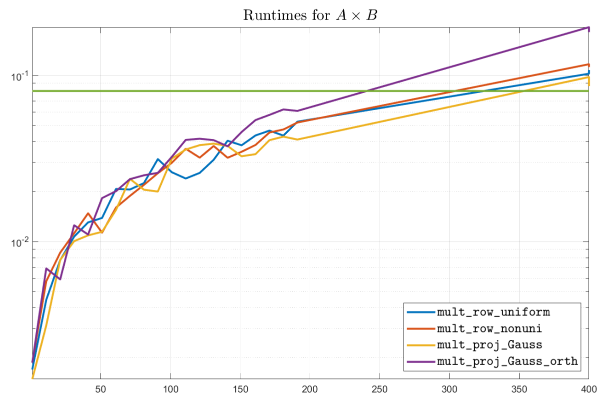
a) The scaling factor is used in RMM methods for ensuring that some properties of the randomized projections are preserved, which improve efficient matrix multiplication.

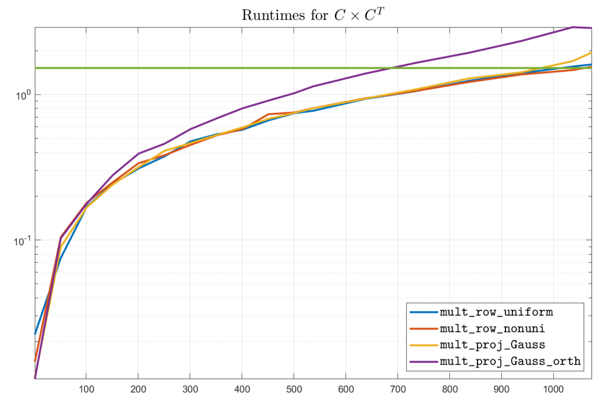
We have applied $\frac{1}{\sqrt{c \cdot probabilities[p_{i_t}]}}$ as scaling factor for the random sampling methods while generating C and R. This scaling aims to normalize each selected column's contribution with respect to the c (number of columns) while considering the probability that it was selected.

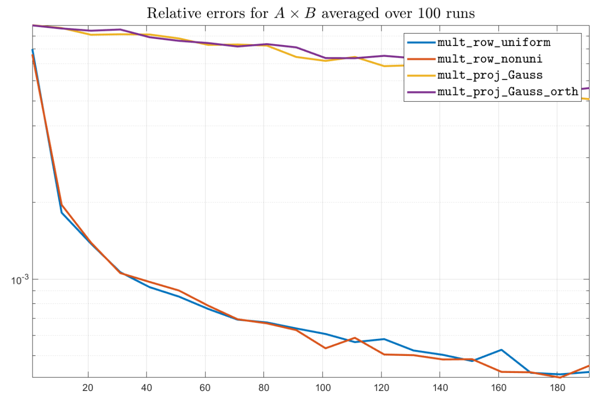
We have applied $\frac{1}{\sqrt{c}}$ as scaling factor for the random projection matrix P which is generated from the random Gaussian values. This allows for the conversion of high-dimensional matrices to lower-dimensional matrices with some information loss due to the random nature of these values. In Gaussian Projection, in order to preserve geometry of original matrices, we also normalize to ensure that each P column has 2-Norm 1. Additionally, we also applied Gram-Schmidt Orthogonalization to orthonormal columns with P to preserve more information from the original matrices.

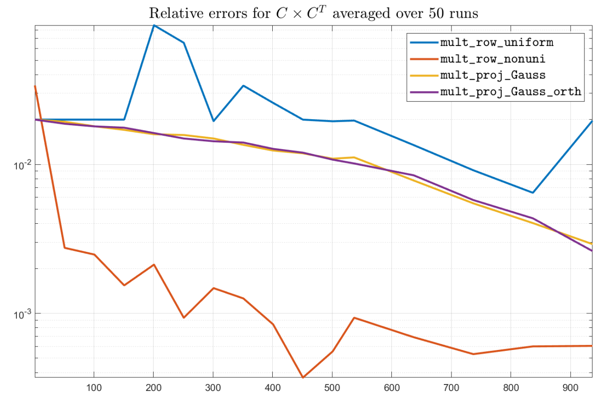
In conclusion, the scaling enhances the resulting matrices' fairness and stability. Therefore, scaling has been implemented to ensure more robust resulting matrices.

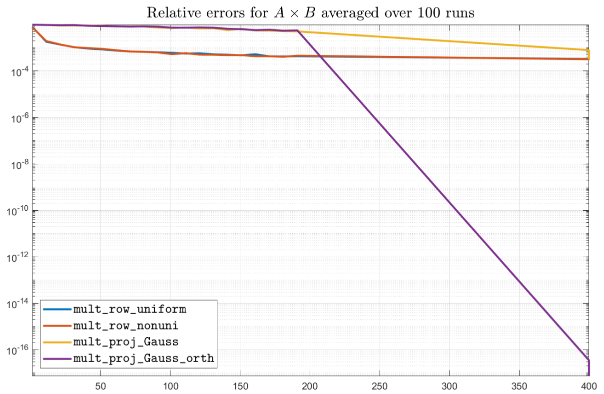
b) The Plotted Figures:

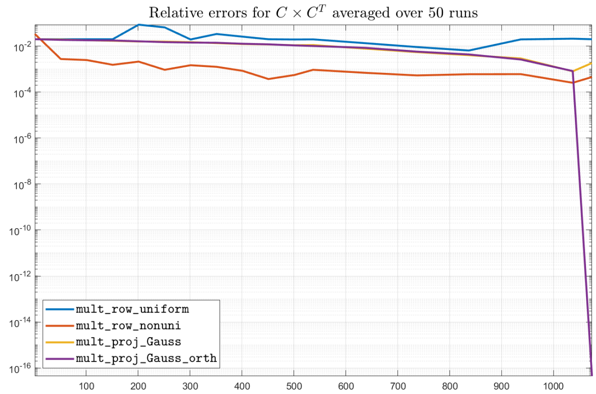












c) When we compared to the run times of algorithms:
We can observe that it grows logarithmically for all algorithms as c increases. They differ slightly from one another. Compared to the other three algorithms, the "*mult_proj_Gauss_orth*" usually takes more time because it has Gaussian Orthogonalization process.

When we compared to the relative errors of algorithms:
For smaller c, the random projection method significantly reduce the data, this results a higher loss of information than random sampling method. So, we can observe that the random projection method has higher relative errors for smaller projection matrix dimension. Also, input matrix dimension increases Non-Uniform Sampling method has the least relative errors than others. As c increases the projection matrix dimension grows, it preserves more features from the original matrix and reduces the information loss . In Gaussian Projection with Orthogonalization method, $c \approx 200$ for AxB and $c \approx 1000$ for CxC^T, the relative errors decreases significantly. Because applying Gram-Schmidt Orthogonalization improves the stability, maintains the original relationship and distances between vectors, and also ensures that each columns captures independent information from original matrix.

When we have small c values we can choose Random Sampling Method. While using the Random Sampling Method we should be careful about if we have a matrix whose columns have similar norms we can apply Uniform Random Sampling over Nonuniform because it is faster and we do not lose much information from original matrices. However, when matrices are sparse and column norms are much vary we should apply Nonuniform Random Sampling to adjust columns probabilities to preserve matrix features while doing selection. On the other hand, when we have a larger c, we can apply the Random Projection Method. The "*mult_proj_Gauss*" method for where randomness is acceptable and distortions are tolerable. Even though "*mult_proj_Gauss_orth*" has a higher cost, we can suggest this method for tasks which require precise relationship preservation. Although $mult\_row\_uniform$ and $mult\_row\_nonuni$ random sampling methods are simple, they may be less suitable for tasks which require exact pattern preservation.

d) CxC^T have bigger matrix sizes and sparser than AxB. Sparse matrices have most of the entries 0 which doesn't hold any information about the matrix features. When using Uniform Random Sampling all columns has same probability to be chosen. Therefore, when we generate C and R primarily using zero columns, we lose too much information about the original matrices, which increases relative errors. This indicates that the Uniform Random Sampling Method is an inappropriate method to use when performing matrix multiplication with sparse matrices, as it leads to decreased performance. As a result, selecting appropriate techniques that are maximized for effectiveness and information preservation is essential.