Languages that are and are not context free

CENG 280

Course outline

- Preliminaries: Alphabets and languages
- Regular languages
- Context-free languages
  - Context-free grammars
  - Parse trees
  - Push-down automaton
  - Push-down automaton - context-free languages
  - Languages that are and that are not context-free, Pumping lemma
- Turing-machines

CFL Closure Properties

Theorem

The context-free languages are closed under union, concatenation, and Kleene-star.

$G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$ with disjoint sets of non-terminals, $(V_1 \setminus \Sigma_1 \cap V_2 \setminus \Sigma_2 = \emptyset)$,

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The proof is constructive. $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$ with disjoint sets of non-terminals, $(V_1 \setminus \Sigma_1 \cap V_2 \setminus \Sigma_2 = \emptyset)$,

- G_U such that $L(G_U) = L(G_1) \cup L(G_2)$
- $G_U = (V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\}, S)$
- G_C such that $L(G_C) = L(G_1)L(G_2)$
- $G_C = (V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \to S_1 S_2\}, S)$
- G_K such that $L(G_K) = L(G_1)^\star)$
- $G_K = (V_1 \cup \{S\}, \Sigma_1, R_1 \cup \{S \to e, S \to SS_1\}, S)$

CFL Closure Properties

- Context-free languages are not closed under intersection or complementation.
- The complementation proof for regular languages requires a deterministic automaton.
- However, not all context free languages are accepted by deterministic push-down automaton.
- There is proof of the closure under intersection based on construction of product of two automata (Problem 2.3.3).
- This construction can be extended to push-down automata, but the product automaton has two stacks.
- However, the same idea words for FA and PDA (only one stack).

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Proof idea: $M_1 = (K_1, \Sigma, \Gamma_2, \Delta_2, s_1, F_1)$, $M_2 = (K_2, \Sigma, \delta, s_2, F_2)$ with $L = L(M_1)$ and $R = L(M_2)$. Define $M = (K, \Sigma, \Gamma, \Delta, s, F)$

- $K = K_1 \times K_2$,
- $\Gamma = \Gamma_1$,
- $s = (s_1, s_2)$, $F = F_1 \times F_2$
- $\Delta = \{((q_1, q_2), a, \beta), ((p_1, \delta(q_2, a)), \gamma) \mid ((q_1, a, \beta), (p_1, \gamma)) \in \Delta_1, q_2 \in K_2\} \cup \{((q_1, q_2), e, \beta), ((p_1, q_2)), \gamma) \mid ((q_1, e, \beta), (p_1, \gamma)) \in \Delta_1, q_2 \in K_2\}$

Examples

Example

CFL or not: L consists of all strings with equal number of a's and b's, and no two consecutive b's is followed by another b.

Examples

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CFL or not: L is CFL, R is regular, then $L \setminus R$.

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CFL or not: L is CFL, R is regular, then $R \setminus L$.