APPLICATION OF FIRST ORDER LOGIC: SYLLOGISMS, v.2

Syllogisms are patterns of argumentation expressed in daily language. Syllogisms were originated by Aristotle (4th century BC), and extended and catalogued by his successors.

Here is an example known as *Modus Felapton*:

> There are flowers.
>
> No flowers are animals.
>
> All flowers are plants.
>
> Therefore, some plants are not animals.

Is this line of reasoning acceptable?

Let us first identify and define the predicates used in the argument.

> $F(x)$: x is a flower.
>
> $P(x)$: x is a plant.
>
> $A(x)$: x is an animal.

Let us rewrite the syllogism in FOL:

$$\begin{array}{l} \exists x\ F(x) \\ \neg\exists x\ (F(x) \land A(x)) \\ \forall x\ (F(x) \to P(x)) \\ \hline \exists x (P(x) \land \neg A(x)) \end{array}$$

Checking the validity of this argument amounts to establishing the sequent

$\exists x\ F(x), \neg\exists x\ (F(x) \land A(x)), \forall x\ (F(x) \to P(x)) \vdash \exists x(F(x) \land \neg A(x))$

We shall be using the basic and derived rules of Natural Deduction to construct a proof.

1. $\exists x\ F(x)$ premise

2. $\neg\exists x\ \big(F(x) \wedge A(x)\big)$ premise

3. $\forall x\ \big(F(x) \rightarrow P(x)\big)$ premise

c fresh name

4. $F(c)$ assumption,1

5. $F(c) \rightarrow P(c)$ 3, $\forall e$

6. $P(c)$ 4,5, $\rightarrow e$

7. $\forall x \neg\big(F(x) \wedge A(x)\big)$ 2, derived$(*)$

8. $\neg\big(F(c) \wedge A(c)\big)$ 7, $\forall e$

9. $\neg F(c) \vee \neg A(c)$ 8, derived$(**)$

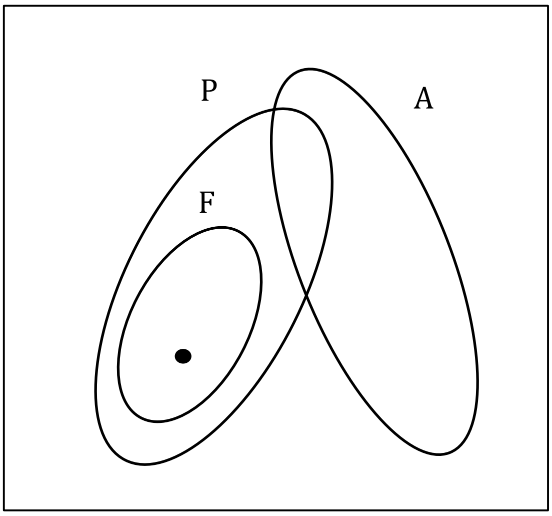
10. $\neg A(c)$ 4,9, derived$(***)$

11. $P(c) \wedge \neg A(c)$ 6,9, $\wedge i$

12. $\exists x\big(P(x) \wedge \neg A(x)\big)$ 11, $\exists i$

13. $\exists x\big(P(x) \wedge \neg A(x)\big)$ 1,4 − 12, $\exists e$

The situation can be illustrated as follows. The predicates F, P and A can be *interpreted* as the sets F, P and A, respectively, as shown in the picture below. The rectangular box represents the universal set. This visual representation of sets is known as a Venn diagram. The constraints on the sets set forth by the premises are also visible from the diagram.



It is interesting to note that an earlier version of *Modus Felapton* omitted the first premise. It goes without saying that there are flowers! In other words, the set F is assumed to be not empty. Without the first premise, however, the proof does not go through. (You may try this as an exercise. You can see this from the diagram as well.)

To benefit from Logic, we need to make our implicit assumptions explicit.

Going back to the proof, the derived rule (*) used in line 7 is based on the sequent

$$\neg\exists x\,(\beta \wedge \gamma) \vdash \forall x \neg\,(\beta \wedge \gamma)$$

Line 9 uses the DeMorgan's law as a derived rule (**).

The rule used in line 10 (***) is directly derived from disjunction elimination, i.e. $\vee e$.

Derive these three rules from the basic rules as an exercise.