Module 27

- Applications of Pumping Lemma
  - General proof template
    - What is the same in every proof
    - What changes in every proof
  - Incorrect pumping lemma proofs
  - Some rules of thumb

Pumping Lemma

Applying it to prove a specific language L is not regular

How we use the Pumping Lemma

- We choose a specific language L
  - For example, $\{a^j b^j \mid j > 0\}$
- We show that L *does not* satisfy the pumping condition
- We conclude that L is not regular

Showing L "does not pump"

- A language L *satisfies* the pumping condition if:
  - *there exists* an integer n > 0 such that
  - *for all* strings x in L of length at least n
  - *there exist* strings u, v, w such that
    - x = uvw and
    - |uv| <= n and
    - |v| >= 1 and
    - *For all* k >= 0, uv^kw is *in* L
- A language L *does not satisfy* the pumping condition if:
  - *for all* integers n of sufficient size
  - *there exists* a string x in L of length at least n such that
  - *for all* strings u, v, w such that
    - x = uvw and
    - |uv| <= n and
    - |v| >= 1
  - *There exists* a k >= 0 such that uv^kw is *not in* L

Example Proof

- A language L *does not satisfy* the pumping condition if:
  - *for all* integers n of sufficient size
  - *there exists a* string x in L of length at least n such that
  - *for all* strings u, v, w such that
    - x = uvw and
    - |uv| <= n and
    - |v| >= 1
  - *There exists* a k >= 0 such that uv^kw is *not in* L
- Proof that L = $\{a^ib^i \mid i>0\}$ does not satisfy the pumping condition
- *Let n be the integer from the pumping lemma*
- *Choose* x = a^nb^n
- Consider *all* strings u, v, w s.t.
  - x = uvw and
  - |uv| <= n and
  - |v| >= 1
- Argue that uv^kw is *not in* L for *some* k >= 0
  - ***Argument must apply to all possible u,v,w***
  - ***Continued on next slide***

Example Proof Continued *

- Proof that L = $\{a^ib^i \mid i>0\}$ does not satisfy the pumping condition
- Let n be the integer from the pumping lemma
- *Choose* x = a^nb^n
- Consider *all* strings u, v, w s.t.
  - x = uvw and
  - |uv| <= n and
  - |v| >= 1
- Argue that uv^kw is *not in* L for *some* k >= 0
  - ***Argument must apply to all possible u,v,w***
  - ***Continued on right***
- ***$uv^0w = uw$*** is not in L
  - uv contains only a's
    - why?
  - uw = $a^{n-|v|}b^n$
    - Follows from previous line and uvw = x = a^nb^n
  - uw contains fewer a's than b's
    - why?
  - Therefore, uw is not in L
- Therefore L does not satisfy the pumping condition

Alternate choice of k *

- Proof that $L = \{a^i b^i \mid i>0\}$ does not satisfy the pumping condition
- Let n be the integer from the pumping lemma
- *Choose* x = $a^n b^n$
- Consider *all* strings u, v, w s.t.
    - x = uvw and
    - |uv| <= n and
    - |v| >= 1
- Argue that $uv^k w$ is *not in* L for *some* k >= 0
    - ***Argument must apply to all possible u,v,w***
    - ***Continued on right***
- $uv^2w = uvvw$ is not in L
    - uv contains only a's
        - why?
    - uvvw = $a^{n+|v|}b^n$
        - follows from previous line and uvw = x = $a^n b^n$
    - uvvw contains more a's than b's
        - why?
    - Therefore, uvvw is not in L
- Therefore L does not satisfy the pumping condition

7

Pumping Lemma

Some bad applications of the pumping lemma

8

Bad Pumping Lemma Applications

- We now look at some examples of bad applications of the pumping lemma
- We work with the language EQUAL consisting of the set of strings over {a,b} such that the number of a's equals the number of b's
- We focus first on bad choices of string x
- We then consider another flawed technique

9

First bad choice of x *

- A language L *does not satisfy* the pumping condition if:
    - Let n be the integer from the pumping lemma
    - *there exists a* string x in L of length at least n such that
    - *for all* strings u, v, w such that
        - x = uvw and
        - |uv| <= n and
        - |v| >= 1
    - *There exists* a k >= 0 such that $uv^k w$ is *not in* L
- Let n be the integer from the pumping lemma
- *Choose* x = $a^{10}b^{10}$
    - What is wrong with this choice of x?

10

Second bad choice of x *

- A language L *does not satisfy* the pumping condition if:
    - Let n be the integer from the pumping lemma
    - *there exists a* string x in L of length at least n such that
    - *for all* strings u, v, w such that
        - x = uvw and
        - |uv| <= n and
        - |v| >= 1
    - *There exists* a k >= 0 such that $uv^k w$ is *not in* L
- Let n be the integer from the pumping lemma
- *Choose* x = $a^n b^{2n}$
    - What is wrong with this choice of x?

11

Third bad choice of x *

- A language L *does not satisfy* the pumping condition if:
    - Let n be the integer from the pumping lemma
    - *there exists a* string x in L of length at least n such that
    - *for all* strings u, v, w such that
        - x = uvw and
        - |uv| <= n and
        - |v| >= 1
    - *There exists* a k >= 0 such that $uv^k w$ is *not in* L
- Let n be the integer from the pumping lemma
- *Choose* x = $(ab)^n$
    - What is wrong with this choice of x?
        - The problem is there is a choice of u, v, w satisfying the three conditions such that for all k >=0, $uv^k w$ is in L
        - What is an example of such a u, v, w?

12

Find the flaw in this proof *

- A language L *does not satisfy* the pumping condition if:
  - Let n be the integer from the pumping lemma
  - *there exists a* string x in L of length at least n such that
  - *for all* strings u, v, w such that
    - x = uvw and
    - |uv| <= n and
    - |v| >= 1
  - *There exists* a k >= 0 such that uv^kw is *not in* L
- Let n be the integer from the pumping lemma
- *Choose* x = ***a^nb^n***
- Let u = a^2, v =a, w = $a^{n-3}b^n$
  - |uv| = 3 <= n
  - |v| = 1
- ***Choose k = 2***
- Argue uv^2w is not in EQUAL
  - uv^2w = uvvw = $a^2aaa^{n-3}b^n$ = $a^{n+1}b^n$
  - There is one more a than b in uv^2w
  - Thus uv^2w is not in L

13

Pumping Lemma

Two rules of thumb

14

Two Rules of Thumb *

- Try to make the first n characters of x identical
  - For EQUAL, choose x = a^nb^n rather than $(ab)^n$
    - Simplifies case analysis as v only contains a's
- Try k=0 or k=2
  - k=0
    - This *reduces* number of occurrences of that first character
  - k=2
    - This *increases* number of occurrences of that first character

15

Summary

- We use the Pumping Lemma to prove a language is not regular
  - Note, does not work for all nonregular languages, though
- Choosing a good string x is first key step
- Choosing a good integer k is second key step
- Must apply argument to all legal u, v, w

16