CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

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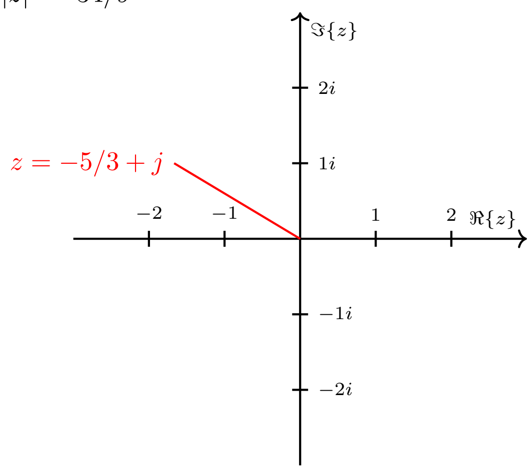
1. (a) Given $z = x + yj$ and $2z + 5 = j - \bar{z}$, finding $\bar{z}$ which is conjugate of z :
Applying the z and $\bar{z}$ to the given equation:
$2(x + yj) + 5 = j–(x–yj)$
$(2x + 5) + 2yj = -x + (y + 1)j$
$2x + 5 = -x$
$y + 1 = 2y$
So $x = -5/3$ and $y = 1$
Putting the values x and y in the z
$z = -5/3 + j$

$|z| = \sqrt{1 + 25/9} = \sqrt{34/9}$

$|z|^2 = 34/9$



(b) Given $z = re^{j\theta}$:
$z^5 = r^5 e^{j5\theta} = 32j$
This equation give us $r = 2$
Remembering the equation $(e^{j\theta} = cos(\theta) + jsin(\theta))$:
$(e^{j5\theta} = j = cos(\pi/2) + jsin(\pi/2))$
Therefore :
$5\theta = \pi/2$
$\theta = \pi/10$
z in polar form:

$z = 2(cos(\frac{\pi}{10}) + jsin(\frac{\pi}{10}))$

$z = 2e^{(j\frac{\pi}{10} + \frac{\pi}{5}k)}$ where $k = 1, 2, 3...$

(c) Multiplying both numerator and denominator of given z with $(1+j)$:

$z = \frac{j-\sqrt{3}}{-2} = \frac{\sqrt{3}-j}{2}$

Finding the magnitude r and angle θ of z:

$r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{-1}{2})^2}$

$r = 1$
$\theta = tan^{-1}(\frac{-1}{\sqrt{3}})$
$\theta = \frac{-\pi}{6}$

(d) $j = e^{j\frac{\pi}{2}}$
$z = e^{j\frac{\pi}{2}} * e^{-j\frac{\pi}{2}}$
$z = e^{j\frac{\pi}{2}} * e^{-j\frac{\pi}{2}} = 1$
$z = 1 + 0j$

2. Below is the signal for $y(t) = x(\frac{1}{2}t + 1)$

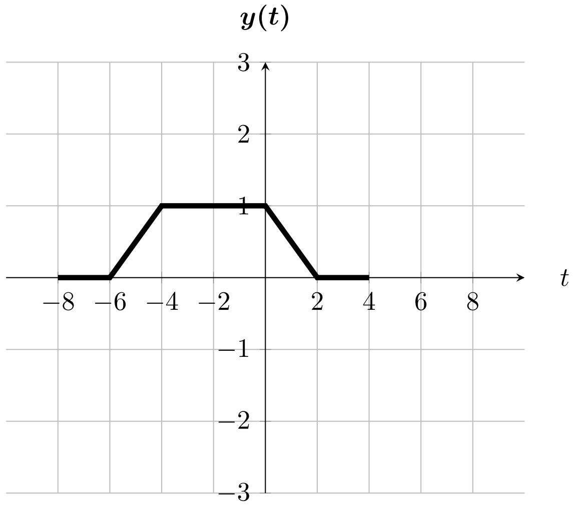


Figure 1: t vs. $y(t)$.

3. (a) Below is the signal for $x[-n] + x[2n-1]$

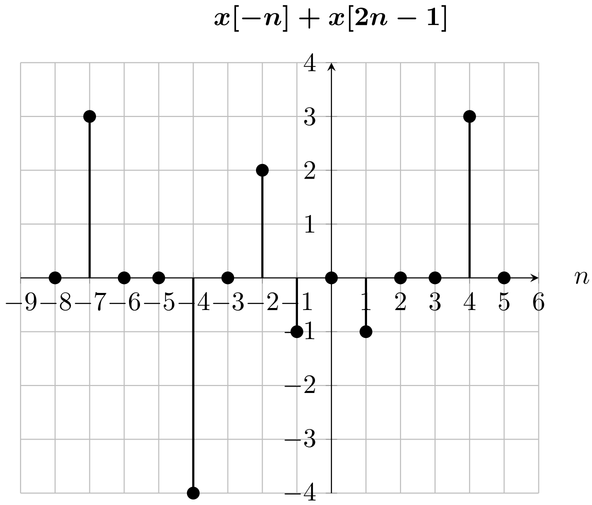


Figure 2: n vs. $x[-n] + x[2n-1]$.

(b) $x[-n] + x[2n-1]$ in terms of the unit impulse function is as follows:
$3\delta(n+7) - 4\delta(n+4) + 2\delta(n+2) - \delta(n+1) - \delta(n-1) + 3\delta(n-4)$

4. (a) $x(t) = 5sin(3t - \frac{\pi}{4})$
This is a continuous signal, fundamental period can be found as:

$$T = \frac{2\pi}{\omega}$$
$$T = \frac{2\pi}{3}$$

$\frac{2\pi}{3}$ is the fundamental period of $x(t)$.

(b) $x[n] = cos[\frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n]$
A discrete signal is periodic only if $\omega_0 = 2\pi\ m/N$ for some integers $N > 0$ and m.
Fundamental period for $cos[\frac{13\pi}{10}n]$ (Since this is a discrete signal we need an integer m to make N integer):

$$N_1 = \frac{2\pi}{\Omega}m$$
$$N_1 = \frac{2\pi}{\frac{13\pi}{10}}m$$
$$N_1 = \frac{20}{13}m \qquad m = 13$$
$$N_1 = 20$$

Fundamental period for $sin[\frac{7\pi}{10}n]$ (Since this is a discrete signal we need an integer m to make N integer):

$$N_2 = \frac{2\pi}{\Omega}m$$
$$N_2 = \frac{2\pi}{\frac{7\pi}{10}}m$$
$$N_2 = \frac{20}{7}m \qquad m = 7$$
$$N_2 = 20$$

To be able to find fundamental period for $x[n]$ we need to find $lcm(N_1, N_2)$:

$$N = lcm(N_1, N_2)$$
$$N = lcm(20, 20)$$
$$N = 20$$

20 is the fundamental period of $x[n]$.

(c) $x[n] = \frac{1}{2}cos[7n - 5]$
This is also a discrete signal fundamental period can be found as:

$$N = \frac{2\pi}{\Omega}m$$
$$N = \frac{2\pi}{7}m$$

There does not exist any integer m which can make $N = \frac{2\pi}{7}m$ integer, so we can exactly say $x[n]$ is an aperiodic signal.

5. (a) Unit step function of $x(t)$ is:
$x(t) = u(t-1) - 3u(t-3) + u(t-4)$

(b) We know that $\delta(\mathrm{t}) = \frac{du(t)}{dt}$
Therefore, we can find $\frac{dx(t)}{dt}$ from the unit step function found above.

$$\frac{dx(t)}{dt} = \frac{d(u(t-1) - 3u(t-3) + u(t-4))}{dt}$$
$$= \frac{du(t-1)}{dt} - 3\frac{du(t-3)}{dt} + \frac{du(t-4)}{dt}$$
$$= \delta(t-1) - 3\delta(t-3) + \delta(t-4)$$

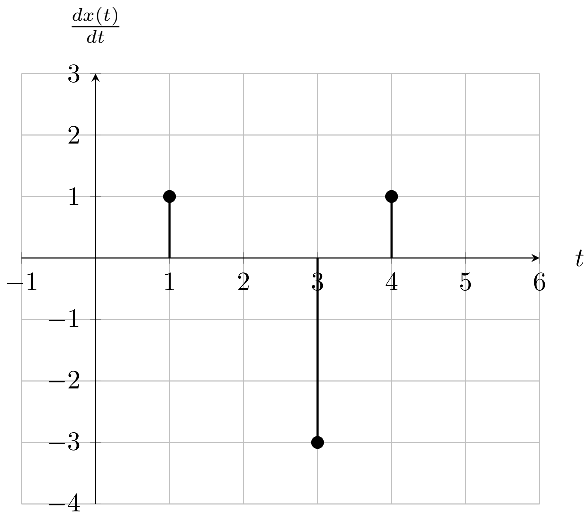


Figure 3: $\frac{dx(t)}{dt}$.

6. (a) $y(t) = tx(2t+3)$:

i. *Memory* : System does not need memory since it does not need to use neither past inputs. So this system is memoryless.

ii. *Stability* : The system is not stable since when we change $x(t)$ with a constant, we can see output is not constant.

iii. *Causality* : The system does not depend on only past and present inputs, it has some future inputs too, so this system is not causal.

iv. *Linearity* : Since The system holds superposition property, so system is linear.

v. *Invertibility* : The system is invertible since distinct inputs lead to distinct outputs and :

$$x(t) = h^{-1}(y(t))$$
$$= (y(\frac{t-3}{2}) * 2)/(t-3)$$

vi. $Time - Invarience$: No time-invariant because time-shifting changes the results.

(b) $y[n] = \sum_{k=1}^{\infty} x[n-k]$:

i. *Memory* : System needs a memory because its output depends on the sum of all past values of input.

ii. *Stability* : The system is not stable since its amplitude is not bounded and it changes due to the inputs. More specifically, each individual signal's sum makes $y[n]$ unbounded since it goes up to ∞ .

iii. *Causality* : The system depends on the past inputs, so this system is causal.

iv. *Linearity* : For system to be linear, it needs to hold superposition property. So we need to check if this system holds superposition property. Let x_1 and x_2 be two input signals:

$$y_1[n] = \sum_{k=1}^{\infty} x_1[n-k]$$
$$y_2[n] = \sum_{k=1}^{\infty} x_2[n-k]$$

When we add these up and multiply by some constants a_1 and a_2, we will have a y_3 as:

$$\begin{aligned} y_3[n] &= a_1 \times y_1[n] + a_2 \times y_2[n] \\ &= a_1 \times \sum_{k=1}^{\infty} x_1[n-k] + a_2 \times \sum_{k=1}^{\infty} x_2[n-k] \end{aligned}$$

and on the other hand when we first perform addition and multiplication then put the signal as input to the system we will have a $y_3^{'}$ such as:

$$\begin{aligned} x_3[n] &= a_1 \times x_1[n] + a_2 \times x_2[n] \\ y_3^{'}[n] &= \sum_{k=1}^{\infty} x_3[n-k] \\ &= a_1 \times \sum_{k=1}^{\infty} x_1[n-k] + a_2 \times \sum_{k=1}^{\infty} x_2[n-k] \end{aligned}$$

Since $y_3 = y_3'$ superposition property holds and system is linear.

v. *Invertibility* : The system is invertible since distinct inputs lead to distinct outputs and:

$$\begin{aligned} x[n] &= h^{-1}(y[n]) \\ &= y[n+1] - y[n] \\ &= \{x[n] + x[n-1] + ...\} - \{x[n-1] + x[n-2] + ...\} \end{aligned}$$

vi. *Time − Invarience* : Since shifting input in time causes an identical shift in the output as well system is time invariant.

7. (a)

```
import numpy as np
import matplotlib.pyplot as plt

filename = "shifted_sawtooth_part_a.csv"
# filename = "sine_part_a.csv"
# filename = "chirp_part_a.csv"

data = np.loadtxt(filename, delimiter=",")
startIndex = int(data[0])
signalList = data[1:]
# Split the signal into its even and odd parts
coordinate = startIndex
length = len(signalList)
limit = length + startIndex
even = []
odd = []
for i in range(0, length):
    negIndex = -coordinate
    if (negIndex < startIndex or negIndex > limit):
        num = 0
    else:
        num = signalList[i-2*coordinate]
    even.append((signalList[i]+num)/2)
    odd.append((signalList[i]-num)/2)
    coordinate += 1

# Plot the original signal, even part, and odd part
n = np.arange(startIndex, startIndex+len(signalList))
plt.plot(n, signalList, label="Signal Part")
plt.plot(n, odd, label="Odd part")
plt.plot(n, even, label="Even part")
plt.legend()
plt.show()
```

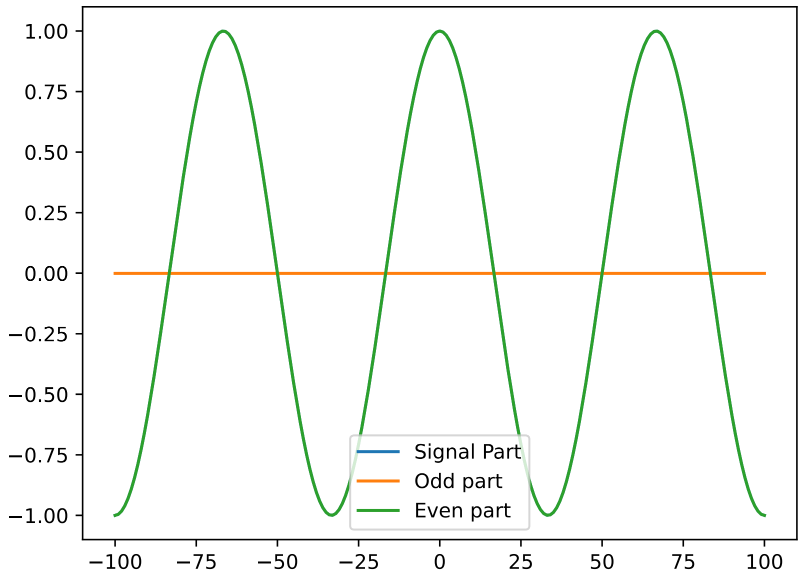


Figure 4: sine part A

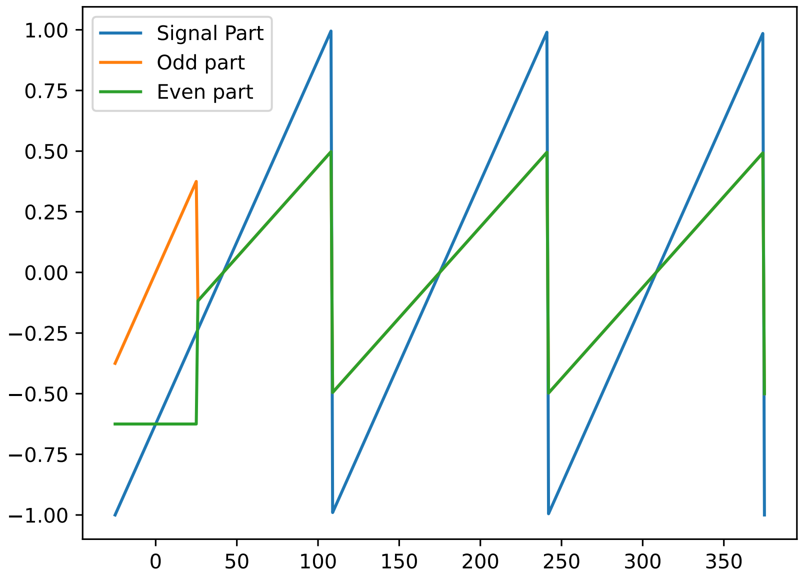


Figure 5: shifted sawtooth part A

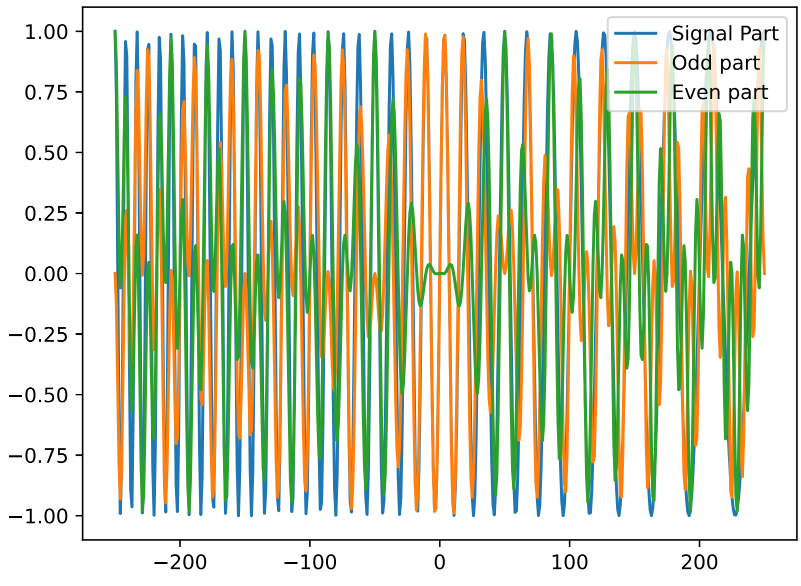


Figure 6: chirp part A

(b)

```
import numpy as np
import matplotlib.pyplot as plt

# filename = "shifted_sawtooth_part_b.csv"
filename = "sine_part_b.csv"
# filename = "chirp_part_b.csv"

data = np.loadtxt(filename, delimiter=",")
startIndex = int(data[0])
a = data[1]
b = data[2]
signalList = data[3:]

n = np.arange(startIndex, startIndex+len(signalList))
start = int(startIndex/a-b)
end = int((startIndex+len(signalList))/a-b)
dimension = len(signalList)/a - (end-start)
newScale = np.arange(start-dimension, end, 1/a)

plt.plot(n, signalList, label="Original signal")
plt.plot(newScale, signalList, label="Shifted and scaled signal")
plt.legend()
plt.show()
```

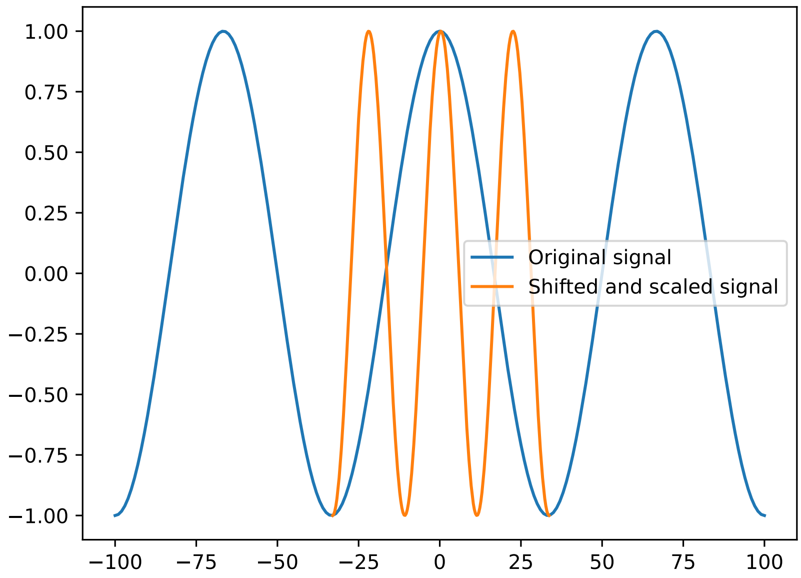


Figure 7: sine part B

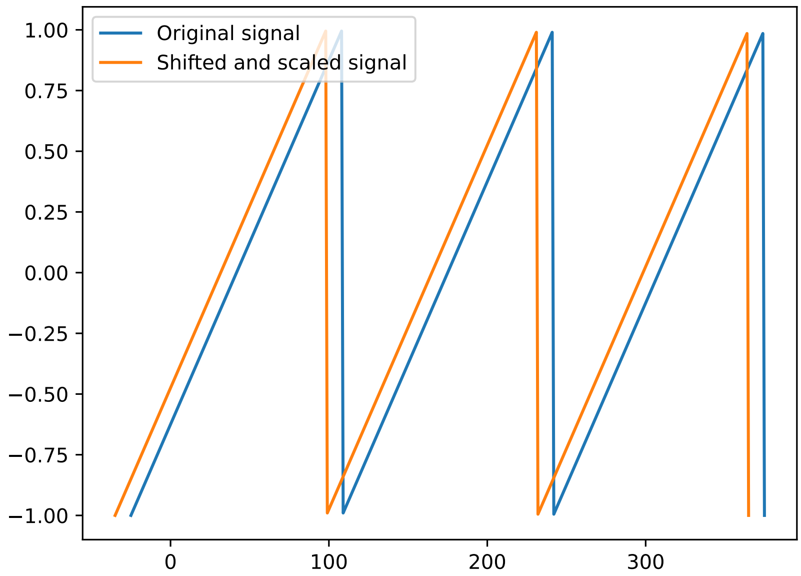


Figure 8: shifted sawtooth part B

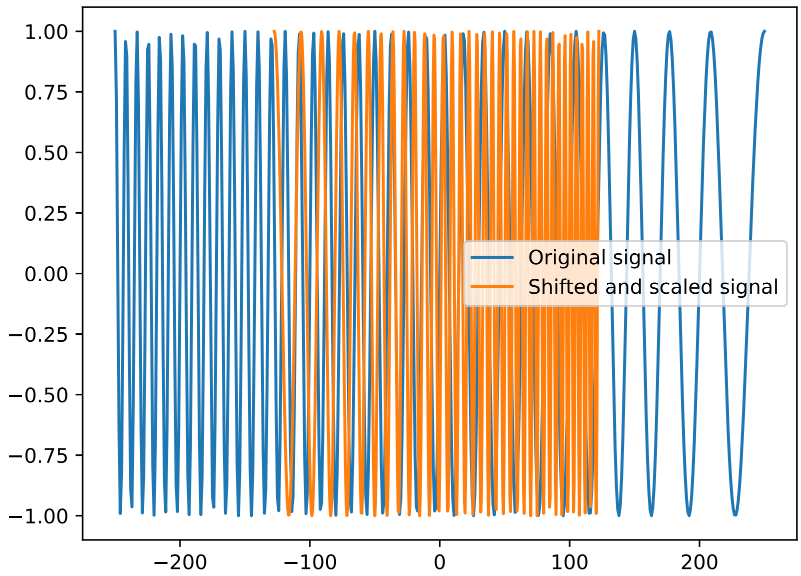


Figure 9: chirp part B