Final Exam, Task-3 Key

Q3. [25 points]

3.a) [10 points]

i) [6 points]

$R_1$ is reflexive. For any x, $xR_1x$, since level(x)=level(x). [ai1. 2 points]

$R_1$ is symmetric. For any x,y x $R_1$y implies y $R_1$x since level(x)=level(y) implies level(y)=level(x). [ai2. 2 points]

$R_1$ is transitive. For any x,y,z, x $R_1$y and y $R_1$z implies x $R_1$z, since level(x)=level(y) and level(y)=level(z) implies level(x)=level(z). [ai3. 2 points]

Having these three properties, $R_1$ is an equivalence relation.

ii) [4 points total]

There are h+1 equivalence classes of $R_1$. [aii1. 2 points]

The set of vertices at each level of the tree constitutes an equivalence class of $R_1$. [aii2. 2 points]

More precisely, the set of equivalence classes is

{ {x ε V| level(x)=i} | i ε I} where I={0,1,…,h}.

3.b) [10 points]

i) [6 points]

$R_2$ is reflexive. For any x, x $R_2$x, since level(x)≤level(x). [bi1. 2 points]

$R_2$ is [not!] anti-symmetric. For any x,y \in V, x $R_2$y and y $R_2$x [not!] implies x=y since level(x) ≤level(y) and level(y) ≤level(x) implies level(x)=level(y), but [not!] x=y. [bi2. 2 points for sensible attempt]

$R_2$ is transitive. For any x,y,z ε V, x $R_2$y and y $R_2$z implies x $R_2$z, since level(x) ≤level(y) and level(y) ≤level(z) implies level(x) ≤level(z). [bi3. 2 points]

Therefore, R2 is [not!] a partial order.

> Corrected version of the question (for information purposes):
>
> $R_2$ is defined as a strict partial order (irreflexive, anti-symmetric, transitive relation) on V:
>
> x $R_2$y if x has a level number less than that of y, i.e. level(x) < level(y).

ii) [4 points for sensible attempt or non-attempt with justification]

> For any non-empty subset U of V, let m be the minimum level of any vertex in U, i.e. m=min{level(x)|x ε U}.

Then, any vertex u with level(u) < m is a lower bound of U (m>0).

The greatest lower bound of U exists if there is only one vertex at level m-1 (m>0).

3.c) [5 points]

Transitive closure of $R_3$ = { (x,y)| x has a child iff y has a child }

Note that “x is an internal vertex”, “x has a child”, “x is the parent of some vertex”, “x has some descendant”, “x is an ancestor of some other vertex” etc. all have the same meaning in this context.