Sample

Duration: 100 minutes, 4 questions, **total:** 100 points

Closed book and notes, only one A4 size handwritten cheat sheet is allowed, please show your work

Name:.....
ID#:.....

PROBLEM	MAXIMUM	SCORE
Problem 1	20	
Problem 2	30	
Problem 3	30	
Problem 4	20	
Total	100	

Question 1: A matrix, A, is called *idempotent* if $A^2 = A$. Show that the eigenvalues of an idempotent matrix are zeros and ones.

Question 2: Given a linear system $Ax = e_1$ where $A \in \mathbb{R}^{n \times n}$ and $e_1 = [1, 0, ..., 0]^T$. Assume that we already have the QR factorization of A and only interested in computing the last element of x (i.e. $x(n)$), describe an efficient method to compute $x(n)$.

Question 3: A pseudocode of an algorithm for computing $\sqrt{a^2+b^2}$ is given:

```
sqrt(a,b)
 s = |a| + |b|
 if (s = 0)
    return 0
 else
    return s * sqrt((a/s)^2 + (b/s)^2)
 end if
```

What is the extra cost compared to directly computing $\sqrt{a^2+b^2}$ and what is the reason for the doing additional computations with s?

Question 4: Let $\mathbf{u}$, $\mathbf{v} \in \mathcal{R}^n$ and $\sigma \neq 0$. Assume that $I - \sigma \mathbf{u}\mathbf{v}^T$ is nonsingular and has an inverse given by $I - \tau \mathbf{u}\mathbf{v}^T$, what is $\mathbf{v}^T\mathbf{u}$?.