Student Information

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Answer 1

a)

Let X be the probability of drawing a white ball from each box.

W = White ball drawn B = Black ball drawn

$$\mathbf{P(X = 1) = P\{W\ B\ B\} + P\{B\ W\ B\} + P\{B\ B\ W\}}$$

$$\mathbf{= [(2/10).(11/15).(9/12) + (8/10).(4/15).(9/12) + (8/10).(11/15).(3/12)] = 5/12}$$

$$\mathbf{P(X = 2) = P\{W\ W\ B\} + P\{W\ B\ W\} + P\{B\ W\ W\}}$$

$$\mathbf{= [(2/10).(4/15).(9/12) + (2/10).(11/15).(3/12) + (8/10).(4/15).(3/12)] = 13/100}$$

$$\mathbf{P(X = 3) = P\{W\ W\ W\}}$$

$$\mathbf{= [(2/10).(4/15).(3/12)] = 1/75}$$

$$\mathbf{P(x)= P(X{=}1) + P(X{=}2) + P(X{=}3) = 5/12 + 13/100 + 1/75 = 14/25}$$

b)

The probability of drawing 3 white ball is

$$\mathbf{P(X = 3) = P\{W\ W\ W\}}$$

$$\mathbf{= [(2/10).(4/15).(3/12)] = 1/75}$$

c)

Drawing two white balls from:

First box = (2/10).(1/9) = 1/45

Second box = (4/15).(3/14) = 2/35

Third box = (3/12).(2/11) = 1/22

As we can conclude, the highest probability of drawing two white balls is from the second box, so I would choose the second box.

d)

Firstly, the probabilities of drawing white ball from each box are:

First box = 2/10 Second box = 4/15 Third box = 3/12

The box with the highest probability of drawing a white ball is the second box. So I'm going to draw from the second box first.

Then, the new probabilites will be:

First box = 2/10 Second box = 3/14 Third box = 3/12

Now the box with the highest probability of drawing a white ball is the third one. So, I will choose third box this time.

e)

Expected value $->$ μ = E(X) = Σ_x x P(x)

P(X = 1) = P{W B B} + P{B W B} + P{B B W}

= 1 . [(2/10).(11/15).(9/12) + (8/10).(4/15).(9/12) + (8/10).(11/15).(3/12)] = 5/12

P(X = 2) = P{W W B} + P{W B W} + P{B W W}

= 2 . [(2/10).(4/15).(9/12) + (2/10).(11/15).(3/12) + (8/10).(4/15).(3/12)] = 13/50

P(X = 3) = P{W W W}

= 3 . [(2/10).(4/15).(3/12)] = 1/25

μ = E(X) = 5/12 + 13/50 + 1/25= 43/60

f)

P(B) = Probability of choosing Box 1 = 1/3

P(W) = Probability of drawing white ball
= (2/10).(1/3)+ (4/15).(1/3)+(3/12).(1/3)= 2/10

According to the Bayes Rule

$$\mathbf{P(B \mid W)} = \frac{P(W \mid B).P(B)}{P(W)}$$

$$P(B| \mathbf{W)} = \frac{(2/10).(1/3)}{(2/10).(1/3) + (4/15).(1/3) + (3/12).(1/3)} = 12/43 = 0.27906$$

Answer 2

a)

P(C) = Probability of Sam is corrupted = 0.1

P(D | C) = Probability of Ring is destroyed when Sam is corrupted = 0.5

P(D) = Probability of Ring is destroyed = (0.9).(0.9)+(0.5).(0.1) = 0.86

According to the Bayes Rule

$$\mathbf{P(C \mid D)} = \frac{P(D \mid C).P(C)}{P(D)}$$

$$P(C| \mathbf{D)} = \frac{(0.5).(0.1)}{(0.9).(0.9) + (0.5).(0.1)} = 5/86 = 0.05813$$

b)

P(C_{SF}) = Probability of Sam And Frodo are corrupted = (0.1).(0.25)

P(D | C_{SF}) = Probability of Ring is destroyed when Sam and Frodo are corrupted

= 0.05

P(D) = Probability of Ring is destroyed in 4 conditions

Sam and Frodo corrupted = (0.25).(0.1).(0.05) = 0.00125

Sam corrupted Frodo not = (0.1).(0.75).(0.5) = 0.0375

Frodo corrupted Sam not = (0.25).(0.9).(0.2) = 0.045

Sam and Frodo not corrupted = (0.9).(0.75).(0.9) = 0.6075

When we sum all these up we get P(D) = 0.69125

According to the Bayes Rule

P$(C_{SF} \mid D) = \frac{P(D \mid C_{SF}).P(C_{SF})}{P(D)} = \frac{(0.05).(0.25).(0.1)}{0.69125} = 0.0018$

Answer 3

a)

There are 2 possible options for a total of four snowy days.

The first option is both Ankara and İstanbul have 2 snowy days.

According to the Table 1 ; P(A=2) . P(I=2) = 0.2

The second option is the Ankara has 3 snowy days and İstanbul has 1 snowy day.

According to the Table 1 ; P(A=3) . P(I=1) = 0.12

So, to get the probability of four snowy days in total, we should add these probabilities.

P(A=2).P(I=2) + P(A=3).P(I=1) = 0.2 + 0.12 = 0.32

b)

Random variables X and Y are independent if

$$P_{(X,Y)}(x,y) = P_X(x)\ .\ P_Y(y) \text{ for all values of x and y.}$$

$P_X(1) = \mathbf{0.30}$ $P_X(2) = \mathbf{0.50}$ $P_X(3) = \mathbf{0.20}$

$P_Y(1) = \mathbf{0.60}$ $P_Y(2) = \mathbf{0.40}$

To decide on the independence of X and Y , we should check if their joint pmf factors into a product of marginal pmfs. We see that all the $P_{(X,Y)}(x,y)$ **values equals to the** $P_X(x)$ **.** $P_Y(y)$ **. We cannot find the a pair of x and y that violates the formula for independent random variables. Therefore, the numbers of errors in two modules are independent.**