

Languages that are and are not context free - Examples

CENG 280

Example

Show that $L = \{a^i b^j c^i d^j \mid i, j \geq 0\}$ is not context free.

Assume that L is CF, and K be its pumping length.

Consider $w = a^K b^K c^K d^K$ $|w| = 4K > K$, $w \in L$

By PT there exists a split for $w = uvxyz$ with $|vy| > 0$
s.t. $uv^i xy^i z \in L$ for any $i \geq 0$. $|vxy| \leq K$

Since $|vxy| \leq K$, vy can not contain

both a 's and c 's or
both b 's and d 's

So $uv^0 xy^0 z$ either contains less a 's than c 's) c1
less b 's than d 's) c1
less c 's than a 's) c2
less d 's than b 's) c2

$a^K b^K c^K d^K$
c1: vxy c2: vxy

Thus $uv^0 xy^0 z \notin L$ for any split. By PT L is not CF

Example

Show that $L = \{ww \mid w \in \{a, b\}^*\}$ is not context free.

Assume that L is CF, and let K be its pumping length.
Consider $t = a^K b a^K b$ $t \in L$ and $|t| = 2K + 2$

$\underbrace{a \dots a}_{uv} \underbrace{b a \dots a}_x \underbrace{b}_{y} \underbrace{}_z$ } For such a split with $v = y = a^n$

The string t is not help-full for PT. $uv^i xy^i z \in L$ for any $i \geq 0$.

Consider another string $t = a^K b^K a^K b^K$, $t \in L$, $|t| = 4K$

By PT, there exists a split $t = uvxyz$ with $|vy| > 0$, $|vxy| \leq K$
such that $uv^i xy^i z \in L$ for any $i \geq 0$.

\Rightarrow continues on the next page.

Example \Rightarrow Homework study.

Show that $L = \{wpw^r \mid w \in \{a, b\}^*, |w| = |p|\}$ is not context free.

$$\underbrace{a^k}_{vxy} \underbrace{b^k}_{vxy} \underbrace{a^k}_{vxy} \underbrace{b^k}_{vxy} = uvxy^0z$$

$$|v| = m \geq 1$$

C1: m is not divisible by 2, $|uv^0xy^0z| = 4k - m$

Thus $uv^0xy^0z \notin L$

odd number, can not be written as ww

C2: m is divisible by 2, sub-cases:

1) vxy is in the first part of the string. $(a^k b^k)$

$$uv^0xy^0z = \underbrace{a^{k-n} b^{k-p}}_{\text{first half}} \underbrace{a^k b^k}_{\text{second half}} \quad \text{with } n+p=m=2r$$

$$|uv^0xy^0z| = 4k - 2r$$

first half ends with a / second half ends with b

can not be in ww form thus $uv^0xy^0z \notin L$

2) vxy is in the second part of the string $(a^k b^k)$

$$uv^0xy^0z = \underbrace{a^k b^k}_{\text{first part}} \underbrace{a^{k-p} b^{k-n}}_{\text{second half}} \quad \text{with } n+p=m=2r$$

$$|uv^0xy^0z| = 4k - 2r$$

First part starts with a
second half starts with b } $uv^0xy^0z \notin L$

3) vxy is in the middle part $(b^k a^k)$

$$uv^0xy^0z = a^k b^{k-p} a^{k-n} b^k \quad \text{with } n+p=m=2r$$

- if $p > 0$ then first part has less b 's
- if $n > 0$ then second part has less a 's

thus $uv^0xy^0z \notin L$.

Thus L is not CF.