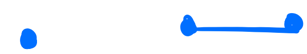
Graph isomorphism

$G = (V_G, E_G)$, $H = (V_H, E_H)$

$f: V_G \to V_H$

$(u, v) \in E_G$, $(f(u), f(v)) \in E_H$

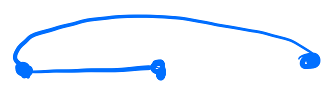


How many non-isomorphic unrooted trees are there

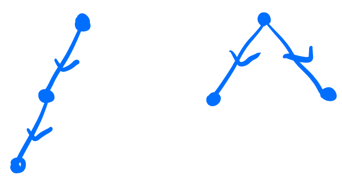
→ with 3 vertices

(1)

$\longleftrightarrow$ isomorphic

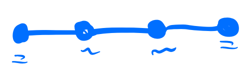


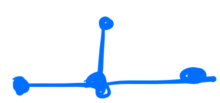
no isomorphic rooted trees – 3 vertices



4 vertices unrooted

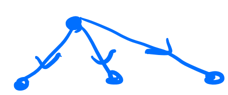
#	degree
2	1
2	2





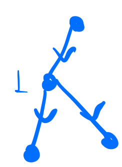
#	degree
3	1
1	3

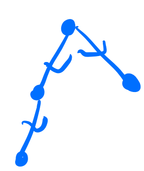
4 vertices rooted



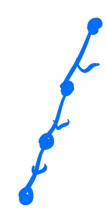
height 1

1





} 2



Exercise : 5 vertices

Ex Show that a full m-ary tree with 76 leaves and height = 3 exists.

$n = l + i \qquad n = m \cdot i + 1$

$l = i(m-1) + 1 = 76$

$i(m-1) = 75 \qquad m = 4, 6, 16$

$m-1 \mid 75$

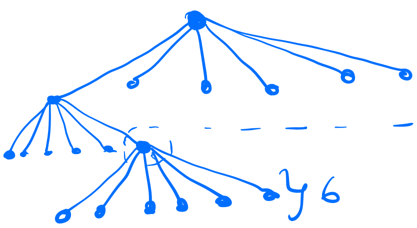
3, 5, 15

$l \leq m^h$ try $m = 4$

$4^3 \to 64 \qquad l = 76$

try $m = 6$

$6^3 = 216 \qquad 76 \leq 216$



h=1

$\to$ h = 2 max 36 leaves

h = 3 max 216 leaves

remove 1 leaf, add 6 new ones

36 ⇝ 76

$\frac{40}{5}$ ⇝ expand to 8 of them (add children)

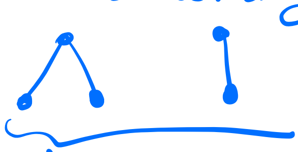
h=3
m=6
l=76

nodes in level 1 = 6

nodes in level 2 = 36 ⇝ 28 leaves
8 internal nodes

nodes in level 3 = 48

ex How many edges are there in a forest of t trees containing a total of n nodes?



forest

	edges
1 tree	$n-1$
2 trees	$n-2$
⋮	
t trees	$n-t$

---

ex How many leaf nodes does a full binary tree with n nodes have?

$$n = i\cancel{m} + 1 = \ell + i \qquad i = \frac{n-1}{2}$$
$$m = 2$$
$$= 2i + 1 = \ell + i \qquad \ell = \frac{n+1}{2}$$
$$\ell = \underline{\underline{i+1}}$$

ex What is the sum of the degrees of a tree with n-vertices?

\# edges $n-1$

total \# degrees $2n-2$

ex Show that every tree is bipartite.

$$G = (V, E) \qquad E \subseteq V \times V$$

$$V = V_0 \cup V_1 \quad \text{s.t.} \quad \underline{E \subseteq V_0 \times V_1 \cup V_1 \times V_0}$$

$$V_0 \cap V_1 = \phi$$

$V_0 \leftarrow$ nodes at even levels $\qquad E \cap V_0 \times V_0 = \phi$

$V_1 \leftarrow$ nodes at odd levels $\qquad E \cap V_1 \times V_1 = \phi$