Take Home Exam 1

Question 1

$\neg(p \wedge q) \Leftrightarrow (\neg q \rightarrow p) \equiv (\neg p \vee \neg q) \Leftrightarrow (\neg q \rightarrow p)$	by the second De Morgan law
$\equiv (\neg p \vee \neg q) \Leftrightarrow (\neg(\neg q) \vee p)$	by the truth table for $\rightarrow$
$\equiv (\neg p \vee \neg q) \Leftrightarrow (q \vee p)$	by the double negation law
$\equiv ((\neg p \vee \neg q) \rightarrow (q \vee p)) \wedge ((q \vee p) \rightarrow (\neg p \vee \neg q))$	by truth table for $\Leftrightarrow$
$\equiv (\neg(\neg p \vee \neg q) \vee (q \vee p)) \wedge (\neg(q \vee p) \vee (\neg p \vee \neg q))$	by the truth table for $\rightarrow$
$\equiv ((p \wedge q) \vee (q \vee p)) \wedge (\neg(q \vee p) \vee (\neg p \vee \neg q))$	by the second De Morgan law
$\equiv ((p \wedge q) \vee (q \vee p)) \wedge ((\neg q \wedge \neg p) \vee (\neg p \vee \neg q))$	by the second De Morgan law
$\equiv ((p \vee q \vee p) \wedge (q \vee q \vee p)) \wedge ((\neg q \wedge \neg p) \vee (\neg p \vee \neg q))$	by the Distributive law (1)
$\equiv ((q \vee p) \wedge (q \vee p)) \wedge ((\neg q \wedge \neg p) \vee (\neg p \vee \neg q))$	by the Idempot. and Comm. law (1)
$\equiv (q \vee p) \wedge ((\neg q \wedge \neg p) \vee (\neg p \vee \neg q))$	by the Idempotent law (1)
$\equiv (q \vee p) \wedge ((\neg q \vee \neg p \vee \neg q) \wedge (\neg p \vee \neg p \vee \neg q))$	by the Distributive law (1)
$\equiv (q \vee p) \wedge ((\neg q \vee \neg p) \wedge (\neg p \vee \neg q))$	by the Idempot. and Comm. law (1)
$\equiv (p \vee q) \wedge (\neg p \vee \neg q)$	by the Comm. law (1) and Idempot. law (2)

Question 2

a. Two different interns in the same faculty cannot have the same employee id number.

$$\forall x \, \forall y \, (\forall z \; (x \neq y) \wedge (I(x,z) \wedge I(y,z)) \rightarrow \exists a \exists b(E(x,a) \wedge E(y,b) \wedge (\boldsymbol{a} \neq b)))$$

b. There are some interns in all faculties who are supervised by no one but themselves.

$$\exists x \; (\forall y \, I(x,y) \rightarrow \forall z((x \neq z) \wedge S(x,x) \wedge \neg S(x,z)))$$

c. At most two interns can be admitted to each job position in the medicine faculty.

$$\exists x \, \exists y \; \forall c \, ((x \neq y) \wedge J(c, medicine) \wedge (A(x,c) \vee A(y,c)) \wedge \forall z(A(z,c) \rightarrow (z = x) \vee (z = y)))$$

Question 3

a.

1: $(p \lor \neg q)$	Premise
2: $(p \lor r)$	Premise
3: $r \rightarrow q$	Assumption
4: p	Assumption
5: $p \rightarrow p$	4, 4, $\rightarrow i$
6: $\neg q$	Assumption
7: r	Assumption
8: q	3, 7, $\rightarrow e$
9: $\bot$	6 , 8, $\neg e$
10: p	9, $\bot e$
11: $r \rightarrow p$	7 – 10, $\rightarrow i$
12: p	2 , 5 , 11, $\lor e$
13: $\neg q \rightarrow p$	6 – 12, $\rightarrow i$
14: p	1 , 5 , 13, $\lor e$
15: $(r \rightarrow q) \rightarrow p$	3 – 14, $\rightarrow i$

b.

1: $(q \rightarrow p) \rightarrow q$	Assumption
2: $\neg q$	Assumption
3: q	Assumption
4: $\bot$	2 , 3, $\neg e$
5: p	4, $\bot$ e
6: $q \rightarrow p$	3 – 5, $\rightarrow i$
7: q	1 ,6, $\rightarrow e$
8: $\bot$	2 - 7, $\neg e$
9: $\neg q \rightarrow \bot$	2 – 8, $\rightarrow i$
10: q	9, $\neg e$
11: $((q \rightarrow p) \rightarrow q) \rightarrow q$	1 – 10, $\rightarrow i$

Question 4

a.

1:	$\neg\forall x\,(P(x) \to Q(x))$	Premise
	t fresh name	
2:	$\neg(P(t) \to Q(t))$	$1, \forall x\ e$
3:	$\neg(P(t) \land \neg Q(t))$	Assumption
4:	$P(t)$	Assumption
5:	$\neg Q(t)$	Assumption
6:	$P(t) \land \neg Q(t)$	$4-5, \land i$
7:	$\bot$	$3 - 6, \neg e$
8:	$\neg\neg Q(t)$	$5-7, \neg i$
9:	$Q(t)$	$8, \neg\neg e$
10:	$P(t) \to Q(t)$	$4-9, \to i$
11:	$\bot$	$3-10, \neg e$
12:	$\neg\neg(P(t) \land \neg Q(t))$	$3-11, \neg i$
13:	$P(t) \land \neg Q(t)$	$12, \neg\neg e$
14:	$\exists x(P(x) \land \neg Q(x))$	$13, \exists x\ i$

b.

1:	$\forall x \forall y (P(x,y) \to \neg P(y,x))$	Premise
2:	$\forall x \exists y P(x,y)$	Premise
	t fresh name	
3:	$\forall y P(t,y) \to \neg P(y,t)$	$1, \forall x\ e$
	c fresh name	
4:	$P(t,c) \to \neg P(c,t)$	$3, \forall y\ e$
5:	$P(t,c)$	Assumption
6:	$\neg P(c,t)$	$4, \to e$
7:	$P(c,t)$	Assumption
8:	$\bot$	$6, 7, \neg e$
9:	$\neg P(c,t)$	$6-8, \neg i$
10:	$\exists z\, \neg P(z,c)$	$5 - 9, \exists z\ i$
11:	$\forall v \exists z\, \neg P(z,v)$	$4 - 10, \forall v\ i$
12:	$\neg\neg\, \forall v \exists z\, \neg P(z,v)$	$2,\ 3 - 11,\ \neg\neg i$
13:	$\neg \exists v \forall z\, P(z,v)$	12