Student Information

Name : Solution

ID :

Answer 1

a)

The probability of getting at least one white ball is the complement of getting three black balls.
$P(W) = 1 - (\frac{8}{10} \times \frac{11}{15} \times \frac{9}{12}) = 1 - 0.44 = 0.56$

b)

$P(3W) = (\frac{2}{10} \times \frac{4}{15} \times \frac{3}{12}) = \frac{1}{75}$, which is approximately 0.013.

c)

Let $P(B1)$ denote the possibility of getting two white balls if two balls are drawn from BOX 1.
$P(B1) = (\frac{2}{10} \times \frac{1}{9}) = \frac{1}{45}$
$P(B2) = (\frac{4}{15} \times \frac{3}{14}) = \frac{2}{35}$
$P(B3) = (\frac{3}{12} \times \frac{2}{11}) = \frac{1}{22}$

To compare easily, let's first convert all of the probabilistic fractions into percentages (values are approximated).
$P(B1) = 0.022$
$P(B2) = 0.057$
$P(B3) = 0.045$
The probability of obtaining two white balls is the higher if we draw them from BOX 2.

d)

Similar to part c, we can use percentages for easier comparison. For the first ball;
$P(B1) = \frac{2}{10} = 0.020\%$
$P(B2) = \frac{4}{15} = 0.027$
$P(B3) = \frac{3}{12} = 0.025\%$
In this case, initially picking from BOX 2 gives the best possibility. For the second ball however, $P(B2') $ becomes $\frac{3}{14} = 0.021$ since the number of white balls in that box is decreased by one. So, the second ball should be picked from BOX 3.

e)

For the expected value E(W), we should calculate expected values for each box and sum them.
$E(W) = E(B1) + E(B2) + E(B3) = 0.2 + 0.27 + 0.25 = 0.72$.

f)

Let $P(A)$ be the probability of taking a ball from BOX 1 and $P(B)$ be the probability of getting a white ball. The given question asks the conditional probability $P(A|B)$. Using Bayes rule;

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Since all boxes have an equal probability, $P(A) = \frac{1}{3}$. We know that $P(B|A) = \frac{2}{10}$. For $P(B)$, all boxes have equal probability and we can calculate probability of getting a white ball from any of the boxes;

$$P(B) = (\tfrac{2}{10})(\tfrac{1}{3}) + (\tfrac{4}{15})(\tfrac{1}{3}) + (\tfrac{3}{12})(\tfrac{1}{3}) = \tfrac{43}{180}$$

This makes;

$$P(A|B) = \frac{(\frac{2}{10})(\frac{1}{3})}{(\frac{43}{180})} = \frac{12}{43} = 0.28$$

If a white ball is taken randomly among all three boxes, there is 28% probability that this ball is taken from BOX 1.

Answer 2

a)

Let $P(S)$ denote the possibility of Sam's corruption and $P(D)$ denote the possibility of destroying the ring.

$$P(S|D) = \frac{P(D|S)P(S)}{P(D)}$$

In the question it is given that $P(S) = 0.1$ and $P(D|S) = 0.5$. Also $P(D)$ can be calculated by considering both cases of Sam's corruption.
$P(D) = (0.1) \times (0.5) + (0.9) \times (0.9) = 0.86$
$P(S|D) = \dfrac{(0.5) \times (0.1)}{0.86} = 0.058$.
Thus if it is known that the ring is destroyed, there is 0.058 probability that Sam is corrupted.

b)

NOTE: In this question there was an ambiguity about the calculation of P(D). The following is one of the solutions, yet one another accepted solution for P(D) is given at the end Q2.

In addition to part a, let us also use $P(F)$ as the probability of Frodo's corruption. Using Bayes Rule combined with Law of Total Probability, following equation must be solved;

$$P(S \cap F|D) = \frac{P(D|S \cap F)P(S \cap F)}{P(D)}$$

Since $P(S)$ and $P(F)$ are independent, $P(S \cap F) = P(S) \times P(F) = 0.1 \times 0.25$.
Also $P(D|S \cap F) = 0.05$, as given in the question.

To calculate $P(D)$ four cases must be considered;

- Both are corrupted; $P_1(D) = 0.1 \times 0.25 \times 0.05 = 0.00125$
- Only Frodo is corrupted; $P_2(D) = 0.25 \times 0.9 \times 0.2 = 0.045$
- Only Sam is corrupted; $P_3(D) = 0.75 \times 0.1 \times 0.5 = 0.0375$
- Both are not corrupted; $P_4(D) = 0.75 \times 0.9 \times 0.9 = 0.6075$

Observe that $P(D) = P_1(D) + P_2(D) + P_3(D) + P_4(D) = 0.69125$. Thus,

$$P(S \cap F|D) = \tfrac{0.00125}{0.69125} = 0.0018$$

This indicated that there is approximately 0.02 chance of both Frodo and Sam being corrupted if the ring is destroyed.

The following alternative calculation of $P(D)$;

$$P(D) = P(D|S)*P(S)+P(D|F)*P(F)+P(D|F'\cap S')*P(F'\cap S')-P(D|F\cap S)*P(F\cap S) = 0.70625$$

Note that in this solution the probability of destroying the ring while both of them are corrupted is calculated twice, so it must be subtracted. The difference in the solutions is due to the interpretation of probabilities and both are valid.

Answer 3

a)

There are two options for four snowy days; $P(A = 3, I = 1)$ and $P(A = 2, I = 2)$.
$P(4) = P(A = 3, I = 1) + P(A = 2, I = 2) = 0.12 + 0.2 = 0.32$

f)

For independence, we need to consider joint probability and marginal probabilities. A single counterexample would be sufficient for dependence. However, for independence all joint probabilities must be checked.

$P(A = 1) = P(A = 1, I = 1) + P(A = 1, I = 2) = 0.3$. Similarly, $P(A = 2) = 0.5$ and $P(A = 3) = 0.2$.
For $P(I)$; $P(I = 1) = 0.6$ and $P(I = 2) = 0.4$. For every joint probability, multiplication of associated marginal probabilities is equal to the joint probability. Thus the two events are independent.
NOTE: To get full points from this part you need to either calculate all individual cases or at least **mention** that all cases must be checked. If you just checked a single case and assumed that the events are independent, you lost points.