Numerical
RECURSION

WORKSHEET

CENG 140
2021

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DIFFICULTY

Greatest Common Divisor ⭐

Description

Write a function, named **`gcd`**, that takes two positive integers (**`k,n`**) as argument and prints their greatest common divisor.

Example

Means:
Function call

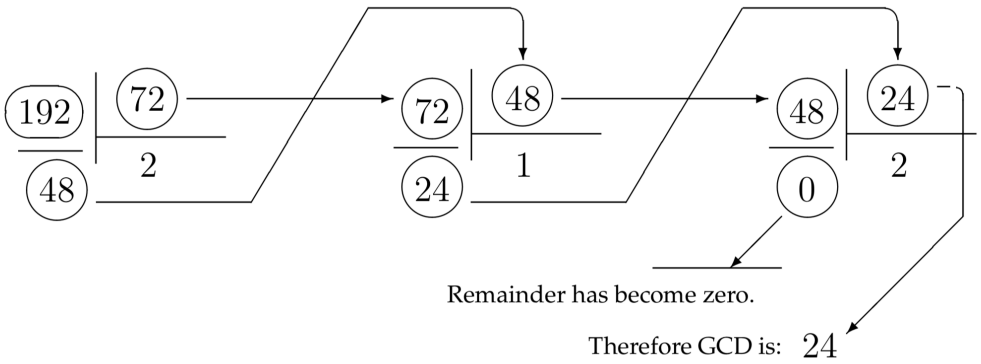
```
>>> gcd(192,72);
24
```

Print out

```
>>> gcd(72,192);
24

>>> gcd(16,15);
1
```

Hint



Warm ups: Simple sequences ⭐

[FOR YOUR OWN BENEFIT: DO NOT CONSULT THE WEB]

- Find recursive definition for the binomial coefficient.
- Find a general recursive formula for arithmetic sequences.
- Find a general recursive formula for geometric sequences.
- $C(n) = \binom{2n}{n} \frac{1}{n+1}$ defines the Catalan number. It appears heavily in Computational Geometry. Find a recursive definition of Catalan numbers.
- Write a C function which computes the n^{th} Fibonacci number in O(n).
  Hint: Use a helper function.

First Recamán's Sequence ⭐

Introduction

The First Recamán's Sequence R is defined as

- R(0) = 0
- R(n) = R(n − 1) − n, if n > 0 ∧ the RHS is positive ∧ R(n) is not already in the sequence,
- R(n) = R(n − 1) + n, otherwise

which can be succinctly defined as "subtract if you can, otherwise add." The first few terms are 0, 1, 3, 6, 2, 7, 13, 20, 12, 21, 11, ..

Description

Write a function, named **`recaman`**, that takes a positive integer (**`n`**) as argument and prints R(n).

Example

```
>>> recaman(6);
13
```

Note

Care about efficiency !

All factorizations ⭐⭐

Description

Prime Factorization, is the representation of any given integer into a product of primes. Since it cannot be further factorized (as all are primes), the Prime Factorization of a number is unique. Write a recursive function, named **`prime_factors`**, that takes a unsigned long integer (**`n`**) as argument and prints out a list of prime factors in increasing order separated by asterisks.

Example

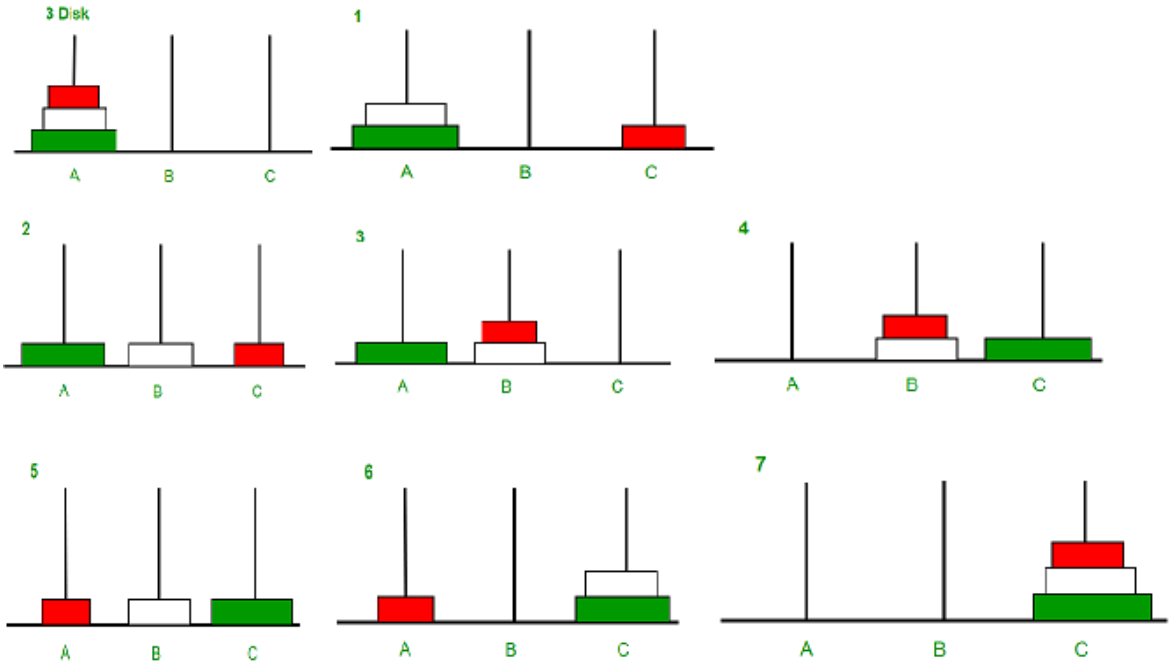
```
>>> all_factorizations(1234567890);
2*3*3*5*3607*3803
```

Tower of Hanoi ⭐⭐⭐

Introduction

Tower of Hanoi is a famous puzzle where we have three rods and N disks. The objective of the puzzle is to move the entire stack to another rod. You are given the number of discs N. Initially, these discs are in the rod A. The objective of the puzzle is to move the entire stack to the rod C, obeying the following simple rules:

1. The discs are arranged such that the top disc is numbered 1 and the bottom-most disc is numbered N.
2. Only one disk can be moved at a time.
3. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
4. No disk may be placed on top of a smaller disk.



Tower of Hanoi solution for N=3

Description

Write a function, named **`TowerOfHanoi`**, that takes a positive integer (**`N`**) as argument and three rod label arguments: (**`from_rod, to_rod, aux_rod`**) and prints out all moves to accomplish the task.

Example

```
>>> TowerOfHanoi(4,'A','C','B');
Move disk 1 from rod A to rod B
Move disk 2 from rod A to rod C
Move disk 1 from rod B to rod C
Move disk 3 from rod A to rod B
Move disk 1 from rod C to rod A
Move disk 2 from rod C to rod B
Move disk 1 from rod A to rod B
Move disk 4 from rod A to rod C
Move disk 1 from rod B to rod C
Move disk 2 from rod B to rod A
Move disk 1 from rod C to rod A
Move disk 3 from rod B to rod C
Move disk 1 from rod A to rod B
Move disk 2 from rod A to rod C
Move disk 1 from rod B to rod C
```

Hint

Believe in ASD.