Middle East Technical University Department of Computer Engineering

CENG371

Scientific Computing

Fall 2023-2024

Homework 4

Due: January 20th, 2024, Saturday 23:59

In this homework you will implement and compare 4 methods for randomized matrix multiplication (RMM).

Question 1 (45 points)

a) (**5 pts**) Implement the naive matrix multiplication algorithm (**Algorithm 2** in Mahoney's notes).

(**filename:** `mult_naive.m` **input:** matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$ **output:** matrix $C = A \times B \in \mathbb{R}^{m\times p}$)

b) (**10 pts**) Implement RMM via uniform row sampling. (see **Algorithm 3** in Mahoney's notes).

(**filename:** `mult_row_uniform.m` **input:** matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$, $c \in \mathbb{Z}^+$, function handle f for matrix multiplication **output:** matrix $D = f(C, R) \in \mathbb{R}^{m\times p}$)

c) (**10 pts**) Implement RMM via non-uniform row sampling where sampling probability p_i of **column** $A^{(i)}$ and **row** $B_{(i)}$ is given by (see p.21 of Mahoney's notes)

$$p_i = \frac{\|A^{(i)}\|\,\|B_{(i)}\|}{\sum_{i'=1}^{n}\|A^{(i')}\|\,\|B_{(i')}\|}.$$

(**filename:** `mult_row_nonuni.m` **input:** matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$, $c \in \mathbb{Z}^+$, function handle f for matrix multiplication **output:** matrix $D = f(C, R) \in \mathbb{R}^{m\times p}$)

d) (**10 pts**) Implement RMM via random projections where elements of the projection matrix P comes from $N(0, 1)$ with the restriction that $\|P^{(i)}\| = 1$. (see **Chapter 5** of Mahoney's notes).

(**filename:** `mult_proj_Gauss.m` **input:** matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$, $c \in \mathbb{Z}^+$, function handle f for matrix multiplication **output:** matrix $D = f(C, R) \in \mathbb{R}^{m\times p}$)

e) (**10 pts**) Implement RMM via random projections where elements of the projection matrix P comes from $N(0, 1)$ with the restriction that $\|P^{(i)}\| = 1$ **and** the columns of P are orthogonal. You can use the Gram-Schmidt process to ensure orthogonality. (see **Chapter 5** of Mahoney's notes).

(**filename:** `mult_proj_Gauss_orth.m` **input:** matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$, $c \in \mathbb{Z}^+$, function handle f for matrix multiplication **output:** matrix $D = f(C, R) \in \mathbb{R}^{m\times p}$)

Question 2 (55 points)

a) (**10 pts**) Reason (formally or informally) about the scalings you have used on C and R while implementing RMM methods in Q1.

b) (**5 pts**) Run the supplementary file `main.m` with your `*.m` files and `*.mat` files in your `PATH` (*i.e.*, either in the same directory as your `main.m` or you have added the directory of the files to your `PATH` via `addpath`). The run will take around 30-40 minutes depending on your CPU. Include the plots into your report.

c) (**30 pts**) Compare the methods based on relative errors and run times with proper references to the plots you have acquired. Which methods would you suggest to be used in which cases? You can include any additional comment/discussion on the methods here.

d) (**10 pts**) The performance of `mult_row_uniform` should reduce while calculating $C \times C^T$ in comparison to $A \times B$. Why do you think this is the case?

Regulations

1. While discussing your findings feel free to refer to the analyses in Mahoney's notes via proper paraphrasing. Just make sure that you reflect **your own reasoning** in a clean and concise manner.
2. If you are using any other programming language/environment than MATLAB, you are expected to produce equivalent results to the provided `main.m`.
3. Your submission should include a single PDF and your `.m` files.
4. Submissions will be done via odtuclass.
5. **Late Submission:** Accepted with a penalty of $-5 \times (day)^2$.