Reasoning about Negation

Rules for Double Negation

$$\frac{A}{\neg \neg A} \quad \neg \neg i$$

$$\frac{\neg \neg A}{A} \quad \neg \neg \epsilon$$

Contradictions: \bot

 \bot is a shorthand for $\phi \land \neg \phi$, where ϕ is any formula.

Observation: For every formula ψ , the argument $\phi \land \neg \phi \models \psi$ is valid.

A	$A \wedge \neg A$
T	F
F	F

(There can be no line of the truth table where $\phi \land \neg \phi$ is true, hence there can be no counterexample to validity.)

Rules for \perp

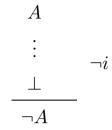
$$A \neg A$$
 \bot

$$\frac{\perp}{A}$$
 $\perp \epsilon$

(Here A can be any formula.)

\neg -Introduction

The rule for introducing negation also makes use of subproofs:



Modus Tollens

We can use $\neg i$ to prove a rule that is very useful for using negative facts:

$$\begin{array}{c|ccc} A \longrightarrow B & \neg B \\ \hline \neg A & & \end{array} MT$$

Proof:

Example

 $1: (S \vee G) \longrightarrow P$ Premise

 $2: P \longrightarrow A$ Premise

 $4-8, \neg i$

A useful variant of $\neg i$:

$$\begin{array}{c}
\neg A \\
\vdots \\
& \text{RAA} \\
\hline
A
\end{array}$$

This rule is also known as proof by contradiction or reductio ad absurdum (RAA).

Law of the excluded Middle

$$A \vee \neg A$$
 LEM