

Q-5) {30 points-2 pts each} Consider the graph G given above to answer the questions below.

- a) What is the number of non-zero entries in the incidence matrix representation of G?

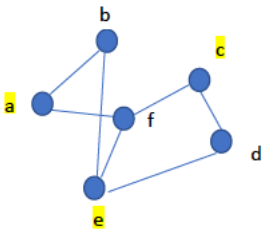
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- b) Does G have a complete graph with at least three vertices as a subgraph? If yes, give this subgraph.

Yes. b-d, d-e, e-b

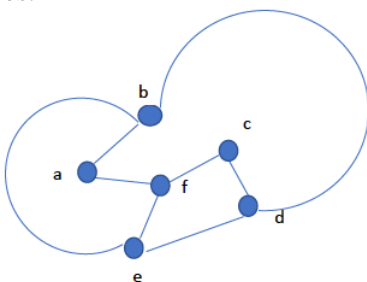
- c) Is G a bipartite graph? Yes / No. If yes explain briefly; If no, which edges need to be removed such that the resulting subgraph of G will be a bipartite graph.

No. It contains an odd cycle and not 2-colorable. Remove b-d edge to make it bipartite (a,e,c and b,d,f)



- c) Is G a planar graph? Yes / No. Explain briefly.

Yes.



e) How many graphs are there that are isomorphic to  $G$  having the same set of vertices as  $G$  (including  $G$  itself)?

$$6! = 720$$

f) How many directed graphs are there that have  $G$  as their underlying undirected graph?

$$2^8 = 256 \text{ (8 edges, 2 possible direction for each)}$$

g) What is the length of the longest simple path in  $G$ ? Give this path.

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e,d,c,f,a,b,e,f

h) What is the number of connected components of  $G$ ? Explain your answer.

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All vertices are connected to at least one other vertex. There is a path between any 2 vertices of the graph.

i) Is there an Euler circuit in  $G$ ? If yes, give such a circuit; if no, state the reason.

No, there are odd degree vertices.

j) Is there an Euler path in  $G$ ? If yes, give such a path; if no, state the reason.

No. More than 2 vertices of odd degree.

k) Does  $G$  have a Hamilton circuit? If yes, find such a circuit; if no, justify your answer.

d,c,f,a,b,e,d

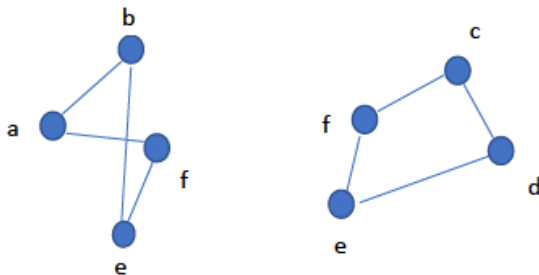
l) Does  $G$  have a Hamilton path? If yes, find such a path; if no, justify your answer.

d,c,f,a,b,e

m) What is the number of non-zero entries in the adjacency matrix representation of  $G$ ?

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n) Draw all subgraphs of  $G$ , if any, that are isomorphic to  $C_4$  (cycle graph with 4 vertices).



o) What is the graph chromatic number of  $G$ ? Explain briefly.

3.

Red: a,c,e

Blue: b,f

Green: d

Not bipartite, not 2-colorable.

