Languages that are and are not context free - Examples

CENG 280

Example

If L_1 is regular, and L_2 is context-free, then $L_1 L_2$ is necessarily

a) [True/False] Context-free

b) [True/False] Regular

a) L_1 is regular, then it is CF

$L_1 L_2$ is CF by closure properties

L_1 = CF, L_2 = CF

b) Not regular.

Consider $L_1 = \{e\}$, then $L_1 L_2 = \{a^n b^n \mid n \geq 0\}$ not regular

$L_2 = \{a^n b^n \mid n \geq 0\}$

Alternatively $L_1 = L(a^*)$ $L_1 L_2 = \{a^n b^m \mid n \geq m\}$ not regular.

$L_2 = \{a^n b^n \mid n \geq 0\}$

Example

Show that $\{a^m b^n \mid m \neq n\}$ is context-free. 1) Write a grammar, 2) use closure properties

1) $S \to aA$
$A \to aAb$
$A \to aA$
$A \to e$
$\{a^m b^n \mid m > n\}$

$S \to Bb$
$B \to aBb$
$B \to Bb$
$A \to e$
$\{a^m b^n \mid m < n\}$

2) $L = \{a^n b^n \mid n \geq 0\}$ $S \to aSb \mid e$ CF

$L_a = \mathcal{L}(a a^*)$ Regular, thus CF

$L_b = \mathcal{L}(b b^*)$ Regular, thus CF

$L_a L \cup L L_b$ is CF by closure properties

*CFLs are closed under union and concatenation

Example

Prove that CFL are closed under Kleene Star using PDA.

L CF, $L = L(M)$ $M = (K, \Sigma, \Gamma, \Delta, s, F)$

Define $M' = (K', \Sigma, \Gamma', \Delta', s', F')$ s.t. $L(M') = L(M)^* = L^*$

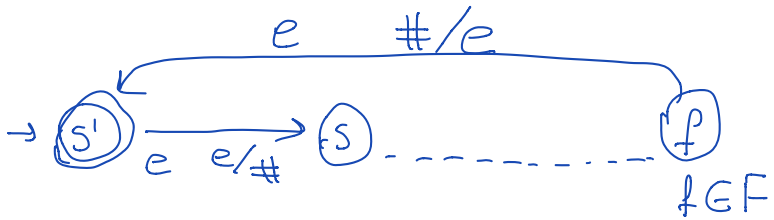
$K' = K \cup \{s'\}$

$F' = \{s'\}$

$\Gamma' = \Gamma \cup \{\#\}$

$\Delta' = \Delta \cup \{(s, e, e), (s', \#)\}$

$\cup \{(f, e, \#), (s', e) \mid f \in F\}$



Example

Show that $L = \{a^m b^n c^p d^q \mid n = q \text{ or } m \leq p \text{ or } m + n = p + q\}$ is context-free.

$L_1 = \{a^m b^n c^p d^q \mid n = q\}$

$L_2 = \{a^m b^n c^p d^q \mid m \leq p\}$

$L_3 = \{a^m b^n c^p d^q \mid m + n = p + q\}$

$L = L_1 \cup L_2 \cup L_3$

L_1: $L_1 = L(G_1)$

G_1:

$S \to AP$

$A \to aA \mid \varepsilon$

$P \to bPd \mid C$

$C \to cC \mid \varepsilon$

Show L_2 and L_3 are also CF

G_2: ----

G_3 ----