Languages that are and are not Regular

CENG 280

Course outline

- Preliminaries: Alphabets and languages
- Regular languages
  - Regular expressions
  - Finite automata: DFA and NFA
  - Finite automata - regular languages
  - Languages that are and that are not regular, Pumping lemma
  - State minimization for DFA
- Context-free languages
- Turing-machines

Languages that are and are not Regular

- Methods to show that a language is regular
- Methods to show that a language is not regular

Regular Languages

To show that a language L is regular, one of the following methods can be used:

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$
- use closure properties, e.g., for regular languages L_1 and L_2, show $L = L_1 \cap L_2$, $L = L_1 \cup L_2$, $L = L_1 L_2$, or $L = \Sigma^\star \setminus L_1$ (more examples can be added)

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$
- use closure properties, e.g., for regular languages L_1 and L_2, show $L = L_1 \cap L_2$, $L = L_1 \cup L_2$, $L = L_1 L_2$, or $L = \Sigma^\star \setminus L_1$ (more examples can be added)

Example

$\Sigma = \{0, 1, \ldots, 9\}$. $L = \{n \mid n \text{ divisible by 2 and 3}\}$ (decimal representations without leading 0's). Show that L is regular.

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$
- use closure properties

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$
- use closure properties

Example

Is L regular or not? L' is regular.

$$L = \{xy \in \Sigma^\star \mid x \in L' \text{ and } y \notin L'\}$$

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$
- use closure properties

Example

Is L^R regular or not? L is regular.

$$L^R = \{w^R \in \Sigma^\star \mid w \in L\}$$

Regular Languages

To show that a language L is regular, one of the following methods can be used:

- write a regular expression α such that $L = \mathcal{L}(\alpha)$
- construct an NFA M such that $L = L(M)$
- use closure properties

Example

Is L regular or not?

$$L = \{w \in \Sigma^\star \mid w \text{ has property } P\}, |L| \text{ is finite}$$

Languages that are not regular

- To show that a language L is not regular, we use the following property of regular languages: as a string is scanned from left to right, the amount of memory required to determine if $w \in L$ or $w \notin L$ must be bounded.

Languages that are not regular

- To show that a language L is not regular, we use the following property of regular languages: as a string is scanned from left to right, the amount of memory required to determine if $w \in L$ or $w \notin L$ must be bounded.
- In RL, infinite languages can be represented with Kleene star (cycle in automata), which induce a periodicity/pattern.

Languages that are not regular

- To show that a language L is not regular, we use the following property of regular languages: as a string is scanned from left to right, the amount of memory required to determine if $w \in L$ or $w \notin L$ must be bounded.
- In RL, infinite languages can be represented with Kleene star (cycle in automata), which induce a periodicity/pattern.

Theorem

Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$ and $xy^i z \in L$ for each $i \geq 0$.

Pumping lemma application

Theorem

Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$ and $xy^iz \in L$ for each $i \geq 0$.

- For each regular language L, $\rightarrow$ *assume that L is regular*

Pumping lemma application

Theorem

Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$ and $xy^i z \in L$ for each $i \geq 0$.

- For each regular language L, $\rightarrow$ *assume that L is regular*
- there exists $n \geq 1$ such that $\rightarrow$ *this is a general term do not pick a number!!!*

Pumping lemma application

Theorem

Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$ and $xy^iz \in L$ for each $i \geq 0$.

- For each regular language L, $\rightarrow$ *assume that L is regular*
- there exists $n \geq 1$ such that $\rightarrow$ *this is a general term do not pick a number!!!*
- for each $w \in L$ with $|w| \geq n$ $\rightarrow$ *write a string with respect to n, use it to reach a contradiction*

Pumping lemma application

Theorem

Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$ and $xy^i z \in L$ for each $i \geq 0$.

- For each regular language L, → *assume that L is regular*
- there exists $n \geq 1$ such that → *this is a general term do not pick a number!!!*
- for each $w \in L$ with $|w| \geq n$ → *write a string with respect to n, use it to reach a contradiction*
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$ → *consider each possible split satisfying these constraints*

Pumping lemma application

Theorem

Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$ and $xy^iz \in L$ for each $i \geq 0$.

- For each regular language L, $\rightarrow$ *assume that L is regular*
- there exists $n \geq 1$ such that $\rightarrow$ *this is a general term do not pick a number!!!*
- for each $w \in L$ with $|w| \geq n$ $\rightarrow$ *write a string with respect to n, use it to reach a contradiction*
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$ $\rightarrow$ *consider each possible split satisfying these constraints*
- for each $i \geq 0$ $xy^iz \in L$ $\rightarrow$ *show that there exists an i such that $xy^iz \notin L$, contradiction*

Pumping lemma application

For each regular language L,

- there exists $n \geq 1$ such that
- for each $w \in L$ with $|w| \geq n$
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$
- for each $i \geq 0$ $xy^iz \in L$

$\rightarrow$ *this is a general term do not pick a number!!!*

$\rightarrow$ *write a string with respect to n, use it to reach a contradiction*

$\rightarrow$ *consider each possible split satisfying these constraints*

$\rightarrow$ *show that there exists an i such that $xy^iz \notin L$, contradiction*

Example

Show that $L = \{a^k b^k \mid k \geq 0\}$ is not regular.

Pumping lemma application

For each regular language L,

- there exists $n \geq 1$ such that
- for each $w \in L$ with $|w| \geq n$
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$
- for each $i \geq 0$ $xy^i z \in L$

$\rightarrow$ *this is a general term do not pick a number!!!*

$\rightarrow$ *write a string with respect to n, use it to reach a contradiction*

$\rightarrow$ *consider each possible split satisfying these constraints*

$\rightarrow$ *show that there exists an i such that $xy^i z \notin L$, contradiction*

Example

Show that $L = \{a^i \mid i \text{ is prime}\}$ is not regular.

Pumping lemma application

For each regular language L,

- there exists $n \geq 1$ such that
- for each $w \in L$ with $|w| \geq n$
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$
- for each $i \geq 0$ $xy^iz \in L$

$\rightarrow$ *this is a general term do not pick a number!!!*

$\rightarrow$ *write a string with respect to n, use it to reach a contradiction*

$\rightarrow$ *consider each possible split satisfying these constraints*

$\rightarrow$ *show that there exists an i such that $xy^iz \notin L$, contradiction*

Example

Show that $L = \{w \in \{a, b\}^\star \mid w \text{ has equal number of } a \text{ and } b\}$ is not regular.

Pumping lemma application

For each regular language L,

- there exists $n \geq 1$ such that
- for each $w \in L$ with $|w| \geq n$
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$
- for each $i \geq 0$ $xy^i z \in L$

$\rightarrow$ *this is a general term do not pick a number!!!*

$\rightarrow$ *write a string with respect to n, use it to reach a contradiction*

$\rightarrow$ *consider each possible split satisfying these constraints*

$\rightarrow$ *show that there exists an i such that $xy^i z \notin L$, contradiction*

Example

Show that $L = \{ww^R \in \{a, b\}^\star\}$ is not regular.

Pumping lemma application

For each regular language L,

- there exists $n \geq 1$ such that
- for each $w \in L$ with $|w| \geq n$
- there exists a split $w = xyz$ with $y \neq e$, $|xy| \leq n$
- for each $i \geq 0$ $xy^iz \in L$

$\rightarrow$ *this is a general term do not pick a number!!!*

$\rightarrow$ *write a string with respect to n, use it to reach a contradiction*

$\rightarrow$ *consider each possible split satisfying these constraints*

$\rightarrow$ *show that there exists an i such that $xy^iz \notin L$, contradiction*

Example

Show that $L = \{wcw \mid w \in \{a, b\}^\star\}$ is not regular.