CENG 384 - Signals and Systems for Computer Engineers
20222
Written Assignment 2 Solutions

May 18, 2023

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1. (a)

$$\int_{\tau=-\infty}^{\infty}(x(\tau)-5y(\tau))d\tau = y(t)$$
$$x(t)-5y(t) = y'(t)$$
$$y'(t)+5y(t) = x(t)$$

(b) char eqn. : $r+5=0 \quad \Rightarrow \quad r=-5 \quad \Rightarrow \quad y_h(t) = K\cdot e^{-5t}$
$y_p(t) = Ae^{-t}+Be^{-3t}$
$y_p'(t) = -Ae^{-t}-3Be^{-3t}$

$$-Ae^{-t}-3Be^{-3t}+5(Ae^{-t}+Be^{-3t}) = e^{-t}+e^{-3t}$$
$$4Ae^{-t}+2Be^{-3t} = e^{-t}+e^{-3t}$$

$4A = 1 \quad \Rightarrow \quad A = 1/4$
$2B = 1 \quad \Rightarrow \quad B = 1/2$

$$y(t) = y_h(t)+y_p(t) = (Ke^{-5t}+\frac{1}{4}e^{-t}+\frac{1}{2}e^{-3t})u(t)$$
$$y(0) = K+\frac{1}{4}+\frac{1}{2} = 0 \quad \Rightarrow \quad K = -\frac{3}{4}$$
$$y(t) = \frac{1}{4}e^{-t}+\frac{1}{2}e^{-3t}-\frac{3}{4}e^{-5t}$$

2. (a)

$$y[n] = x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

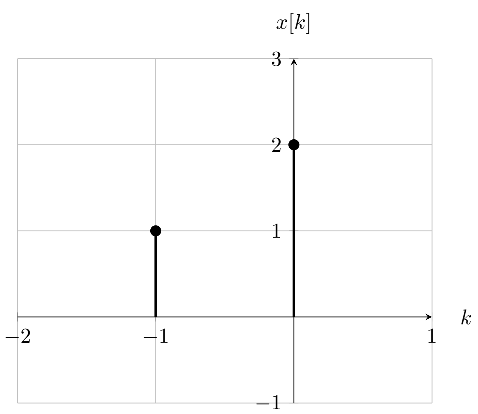

Figure 1: k vs. $x[k]$.

$$
\begin{aligned}
&= x[-1]h[n+1] + x[0]h[n] \\
&= (\delta[n] + 2\,\delta[n+2]) \\
&\quad + (2\,\delta[n-1] + 4\,\delta[n+1]) \\
&= 2\,\delta[n-1] + \delta[n] + 4\,\delta[n+1] + 2\,\delta[n+2]
\end{aligned}
$$

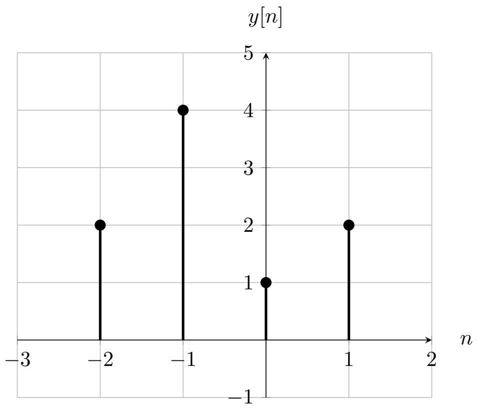

Figure 2: n vs. $y[n]$.

(b)

$$
\begin{aligned}
\frac{dx(t)}{dt} &= \delta(t-1) + \delta(t+1) \\
y(t) &= \int_{-\infty}^{\infty} (\delta(\tau-1) + \delta(\tau+1))e^{-(t-\tau)} sin(t-\tau)u(t-\tau)d\tau \\
&= e^{-t+1} sin(t-1)u(t-1) + e^{-t-1} sin(t+1)u(t+1)
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_0^t e^{-\tau}e^{-2(t-\tau)}d\tau \\
&= e^{-2t}\int_0^t e^{\tau}d\tau \;=\; e^{-2t}(e^t-1)u(t) \;=\; (e^{-t}-e^{-2t})u(t)
\end{aligned}
$$

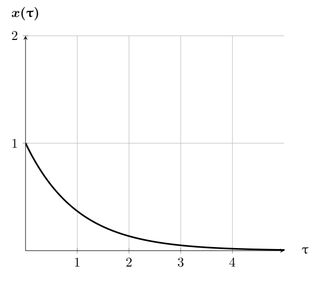

Figure 3: τ vs. $x(\tau)$.

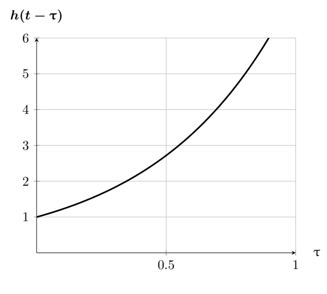

Figure 4: τ vs. $h(t-\tau)$.

(b)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

For $0 < t < 1$:

$$\int_0^t e^{3(t-\tau)}d\tau \;=\; e^{3t}\int_0^t e^{-3\tau}d\tau \;=\; -\frac{1}{3}e^{3t}(e^{-3t}-1) \;=\; \frac{1}{3}(e^{3t}-1)(u(t)-u(t-1))$$

For $t > 1$:

$$\int_0^1 e^{3(t-\tau)}d\tau \;=\; -\frac{1}{3}e^{3t}(e^{-3}-1) \;=\; \frac{1}{3}(e^{3t}-e^{3t-3})u(t-1)$$

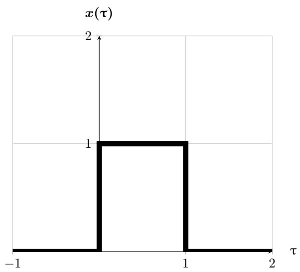

Figure 5: τ vs. $x(\tau)$.

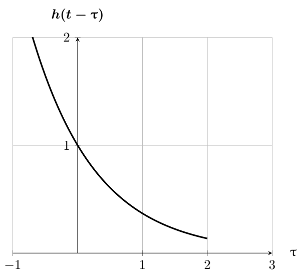

Figure 6: τ vs. $h(t-\tau)$.

4. (a) char eqn. : $r^2 - r - 1 = 0$
   Roots are $\frac{1+\sqrt{5}}{2}$, $\frac{1-\sqrt{5}}{2}$
   General solution : $y[n] = A(\frac{1+\sqrt{5}}{2})^n + B(\frac{1-\sqrt{5}}{2})^n$
   Plugging in the initial values:

$$y[0] = A + B = 1$$
$$y[1] = (\frac{1+\sqrt{5}}{2})A + (\frac{1-\sqrt{5}}{2})B = 1 \textit{ thus:}$$
$$A = \frac{1+\sqrt{5}}{2\sqrt{5}}, B = \frac{-1+\sqrt{5}}{2\sqrt{5}}$$

   $y[n] = (\frac{1+\sqrt{5}}{2\sqrt{5}})(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2\sqrt{5}})(\frac{1-\sqrt{5}}{2})^n$

   $y[n] = \frac{1}{\sqrt{5}}\left[(\frac{1+\sqrt{5}}{2})^{n+1} - (\frac{1-\sqrt{5}}{2})^{n+1}\right]$

   (b) char eqn. : $r^3 - 6r^2 + 13r - 10 = 0$
   Roots are 2, $2-j$, $2+j$
   General solution : $y(t) = Ae^{2t} + B\cos(t)e^{2t} + C\sin(t)e^{2t}$
   Plugging in the initial values we get:

$$A = 2, B = -1, C = \frac{-1}{2}$$

   $y(t) = e^{2t}(-\frac{1}{2}\sin(t) - \cos(t) + 2)$

5. (a)

$$y_p(t) = c_1 \sin(5t) + c_2 \cos(5t)$$
$$y_p'(t) = 5c_1 \cos(5t) - 5c_2 \sin(5t)$$
$$y_p''(t) = -25c_1 \sin(5t) - 25c_2 \cos(5t)$$

Plug them into the equation:

$(-19c_1 - 25c_2)\sin(5t) + (25c_1 - 19c_2)\cos(5t) = \cos(5t)$

$c_1 = \frac{25}{986}$ and $c_2 = \frac{-19}{986}$

$y_p(t) = \frac{25}{986}\sin(5t) - \frac{19}{986}\cos(5t)$

(b) char eqn. : $r^2 + 5r + 6 = 0$

Roots are -2, -3

Homogeneous solution: $y_h(t) = ae^{-2t} + be^{-3t}$

(c) $y(t) = y_h(t) + y_p(t) = ae^{-2t} + be^{-3t} + \frac{25}{986}\sin(5t) - \frac{19}{986}\cos(5t)$

$y(0) = 0 = a + b - \frac{19}{986} \longrightarrow a + b = \frac{19}{986}$

$y'(t) = -2ae^{-2t} - 3be^{-3t} + \frac{125}{986}\cos(5t) + \frac{95}{986}\sin(5t)$

$y'(0) = 0 = -2a - 3b + \frac{125}{986} \longrightarrow 2a + 3b = \frac{125}{986}$

$a = -\frac{68}{986}$ and $b = \frac{87}{986}$

$y(t) = -\frac{68}{986}e^{-2t} + \frac{87}{986}e^{-3t} + \frac{25}{986}\sin(5t) - \frac{19}{986}\cos(5t)$

6. (a) $x[n] * h_0[n] = w[n]$

$h_0[n] - \frac{1}{2}h_0[n-1] = \delta[n]$

$h_0[0] - \frac{1}{2}h_0[-1] = \delta[0] = 1$ and we know $h_0[-1] = 0$ since the system is initially at rest. Therefore, $h_0[0] = 1$.

$h_0[1] - \frac{1}{2}h_0[0] = \delta[1] = 0$ Therefore, $h_0[1] = \frac{1}{2}$.

$h_0[2] = \frac{1}{4}$ ... $h_0[n] = (\frac{1}{2})^n u[n]$.

(b) $h[n] = h_0[n] * h_0[n]$

$h[n] = \sum_{k=0}^{n}(\frac{1}{2})^k(\frac{1}{2})^{n-k} = \sum_{k=0}^{n}(\frac{1}{2})^n$

$h[n] = (n+1)(\frac{1}{2})^n$

(c) $w[n] - \frac{1}{2}w[n-1] = x[n]$

$y[n] - \frac{1}{2}y[n-1] = w[n] = a$

$y[n-1] - \frac{1}{2}y[n-2] = w[n-1] = b$

$a - \frac{1}{2}b = x[n]$

$y[n] - \frac{1}{2}y[n-1] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2] = x[n]$

$y[n] - y[n-1] + \frac{1}{4}y[n-2] = x[n]$

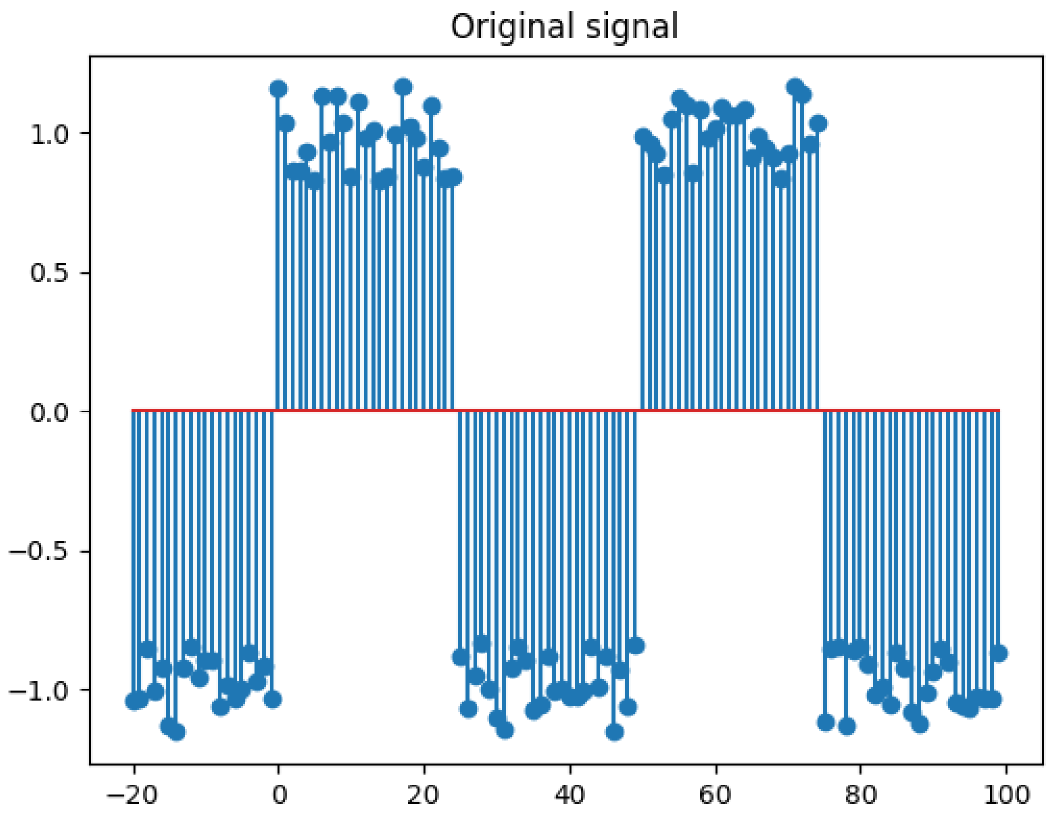

Figure 7: Original signal

7. (a) Convolution with $delta[n-5]$ shifts the input signal right by 5.

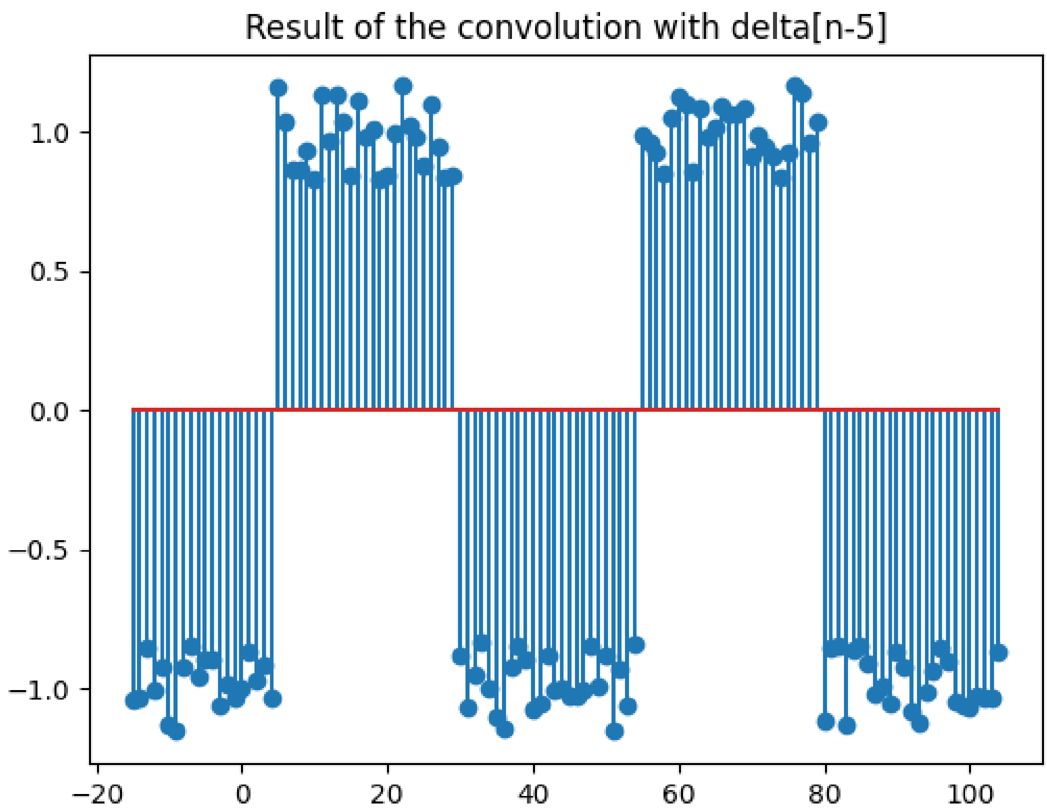

Figure 8: Output signal of convolution with $\delta[n-5]$

(b) Moving average filter smooths the signal. When N increases, the signal will be more smoothed.

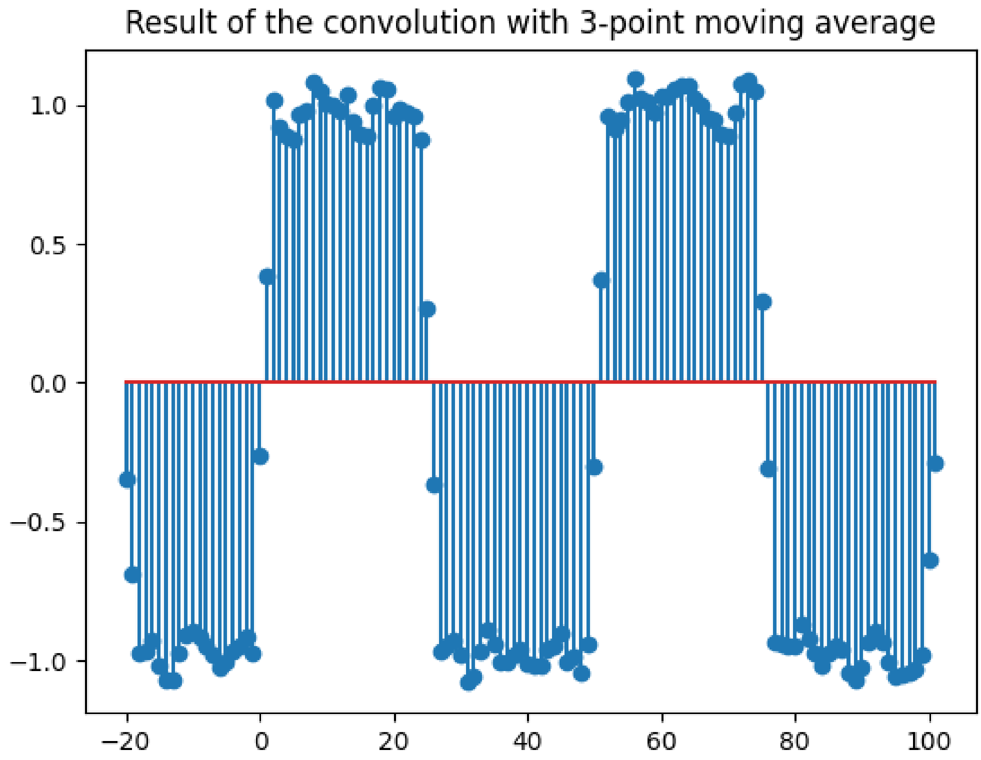

Figure 9: Output signal of convolution with 3-point moving average filter.

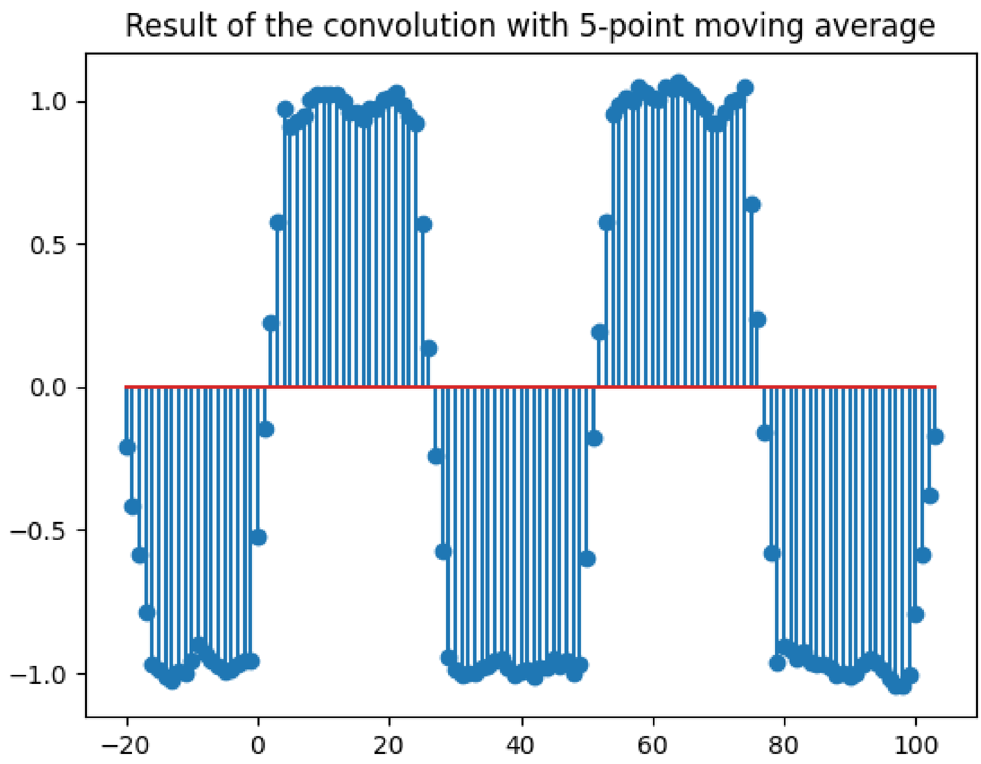

Figure 10: Output signal of convolution with 5-point moving average filter.

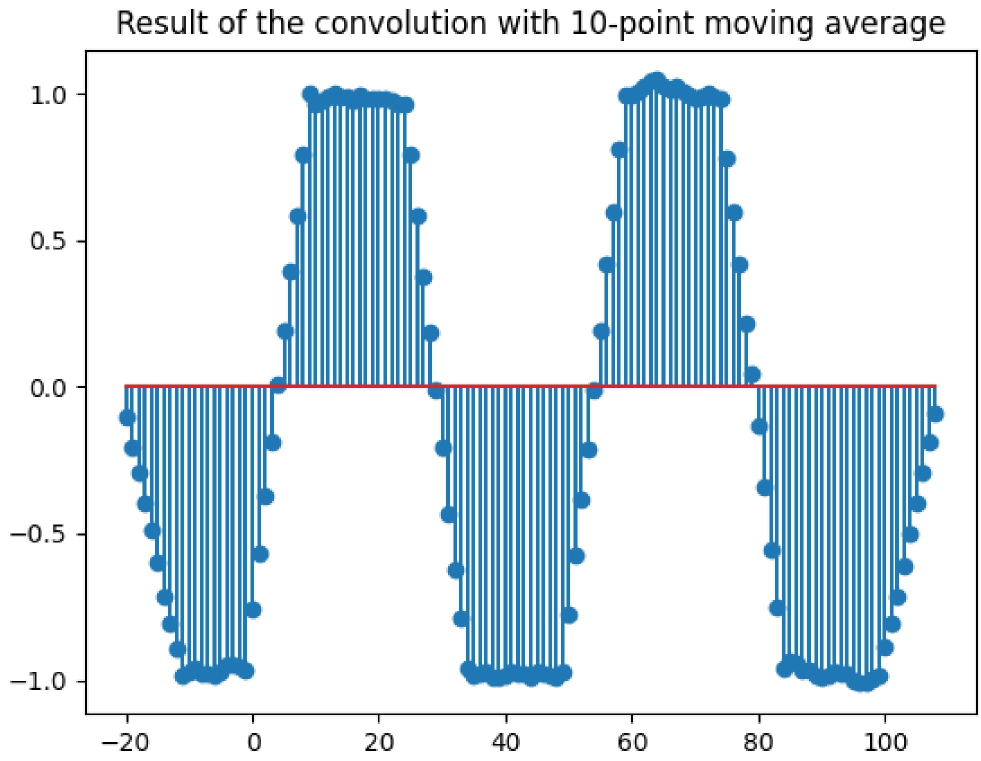

Figure 11: Output signal of convolution with 10-point moving average filter.

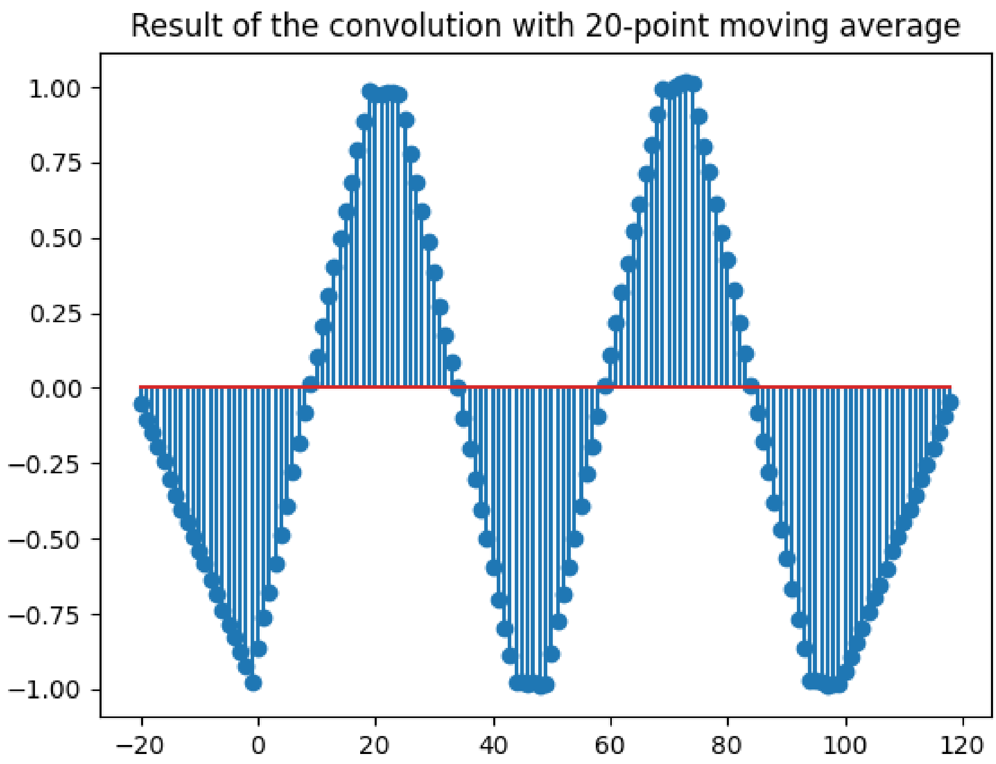

Figure 12: Output signal of convolution with 20-point moving average filter.