PART B

Let $M = (K, \Sigma, \Delta, s, F)$ be a FA

Construct a reg. exp. R such that

$$L(R) = L(M)$$

represented as the union of many (but finite no of) languages

Let $K = \{q_1, \dots, q_n\}$ and $s = q_1$.

For $i, j = 1, \dots, n$ and $k = 0, \dots, n$

define $R(i, j, k)$ as the set of all strings in Σ^* that may drive M from state q_i to state q_j without passing through any intermediate state numbered $k+1$ or greater.

q_i & q_j may be numbered higher than k

$R(i, j, k)$ = set of strings spelled by all paths from q_i to q_j of rank k.

When $k = n$:

$$R(i, j, n) = \{w \in \Sigma^* : (q_i, w) \vdash_M^* (q_j, e)\}$$

Therefore:

$$L(M) = \bigcup \{R(1, j, n) : q_j \in F\}$$

If All of these sets $R(i,j,k)$ are regular, hence so is $L(M)$

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Prove that each $R(i,j,k)$ is regular

By induction on k:

For $k=0$

$R(i,j,0)$ is either $\{a \in \Sigma \cup \{e\} : (q_i, a, q_j) \in \Delta\}$ if $i \neq j$

or it is $\{e\} \cup \{a \in \Sigma \cup \{e\} : (q_i, a, q_j) \in \Delta\}$. if $i = j$

Induction Hypothesis

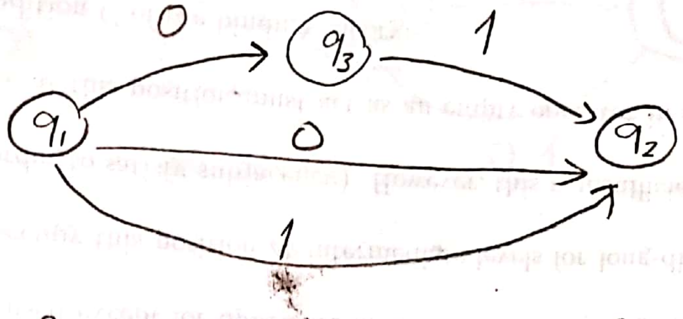
Suppose $R(i,j,k-1)$ for all i,j are regular languages

$R(i,j,k)$ can be defined combining prev. defined reg. lang. by the reg. operations of $\cup, *, \cdot$

$$R(i,j,k) = R(i,j,k-1) \cup R(i,k,k-1) . R(k,k,k-1)^* R(k,j,k-1)$$

$R(i,j,k) \equiv R_{ij}^{k}$ {in some other books} $\xrightarrow{\text{Corr. reg. exp}} r_{ij}^{k}$

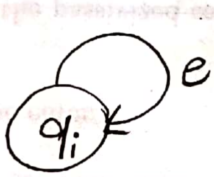
ex:

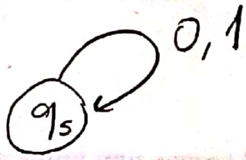


- $R_{12}^{2} = \{0, 1\}$ $\quad R_{12}^{3} = \{0, 1, 01\}$

$$R_{ij}^{0} = \{ a \mid \delta(q_i, a) = q_j,\ a \in \Sigma \}$$

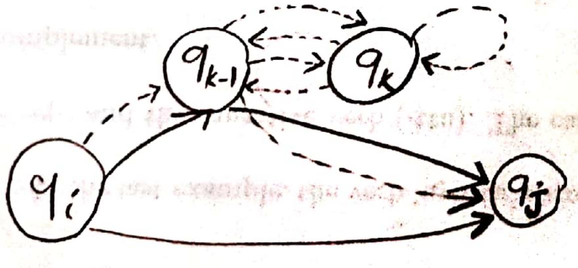
R_{ij}^{0} contains e if $i = j$ (automatically understood)



- 

$R_{55} = \{e, 0, 1\}$

$$R_{ij}^{k} = R_{ij}^{k-1} + R_{ik}^{k-1} \left(R_{kk}^{k-1}\right)^{*} R_{kj}^{k-1}$$



$$r_{ij}^{k} = r_{ij}^{k-1} + r_{ik}^{k-1} \left(r_{kk}^{k-1}\right)^{*} r_{kj}^{k-1}$$

$\boxed{rr^{*} = r^{+}}$

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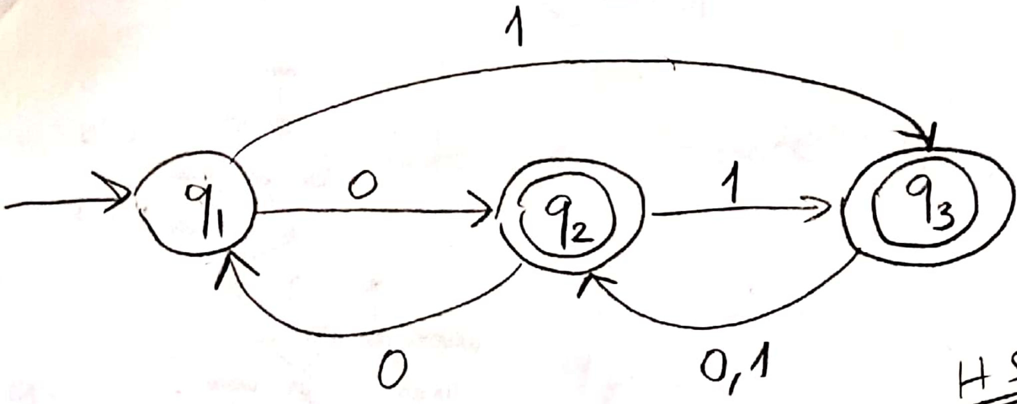


HS

$r_{13}^{2} = \dots = 0^*1$

$r_{33}^{2} = e+(0+1)0^*1$

$r = r_{12}^{3} + r_{13}^{3}$

k=3) $r_{12}^{3} = r_{12}^{2} + r_{13}^{2} \left(r_{33}^{2}\right)^* r_{32}^{2}$

k=2) $r_{12}^{2} = r_{12}^{1} + r_{12}^{1} \left(r_{22}^{1}\right)^* r_{22}^{1} = 0 + 0(00+e)^*(00+e)$

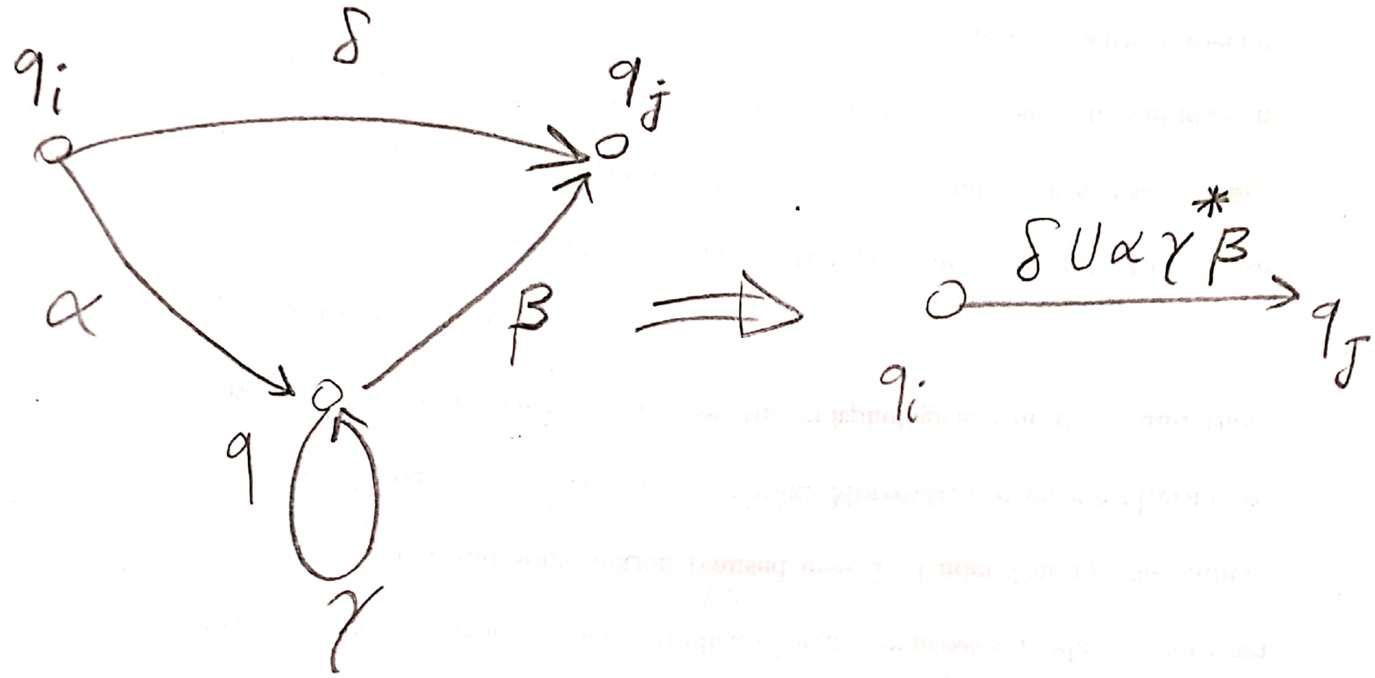
$= 0(00)^*$ (all odd number of zeros)

$r_{12}^{1} = 0$

$r_{22}^{1} = 00+e$

$r = 0^*1\left((0+1)0^*1\right)^* + \left(e+(0+1)(00)^*\right) + 0(00)^*$

To eliminate a state q



- Reg. langs. are closed under a variety of operations.
- Reg. langs. may be specified either by reg. expressions or by det. or nondet. finite automata.