Student Information

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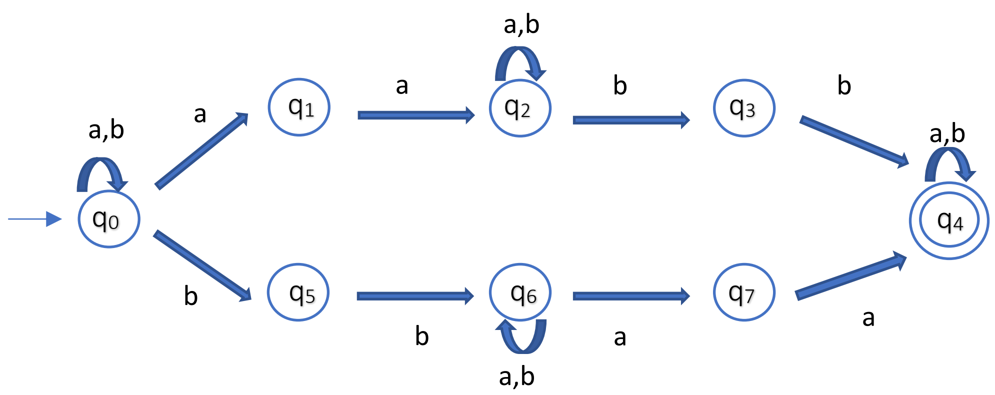
Answer 1

a)

(a*b*)* aa (a*b*)* bb (a*b*)* ∪ (a*b*)* bb (a*b*)* aa (a*b*)*

b)

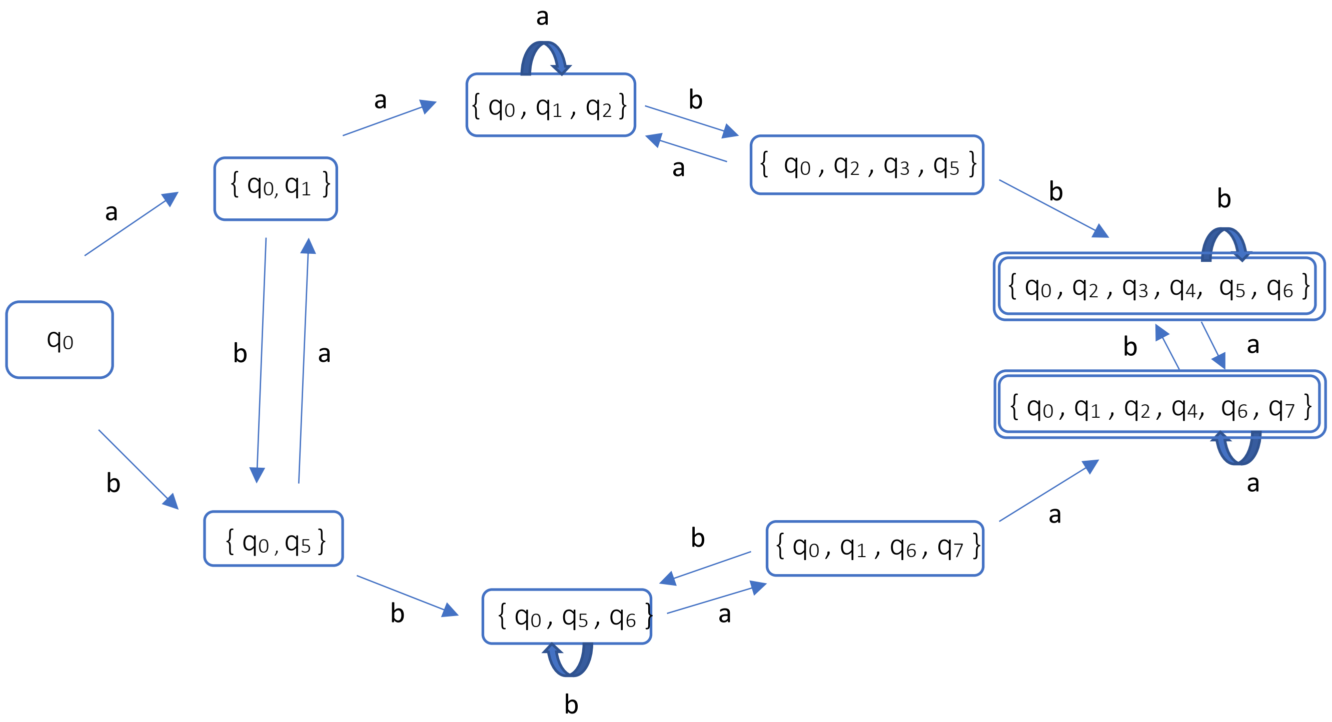


c)

State / Alphabet	a	b
-> q_0	q_0, q_1	q_0, q_5
q_1	q_2	-
q_2	q_2	q_2 , q_3
q_3	-	q_4
*q_4	q_4	q_4
q_5	-	q_6
q_6	q_6 , q_7	q_6
q_7	q_8	-

State / Alphabet	a	b
-> q_0	{ q_0, q_1 }	{ q_0, q_5 }
{ q_0, q_1 }	{ q_0, q_1, q_2 }	{ q_0, q_5 }
{ q_0, q_5 }	{ q_0, q_1 }	{ q_0, q_5, q_6 }
{ q_0, q_1, q_2 }	{ q_0, q_1, q_2 }	{ q_0, q_2, q_3, q_5 }
{ q_0, q_5, q_6 }	{ q_0, q_1, q_6, q_7 }	{ q_0, q_5, q_6 }
{ q_0, q_2, q_3, q_5 }	{ q_0, q_1, q_2 }	{ $q_0, q_2, q_3, q_4, q_5, q_6$ }
{ q_0, q_1, q_6, q_7 }	{ $q_0, q_1, q_2, q_4, q_6, q_7$ }	{ q_0, q_5, q_6 }
{ $q_0, q_2, q_3, q_4, q_5, q_6$ }	{ $q_0, q_1, q_2, q_4, q_6, q_7$ }	{ $q_0, q_2, q_3, q_4, q_5, q_6$ }
{ $q_0, q_1, q_2, q_4, q_6, q_7$ }	{ $q_0, q_1, q_2, q_4, q_6, q_7$ }	{ $q_0, q_2, q_3, q_4, q_5, q_6$ }

Equivalent DFA



d)

For Deterministic Finite Automata

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (\{q_0, q_5\}, babb) \\
&\vdash_M (\{q_0, q_5, q_6\}, abb) \\
&\vdash_M (\{q_0, q_1, q_6, q_7\}, bb) \\
&\vdash_M (\{q_0, q_5, q_6\}, b) \\
&\vdash_M (\{q_0, q_5, q_6\}, e)
\end{aligned}
$$

Since $(q_0, bbabb) \vdash_{M^*} (\{q_0, q_5, q_6\}, e)$, and so there is no acceptance state in last step “bbabb” is not accepted by DFA.

For Non-Deterministic Finite Automata

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (q_0, babb) \\
&\vdash_M (q_0, abb) \\
&\vdash_M (q_0, bb) \\
&\vdash_M (q_0, b) \\
&\vdash_M (q_0, e)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (q_5, babb) \\
&\vdash_M (q_6, abb) \\
&\vdash_M (q_6, bb) \\
&\vdash_M (q_6, b) \\
&\vdash_M (q_6, e)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (q_0, babb) \\
&\vdash_M (q_0, abb) \\
&\vdash_M (q_0, bb) \\
&\vdash_M (q_0, b) \\
&\vdash_M (q_5, e)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (q_0, babb) \\
&\vdash_M (q_0, abb) \\
&\vdash_M (q_0, bb) \\
&\vdash_M (q_5, b) \\
&\vdash_M (q_6, e)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (q_5, babb) \\
&\vdash_M (q_6, abb) \\
&\vdash_M (q_7, bb)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, bbabb) &\vdash_M (q_0, babb) \\
&\vdash_M (q_0, abb) \\
&\vdash_M (q_1, bb)
\end{aligned}
$$

Since a string is not accepted by a nondeterministic finite automaton, there is no one sequence of moves leading to a final state, it follows that “ bbabb” $\notin L(M)$.

Answer 2

a) Assume that L_1 is regular and let 'p' be the pumping length such that any string w ∈ L where w = $a^m b^n$ when m > n and |w| ≥ p should satisfy these conditions :

i. w = $xy^{\alpha}z \in L$ for every $\alpha \geq 0$

ii. |y| > 0

iii. |xy| ≤ p

$\underbrace{\text{aa........aaa}}_{xy}\text{b.........bb}$ where $x = a^i$, $y = a^j$, $z = a^k b^p$

such that i +j + k = p+1, and j > 0.
We pump with α = 0 and get the word xz = $a^i z = a^i a^k b^p$.
Since i + j + k = p+1 and j > 0, then i + k ≤ p.
So there is a contradiction. $xy^0z \notin L_1$.

Therefore, L_1 is not a regular language.

Assume L_2 is A regular language, if $L_2 = \overline{L_1}$, then $\overline{L_1}$ should be a regular language.

By Complementation Theorem, complement of regular language must be regular.

So, $\overline{(\overline{L_1})}$ should be regular. Since $\overline{(\overline{L_1})} = L_1$, $\overline{(\overline{L_1})}$ cannot be regular.

As a result, there is a contradiction. L_2 is not a regular language.

b)

$L_4 = \{ a^n b^n \mid n \in \mathbb{N}^+ \}$ $L_5 = \{ a^m b^n \mid m, n \in \mathbb{N} \}$ $L_6 = b^* a (ab^*a)^*$

When we look at the L_5 m and n valus can take all the natural number values, but for the L_4 n can take only the positive natural numbers. When we set the both m and n values to n in L_5, we can observe L_4 is a subset of L_5.

Since L_4 is the subset of L_5 , let $L = L_4 \cup L_5 = L_5$. Then, we can say that L should be regular because L_5 is regular.

As we already know, if we can write language in the form of a regular expression, this language is a regular language. So, L_6 is a regular language.

According to the theorem , for any regular languages A and B, then A ∪ B should be regular.

As a result $L \cup L_6 = L_4 \cup L_5 \cup L_6$ should be regular.