CENG 384 - Signals and Systems for Computer Engineers
20222
Written Assignment 4 Solutions

June 13, 2023

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1. (a) The D.E. of the system can be derived from the frequency response as follows:

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega - 1}{j\omega + 1}$$

$$j\omega Y(j\omega) + Y(j\omega) = j\omega X(j\omega) - X(j\omega)$$

Using differentiation in time property of the Fourier Transform, we get the result:

$$y'(t) + y(t) = x'(t) - x(t).$$

(b) The F.R. can be expanded as follows:

$$H(j\omega) = \frac{j\omega + 1 - 2}{j\omega + 1} = 1 - \frac{2}{j\omega + 1}$$

Using IFT (Table 4.2) we obtain:

$$h(t) = \delta(t) - 2e^{-t}u(t).$$

(c) The transform of $x(t)$ is $X(j\omega) = 1/(j\omega + 2)$. Multiplying this with $H(j\omega)$ gives:

$$Y(j\omega) = \frac{1}{j\omega + 2} \times \frac{j\omega - 1}{j\omega + 1} = \frac{A}{j\omega + 1} + \frac{B}{j\omega + 2}$$

Solving for A and B, we obtain:

$$Y(j\omega) = \frac{-2}{j\omega + 1} + \frac{3}{j\omega + 2}$$

whose inverse (by Table 4.2) is then:

$$y(t) = (-2e^{-t} + 3e^{-2t})u(t).$$

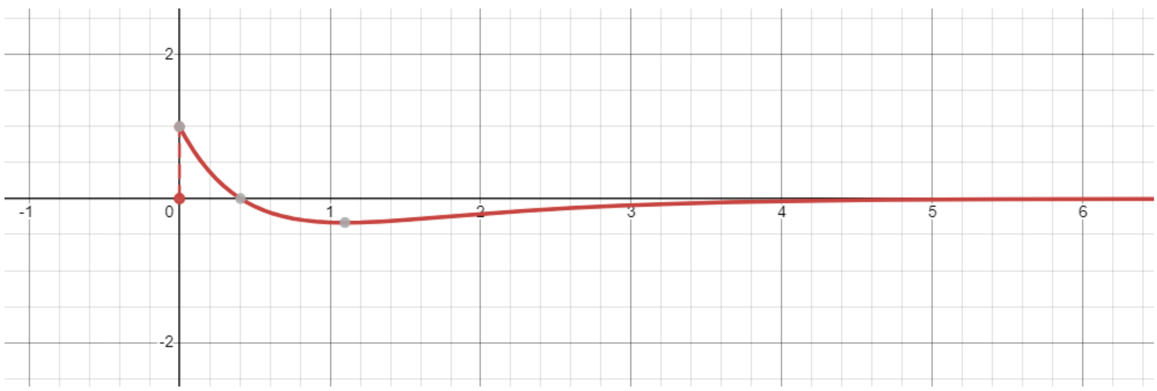
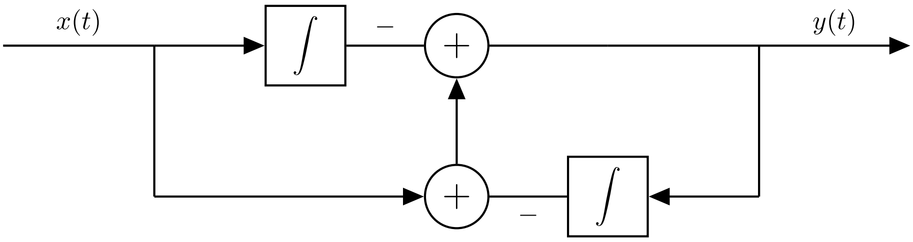


Figure 1: t vs y(t)

(d) The block diagram can be drawn like this:



2. (a) Using time shifting property of the Fourier Transform, we obtain:

$$e^{j\omega}Y(e^{j\omega}) - \frac{1}{2}Y(e^{j\omega}) = e^{j\omega}X(e^{j\omega})$$

$$\left(e^{j\omega} - \frac{1}{2}\right)Y(e^{j\omega}) = e^{j\omega}X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}} = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}.$$

(b) Using IFT (Table 5.2) we obtain:

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

(c) From Table 5.2 we obtain:

$$X(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}e^{-j\omega}}$$

$$Y(e^{j\omega}) = H(e^{j\omega}) \times X(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} \times \frac{1}{1 - \frac{3}{4}e^{-j\omega}}$$

$$Y(e^{j\omega}) = \frac{A}{1 - \frac{1}{2}e^{-j\omega}} + \frac{B}{1 - \frac{3}{4}e^{-j\omega}}$$

Solving for A and B, we obtain:

$$Y(e^{j\omega}) = \frac{-2}{1 - \frac{1}{2}e^{-j\omega}} + \frac{3}{1 - \frac{3}{4}e^{-j\omega}}$$

whose inverse (by Table 5.2) is then:

$$y[n] = -2\left(\frac{1}{2}\right)^n u[n] + 3\left(\frac{3}{4}\right)^n u[n].$$

3. (a) The D.E. of the system can be derived from the frequency response as follows:

$$H(j\omega) = H_1(j\omega) \times H_2(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{(j\omega)^2 + 3j\omega + 2}$$

$$(j\omega)^2 Y(j\omega) + 3j\omega Y(j\omega) + 2Y(j\omega) = X(j\omega)$$

Using differentiation in time property of the Fourier Transform, we get the result:

$$y''(t) + 3y'(t) + 2y(t) = x(t).$$

(b) The F.R. can be expanded as follows:

$$H(j\omega) = \frac{A}{j\omega+1} + \frac{B}{j\omega+2}$$

Solving for A and B, we obtain:

$$Y(j\omega) = \frac{1}{j\omega+1} - \frac{1}{j\omega+2}$$

Using IFT (Table 4.2) we obtain:

$$h(t) = e^{-t}u(t) - e^{-2t}u(t).$$

(c) The transform of $x(t)$ is $X(j\omega) = j\omega$. Multiplying this with $H(j\omega)$ gives:

$$Y(j\omega) = \frac{j\omega}{(j\omega+1)(j\omega+2)} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2}$$

Solving for A and B, we obtain:

$$Y(j\omega) = \frac{-1}{j\omega+1} + \frac{2}{j\omega+2}$$

whose inverse (by Table 4.2) is then:

$$y(t) = (-e^{-t} + 2e^{-2t})u(t).$$

4. (a)

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$
$$H(e^{j\omega}) = \frac{3}{3+e^{-j\omega}} + \frac{2}{2+e^{-j\omega}}$$
$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{12+5e^{-j\omega}}{6+5e^{-j\omega}+e^{-2j\omega}}$$

Using time shifting property of the Fourier Transform, we obtain:

$$y[n-2] + 5y[n-1] + 6y[n] = 5x[n-1] + 12x[n].$$

(b) From part a we have the frequency response:

$$H(e^{j\omega}) = \frac{12+5e^{-j\omega}}{6+5e^{-j\omega}+e^{-2j\omega}}$$

(c)

$$H(e^{j\omega}) = \frac{3}{3+e^{-j\omega}} + \frac{2}{2+e^{-j\omega}}$$
$$H(e^{j\omega}) = \frac{1}{1+\frac{1}{3}e^{-j\omega}} + \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

Using IFT (Table 5.2) we obtain:

$$h[n] = \left(\frac{-1}{3}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n].$$

5. .

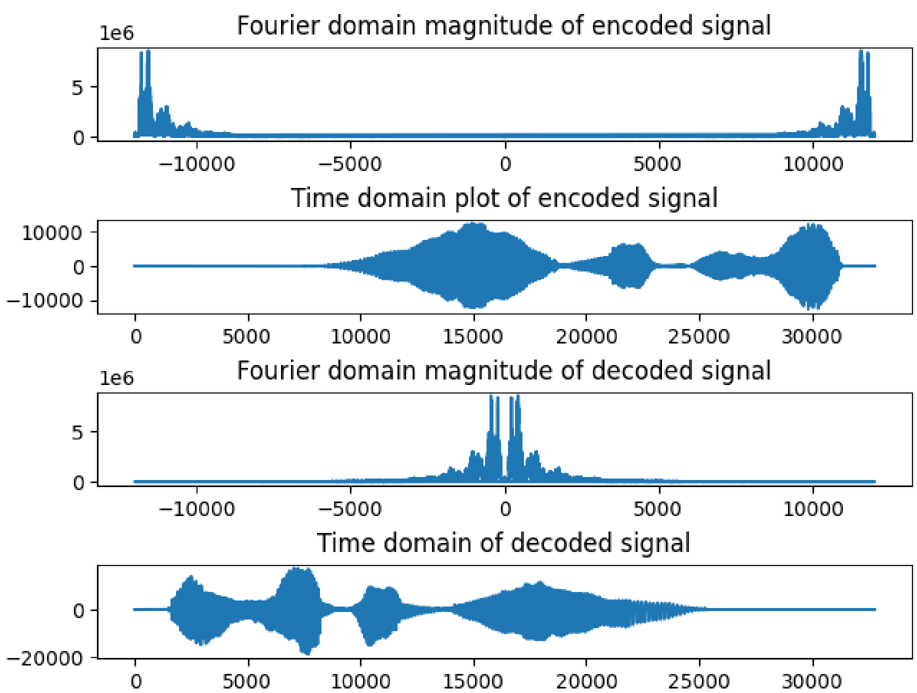


Figure 2: Secret message is "I have a dream"