

Example 4.6.2: The following grammar G generates the language $\{a^n b^n c^n : n \geq 1\}$. $G = (V, \Sigma, R, S)$, where

$$V = \{S, a, b, c, A, B, C, T_a, T_b, T_c\},$$

$$\Sigma = \{a, b, c\}, \text{ and}$$

$$R = \{S \rightarrow ABCS,$$

$$S \rightarrow T_c,$$

$$CA \rightarrow AC,$$

$$BA \rightarrow AB,$$

$$CB \rightarrow BC,$$

$$CT_c \rightarrow T_c c,$$

$$CT_c \rightarrow T_b c,$$

$$BT_b \rightarrow T_b b,$$

$$BT_b \rightarrow T_a b,$$

$$AT_a \rightarrow T_a a,$$

$$T_a \rightarrow e\}.$$

The first three rules generate a string of the form $(ABC)^n T_c$. Then the next three rules allow the A 's, B 's, and C 's in the string to “sort out” themselves correctly, so that the string becomes $A^n B^n C^n T_c$. Finally, the remaining rules allow the T_c to “migrate” to the left, transforming all C 's to c 's, and then becoming T_b . In turn, T_b migrates to the left, transforming all B 's into b 's and becoming T_a , and finally T_a transforms all A 's into a 's and then is erased.