CENG 384 - Signals and Systems for Computer Engineers
20222
Written Assignment 1 Solutions

April 5, 2023

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1. (a) (5 pts)

$$2x + 5 + 2yj = j - (x - yj) \tag{1}$$
$$2x + 5 + 2yj = -x + (y+1)j \tag{2}$$
$$2x + 5 = -x \tag{3}$$
$$x = -\frac{5}{3} \tag{4}$$
$$2y = y + 1 \tag{5}$$
$$y = 1 \tag{6}$$
$$z = -\frac{5}{3} + j \tag{7}$$
$$\bar{z} = -\frac{5}{3} - j \tag{8}$$
$$|z|^2 = z\bar{z} = (-\frac{5}{3} + j)(-\frac{5}{3} - j) = \frac{25}{9} + 1 = \boxed{\frac{34}{9}} \tag{9}$$

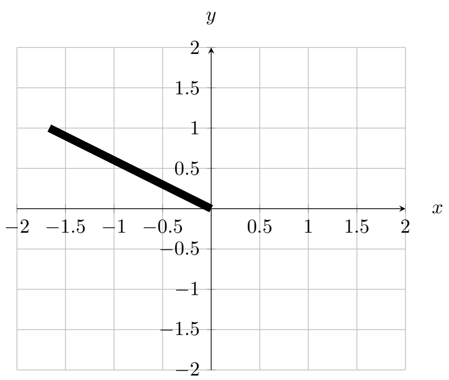

Figure 1: z on the complex plane.

(b) (5 pts)

$$z^5 = r^5 e^{5j\theta} = 32j \tag{10}$$
$$r^5 e^{5j\theta} = 32e^{j\frac{\pi}{2}} \tag{11}$$

$$r = 2, 5\theta = \frac{\pi}{2} + 2\pi k, k \in Z \tag{12}$$
$$z_1 = \boxed{2e^{-j\frac{7\pi}{10}}}, z_2 = \boxed{2e^{-j\frac{3\pi}{10}}}, z_3 = \boxed{2e^{j\frac{\pi}{10}}}, z_4 = \boxed{2e^{j\frac{\pi}{2}}}, z_5 = \boxed{2e^{j\frac{9\pi}{10}}} \tag{13}$$
$$k = -2, \theta = -\frac{7\pi}{10} = -126^\circ \tag{14}$$
$$k = -1, \theta = -\frac{3\pi}{10} = -54^\circ \tag{15}$$
$$k = 0, \theta = \frac{\pi}{10} = 18^\circ \tag{16}$$
$$k = 1, \theta = \frac{\pi}{2} = 90^\circ \tag{17}$$
$$k = 2, \theta = \frac{9\pi}{10} = 162^\circ \tag{18}$$

(c) (5 pts)

$$z_1 = 1 + j = \sqrt{2}e^{j\frac{\pi}{4}} \tag{19}$$

$$z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}j = e^{j\frac{\pi}{3}} \tag{20}$$

$$z_3 = -1 + j = \sqrt{2}e^{j\frac{3\pi}{4}} \tag{21}$$

$$z = \frac{z_1 z_2}{z_3} = \frac{\cancel{\sqrt{2}}e^{j\frac{\pi}{4}}e^{j\frac{\pi}{3}}}{\cancel{\sqrt{2}}e^{j\frac{3\pi}{4}}} = \frac{e^{j(\frac{\pi}{4}+\frac{\pi}{3})}}{e^{j\frac{3\pi}{4}}} = e^{-j\frac{\pi}{6}} \tag{22}$$

$$|z| = \boxed{1}, \quad \angle z = \boxed{-\frac{\pi}{6}}\ radians = -30° \tag{23}$$

(d) (5 pts)

$$z = je^{-j\frac{\pi}{2}} = e^{j\frac{\pi}{2}}e^{-j\frac{\pi}{2}} = e^0 = 1 = \boxed{1e^{j0}} \tag{24}$$

2. (10 pts)

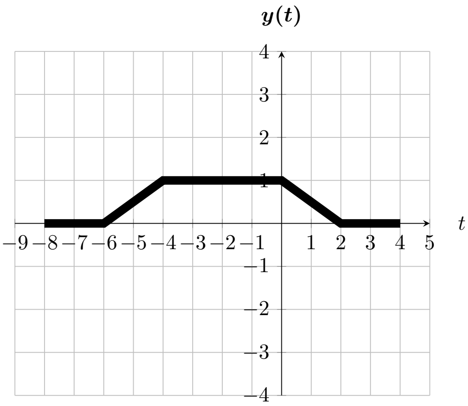

Figure 2: t vs. $y(t)$.

3. (a) (10 pts) $x[-n]$ is the reflection of $x[n]$ about y-axis. For $x[2n-1]$, we first shrink $x[n]$ by 2 and then shift to the right by $\frac{1}{2}$ and take the integer n values. At the end we sum $x[-n]$ and $x[2n-1]$.

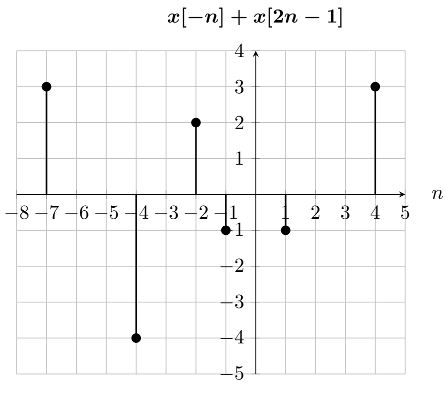

Figure 3: n vs. $x[-n] + x[2n-1]$.

(b) (5 pts) $x[-n] + x[2n-1] = 3\delta[n+7] - 4\delta[n+4] + 2\delta[n+2] - \delta[n+1] - \delta[n-1] + 3\delta[n-4]$

4. (a) (5 pts) $x(t) = 5\sin(3t - \frac{\pi}{4}) = A\sin(\omega_0 t + \phi) \implies T_0 = \frac{2\pi}{|\omega_0|} \implies T_0 = \boxed{\frac{2\pi}{3}}$

(b) (5 pts) $x[n] = x_1[n] + x_2[n] = \cos\left(\frac{13\pi}{10}n\right) + \sin\left(\frac{7\pi}{10}n\right)$

$x_1[n] \implies N_1 = \frac{2\pi}{\omega_0}m = \frac{20}{\cancel{13}}\cancel{13}$

$x_2[n] \implies N_2 = \frac{2\pi}{\omega_0}m = \frac{20}{\cancel{7}}\cancel{7}$

So the common period is 20, $N_0 = \boxed{20}$

(c) (5 pts) $x[n] = \frac{1}{2}\cos(7n-5) \implies N_0 = \frac{2\pi}{\omega_0}m = \frac{2\pi}{7}m$

We don't have an integer m value that makes N_0 integer. So $x[n]$ is not periodic.

5. (a) (5 pts) $x(t) = u(t-1) - 3u(t-3) + u(t-4)$

(b) (5 pts) $\frac{dx(t)}{dt} = \delta(t-1) - 3\delta(t-3) + \delta(t-4)$

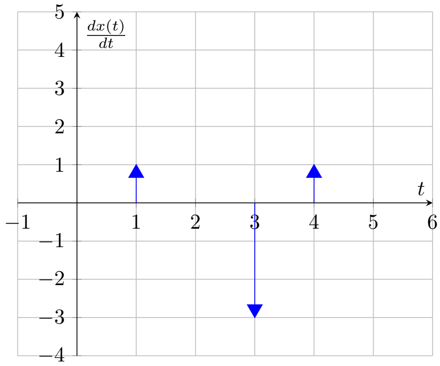

Figure 4: t vs. $\frac{dx(t)}{dt}$.

6. (a) (6 pts) $y(t) = tx(2t+3)$

i. **Memory** Has memory; output is dependent on the input at a different time. $y(1) = x(5)$.

ii. **Stability** Not stable; if input is constant, $y(t)$ depends on t, thus unbounded.

iii. **Causality** Non Causal; $y(1) = x(5)$, output depends on future value of input.

iv. **Linearity** Linear; Superposition holds.

v. **Invertibility** Not invertible; $x(t) = \frac{2y(\frac{t-3}{2})}{t-3}$ if $t = 3$, the denominator is zero.

vi. **Time Invariance** Time varying; $tx(2t+3-t_0) \neq (t-t_0)x(2t+3-2t_0)$, shift in input does not result in shift in output.

(b) (6 pts) $y[n] = \sum_{k=1}^{\infty} x[n-k]$

i. **Memory** Has memory, output depends on the past values of input.

ii. **Stability** Not stable, system response grows without bound in response to small inputs.

iii. **Causality** Causal, output does not depend on future input values.

iv. **Linearity** Linear, superposition holds.

v. **Invertibility** Invertible, $x[n] = y[n+1] - y[n]$.

vi. **Time Invariance** Time invariant, a time shift in input results in an identical time shift in output.

7. (a) (9 pts)

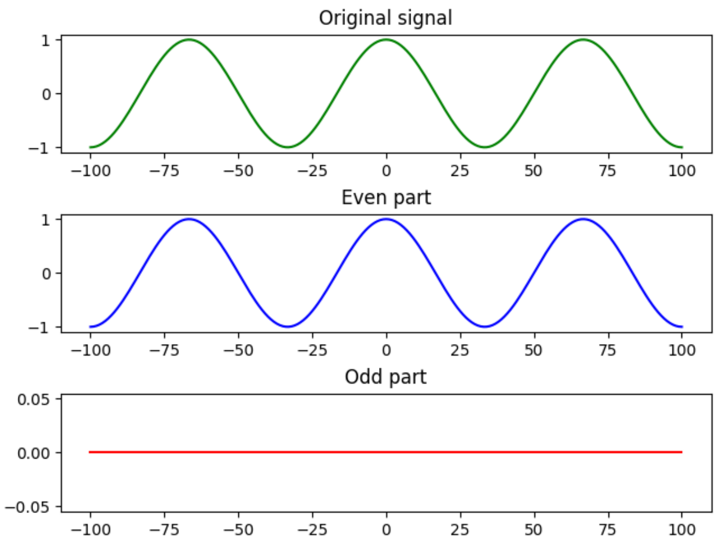

Figure 5: Even and odd parts of sine.png

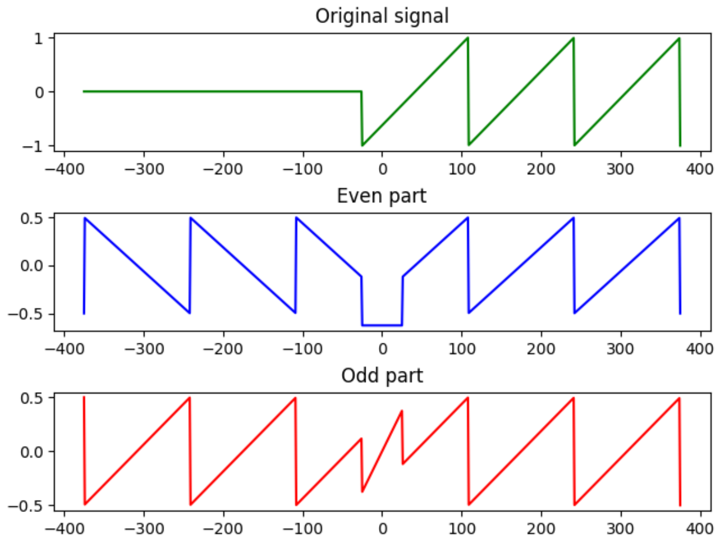

Figure 6: Even and odd parts of shifted-sawtooth.png

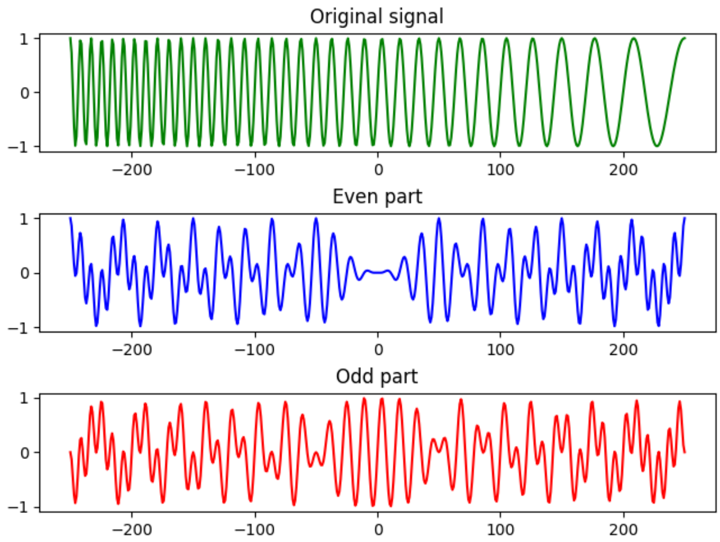

Figure 7: Even and odd parts of chirp.png

(b) (9 pts)

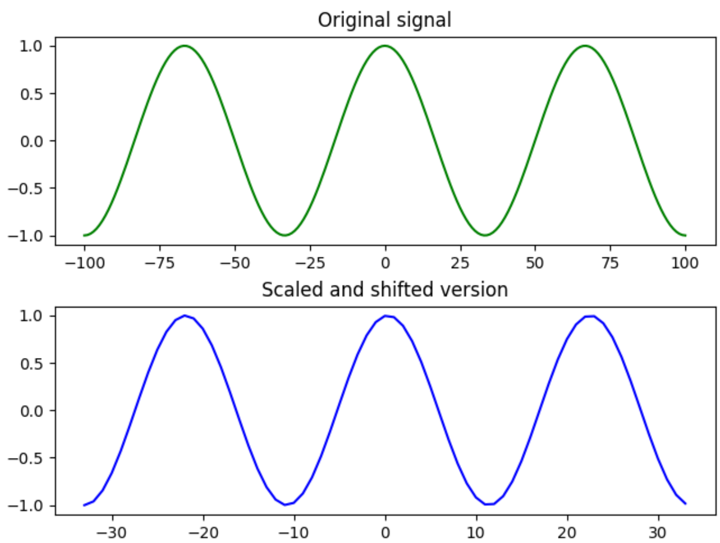

Figure 8: Shifted and scaled version of sine.png

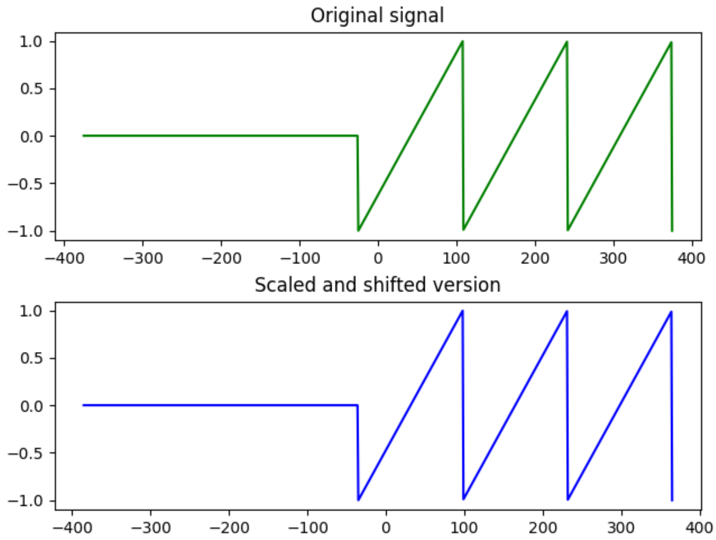

Figure 9: Shifted and scaled version of shifted-sawtooth.png

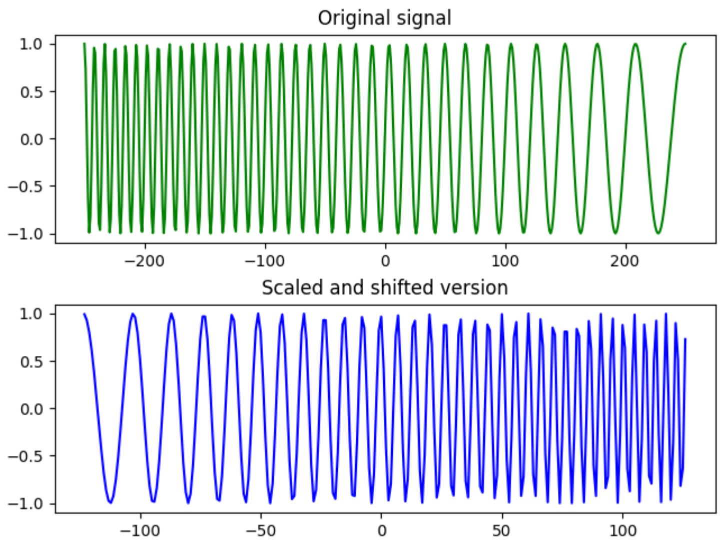

Figure 10: Shifted and scaled version of chirp.png