

$$G = (V, \Sigma, R, s)$$

$$V = \{S, a, b\}$$

$$\Sigma = \{a, b\}$$

$$R: S \rightarrow aSb \mid \epsilon$$

$$L(G) = \{a^n b^n \mid n \geq 0\}$$

Some Derivations.

$$S \Rightarrow \epsilon$$

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaasbb \Rightarrow aaaaSbbbb \Rightarrow aaaaabbbb$$

$$G = (V, \Sigma, R, S)$$

$$V = \{S, NP, VP, N, V, \text{john}, \text{ate}, \text{slept}, \text{apple}\}$$

$$\Sigma = \{\text{john}, \text{ate}, \text{apple}, \text{slept}\}$$

$$R: 1) S \rightarrow NP VP$$

$$5) N \rightarrow \text{john}$$

$$2) NP \rightarrow N$$

$$6) N \rightarrow \text{apple}$$

$$3) VP \rightarrow V NP$$

$$7) V \rightarrow \text{ate}$$

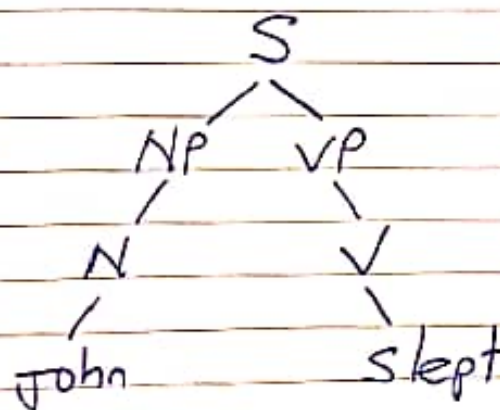
$$4) VP \rightarrow V$$

$$8) V \rightarrow \text{slept} \left. \begin{array}{l} V \rightarrow \text{ate} \\ V \rightarrow \text{slept} \end{array} \right\}$$

Derivation of "ali slept"

$$S \xRightarrow{1} NP VP \xRightarrow{2} N VP \xRightarrow{5} \text{john} VP \xRightarrow{4} \text{john} V \xRightarrow{8} \text{john slept}$$

Parse Tree:



$$S \xRightarrow[G]{*} \text{john slept}$$

start symbol
↙

$$G = (V, \Sigma, R, E)$$

$$V = \{+, \times, (,), id, T, F, E\}$$

$$\Sigma = \{+, \times, (,), id\}$$

$$R: 1) E \rightarrow E + T$$

$$5) T \rightarrow F$$

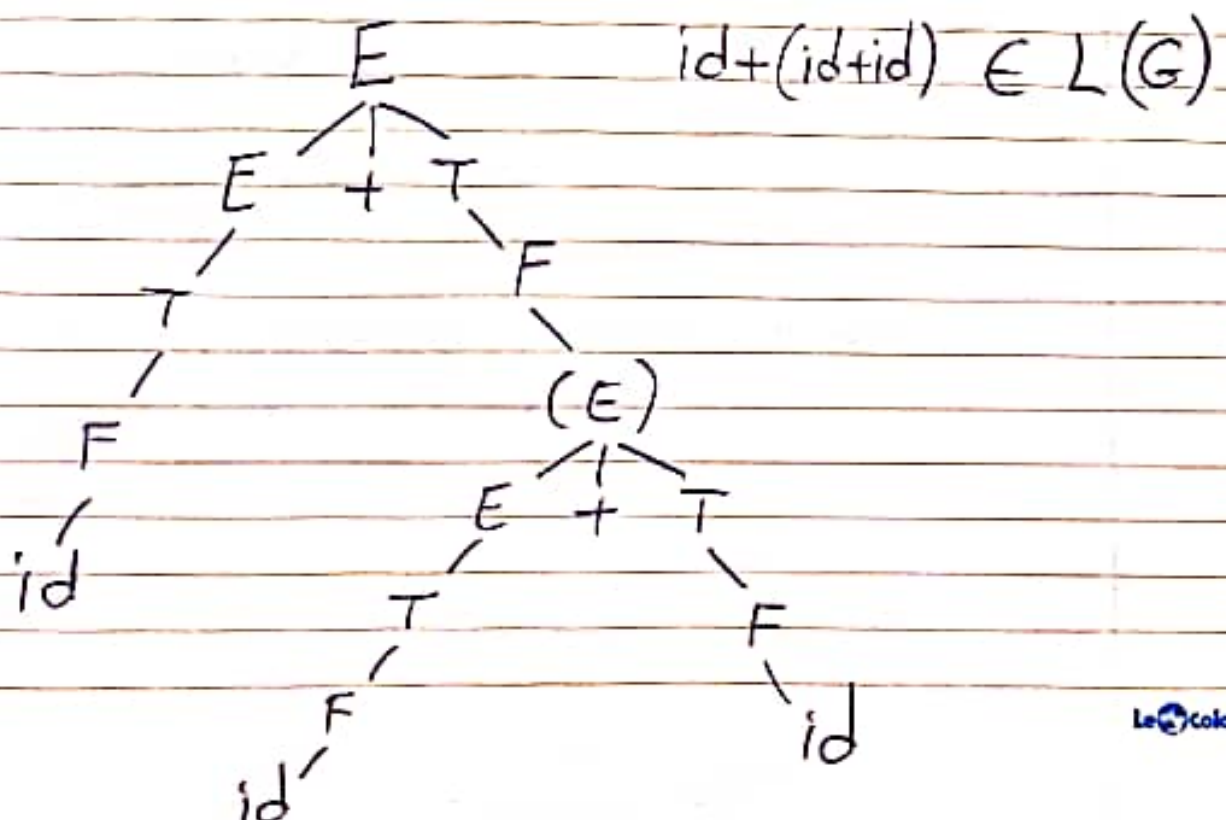
$$2) T \rightarrow T \times F$$

$$6) F \rightarrow id$$

$$3) F \rightarrow (E)$$

$$4) E \rightarrow T$$

Derivation of $id + (id \times id)$

$$\begin{aligned} S &\Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow id + T \Rightarrow id + F \\ &\Rightarrow id + (E) \Rightarrow id + (E + T) \Rightarrow id + (T + T) \Rightarrow id + (E + T) \\ &\Rightarrow id + (id + T) \Rightarrow id + (id + F) \Rightarrow id + (id + id) \end{aligned}$$


DFA to CFG construction:

$$M = (K, \Sigma, \delta, s, F)$$

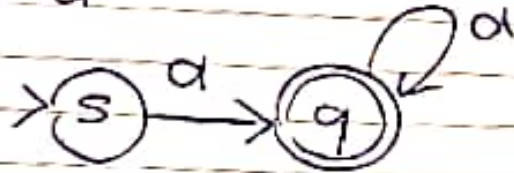
$$G = (V, \Sigma, R, S)$$

$$V = K \cup \Sigma$$

$$S = s$$

$$R = \{ q \rightarrow ap \mid \delta(q, a) = p \} \cup \{ q \rightarrow \epsilon \mid q \in F \}$$

$$L = a^+$$



$$G = (\{s, q, a\}, \{a\}, R, s)$$

$$R: 1) s \rightarrow aq$$

$$2) q \rightarrow aq$$

$$3) q \rightarrow \epsilon$$

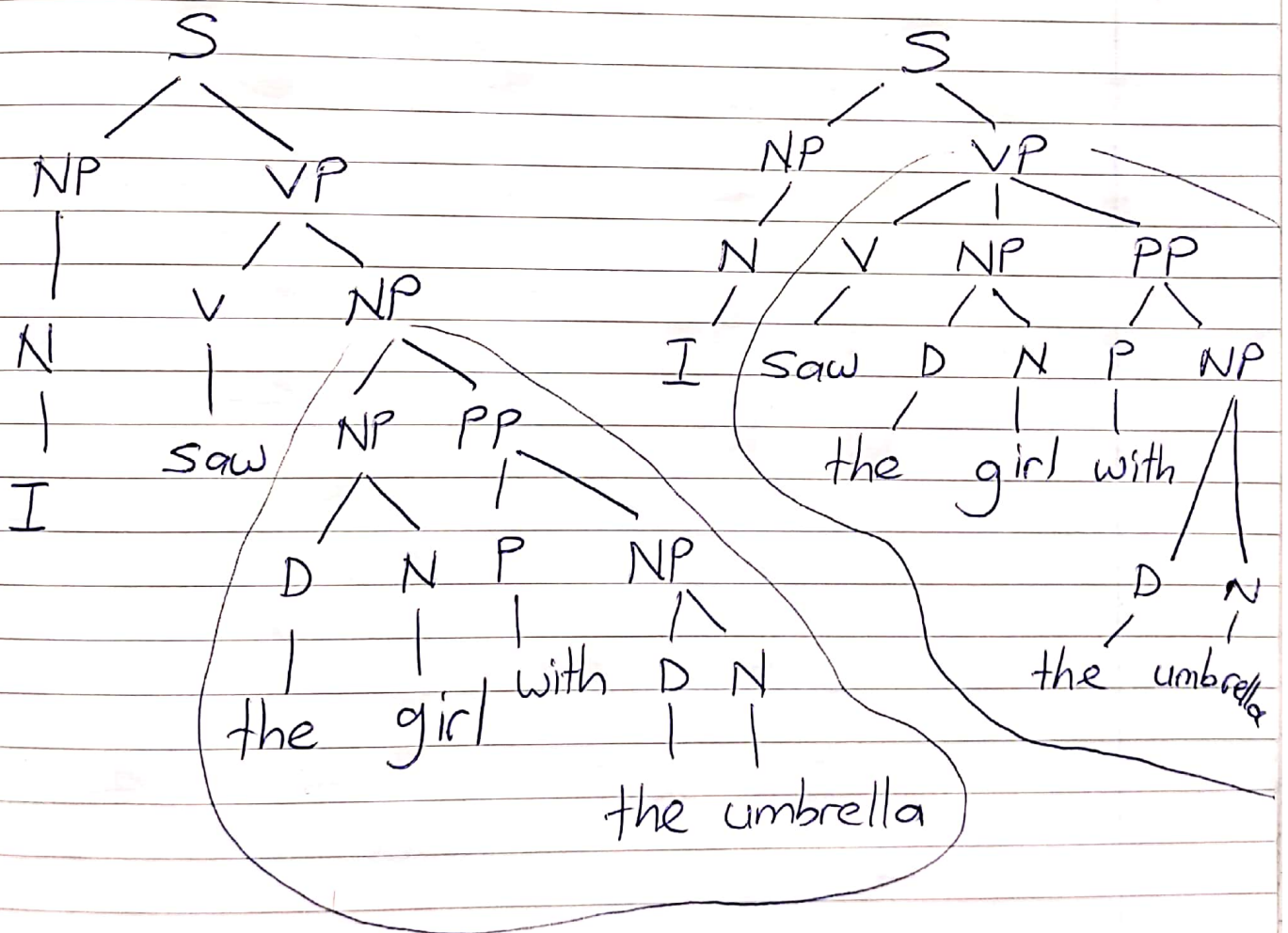
Derivation:

$$s \xRightarrow[1]{a} aq \xRightarrow[2]{a} aaq \xRightarrow[2]{a} aaaaq \xRightarrow[3]{\epsilon} aaaa$$

$S \rightarrow NP VP$
 $NP \rightarrow D N \mid NP PP \mid N$
 $VP \rightarrow V$
 $VP \rightarrow V NP$
 $VP \rightarrow V NP PP$
 $PP \rightarrow P NP$

$N \rightarrow I$
 $N \rightarrow \text{girl}$
 $N \rightarrow \text{telescope}$
 $V \rightarrow \text{saw}$
 $D \rightarrow \text{the}$
 $P \rightarrow \text{with}$

I saw the girl with the telescope



Ambiguous

Different parse trees

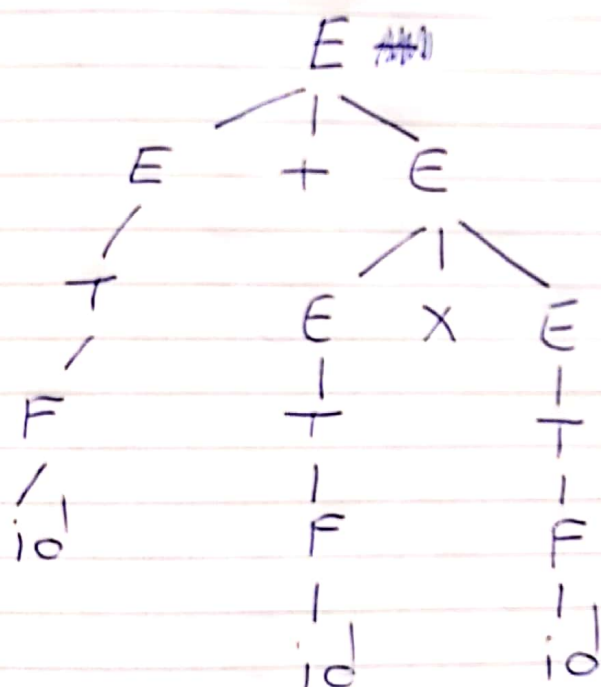
G:

$$E \rightarrow E + E$$
$$E \rightarrow E \times E$$
$$E \rightarrow T$$
$$T \rightarrow F$$
$$F \rightarrow id$$

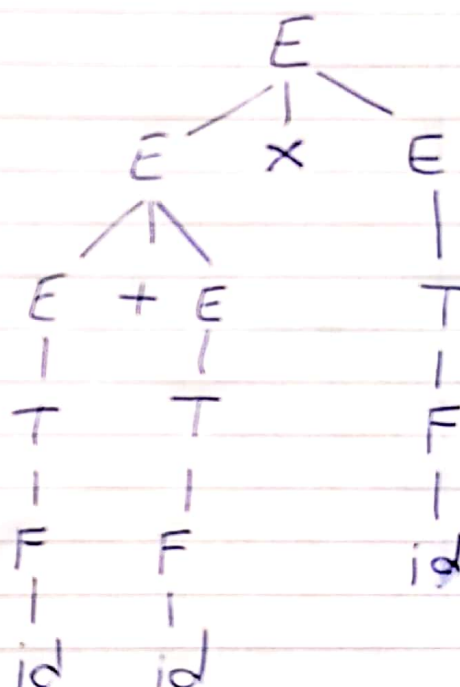
start symbol: E

$$\Sigma = \{+, \times, id\}$$

w: $id + id \times id$



Evaluation order
First \times
Then $+$



Evaluation Order
First $+$
Then \times

- Ambiguous - Different parse trees

need to change grammar
to remove ambiguity

$S \rightarrow A B$

$A \rightarrow C D$

$B \rightarrow E F$

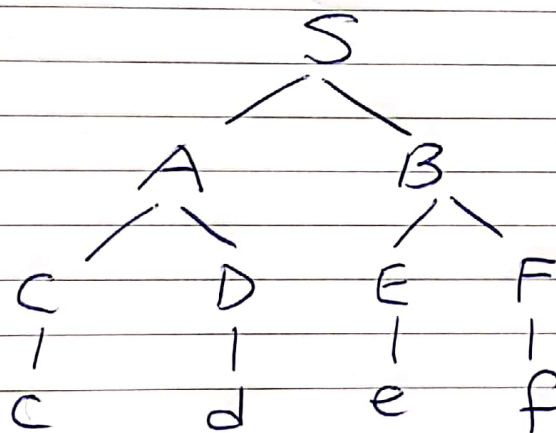
$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow c$

$D \rightarrow d$

$E \rightarrow e$



Many
Derivations
for the
same parse
tree.
(Equivalent
Derivations)

Leftmost Derivation:

$S \Rightarrow \underline{A} B \Rightarrow \underline{C} D B \Rightarrow c D B \Rightarrow c D \underline{E} F \Rightarrow c d e \underline{F} \Rightarrow c d e f$

Rightmost Derivation:

$S \Rightarrow A \underline{B} \Rightarrow A \underline{E} F \Rightarrow A \underline{E} f \Rightarrow A e \underline{f} \Rightarrow C D e \underline{f} \Rightarrow C d e \underline{f} \Rightarrow c d e f$

Another Derivation:

$S \Rightarrow A B \Rightarrow A E F \Rightarrow C D E F \Rightarrow C D e F \Rightarrow C d e F \Rightarrow c d e F \Rightarrow c d e f$