# CENG 384 - Signals and Systems for Computer Engineers
# Spring 2023
# Homework 4

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1. (a)

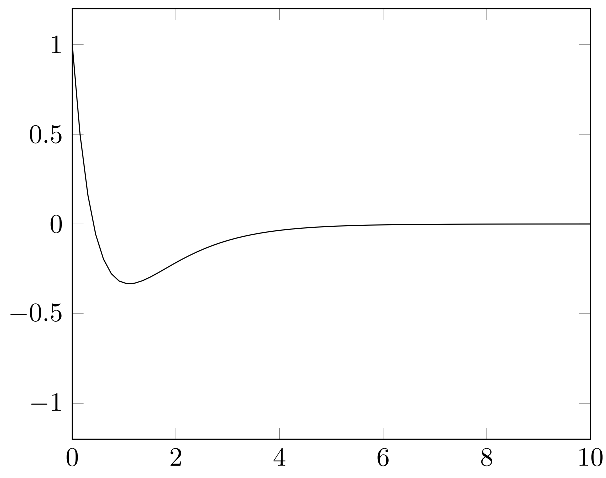
$$H(jw) = \frac{jw - 1}{jw + 1}$$
$$\frac{Y(jw)}{X(jw)} = \frac{jw - 1}{jw + 1}$$
$$jwX(jw) - X(jw) = jwY(jw) + Y(jw)$$
$$x'(t) - x(t) = y'(t) + y(t)$$

(b)

$$H(jw) = \frac{jw - 1}{jw + 1}$$
$$H(jw) = 1 + \frac{-2}{jw + 1}$$
$$h(t) = \delta(t) - 2e^{-t}u(t)$$

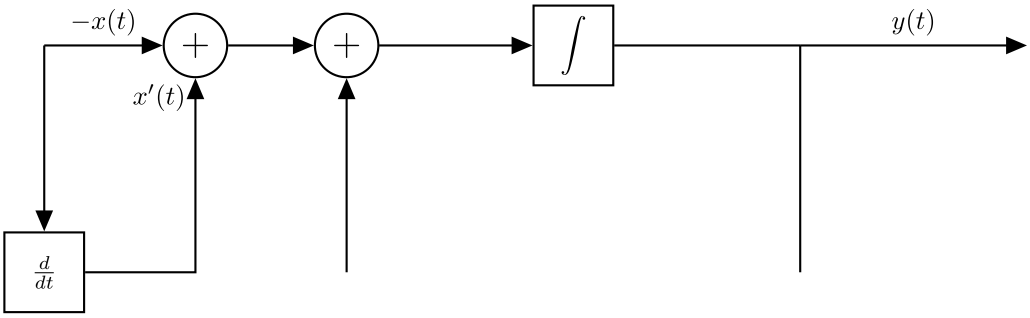
(c)

$$X(jw) = \frac{1}{jw + 2}$$
$$Y(jw) = X(jw).H(jw)$$
$$Y(jw) = \frac{1}{jw + 2}.\frac{jw - 1}{jw + 1}$$
$$Y(jw) = \frac{3}{jw + 2} - \frac{2}{jw + 1}$$
$$y(t) = (3e^{-2t} - 2e^{-t})u(t)$$




$$y(t) = (3e^{-2t} - 2e^{-t})u(t)$$

(d)




2. (a)

$$e^{jw}Y(e^{jw}) - \frac{1}{2}Y(e^{jw}) = e^{jw}X(jw)$$

$$Y(jw)(e^{jw} - \frac{1}{2}) = e^{jw}X(jw)$$

$$\frac{Y(jw)}{X(jw)} = \frac{e^{jw}}{e^{jw} - \frac{1}{2}}$$

$$H(e^{jw}) = \frac{1}{1 - \frac{1}{2}e^{-jw}}$$

(b)

$$h[n] = (\frac{1}{2})^n u[n]$$

(c)

$$y[n] = x[n] * h[n]$$

$$y[n] = x[n] - \frac{1}{2}x[n-1]$$

When $x[n] = \left(\frac{3}{4}\right)^n u[n]$:

$$y[n] = -2\left(\frac{1}{2}\right)^n u[n] + 3\left(\frac{3}{4}\right)^n u[n]$$

3. (a)

$$H(jw) = H_1(jw).H_2(jw)$$

$$H(jw) = \frac{1}{jw+1}.\frac{1}{jw+2}$$

$$\frac{Y(jw)}{X(jw)} = \frac{1}{(jw)^2 + 3(jw) + 2}$$

$$(jw)^2Y(jw) + 3(jw)Y(jw) + 2Y(jw) = X(jw)$$

$$(jw)^2Y(jw) \xleftrightarrow{FT} y''(t)$$

$$3(jw)Y(jw) \xleftrightarrow{FT} 3y'(t)$$

$$2Y(jw) \xleftrightarrow{FT} 2y(t)$$

$$X(jw) \xleftrightarrow{FT} x(t)$$

$$y''(t) + 3y'(t) + 2y(t) = x(t)$$

(b)

$$H(jw) = \frac{1}{jw+1}.\frac{1}{jw+2}$$

$$H(jw) = \frac{1}{jw+1} - \frac{1}{jw+2}$$

$$H_3(jw) = \frac{1}{jw+1} \xleftrightarrow{FT} h_1(t) = e^{-t}u(t)$$

$$H_4(jw) = \frac{1}{jw+2} \xleftrightarrow{FT} h_2(t) = e^{-2t}u(t)$$

$$h(t) = h_1(t) + h_2(t)$$

$$h(t) = (e^{-t} + e^{-2t})u(t)$$

(c)

$$H(jw) = \frac{1}{jw+1} - \frac{1}{jw+2}$$
$$\frac{Y(jw)}{X(jw)} = \frac{1}{jw+1} - \frac{1}{jw+2}$$
$$Y(jw) = \frac{1}{jw+1}X(jw) - \frac{1}{jw+2}X(jw)$$

When $X(jw) = jw$ :

$$Y(jw) = \frac{jw}{jw+1} - \frac{jw}{jw+2}$$
$$Y(jw) = \left(1 - \frac{1}{jw+1}\right) - \left(1 - \frac{2}{jw+2}\right)$$
$$Y(jw) = \frac{2}{jw+2} - \frac{1}{jw+1}$$
$$y(t) = (2e^{-2t} - e^{-t})u(t)$$

4. (a)

$$H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$$
$$H(e^{jw}) = \frac{3}{3+e^{-jw}} + \frac{2}{2+e^{-jw}}$$
$$\frac{Y(e^{jw})}{X(e^{jw})} = \frac{5e^{-jw}+12}{e^{-2jw}+5e^{-jw}+6}$$
$$e^{-2jw}Y(e^{jw}) + 5e^{-jw}Y(e^{jw}) + 6Y(e^{jw}) = 5e^{-jw}X(e^{jw}) + 12X(e^{jw})$$
$$e^{-2jw}Y(e^{jw}) \xleftrightarrow{FT} y[n-2]$$
$$5e^{-jw}Y(e^{jw}) \xleftrightarrow{FT} 5y[n-1]$$
$$6Y(e^{jw}) \xleftrightarrow{FT} 6y[n]$$
$$5e^{-jw}X(e^{jw}) \xleftrightarrow{FT} 5x[n-1]$$
$$12X(e^{jw}) \xleftrightarrow{FT} 12x[n]$$
$$y[n-2] + 5y[n-1] + 6y[n] = 5x[n-1] + 12x[n]$$

(b)

$$H(e^{jw}) = \frac{1}{1+\frac{1}{3}e^{-jw}} + \frac{1}{1+\frac{1}{2}e^{-jw}}$$

(c)

$$h[n] = h_1[n] + h_2[n]$$
$$H_1(e^{jw}) = \frac{1}{1+\frac{1}{3}e^{-jw}} \xleftrightarrow{FT} h_1[n] = \left(\frac{-1}{3}\right)^n u[n]$$
$$H_2(e^{jw}) = \frac{1}{1+\frac{1}{2}e^{-jw}} \xleftrightarrow{FT} h_2[n] = \left(\frac{-1}{2}\right)^n u[n]$$
$$h[n] = \left(\left(\frac{-1}{2}\right)^n + \left(\frac{-1}{3}\right)^n\right) u[n]$$

5.

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.io import wavfile

def fft(signal):
    N = len(signal)
    if (N <= 1):
        return signal
    even = fft(signal[::2])
    odd = fft(signal[1::2])
    T = [np.exp(-2j * np.pi * i / N) * odd[i] for i in range(N // 2)]
    return [even[i] + T[i] for i in range(N // 2)]
            +[even[i] - T[i] for i in range(N // 2)]

def ifft(signal):
    N = len(signal)
    conjugate_signal = np.conjugate(signal)
    fft_signal = fft(conjugate_signal)
    return np.conjugate(fft_signal) / N


sample_rate, encoded_signal = wavfile.read('encoded.wav')


encoded_data = fft(encoded_signal)


positive_freq = encoded_data[:len(encoded_data)//2]
negative_freq = encoded_data[len(encoded_data)//2:]


reversed_positive_freq = positive_freq[::-1]
reversed_negative_freq = negative_freq[::-1]


reversed_fft = np.concatenate((reversed_positive_freq, reversed_negative_freq))

# Perform IFFT on the reversed FFT
decoded_data = ifft(reversed_fft).real

# Convert the decoded signal to integer type
decoded_data = np.int16(decoded_data)

# Save the decoded audio as "decoded.wav" file
wavfile.write('decoded.wav', sample_rate, decoded_data)

plt.figure(figsize=(10, 6))
plt.subplot(2, 1, 1)
plt.magnitude_spectrum(encoded_data, Fs=sample_rate, scale='dB')
plt.title('Encoded Signal')
plt.subplot(2, 1, 2)
plt.magnitude_spectrum(decoded_data, Fs=sample_rate, scale='dB')
plt.title('Decoded Signal')
plt.tight_layout()
plt.show()

plt.figure(figsize=(10, 6))
plt.subplot(2, 1, 1)
plt.plot(encoded_data)
plt.title('Encoded Signal')
plt.subplot(2, 1, 2)
plt.plot(decoded_data)
plt.title('Decoded Signal')
plt.tight_layout()
plt.show()
```

The message is : "I HAVE A DREAM".

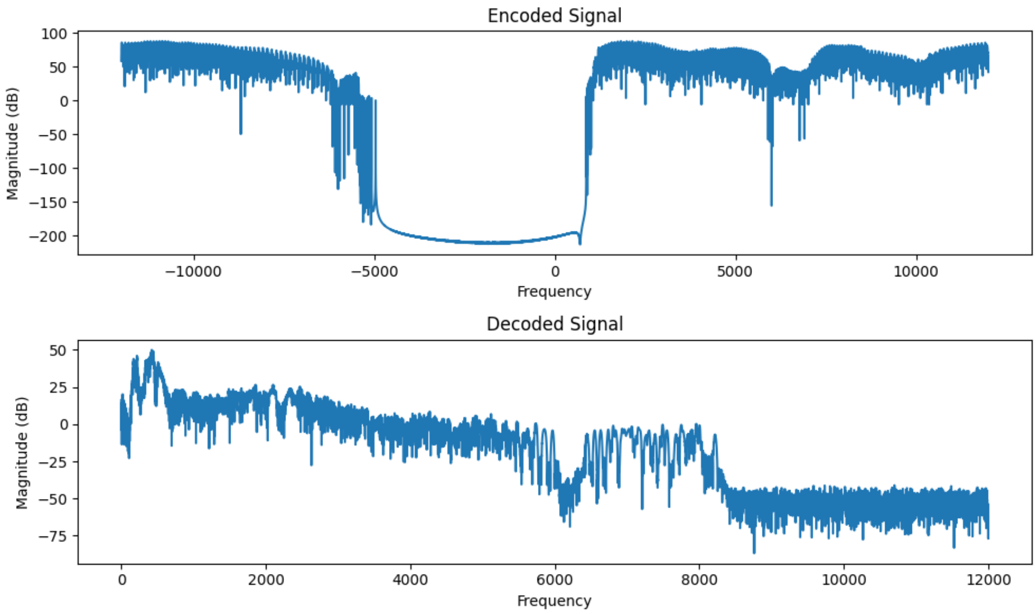



Figure 1: Frequency Domain Magnitude of Encoded and Decoded Signal

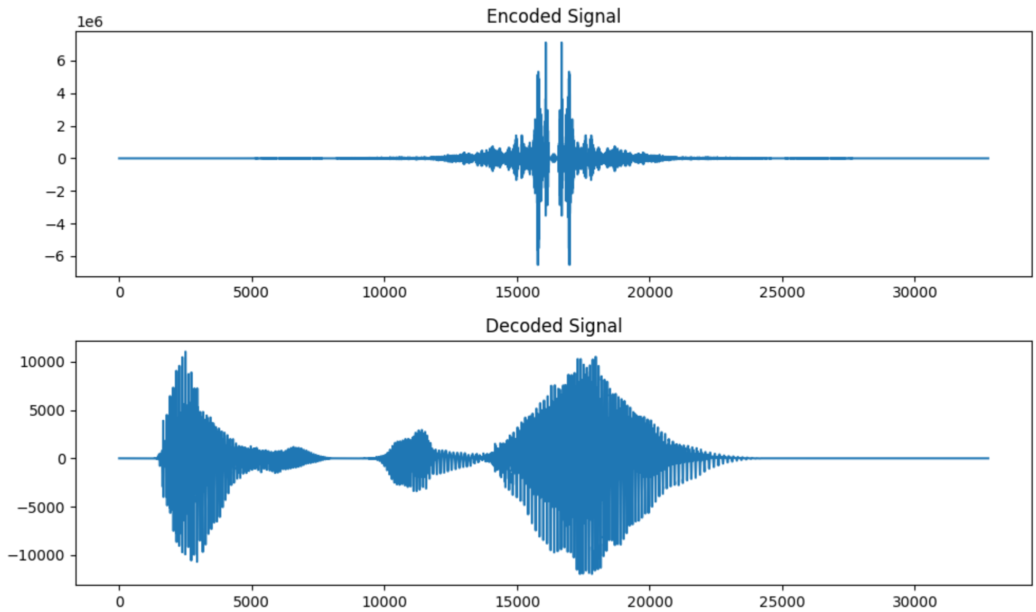



Figure 2: Time Domain Magnitude of Encoded and Decoded Signal