CENG 371 - Scientific Computing
Fall 2023
Homework 2

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Answer 1

LU Factorization algorithm files has been uploaded.

Answer 2

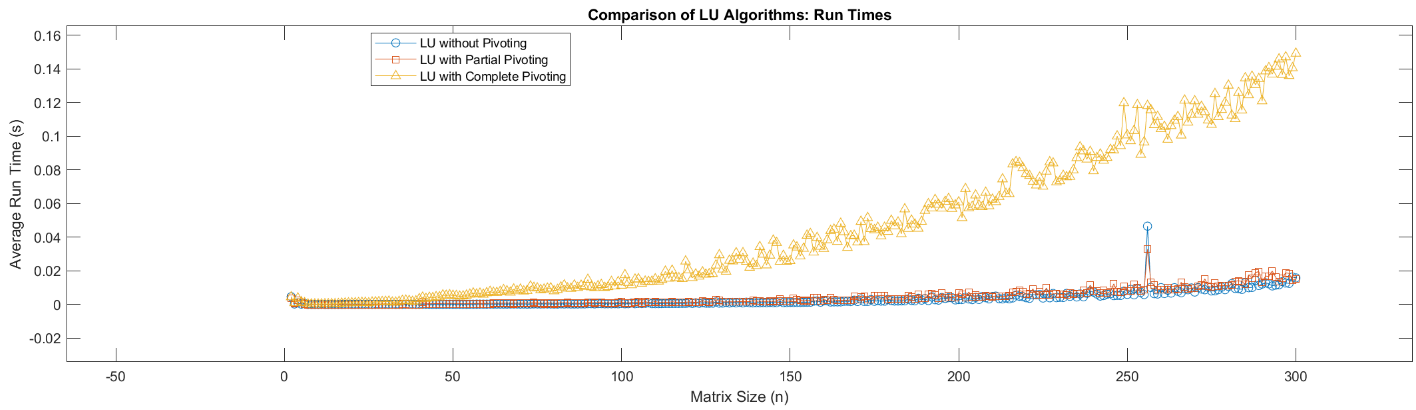

Figure 1: Comparison of LU Algorithms : Run Times 2-Norm

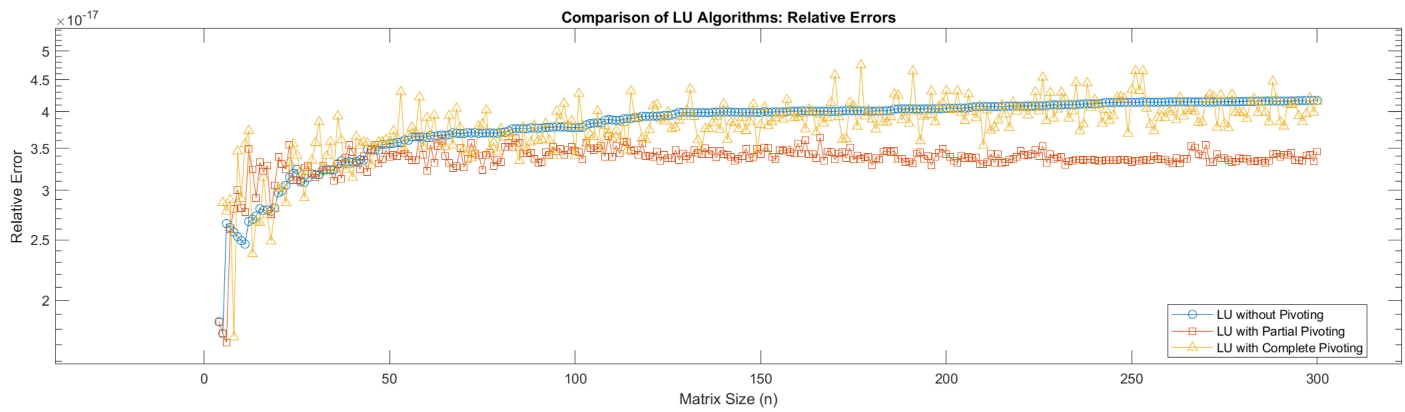

Figure 2: Comparison of LU Algorithms : Relative Errors 2-Norm

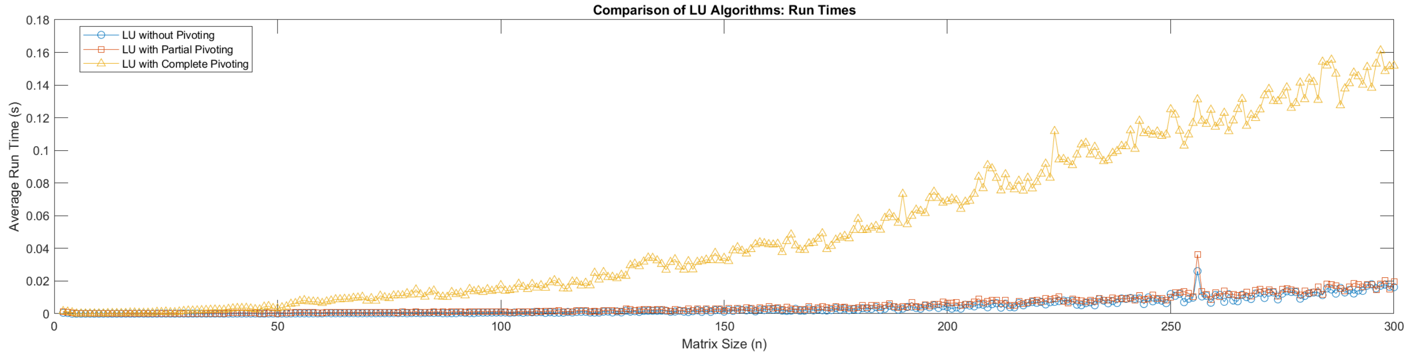

Figure 3: Comparison of LU Algorithms : Run Times Frobenius Norm

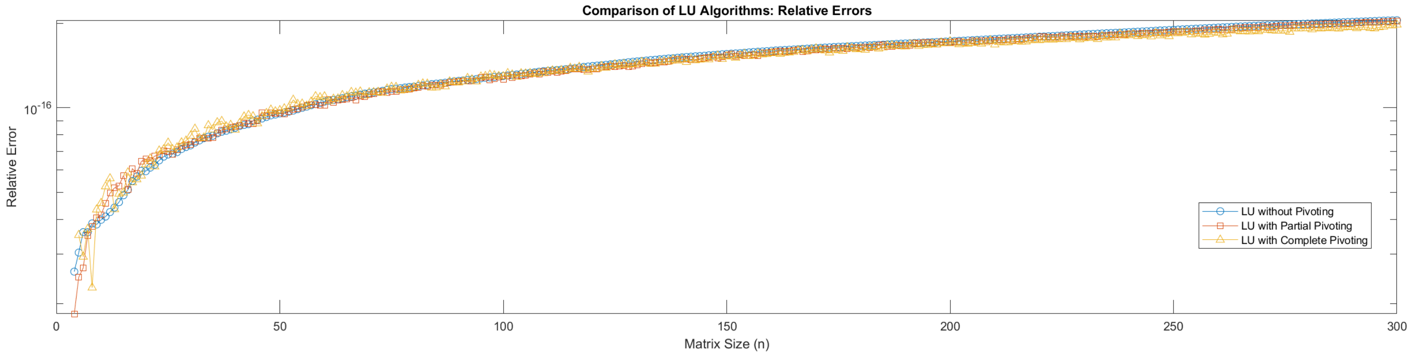

Figure 4: Comparison of LU Algorithms : Relative Errors Frobenius Norm

a) Hilbert matrices are widely known for their ill-conditioned nature, characterized by exceptionally large condition numbers. A high condition number implies that slight changes in the input or numerical errors can result in significantly magnified changes in the output.

The choice of norm significantly influences the accuracy observed in various LU decomposition algorithms. The 2-norm and the Frobenius norm, commonly used for assessing numerical error, exhibit distinct characteristics that can lead to differing results in certain situations.

Here are key observations regarding different LU decomposition algorithms applied to Hilbert matrices:

1. Without Pivoting:
   i. This method is generally the fastest due to fewer operations (no row exchanges).
   ii. However, it can be more susceptible to numerical instability, particularly with ill-conditioned matrices. The choice of norm does not significantly impact the relative errors in this algorithm.
2. With Partial Pivoting:
   i. Introducing partial pivoting, with additional row exchanges, may increase computational cost compared to the no-pivoting method.
   ii. Partial pivoting improves numerical stability, resulting in more accurate results. In 2-Norm relative error computation, it demonstrates greater accuracy and fewer errors than complete pivoting.
3. With Complete Pivoting:
   i. Complete pivoting involves both row and column exchanges, making it computationally more expensive due to additional permutations.
   ii. It is generally the most stable method. In Frobenius Norm relative error computation, it exhibits fewer errors than partial pivoting. However, in 2-Norm, it shows larger relative errors.

In summary, both relative errors and runtimes tend to increase as the size of the Hilbert matrix grows. Complete Pivoting experiences a more significant increase in runtimes due to its row and column exchanges. Moreover, the choice of norm affects relative errors in algorithms. In 2-Norm, partial pivoting is faster and has fewer errors, while in Frobenius Norm, complete pivoting, despite being slower, is more accurate.

b) Square matrices, including ill-conditioned Hilbert matrices, appear to be factorized successfully by my LU decomposition algorithms. Nevertheless, numerical errors determine the success, and additional factors like pivoting and norm selection are critical to preserving numerical stability for ill-conditioned matrices. Experimenting with different matrices—including ones with varying conditions and sizes—allows us to get an excellent understanding of how well the algorithms perform overall.