Pushdown Automata

CENG 280

Course outline

- Preliminaries: Alphabets and languages
- Regular languages
- Context-free languages
  - Context-free grammars
  - Parse trees
  - Push-down automaton
  - Push-down automaton - context-free languages
  - Languages that are and that are not context-free, Pumping lemma
- Turing-machines

Pushdown Automata

- What feature do we need to add to FSA so that it can recognize a CFL?
- Consider $a^n b^n$, ww^R.

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- MEMORY !

Input tape, reading head, internal state, STACK (or pushdown store)

- Read at most one symbol at a time
- Read/write only the top of the stack
- Remove from top check against the current input
- The word is accepted, if when it is read, the automaton is in accepting state and the stack is empty.

Definition

Pushdown automaton is a sextuple $M = (K, \Sigma, \Gamma, \Delta, s, F)$ where

- K is a finite set of states,
- Σ is an alphabet (input symbols)
- Γ is an alphabet (stack symbols)
- $s \in K$ is the initial state
- $F \in K$ is the set of final states, and,
- $\Delta \subset (K \times (\Sigma \cup \{e\}) \times \Gamma^{\star}) \times (K \times \Gamma^{\star})$ is a finite transition relation.

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- If $((p, a, \delta), (q, \gamma)) \in \Delta$, then when M is in state p, if it reads $a \in \Sigma$ (or if a is e without reading a symbol) and if the top of the stack is δ, it enters state q and replaces δ with γ.
- $((p, a, \delta), (q, \gamma))$ is called a transition of M.
- Since Δ is a relation, several transitions can be applicable at a point. The machine chooses non-deterministically from the applicable transitions

Push/Pop transitions

- $((p, a, e), (q, b))$: **push** transition, read a push b to the top of the stack.

- $((p, a, b), (q, e))$: **pop** transition, read a pop b from the top of the stack.

- The **configuration** of a pushdown automaton is a member of $K \times \Sigma^\star \times \Gamma^\star$: the current state, unread part of the input type, the contents of the stack (read top-down).
- A configuration (p, x, α) of M **yields** (q, y, ζ) (shown as $(p, x, \alpha) \vdash_M (q, y, \zeta)$) if there is a transition $((p, a, \beta), (q, \gamma))$ such that
  - $x = ay$,
  - $\alpha = \beta\nu$,
  - $\zeta = \gamma\nu$ for some $\nu \in \Gamma^\star$.

- The reflexive transitive closure of $\vdash_M$ is denoted by $\vdash_M^\star$.
- M accepts a word $w \in \Sigma^\star$ if and only if for some $f \in F$

$$(s, w, e) \vdash_M^\star (f, e, e)$$

- Any sequence $C_0, \ldots, C_n$ with $C_i \vdash_M C_{i+1}$ is called a **computation** of M. It has length n (or n steps).
- The language accepted by M, $L(M)$ is the set of strings accepted by M.

Transition diagram

Show $((p, a, \alpha), (q, \beta))$ using an arrow from p to q with label "$a \quad \alpha/\beta$"

Pushdown automata examples

Example

PDA for $L = \{wcw^R \mid w \in \{a, b\}^\star\}$. Accepting computation over *abcba*

Pushdown automata examples

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PDA for $L = \{wcw^R \mid w \in \{a, b\}^\star\}$. Accepting computation over *abcba*

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Example

Write a PDA for $L = \{ww^R \mid w \in \{a, b\}^\star\}$, and computations over *abba*

Pushdown automata examples

Example

Write a PDA M such that $L(M) = \{w \in \{a, b\}^\star \mid$
w has the same number of $a's$ and $b's\}(Grammar?)$.

Pushdown automata examples

Example

Write a PDA M such that $L(M) = \{w \in \{0,1\}^\star \mid \sum_{i=0}^{|w|} w_i \leq \frac{|w|}{2}\}$.

Pushdown automata examples

Example

Given $G = (V, \Sigma, R, S)$ with $V = \{S, (,), [,]\}$, $\Sigma = \{(,), [,]\}$, and R:

$$S \rightarrow e \mid (S) \mid [S] \mid SS$$

Construct a PDA M such that $L(M) = L(G)$.