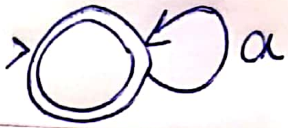
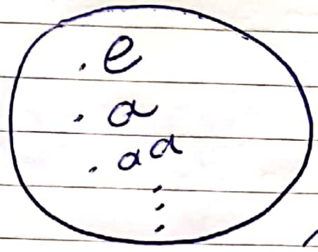


1) $L = \mathcal{L}(a^*) = \{e, a, aa, aaa, \ldots\}$

$\Sigma = \{a\}$

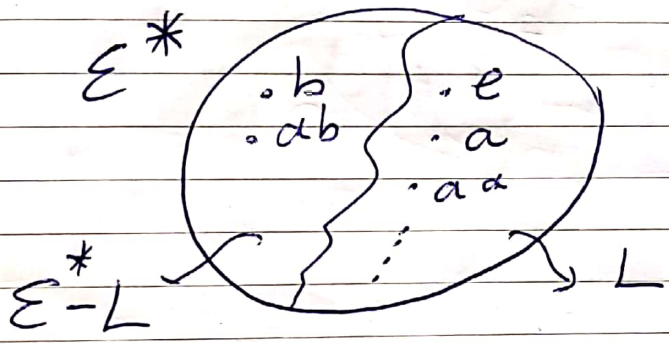
Σ^*



Only 1 equivalence class wrt $\approx_L$ which is L.

All strings in L are equivalent wrt $\approx_L$

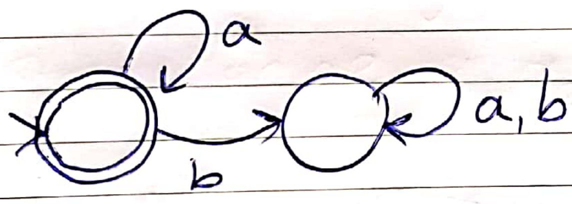
2) If $\Sigma = \{a, b\}$ $\Sigma^* = \{a, b\}^*$
$L = \mathcal{L}(a^*)$

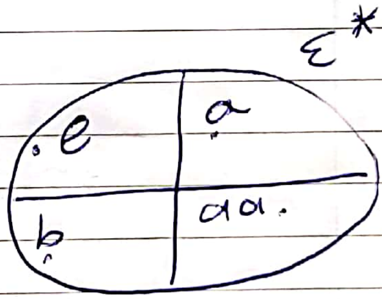


2 equivalence classes wrt $\approx_L$

$[e] = L$

$[b] = \Sigma^* - L$





3) $L = \mathcal{L}(ab \cup ba)^*$

Equivalence Classes wrt L;

$[e] = L$, the z's that make $xz \in L$; $z \in L$

$[a] = La$, " " " " " ; $z \in bL$

$[b] = Lb$, " " " " " ; $z \in aL$

$[aa] = L(aa \cup bb)\Sigma^*$; no z can restore an x in this class to $xz \in L$

$\sim_M$

$$s \xrightarrow{x} q$$

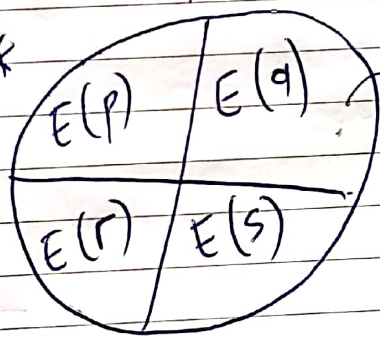
$$s \xrightarrow{y} q'$$

If $q = q'$ then $x \sim_M y$

\# of equivalence classes : \# of reachable states

ex: $K = \{p, q, r, s\}$

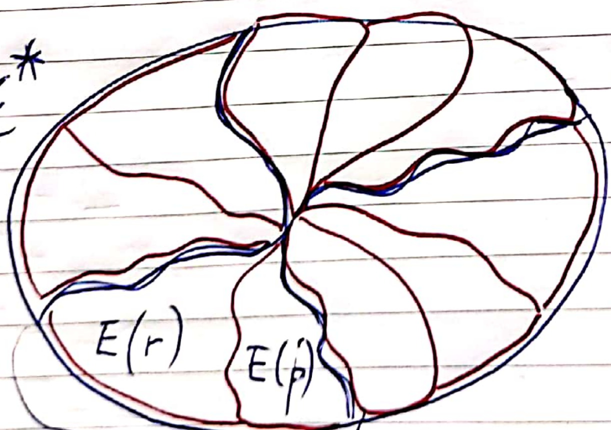
Σ^*



If q is unreachable, this eq. class is empty

$\sim_M$ refine $\approx_{L(M)}$ (If $x \sim_M y$ then $x \approx_{L(M)} y$.)

Σ^*



$\approx_{L(M)}$ 4 eq. classes

$\sim_M$ 10 eq. classes

p&r can be merged

(Minimal)
Standard DFA M' for a reg. lang L

Direct Construction

Determine equivalence classes of $\approx_L$

M':

$K = \{[x] : x \in \Sigma^*\}$

$s = [e]$

$F = \{[x] : x \in L\}$

For any $[x] \in K$ and any $a \in \Sigma$, define $\delta([x], a) = [xa]$.

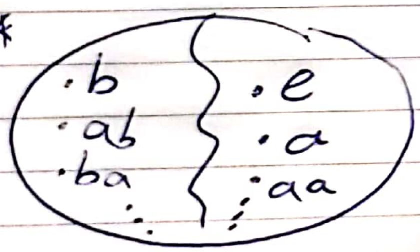
Indirect Construction

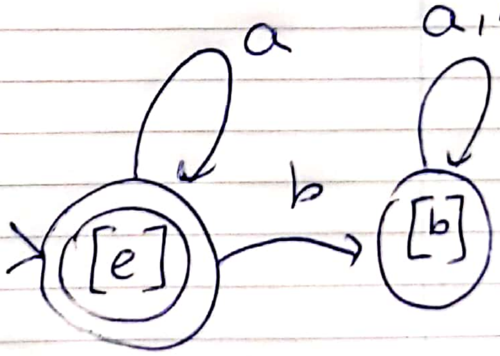
Any DFA M s.t. $L(M) = L$

$\downarrow$ state minimization
- eliminate unreachable states
- merge equivalent states

M'

ex: $\Sigma = \{a, b\}$ $\quad L = a^*$

Σ^*




$K = \{[e], [b]\}$

$s = [e]$

$F = \{[e]\}$

$\delta([e], a) = [a] = [e]$

$\delta([e], b) = [b]$

$\delta([b], a) = [ba] = [b]$

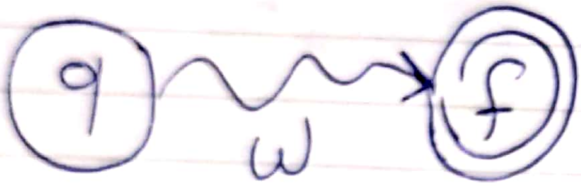
$\delta([b], b) = [bb] = [b]$

Equivalent States:

$A_M \subseteq K \times \Sigma^*$

$(q, \omega) \in A_M$ iff $(q, \omega) \vdash_M^* (f, e)$

$f \in F$

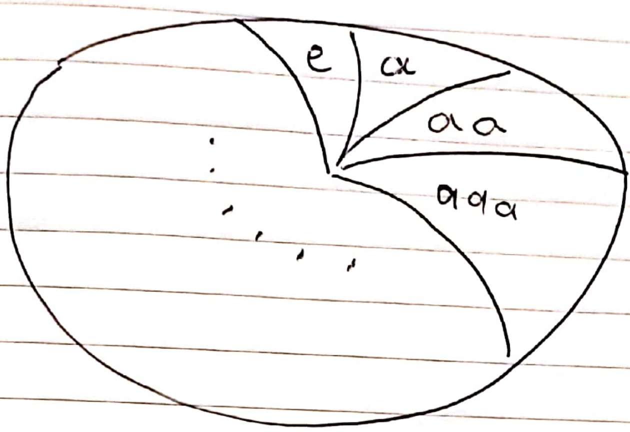


$q, p \in K \quad q \equiv p$ if

$\forall z \in \Sigma^* : (q, z) \in A_M$ iff $(p, z) \in A_M$

$$L = \{ a^n b^n \mid n \geq 0 \}$$

Σ^*



$$\underbrace{[e] \not\approx_L [a]}_{} \not\approx_L [aa] \not\approx_L [aaa] \cdots$$

choose $z = b$

$x = e$, $y = a$

$xz \notin L$, $yz \in L$

infinitely many equivalence classes of $\approx_L$

L is not regular

since # of states in a FSA for L
there must be infinitely many states
but # of states of a FSA
is finite.