Write T (for True) or F (False) at the end of each statement below. Leave it blank if you don't know the answer.

1 The barycentric coordinates of a point inside a triangle will add up to 1 even if some of them may be negative. False

2 The surface of a unit sphere can be modeled by a parametric equation with two parameters only. True

3 A rectangular image cannot be used for texture mapping of a triangle. False

4 The ambient component of a sphere's illumination is not dependent on the sphere's material (i.e., sphere's reflectance coefficient). False ambient reflectance k_a

5 The surface color obtained by texture mapping can be used as an object's reflectance coefficient in ray tracing computations. True diffuse reflectance coef

6 The specular (Blinn-Phong shading) component of the ray tracing illumination model depends on the viewer's position and the light position, but it does not depend on the normal vector of the surface. False $k_s (\hat{n} \cdot \hat{h})^p$ (n DOT h)^p

7 Diffuse shading components of a surface point are the same for cameras located at different points. True $k_d (\vec{n} \cdot \vec{w}_i) \frac{I_i}{r^2}$

8 When we increase the specular exponent (shininess), the specular highlight on the sphere will get larger. False

9 The resolution of the texture map can be larger than the resolution of the image generated by ray tracing. True

10 Having a 2D texture image is one of the requirements of texture mapping. False

11 Due to mirror reflections, in ray tracing it is possible to see objects that are behind the camera. True

12 The cells in a k-d tree can have high aspect ratio. True

13 The cells in a quadtree are cubical. False

14 The dot product of any two vectors gives the cosine of the angle between them. False (unit)

15 If you apply displacement mapping to a sphere, then its shadow will have bumps. True

16 It is not possible to represent a non-manifold mesh using Indexed Face-Set data structure. False

17 Average vertex degree in a triangle mesh is 3. False

18 Rotation is a nonlinear transformation. False

Fill in the blanks of each statement below.

19 The main difference between bump mapping and displacement mapping is that bump mapping displaces the normals only NOT the geometry of the object.

20 In ray tracing we determine whether a point is under shadow or not by Casting another ray from the point towards the light, if it intersects an object before the light, it is in shadow.

21 Homogenous coordinates enable us to Compose multiple transformations together and translate using matrix multiplication.

22 The main difference between face-set structure and indexed face-set structure is Face-set repeats some verties causing redundancies and anomalies, indexed Prevents repetetion by referencing vertices with indexes.

23 The output of the Marching Cubes algorithm is polygon/triangle mesh (of isosurface).

24 The term AABB stands for axis-aligned bounding box.

25 The steps required to reflect a point over an arbitrary plane are 1) translate a pnt on plane to origin 2) rotate plane normal so it aligns w/ z-axis 3) reflect over xy-plane 4) undo 2 5) undo 1.

26 The implicit formula for a sphere is $(x-C_x)^2+(y-C_y)^2+(z-C_z)-R^2=0$

$(P-C)^2-R^2=0$

Solve the following problems. Show your work.

27 Compose the matrix that doubles the size a 2D model w.r.t. the fixed point $(\overset{a}{2},\overset{b}{7})$.

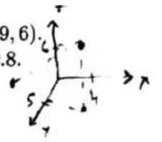
$$M = \underbrace{\text{Translate Back}}_{T_B} \times \underbrace{\text{Scale}}_{S} \times \underbrace{\text{Translate}}_{T}$$

$$T = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} ; \quad S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad T_B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S \times T : \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -14 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = T_B \times S \times T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -14 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -4+2 \\ 0 & 2 & -14+7 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

28 Rotate the point (3, 1, 7) 30 degrees around the axis passing through (4, 5, 6) and (4, 9, 6). Show the transformation matrix and the rotated point. Recall $sin30 = 0.5, sin60 = 0.8$.



Rotation along axis parallel to the y axis

(1) translate to y axis $\rightarrow T = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(2) Rotate along y axis $\rightarrow R = \begin{bmatrix} \cos 30 & 0 & \sin 30 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 30 & 0 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(3) undo translation $\rightarrow T_u$

$M = T_u \times R \times T$

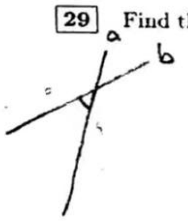
$P' = M P$

$$R \times T = \begin{bmatrix} \cos 30 & 0 & \sin 30 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 30 & 0 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & \frac{\sqrt{3}}{2} \times -4 - 3 \\ 0 & 1 & 0 & -5 \\ -0.5 & 0 & \frac{\sqrt{3}}{2} & 2 - 6 \times \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = T_u \times R \times T = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & \frac{4\sqrt{3}-6}{2} \\ 0 & 1 & 0 & -5 \\ -0.5 & 0 & \frac{\sqrt{3}}{2} & \frac{4-6\sqrt{3}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & \frac{4\sqrt{3}-6}{2} \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & \frac{\sqrt{3}}{2} & \frac{4-6\sqrt{3}}{2}+6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\frac{\sqrt{3}}{2}$

29 Find the distance between line $(3, 3, 3) + t(2/3, 2/3, 1/3)$ and line $(5, 4, 3) + s(1/3, 2/3, 2/3)$.



$\vec{a} = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$

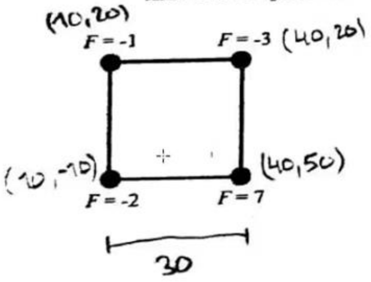
$\begin{array}{r} 81 \\ -\,64 \\ \hline 17 \end{array}$

$\vec{a} \cdot \vec{b} = \frac{2}{9} + \frac{4}{9} + \frac{2}{9} = \frac{8}{9}$

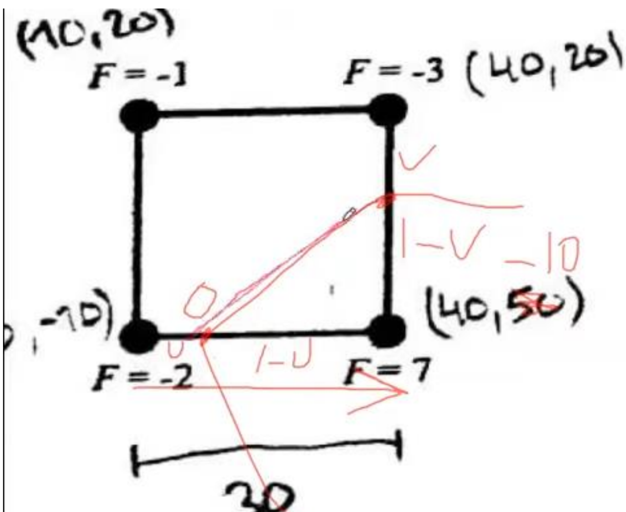
$8^2 + d^2 = 9^2 \rightarrow 64 + d^2 = 81$

$d = \sqrt{81 - 64} = \sqrt{17}$

30 Suppose the function F is linear on the edges of the cell below, whose top left corner (where $F = -1$) has the coordinate $(10, 20)$. Let the side length of the cell be 30. First and second coordinates increase as we go right and up, respectively. Extract the shape from this cell by showing the intersection point(s) along with the corresponding coordinate values, assuming that the shape is defined at $F = 1$.



$F = -1(-3-1) \cdot \quad +$



$-2(1-v)+7v=0 \Rightarrow v=\frac{2}{9}$

$(10,-10)+\frac{2}{9}(30,0)=(x,y)$

$\frac{150}{9}, -10$