METU **Department of Computer Engineering** **Fall 2020**

CENG 223—Discrete Computational Structures

FINAL Jan 18, 2021
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Q-3){*25 points*} Let $T = (V, E)$ be a rooted tree.

a) The binary relation R_1 on V is defined as follows. For any $x, y \in V$, xR_1y if the vertices x and y are at the same level (i.e., $level(x) = level(y)$).

i) Show that R_1 is an equivalence relation. Give details.

ii) Suppose the height of T is h. What can you say about the number of equivalence classes of R_1? Justify your answer in one or two sentences.

b) The binary relation R_2 on V is defined as follows. For any $x, y \in V$, xR_2y if x has a level number not larger than that of y (i.e., $level(x) \leq level(y)$).

i) Show that R_2 is a partial order. Give details.

ii) Let U be an arbitrary non-empty subset of V.

1) Does a lower bound of U exist?

2) Does the greatest lower bound of U exist?

Justify your answers in detail.

c) The binary relation R_3 on V is defined as follows. For any $x, y \in V$, xR_3y if x and y are both leaves or both non-leaves, and the levels of x and y differ by at most one (i.e. $|level(x)–level(y)| \leq 1$). Describe the transitive closure of R_3 in terms of parent/child or ancestor/descendant relationships (i.e. without using such terms as leaf or level, which have been used in the definition of R_3). Your description must fit in the blank portion in the following equality:

Transitive closure of $R_3 = \{(x, y) \in V \times V |$ $\cdots$ $\}$