

Boolean Algebra Minimization Examples

Using Boolean algebra techniques, simplify this expression:

$AB + A(B + C) + B(B + C)$

$AB + AB + AC + BB + BC$
 $AB + AB + AC + B + BC$
 $AB + AC + B + BC$
 $AB + AC + B$
 $B+AC$

| | | |
|----------------|---|-----------------|
| Involution | $X = \overline{(\overline{X})}$ | |
| Identity | $X + 1 = 1$ | $X \cdot 0 = 0$ |
| | $X + 0 = X$ | $X \cdot 1 = X$ |
| Idempotence | $X + X = X$ | $X \cdot X = X$ |
| | $X + (Y + Z) = (X + Y) + Z$ | |
| Associativity | $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$ | |
| | $X \cdot Y + X \cdot \overline{Y} = X$ | |
| Adjacency | $(X + Y) \cdot (X + \overline{Y}) = X$ | |
| | $X + (X \cdot Y) = X$ | |
| Absorption | $X \cdot (X + Y) = X$ | |
| | $X + (\overline{X} \cdot Y) = X + Y$ | |
| Simplification | $X \cdot (\overline{X} + Y) = X \cdot Y$ | |
| | $X \cdot Y + \overline{X} \cdot Z + Y \cdot Z = X \cdot Y + \overline{X} \cdot Z$ | |
| Consensus | $(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\overline{X} + Z)$ | |
| | $\overline{X + Y} = \overline{X} \cdot \overline{Y}$ | |
| DeMorgan's | $\overline{X \cdot Y} = \overline{X} + \overline{Y}$ | |

$$(a+c)(a'+b+c)(a'+b'+c)$$

$$= (aa' + ab + ac + a'c + bc + cc) (a' + b' + c)$$

distributive

$$= (ab + ac + a'c + bc + c)(a' + b' + c)$$

$$= (\mathbf{ab} + ac + \mathbf{a'c} + \mathbf{bc} + c)(a' + b' + c)$$

consensus

$$= (ab + \mathbf{ac} + \mathbf{a'c} + c)(a' + b' + c)$$

adjecency

$$= (ab + \mathbf{c} + \mathbf{c})(a' + b' + c)$$

Idempotence

$$= (ab + c)(a' + b' + c)$$

distributive

$$= \mathbf{aa'b} + \mathbf{abb'} + abc + a'c + b'c + \mathbf{cc}$$

complement and identity

$$= abc + a'c + b'c + c$$

$$= c(ab + a' + b' + 1)$$

$$= c$$

$$A'BC + AB'C' + A'B'C' + AB'C + ABC$$

$$\mathbf{A'BC} + \underline{AB'C'} + A'B'C' + \underline{AB'C} + \mathbf{ABC} + AB'C'$$

$$BC + AB' + B'C'$$

---- OR ---|

$$\mathbf{A'BC} + \underline{AB'C'} + \underline{A'B'C'} + AB'C + \mathbf{ABC} + ABC$$

$$BC + B'C' + AC$$

$$ABC + A' + AB'C$$

$$AC + A'$$

$$A' + C$$