Take Home Exam 2

**Question 1**

AB\ AB = { x | x AB x AB } by definition of set difference

= { A AB } by definition of union

={ A AB } by definition of

={ A x x B } by definition of instersection

={ A x x B } by the first De Morgan’s law

= { A x x B } by definition of

= { A x x B } by the Distributive law

2

1

**1**

= A x x by the Distributive law

= ( F x ) by the Negation law

= x by the Identity law

**2**

= x B x B by the Distributive law

= x B by the Negation law

= x B by the Identity Law

= x x B } by union of 1 and 2

= { (B\A) (A\B) } by definition of set difference

= (B\A) (A\B) by definition of union

= (A\B) (B\A) by the Commmutative law for union

**Question 2**

{ | : is a function} \ { | : {0, 1} is a function} is uncountable.

{ | : is a function}

1 ...

2 ...

3 ...

... continues

x = ... ( x )

… continues

By design x and it is missed by the enumaration.

So, there does not exist on enumeration counting each element in

Therefore, { | : is a function} is an uncountably infinite.

{ | : {0, 1} is a function}

: {0, 1}

0 1 2 3 …

0 (0,0) (0,1) (0,2) (0,3) …

1 (1,0) (1,1) (1,2) (1,3) …

We have a tuple (a,b) which is a subset of {0, 1}

(0,0) (0,1), (1,0) (1,1), (0,2) ….

a+b= 0 a+b = 1 a+b= 2

We have an enumaration algorithm for visiting each element in the set of {0, 1}.

We cannot find a set which cannot be visited by using this method.

So, if we count each set exactly once, will be a bijection.

Therefore, { | : {0, 1} is a function} is countably infinite.

Assume A is uncountable and B is countable sets and A\B is countable set.

Since A\B is countable and B is countable , (A\B) B is also countable.

(A\B) B = {x | x (A\B) x } by definition of union

{x | (xA B) x } by definition of set difference

{x | (xA x B x)} by the Distributive law

{x | (xA x } by the Negation law

{x | (xA x } by the Identity law

AB by definition of union

Thus, (A\B) B = AB

According to the AB , if AB is countable , A should also be countable set.  
 There is a contradiction.

Therefore, A\B is uncountably infinite.

**Question 3**

4n + 5n2. log(n) O(2n) for x>2 and n>2

Assume 4n + 5n2. log(n) O(2n) ,

By definition of BIG-O notation is O() , if there are constants C and k such that :

| | C . | | , n k

|4n + 5n2. log(n)| C . |2n|

Since 5n2. log(n) is positive for all n>2, |4n + 5n2. log(n)|

|4n + 5n2. log(n)| C . |2n|

|4n | C . | 2n | , n k

4n / 2n  C

(4/2)n C

2n  C

n. log(2) log(C)

n log(C) / log(2)

'C' is a positive constant, but we need to find:

k such that, for any n k should satisfy n log(C) / log(2)

It is not the case that n log(C) / log(2) for all n k, because *n* can be arbitrarily large.

So, 4n + 5n2 log(n) O(2n).

**Question 4**

(2x - 1)n  - x2 -x-1 mod ( x-1 )

(2x - 1)n  - x2 +x +1 0 mod (x-1)

(2x - 1)n  - x2 +x +1 0 mod (x-1)

If a b mod m , then ak  bk mod m for any positive integer k

If (2x-1) 1 mod (x-1)

Then, (2x-1)n 1n mod (x-1)

So, (2x-1)n 1 mod (x-1)

(2x - 1)n  - (x2 – x -1 ) 0 mod (x-1)

x2 -x -1 mod (x-1) -1

(2x - 1)n  - (x2 – x -1 ) 0 mod (x-1)

1 - (-1) 0 mod (x-1)

2 0 mod (x-1)

For given two positive integers x and n such that x > 2 and n > 2,

If (x-1) | 2

(x-1) can be ‘2’ or ‘-2’

As a result, x can be ‘3’ or ‘-1’.

If x>2, then x = 3.