Take Home Exam 4

**Question 1**

**= + 2n , n 1**

**= 1**

=

A(x) – = +

A(x) – = x . A(x) +

B(x) =

A(x) – = x . A(x) +

B(x) – b0

b0 = (2x)0 =1

A(x) – 1 = x . A(x) + -1

A(x) . (1-x) =

A(x) =

A(x) = , A = -1 B = 2

A(x) =

(-1, -1, -1, -1 …. , -1, ….)

(2, 4, 8, 16, …., 2n+1, ….)

(1, 3, 7, 15, …., 2n+1-1, …) = A(x)

A(n) = 2n+1-1

**Question 2**

R = { (a,b) | a divides b } on A = { 1, 2, 3, 9, 18}

1. Hasse diagram of R

18

9

2

3

1

1 2 3 9 18

1

2

3

9

18

1. Matrix representation of R is MR =
2. For the poset to be called a lattice, each pair of elements must have both a unique least upper bound and a unique greatest lower bound.

Then, in poset (A,R), we can see every pair of elements has both a least upper bound and a greatest lower bound. Hence, this poset can be called a lattice.

d) Symetric Closure of R = R R-1 { (b,a) | (a,b) R }

MR =  MR-1  =

MS(R) = + =

e) The integers 2 and 9 are incomparable, because 2 9 and 9 2. The integers 3 and 18 are comparable, because 3 | 18.

**Question 3**

1. A relation has ordered pairs (a,b). For anti-symmetric relation,

,

A = {a1 , a2, a3, … an} , R AA

a11 , a22, a33, …. , ann  can be both 0 or 1.

So, for (an,an), total number of ordered pairs = n,

The total number of relation of diagonal part = 2n.

From the remaining part,

For , and , aij and aji cannot be both 1,

To find number of different aij,(total number of ordered pairs– diagonal pairs)/2

(total number of ordered pairs – diagonal pairs ) / 2

n2 - n = n . (n-1) /2

There are 3 possible interpretation that makes aij symetric.

(aij = 0 aji = 0 , aij = 1 aji = 0 , aij = 0 aji = 1 , aij = 1 aji = 1 ) .

This is not anti-symmetric.

So, there are option from here.

As a result, total number of anti-symmetric relation is .



A = {a1 , a2, a3, … an} , R AA

Since all diagonal elements are part of the reflexive relation, (a11 , a22, a33, …. , ann ) are all 1.

From the remaining part,

For , and , there are (n2 − n)/2 many different aij.

For anti-symmetric relation, there are 3 possibilities for each of the remaining (n2 −n)/2 elements.

(aij = 0 aji = 0 , aij = 1 aji = 0 , aij = 0 aji = 1 , aij = 1 aji = 1 ) .

This is not anti-symmetric.

Thus, we get binary relations which are reflexive and antisymmetric.