CHAPTER 14

MORTGAGE LOAN REPAYMENT AND REFINANCING OPTIONS

Learning Objectives

After studying this chapter, a student should be able to:

- ☑ Explain variable rate mortgages
- ☑ Calculate payments and outstanding balances on variable rate mortgages and reverse mortgages
- ☑ Describe graduated payment mortgages, sinking fund assisted mortgages, and reverse mortgages
- ☑ Calculate the cost of borrowing upon refinancing
- ☑ Describe the refinancing options available
- ☑ Calculate maximum additional funds under various refinancing options
- ☑ Describe wrap-around mortgages

INTRODUCTION

The fundamental financial element of a mortgage contract is the borrower's promise to repay. This covenant is almost universally comprised of a promise to repay the principal money borrowed and to pay interest on the borrowed capital. In this very narrow context, the contract will specify several items: the face amount of the loan, the interest rate, and the terms of repayment.

The face amount or face value[1] of the loan (the number of dollars the borrower is promising to repay at the contract rate of interest) must be clearly stated. The contract will specify the interest rate that will apply throughout the term of the contract and how the interest will be calculated. These details must be stated very clearly as slight variations can lead to significantly different outcomes. The contract will also specify how the principal will be repaid and the interest will be paid. Included here will be the size of payment(s), the frequency of payments, and the number of payments.

In designing the details of the repayment plan, the lender and the borrower may have conflicting interests and concerns. For example, residential borrowers generally prefer a repayment scheme that fits their long-term household budget and provides a high degree of certainty. The certainty is provided whenever payments are known in advance, i.e., the standard fixed or constant payment mortgage. An ideal payment plan could be one in which mortgage payments represent a constant portion of the borrower's income. Given most borrowers expect their incomes to rise, at least in nominal terms, this may suggest a repayment plan with increasing mortgage payments.

However, lenders have a different set of concerns. Lenders will only accept risks up to their maximum acceptable level, although this acceptable level will vary from lender to lender. Lenders ideally prefer to keep the loan-to-value ratio within acceptable limits. Therefore, lenders prefer that owners' equity should increase or, at worst, remain constant. The lender also wishes to ensure that the debt service ratio is always manageable and wants to minimize interest rate risk. This is the risk that the interest spread between the cost of raising funds and the return on the investment of funds will decrease over time due to unforeseen changes in the cost of raising funds, which are not or cannot be matched by changes in the lending rate. Finally, the lender, as the possessor of a debt instrument, must be concerned with the preservation of purchasing power if facing high rates of inflation.

To meet the requirements of borrowers and lenders, several different repayment programs have been either implemented or at least seriously discussed. Each of the plans has been promoted to solve particular problems.

The various mortgage repayment plans introduced to address borrowers' and lenders' concerns may be grouped as follows:

Fixed Interest Rate Loans

- Interest accruing loans
- Interest only loans
- Straight line principal reduction loans
- Fully amortized constant payment loans
- Partially amortized constant payment loans; the standard Canadian mortgage

Variable Rate Mortgages (VRMs)

- Standard VRMs with amortization, payment, or outstanding balance adjustment

- Dual rate VRMs

Alternative Repayment Plans

- Graduated payment mortgages
- Sinking fund assisted mortgages
- Reverse annuity mortgages

Interest accruing, interest only, straight line principal reduction, and fully and partially amortized constant payment loans have been discussed in previous chapters. Variable rate mortgages, graduated payment mortgages, sinking fund assisted mortgages, and reverse annuity mortgages are the focus in this chapter.

This chapter will also look at refinancing options, examining how borrowers might evaluate options at the end of the loan term or in deciding to prepay the loan balance before the term's expiry. There are several alternatives that may be explored with respect to refinancing, each potentially involving significant implications in terms of risk, yield (or cost), and cash flow to the lender and borrower.

Balancing Concerns of Lenders and Borrowers

This chapter evaluates several mortgage loan repayments plans to illustrate the wide range of loan options and their continuing evolution. The repayment plans are described and evaluated with the following criteria in mind:

- **Lenders:** balancing maximal return with minimized capital risk (default, loss of principal) and income risk (loss of interest income)
- **Borrowers:** maximizing affordability, leading to maximum borrower capability while minimizing interest cost

VARIABLE RATE MORTGAGES (VRMs)

A variable rate mortgage is a modification of the constant payment partially amortized mortgage plan. As the name implies, variable rate mortgages differ from conventional mortgages in that the rate of interest payable may be adjusted periodically throughout the contractual term of the mortgage. There are different forms of variable rate mortgages available, including varied prepayment privileges in an open or closed format.

Variable rate mortgages have become an increasingly popular choice in recent years. In 2021, VRM's percentage of total mortgage initiations was only 10% of the market. By early 2022, this advanced to a peak of 57%. However, with the rising interest rates in 2022, the popularity of VRMs declined greatly. As of April 2023, VRMs accounted for approximately 8% of new mortgage initiations. Borrowers are most interested in VRMs when the lower rates and risk of VRMs outweigh the security of fixed rate mortgages. Presumably once VRM rates decline relative to fixed rate mortgages, VRMs will once again become a more popular option.

Common Features of Variable Rate Mortgages

Variable mortgage rates are usually linked to the chartered banks' prime rates. The *prime rate*, also known as the prime lending rate, is an annual interest rate used by Canada's major banks and financial institutions to set rates for variable rate mortgages and lines of credit. The prime rate varies with changes in the Bank of Canada's overnight lending rate.

prime rate (prime lending rate)
an annual interest rate used by Canada's major banks and financial institutions to set rates for variable rate mortgages and lines of credit

Historically, the rates on closed VRMs tend to be slightly lower than the equivalent term fixed rate mortgage. This compensates borrowers for taking on additional risk in foregoing the interest rate stability of a fixed rate mortgage. However, towards the end of 2022, the spread between VRMs and fixed rates mortgages was virtually non-existent. In early 2023, VRM rates even exceeded fixed mortgage rates, which is a sign of short-term uncertainty in finance markets. This situation is not expected to prevail for long, with a return to the traditional "spread" in rates between VRMs and fixed rate mortgages.

Similar to a standard constant blended payment mortgage, a variable rate mortgage may be "open" or "closed". If the mortgage is open, it can be paid off at any time without an interest penalty. If the mortgage is closed, then any principal prepayment above an agreed upon threshold will attract an interest penalty. Open VRM rates are usually higher than those for closed VRMs. Variable rate mortgages typically have terms ranging from one to five years. Many VRMs are convertible into fixed rate mortgages. If interest rates begin to rise, the borrower can choose to lock in at the current posted rate for a term that is equal to or longer than the remaining term on the VRM. This reduces some of the interest rate risk faced by borrowers with VRMs.

The primary advantage to lenders with VRMs is reducing risk by better matching interest rates on their asset and liability portfolios. VRMs reduce the likelihood of lenders being caught in a position where decreases in the spread between the rates attached to assets (e.g., mortgages and loans) and deposit liabilities (e.g., GICs, term deposits, and variable rate deposits) compromise their profit position. Reducing mismatching risk lessens lenders' interest rate risk and should lead to reduced overall mortgage interest rates over time. The level of borrower uncertainty increases with the use of VRMs while the lender's risk of mismatching decreases.

A VRM is initiated as a standard amortizing loan based on the prevailing mortgage rate. The loan is periodically reviewed, with the interest rate changed as necessary to reflect the bank's prime rate. Interest rate changes are reflected in one or more changes to the amortization period, the outstanding balance, or the repayment schedule. Each is explained in the following sections.

VRMs with Amortization Period Adjustment

Changes in the market interest rate may be reflected by adjustments to the amortization period, with the amount of the payments kept constant. A decrease in interest rates would lead to a shortening of the remaining amortization period and an increase would result in a lengthening of the remaining amortization period.

A risk to the lender is that large increases in mortgage rates may lead to a mortgage with an infinite amortization period – meaning the constant regular payments are insufficient to ever pay off the loan. Lenders may protect themselves by stipulating a maximum amortization period, above which the loan terms are revisited, and the payment adjusted as needed or a prepayment made to keep the payments constant, e.g., the amortization period may be specified not to exceed 30 years.

The following illustration introduces the symbolic notation for the present value relationship for a stream of regular payments:

$$PV = PMT \times a⟦n, j_m⟧$$

where:

PV = Present value of the stream of regular cash flows

PMT = Dollar amount of the regular payment or cash flow

$a⟦n, j_m⟧$ = Present value of $1 per period for n periods, discounted at a nominal rate of j_m

n = Number of compounding periods

j = Interest rate per annum; requires compounding frequency

It is important to understand that $a⟦n, j_m⟧$ is not a mathematical formula; it is simply a shorthand or symbolic way of writing something that is otherwise very cumbersome. It represents the mathematical relationship to be used in solving problems where streams of regular payments are involved. The double square brackets "⟦, ⟧" are traditionally used to separate the values for n and j.

The following example illustrates the change in amortization period resulting from an increase and a decrease in interest rates.

Illustration 14.1

A $150,000 variable rate mortgage was written one year ago at 8% per annum, compounded semi-annually, to be amortized over 25 years by monthly payments. The mortgage contract specified that the interest rate could be adjusted, on each anniversary of the mortgage, to the current market rate.

Calculate the amortization at the start of Year 2 under each of the following situations:

a. The interest rate on the first anniversary date is 9% per annum, compounded semi-annually.

b. The interest rate on the first anniversary date is 6.5% per annum, compounded semi-annually.

c. The interest rate on the first anniversary date is 9.5% per annum, compounded semi-annually.

Solution:

Face Amount:	\$150,000
Amortization:	25 years (300 months)
Initial Contract Rate:	$j_2 = 8\%$
Annual Interest Rate Adjustments	

Year 1

$$PV = PMT \times a[\![n, j_2 = 8\%]\!]$$

$$\$150{,}000 = PMT \times a[\![300, j_{12} = 7.869836\%]\!]$$

where: n = Original amortization

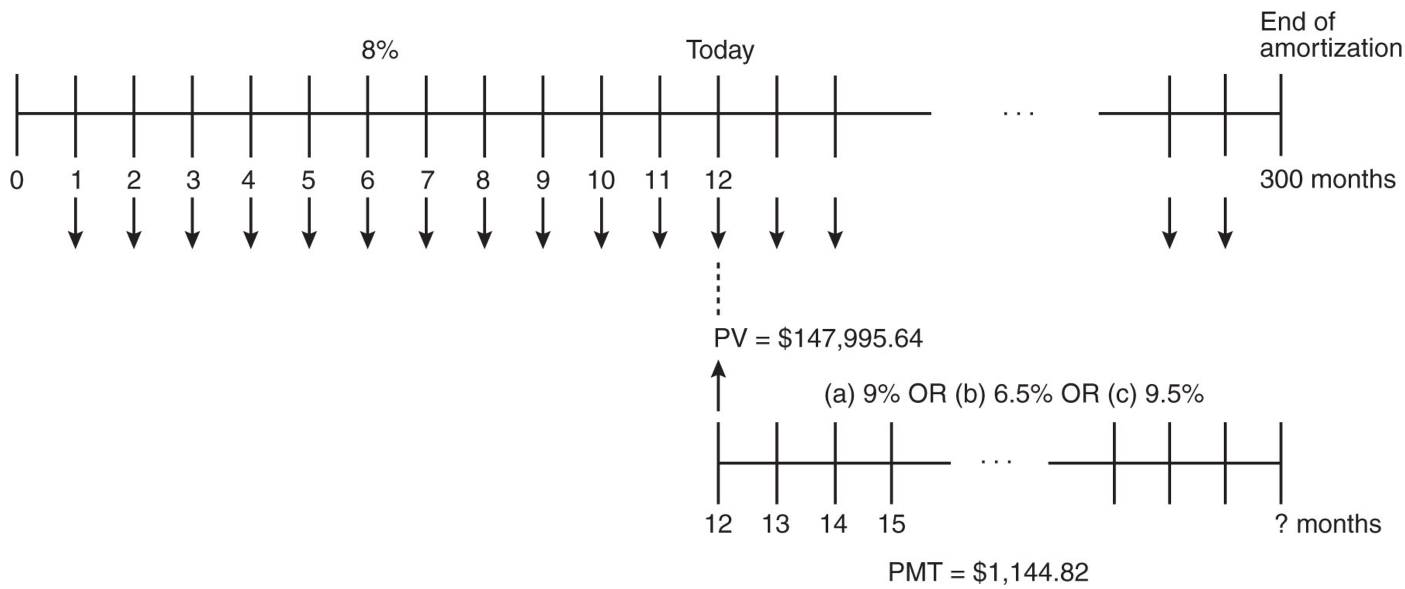
PV = Original loan

PMT = Year 1 payment

Calculation

Press	Display	Comments
8 ■ NOM%	8	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	8.16	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	7.869836	Equivalent j_{12} rate
150000 PV	150,000	Loan amount
300 N	300	Amortization period
0 FV	0	
PMT	–1,144.820182	Unrounded monthly payment
1144.82 +/– PMT	–1,144.82	Rounded monthly payment
N	300.000149	Amortization period based on rounded payments
12 INPUT ■ AMORT	12–12	
= = =	147,995.640934	OSB_{12}

The Year 1 payment is \$1,144.82 and the amortization based on the rounded payment is 300.000149 months. The outstanding balance at the end of the first year is \$147,995.64.



a. The variable interest rate in the second year is 9% per annum, compounded semi-annually.

Year 2

$$PV = PMT \times a[\![R, j_2 = 9\%]\!]$$

where: PV = Outstanding balance after 12 months

R = Revised amortization

j_2 = Interest rate for Year 2

PMT = Year 1 payment

$$\$147{,}995.64 = \$1{,}144.82 \times a[\![R, j_{12} = 8.835748\%]\!]$$

Calculation

Press	Display	Comments
9 ■ NOM%	9	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	9.2025	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	8.835748	Equivalent j_{12} rate
147995.64 PV	147,995.64	New loan amount
1144.82 +/– PMT	–1,144.82	Monthly payment
0 FV	0	
N	413.524043	New amortization period (months)

The amortization at the start of Year 2 is 413.524 months (34.46 years). The increase in interest rate is reflected as an increase in the amortization, from approximately 24 years to 34.5 years.

b. The variable interest rate in the second year is 6.5% per annum, compounded semi-annually.

Year 2

$$PV = PMT \times a⟦R,\ j_2 = 6.5\%⟧$$

where: PV = Outstanding balance after 12 months
R = Revised amortization
j_2 = Interest rate for Year 2
PMT = Year 1 payment

$$\$147{,}995.64 = \$1{,}144.82 \times a⟦R,\ j_{12} = 6.413688\%⟧$$

Calculation

Press	Display	Comments
6.5 ■ NOM%	6.5	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	6.605625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	6.413688	Equivalent j_{12} rate
147995.64 PV	147,995.64	New loan amount
1144.82 +/– PMT	–1,144.82	Monthly payment
0 FV	0	
N	220.280968	New amortization period (months)

The amortization at the start of Year 2 is 220.28 months (18.357 years). The decrease in interest rate is reflected as a decrease in the amortization, from approximately 24 years to 18.4 years.

c. The variable interest rate in the second year is 9.5% per annum, compounded semi-annually.

Year 2

$$PV = PMT \times a⟦R,\ j_2 = 9.5\%⟧$$

where: PV = Outstanding balance after 12 months
R = Revised amortization
j_2 = Interest rate for Year 2
PMT = Year 1 payment

$$\$147{,}995.64 = \$1{,}144.82 \times a⟦R,\ j_{12} = 9.31726\%⟧$$

Calculation

Press	Display	Comments
9.5 ■ NOM%	9.5	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	9.725625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	9.31726	Equivalent j_{12} rate
147995.64 PV	147,995.64	New loan amount
1144.82 +/– PMT	–1,144.82	
0 FV	0	
N	no solution	New amortization period

The amortization at the start of the second year is infinite; that is, the payments are not large enough to ever repay the loan at this interest rate. Note that the calculator shows a "no solution" message, which means that the payments are insufficient to ever repay this loan. Changing the interest rate between 9% and 9.5% by trial and error shows that a rate above 9.28% per annum, compounded monthly (or 9.46% per annum, compounded semi-annually) will lead to an infinite amortization period.

This illustration demonstrates the sensitivity of the amortization period to changes in interest rates. The example above shows that a 1% increase in the interest rate leads to a 10-year increase in the amortization period. A 1.5% increase in rates creates a situation where the loan will never be repaid based on the set payment. As a result, lenders typically rely on other methods to adjust for changes in rates. With the recent increase in interest rates, some financial institutions have temporarily extended amortization periods to 30-35 years, or even longer, to help borrowers manage the higher rates and borrowing cost. It is expected that, upon renewal, most of these mortgages will revert to the original amortization schedule, which will generally result in higher mortgage payments.

VRMs with Outstanding Balance Adjustment

The prior section illustrated how interest rate changes can be reflected in changes in amortization. Alternatively, in a partially amortized VRM, an interest rate change may be reflected in keeping payments constant but changing the outstanding balance at the end of the loan term. This is simpler for loan administration and for affordability for borrowers. If interest rates increase, more of the payment goes to cover interest and less to principal, meaning the outstanding balance will increase. If interest rates fall, more of the payment is paid toward the principal portion of the loan and the outstanding balance will decrease. Illustration 14.2 illustrates the impact of changes in interest rates on the outstanding balance.

Illustration 14.2

Assume a borrower has applied for a $100,000 closed variable rate mortgage. The interest rate to be charged on this loan is at prime. At the time of loan initiation, prime rate is 4.25% per annum, compounded semi-annually. The borrower will make monthly payments on a 25-year amortization and a 5-year term. Interest rate adjustments are made annually on the anniversary date of the mortgage.

Determine the outstanding balance owing on this loan at the end of Year 1 and Year 2 (and determine the breakdown of principal and interest paid on the 12th and 24th payments) under the following scenarios:

- Prime rate remains constant at j_2 = 4.25% for the period.
- Prime rate increases to j_2 = 6.5% at the end of Year 1 (after 12 monthly payments have been made).
- Prime rate decreases to j_2 = 3.5% at the end of Year 1.

Solution:

Option 1: Prime rate remains constant at j_2 = 4.25% for the period.

$$PV = PMT \times a[\![n, j_2 = 4.25\%]\!]$$
$$\$100{,}000 = PMT \times a[\![300, j_{12} = 4.212851\%]\!]$$

where:
n = Original amortization
PV = Original loan
PMT = Payment

Calculation

Press	Display	Comments
4.25 ■ NOM%	4.25	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	4.295156	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	4.212851	Equivalent j_{12} rate
100000 PV	100,000	Loan amount
300 N	300	Amortization period
0 FV	0	
PMT	–539.660164	Monthly payment
539.66 +/– PMT	–539.66	Rounded monthly payment
N	300.000161	Revised amortization based on rounded payment
12 INPUT ■ AMORT	12–12	
=	–196.001157	Principal paid on 12th payment
=	–343.658843	Interest paid on 12th payment
=	97,692.718427	OSB at end of Year 1
24 INPUT ■ AMORT	24–24	
=	–204.419712	Principal paid on 24th payment
=	–335.240288	Interest paid on 24th payment
=	95,286.335507	OSB at end of Year 2

The Year 1 payment is $539.66 and the amortization based on the rounded payment is just over 300 months. The outstanding balance at the end of Year 1 is $97,692.72. The principal portion of the 12th payment is $196.00 and the interest portion is $343.66.

In Option 1, the base case, the outstanding balance at the end of Year 2 is $95,286.34. The principal paid on the 24th payment is $204.42 and the interest paid is $335.24.

Option 2: Prime rate increases to j_2 = 6.5% at the end of Year 1.

Year 2

$$\$97{,}692.72 = \$539.66 \times a[\![n, j_2 = 6.5\%]\!]$$

$$\$97{,}692.72 = \$539.66 \times a[\![n, j_{12} = 6.413688\%]\!]$$

where: n = Revised amortization

PV = OSB_{12}

PMT = Year 1 payment

Calculation (continued)

Press	Display	Comments
6.5 ■ NOM%	6.5	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	6.605625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	6.413688	Equivalent j_{12} rate
97692.72 PV	97,692.72	New loan amount
N	643.038485	Amortization period remaining
12 INPUT ■ AMORT	12–12	
=	–18.575683	Principal paid on 24^{th} payment
=	–521.084317	Interest paid on 24^{th} payment
=	97,476.215465	OSB at end of Year 2

The revised amortization (to reflect the increase in the prime rate) is 643.04 months (53.6 years). As a result, the outstanding balance owing at the end of Year 2 is $97,476.22 (higher than in Option 1), the principal paid on the 24^{th} payment is only $18.58, and the interest paid on the 24^{th} payment increases to $521.08.

Option 3: Prime rate decreases to j_2 = 3.5% at the end of Year 1.

Year 2

$$\$97{,}692.72 = \$539.66 \times a[\![n, j_2 = 3.5\%]\!]$$
$$\$97{,}692.72 = \$539.66 \times a[\![n, j_{12} = 3.474749\%]\!]$$

where: n = Revised amortization

$PV = OSB_{12}$

PMT = Year 1 payment

Calculation (continued)

Press	Display	Comments
3.5 ■ NOM%	3.5	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	3.530625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	3.474749	Equivalent j_{12} rate
N	256.870305	Remaining amortization period
12 INPUT ■ AMORT	12–12	
=	–265.076946	Principal paid on 24th payment
=	–274.583054	Interest paid on 24th payment
=	94,561.826551	OSB at end of Year 2

In Option 3, the revised amortization (to reflect the decrease in the prime rate) is 256.87 months (21.4 years). As a result, the outstanding balance owing at the end of 2 years is $94,561.83 (lower than in Option 1), the principal paid on the 24th payment increases to $265.08, and the interest paid on the 24th payment decreases to $274.58.

An advantage of this method is that no changes are required of the borrower during the loan term, with respect to changing the required payments for the loan – the changes are all easily accounted for at the end of the loan term. However, if rates increase dramatically, the lender risks having the constant payments insufficient to pay off the interest due in each period. In Option 2, the rate increase results in a significant decline in the principal portion and as a result, the outstanding balance on the mortgage at the end of Year 2 does not decline much (relative to Option 1). If the rate increases to 6.75%, the $539.66 payments are insufficient to repay the interest owing each month, and the loan has no principal repayment, with interest accruing and adding on to the outstanding balance owing. In this situation, lenders will generally require the borrower to increase the payment, pay a lump sum, or pay off the remaining balance. In the situation where the prime rate declines (as shown in Option 3), the borrower's outstanding balance decreases considerably relative to the base case (Option 1).

TRIGGER RATE and TRIGGER POINT

If a VRM has fixed payments, rising interest rates means that a larger portion of the borrower's payment goes toward interest and a smaller portion of the payment goes toward principal. With no payment adjustment, this increases the loan's amortization period (as shown previously).

With sharp increases in interest rates, borrowers may get to a point where very little of their monthly payment is used toward paying down the principal. If this gets to the point where the regular payment is insufficient to cover interest – i.e., the entire mortgage payment is going to interest and additional interest must be added to principal – lenders may require borrowers to increase their payments. Otherwise, if no action is taken, the outstanding balance on the mortgage will increase. Since the regular payment is not large enough to cover the cost of borrowing, the additional amount is known as deferred interest and added to the outstanding balance on the mortgage. This is known as *negative amortization*, where the mortgage effectively becomes an interest accruing loan.

A *trigger point* is the time at which the borrower can no longer carry on with the same payment; this is specified in the mortgage contract. The *trigger rate* is the interest rate that prompts this trigger point. A common trigger point is where the outstanding balance of the mortgage exceeds the original amount borrowed. For example, say you initiated a $400,000 VRM and after a year paid down the OSB to $375,000. If interest rates rise, but payments are held constant, the additional unpaid interest would be added to the OSB. The trigger point is when the OSB increases to $400,000. Without a change, the loan balance could continue to grow and potentially exceed the property's total value. This trigger point can also be described in loan-to-value terms: once you exceed the loan's initial underwriting criteria, the lender will likely seek action. Increasing the monthly payments is most common, but the lender could also demand a lump sum prepayment or require the VRM be moved to a fixed interest rate mortgage.

trigger rate
the interest rate level at which the borrower's regular payment is not large enough to cover the interest portion of the payment

trigger point
when VRM payments are fixed, but the VRM rate increases to a point that the payments are entirely interest and the loan's OSB is increasing, i.e., the loan has negative amortization

Source: Lavin, J. "The trigger rate: Everything you need to know". [ratehub.ca]

VRMs with Payment Adjustment

Another option for VRM lenders to reflect interest rate changes is to change the periodic payment owing. This is administratively more complex, so it likely is used only when the amortization period or the outstanding balance changes are unreasonable. If interest rates decrease, the borrower's payment declines; if interest rates increase, the payment increases; the loan's amortization period does not change, nor does the outstanding balance. This is demonstrated in the following illustration.

Illustration 14.3

A \$450,000 variable rate mortgage was written two years ago at 5% per annum, compounded semi-annually, to be amortized over 25 years by monthly payments. The mortgage contract specified that the interest rate could be adjusted on each anniversary of the mortgage, to the current market rate. The interest rate on the first anniversary date was 6% per annum, compounded semi-annually and the interest rate on the second anniversary date is 7% per annum, compounded semi-annually. Changes in rates are to be reflected by changes in the payment (and the original amortization schedule is to be maintained).

Calculate the required payments for Years 1, 2, and 3.

Solution:

Face Amount:	\$450,000
Amortization:	25 years
Initial Contract Rate:	$j_2 = 5\%$
Rate at End of Year 1:	$j_2 = 6\%$
Rate at End of Year 2:	$j_2 = 7\%$

Year 1

$$PV = PMT \times a[\![n, j_2 = 5\%]\!]$$
$$\$450{,}000 = PMT \times a[\![300, j_{12} = 4.948699\%]\!]$$

where:
n = Original amortization
PV = Original loan
PMT = Year 1 payment

Calculation

Press	Display	Comments
5 ■ NOM%	5	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	5.0625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	4.948699	Equivalent j_{12} rate
450000 PV	450,000	Loan amount
300 N	300	Amortization period
0 FV	0	
PMT	–2,617.222433	Year 1 payment
2617.22 +/– PMT	–2,617.22	Year 1 rounded payment
N	300.00055	Amortization period based on rounded payments
12 INPUT ■ AMORT	12–12	
= = =	440,652.375482	OSB_{12}

The Year 1 payment is $2,617.22 and the amortization based on the rounded payment is 300.00055 months. The outstanding balance at the end of the first year is $440,652.38.

Year 2

$$PV = PMT \times a⟦R, j_2 = 6\%⟧$$

where: PV = Outstanding balance after 12 months

R = Revised amortization

j_2 = Interest rate for Year 2

PMT = Year 2 constant payment

$$PV = PMT \times a⟦n, j_2 = 6\%⟧$$

$$\$440{,}652.38 = \$2{,}617.22 \times a⟦R, j_{12} = 5.926346\%⟧$$

The impact of the increase of the nominal interest rate from 5% to 6% is an extension of the remaining amortization period from 288 months (300 – 12) to over 361 months (R = 361.48 months), an increase of an additional 6 years of payments. However, to maintain the original amortization schedule, the amortization is set at 288 months, and as a result, the payment will increase to $2,870.99.

$$\$440{,}652.38 = PMT \times a⟦288, j_{12} = 5.926346\%⟧$$

$$PMT = \$2{,}870.99$$

$$OSB_{24} = \$432{,}084.86$$

HELPFUL HINT!

Note that in Year 2 of this example, we calculate the outstanding balance after 12 months, not 24 months, because the loan starting date is now the beginning of Year 2. The outstanding balance is calculated 12 months from that point, not from the original loan starting date. In addition, in Year 3 of this example, the process is the same. We calculate the outstanding balance 12 months from the beginning of Year 3, which represents the outstanding balance 36 months from the original loan starting date.

Calculation (continued)

Press	Display	Comments
6 ■ NOM%	6	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	6.09	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	5.926346	Equivalent j_{12} rate
440652.38 PV	440,652.38	New loan amount
N	361.478721	New amortization period
288 N	288	Enter remaining amortization
PMT	–2,870.992229	Required payments
2870.99 +/– PMT	–2,870.99	Rounded payment
N	288.000494	Amortization period based on rounded payments
12 INPUT ■ AMORT	12–12	
= = =	432,084.855777	OSB_{24}

Year 3

The same process applies to the Year 3 calculations. The amortization in Year 3 with a rate of 7% per annum, compounded semi-annually will increase the amortization to over 349 months.

$$PV = PMT \times a\llbracket R, j_2 = 7\% \rrbracket$$

where:
PV = Outstanding balance after 24 months
R = Revised amortization
j_2 = Interest rate for Year 3
PMT = Year 3 constant payment

$$PV = PMT \times a\llbracket n, j_2 = 7\% \rrbracket$$
$$\$432{,}084.86 = \$2{,}870.99 \times a\llbracket R, j_{12} = 6.900047\% \rrbracket$$
$$R = 349.750466 \text{ months}$$

To maintain the original amortization schedule, the monthly payment in the third year will increase from $2,870.99 to $3,127. The outstanding balance owing at the end of the third year is \$424,126.36.

$\$432{,}084.86 = PMT \times a[\![276, j_{12} = 6.900047\%]\!]$

$PMT = \$3{,}127$

$OSB_{36} = \$424{,}126.36$

Calculation (continued)

Press	Display	Comments
7 ■ NOM%	7	New stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	7.1225	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	6.900047	Equivalent j_{12} rate
432084.86 PV	432,084.86	New loan amount
N	349.750466	New amortization period
276 N	276	Enter remaining amortization
PMT	–3,127.00381	Required payment
3127 +/– PMT	–3,127	Rounded payment
N	276.000822	Amortization based on rounded payments
12 INPUT ■ AMORT	12–12	
= = =	424,126.357177	OSB_{36}

The increase in the payment amount will ensure that the loan is repaid over the remaining amortization period, thereby overcoming one of the concerns introduced by the constant payment VRM. However, this variant introduces the risk that the increase in payments may increase the risk of default if the borrower's income does not increase sufficiently to make the larger payments. Lenders may be forced to establish more conservative estimates of the maximum permissible loan amount to any one borrower.

Discussion Point: Lenders' Risks in Variable Rate Mortgages

In a variable rate mortgage loan where interest rate changes are reflected in payment adjustments, what risks does the lender undertake?

- The lender is protected from the risk of the loan resulting in an infinite amortization period (payments inadequate to ever pay off the loan).
- The lender now runs the risk of payments increasing to the point that the borrower can no longer afford them, and this leads to default.
- Consider the subprime mortgage crisis in the United States and in particular how variable rate mortgages can be abused with predatory lending: making loans to borrowers knowing the loan payments cannot be maintained once the variable rate increases; see the dual rate/teaser rate example later in this section.

Dual Rate Variable Rate Mortgages (Teaser Rate)

A variable rate mortgage alternative involves using a dual rate, or a rate that changes during the loan term at set intervals. A variation of this is a teaser rate mortgage, where a lender offers a low upfront rate, typically as a marketing incentive, and then raises the rate to market levels in six months to two years.

Illustration 14.4

A lender offers a dual rate mortgage as a marketing incentive. This could be a developer looking for a way to market subdivision lots without lowering the list price; or it could be a lender targeting low-income individuals to improve home ownership affordability.

The dual rate variable mortgage is for $400,000 with monthly payments (rounded up to the next higher dollar), a 25-year amortization, and a 2-year term.

- Starting interest rate is j_{12} = 2% for Year 1
- Interest rate increases to j_{12} = 6% for Year 2

Calculate the payment for Year 1 and Year 2 and the outstanding balance owing at the end of Year 2. How much principal is repaid during Year 1 and Year 2?

Solution:

Year 1

$$PV = PMT \times a⟦n, j_{12} = 2\%⟧$$

$$\$400{,}000 = PMT \times a⟦300, j_{12} = 2\%⟧$$

where:
n = Original amortization
PV = Original loan
PMT = Year 1 payment

Calculation

Press	Display	Comments
2 I/YR	2	Stated nominal rate
12 ■ P/YR	12	Stated compounding frequency
400000 PV	400,000	Loan amount
300 N	300	Amortization period
0 FV	0	
PMT	–1,695.417355	Contractual payment
1696 +/– PMT	–1,696	Actual monthly payment made
N	299.866328	Revised amortization based on rounded payment
1 INPUT 12 ■ AMORT	1–12	
=	–12,465.858069	Principal repaid Year 1
=	–7,886.141931	Interest paid Year 1
=	387,534.141931	OSB at end of Year 1

The Year 1 payment is $1,696 and the amortization based on the rounded payment is 299.866 months. The outstanding balance at the end of the first year is $387,534.14.

Year 2 $\quad PV = PMT \times a⟦R, j_{12} = 6\%⟧$

where: $\quad PV$ = Outstanding balance after 12 months

R = Revised amortization

j_{12} = Interest rate for Year 2

PMT = Year 2 payment

$$PV = PMT \times a⟦n, j_{12} = 6\%⟧$$

$$\$387{,}534.14 = \$1{,}696 \times a⟦R, j_{12} = 6\%⟧$$

$$R = \text{infinite}$$

The amortization in Year 2 is infinite, that is, the payments are not large enough to ever repay the loan at this interest rate. Therefore, the payment must be adjusted to reflect the original amortization schedule (N = 288 months).

$\$387{,}534.14 = PMT \times a⟦288, j_{12} = 6\%⟧$

$PMT = \$2{,}543$

$OSB_{24} = \$380{,}067.06$

Calculation (continued)

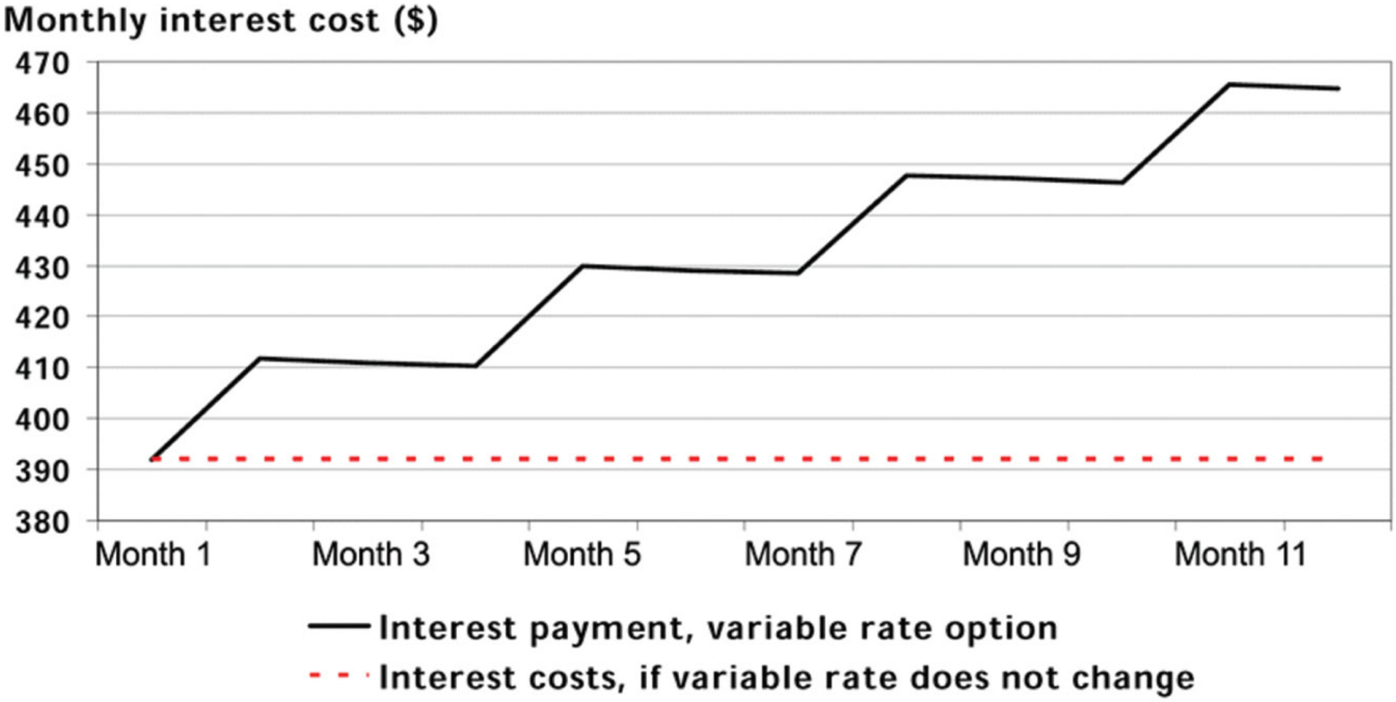
Press	Display	Comments
6 I/YR	6	New stated nominal rate
12 ■ P/YR	12	Compounding frequency
387534.14 PV	387,534.14	OSB at end of Year 1
0 FV	0	
N	no solution	No solution result means the amortization is infinite
288 N	288	Amortization must be set to remaining amortization
PMT	–2,542.138959	Unrounded payment
2543 +/– PMT	–2,543	Rounded payment
1 INPUT 12 ■ AMORT	1–12	
=	–7,467.077334	Principal repaid Year 2
=	–23,048.922666	Interest paid Year 2
=	380,067.062666	OSB at end of Year 2

The Year 1 payment is $1,696 and the Year 2 payment is significantly higher at $2,543. The outstanding balance at the end of Year 1 is $387,534.14 and the outstanding balance at the end of Year 2 is $380,067.06. Considerably less principal is repaid during Year 2 when the rate increases: $12,465.86 principal is repaid during Year 1 and only $7,467.08 is repaid during Year 2.

Consider the risks to the lender of this increased payment. While it has the advantage of opening home ownership to a wider pool of applicants who can qualify for the 2% payment but not for the market 6% payment, the question is what happens when the loan reverts to the market rate? Unless these people suddenly have a dramatic increase in their incomes such that they can afford a much larger income payment, they may be in financial trouble. During the subprime crisis, these loan maturities and rate bumps coincided with a drop in real estate values, and the resulting wave of foreclosures sparked a worldwide financial meltdown.

Figure 14.1 illustrates the increase in interest cost for a $100,000 VRM that starts at 4.75% and increases 0.25% per quarter during the year.

FIGURE 14.1: Monthly Interest Costs Rise with Mortgage Rate



Source: Canada Mortgage and Housing Corporation. (2005). *Housing Now Canada*. Canada: CMHC.

Conclusion: Fixed or Variable?

Variable rate mortgages offer advantages and disadvantages to both lenders and borrowers. Borrowers have the advantage of lower interest costs when mortgage rates stay flat or decline but at the risk of greater costs should mortgage rates rise. For lenders, VRMs allow them to better match assets and liabilities, reducing interest rate risk. However, this comes at the risk of increased defaults, should interest rates rise significantly.

The choice between fixed and variable rate mortgages depends on borrowers' personal circumstances and preferences. The potential interest rate savings from VRMs are directly related to increased risk of interest rate costs. Therefore, borrowers must consider "downside risk" – are they financially capable to withstand a dramatic increase in mortgage carrying costs? A borrower with limited financial resources to absorb potential interest rate shocks may be risk averse and choose the assurance of a fixed payment. Alternatively, borrowers willing and able to withstand these risks may realize significant interest savings.

The answer to the question depends on the borrower's risk preferences, beliefs about interest rate trends, and terms of the loan. See the Convertible

VRM discussion below for an example of how loan terms impact the decision.

Convertible VRMs: Hedging Interest Rate Risk for Borrowers

Variable rate mortgages have become increasingly popular, in a large part due to conversion privileges – meaning the borrower may lock into a fixed rate mortgage at any time. This reduces the interest rate risk significantly, because if rates vary slightly, the borrower can "ride the interest rate curve" to their advantage, but if there is a dramatic increase in interest rates, the borrower can opt to convert to a fixed payment.

This mitigates risk but does not eliminate it because the rate on the conversion tends to be at the lender's posted rates, rather than the 0.5% to 1.5% discounts off posted fixed rates that most borrowers can negotiate. This means that if you negotiated a 5-year term fixed rate mortgage, the posted rate is 5% and the discounted rate is 4%. The variable rate mortgage is at 3.5%, but if you lock in, it will be at the 5% rate, effectively "losing" the 1% discount over the loan term that the borrower could have achieved.

The answer to the question of whether convertible variable rate mortgages make sense depends on the borrower's best guess on if rates will rise, when they will rise, and by how much. What is the penalty for locking in and what is the opportunity cost of not choosing a fixed rate loan from the start? As a borrower or investor, how well are you able to handle uncertainty – are you comfortable with keeping an eye on your mortgage rates or will it be a daily stressor?

The economic turbulence of the late 2000s reduced the funds available for mortgage loans and led to lenders reassessing the highly attractive terms of VRMs. With the long-term "borrower's market" becoming increasingly competitive, lenders are adjusting their mortgages to reduce risk: for instance, the 1% plus discounts below prime rate have disappeared and VRMs have become "prime plus". It is possible that some of the other elements making VRMs attractive to borrowers may be altered or eliminated completely. At the time of publishing, conversion privileges remain in VRMs.

There are some legal concerns that must be addressed in the context of VRMs. Canada's *Interest Act* stipulates that, in the case of loans with blended repayment schedules, "the amount of such principal money and the rate of interest chargeable thereon" must be stated on the face of the mortgage document. Hence VRMs, in their simpler form as outlined above, may contravene the requirements of the *Interest Act*, as the lender is not able to ascertain the interest rate at the time the contract is created.

To overcome the inflexibility of the *Interest Act*, some mortgage lenders have devised a two-part variable payment, fixed maturity VRM system under which a registerable mortgage document is drawn with a contract rate specified. The stated rate is a number of percentage points above the current market rate for a comparable mortgage investment. This document is designed to conform to the provisions of the *Interest Act* and establishes the maximum rate of interest that the borrower may be called on to pay during the term of the agreement. This document is accompanied by an agreement between the lender and borrower where the parties agree that the borrower will pay periodic instalments in an amount that will amortize the loan at the market rate prevailing at the time of periodic interest rate revisions, rather than at the registered contract rate. This side contract may specify the maximum interest rate chargeable and the frequency of rate revision. In essence, borrowers are assured that they will never pay more than the registered contract rate but otherwise face the consequences and benefits of changes in interest rates that may occur over the life of the mortgage. Whether these changes are implemented by varying the amount of the monthly payment or by adjusting the maturity date, or by a combination of both, does not appear to be a critical issue subject to the considerations mentioned above.

GRADUATED PAYMENT MORTGAGES (GPMs)

Graduated payments mortgages are another modification to the standard mortgage repayment scheme where the size of payments is changed, in some regular way, during the term of the loan. In GPMs, payments are increased during the loan term. This modification results primarily from a concern with housing affordability and an attempt to match a growing payment with a borrower's increasing income.

Over the years, a great deal of political and governmental attention has been given to stimulating or subsidizing the demand for owner-occupied affordable housing. The method that these government housing programs have generally followed has been to alter the debt financing aspects of home purchase (for selected groups) from those practices, which generally exist in the marketplace. Historically, one area that has been widely used is the subsidized reduction of the front end or down payment costs of home ownership, through such measures as high loan-to-value ratios or initial acquisition grants. As these entry costs were progressively reduced relative to house prices using subsidy programs, the ability of purchasers' incomes to service the debt to make monthly payments became viewed as the effective constraint on additional potential purchasers to which government officials wished to extend the opportunity of home ownership.

In inflationary times, the constant payment mortgage may present an impediment to affordability of home ownership. If a borrower's income increases over time, the actual gross debt service ratio will decline over time (if the income increases exceed increases in property taxes). Consequently, the borrower's long-run housing consumption will be constrained to a level based on income at the start of the loan. If gross debt service ratios could be adjusted to reflect these expected changes in borrowers' incomes, households would be able to obtain a larger mortgage, thereby facilitating a home purchase sooner than would otherwise occur (and, parenthetically, the purchase of a more expensive house).

In response to these considerations, several mortgage repayment schemes have been developed that provide for increasing, rather than constant, monthly payments. These repayment plans are known generally as graduated mortgage programs and are primarily offered under government and non-profit society auspices. In the United States, under the Federal Housing Administration (FHA), there is an FHA graduated payment mortgage available for homebuyers who currently have low to moderate incomes but expect them to increase substantially over the next 5 to 10 years. Through this FHA loan program, also referred to as Section 245, those who have limited incomes are able to purchase a home and make mortgage payments that will grow along with their earning potential.

In Canada, CMHC developed programs in the 1970s and 1980s to assist with affordability in Canada's high inflation and high interest rate environment. In the 1990s and mid-2000s, this assistance has proved unnecessary, with historically low interest rates. Should interest rates rise once again, these government programs may become available. In the meantime, graduated payment loans continue to be used on a small-scale for social housing, typically with non-profit organizations, and with some home equity loan providers (teaser rate mortgages).

General Characteristics

The general characteristics of standard mortgages include constant monthly payments of blended principal and interest and a consequent reduction in total indebtedness throughout the term of the loan. In the early stages of the repayment, the outstanding balance is reduced at a very slow rate, as the constant payments include all interest due plus some small portion of principal. However, over time the slow reduction in the outstanding balance results in a small reduction in interest due and therefore, given the constant payment size, an increasing rate of principal repayment (debt reduction).

On the other hand, graduated payment mortgages have payments that increase over some, or all, of the life of the loan. Consequently, the initial payments are lower, and the later payments higher, than the payments that would exist on a constant payment loan of similar amount and terms. It is the reduction of the initial payments that creates both the advantages and the disadvantages of GPMs. The reduction in early payments does increase the size of mortgage that can be serviced with a given income (or reduces the income necessary to service a mortgage of a given amount) if the same gross debt service ratio is used on the initial payments.

However, the interest due at the end of the first month on a loan is slightly smaller than the amount of the payment on the standard mortgage, and most often larger than the first payment on the graduated payment scheme. Thus, the initial payment on the GPM is not sufficient to pay the interest that is due. The unpaid interest is added to the outstanding balance and accrues interest at the contract rate. Therefore, the borrower's indebtedness increases over the initial payment periods. The total indebtedness of the borrower continues to grow until the gradually increasing periodic payments become large enough to pay all interest charged in a payment period and make some contribution to debt repayment.

Elements of risk on GPMs stem from both the increasing indebtedness that occurs in the initial years of the loan and the increasing size of payments over time. If the inflation rate of property values exceeds the growth rate of the borrower's indebtedness, then the lender is protected from the risk of holding a mortgage with an outstanding balance equal to or greater than the property value (and borrowers are protected from losing all their initial equity). Therefore, GPMs represent a high degree of risk of *capital loss* unless extremely conservative loan-to-value ratios are used. Similarly, the graduated increases in payments will require a larger and larger portion of the borrowers' incomes unless their incomes grow faster than the rate of increase of payments. If such income inflation does not occur, the risk of arrears and/or default on payments will also increase greatly. Clearly, the advocates of GPMs must be strongly committed to the belief of continuing inflation in borrowers' incomes.

capital loss
loss incurred on an asset due to depreciation and is realized when the asset is sold for less than the original purchase price

Even if such inflation could be unquestionably assumed, it does not necessarily justify GPMs. One could argue that inflation in prices of consumer goods (e.g., food, clothing, etc.), plus other property costs (e.g., insurance, taxes, energy, etc.) will also occur along with inflation in incomes. If this is the case, a GPM scheme could only reflect increased incomes without increasing default risk if they were based on constant dollars (i.e., purchasing power) rather than on income inflation, which ignores cost of living inflation.

Illustration 14.5

- A non-profit housing society is assisting a low-income family to purchase a house.
- For a $200,000 mortgage, market-rate payments are $1,100 per month; the family can only afford $500.
- A year from now, once the family has more financial security, they should be able to afford $1,100 per month.
- The society will set up a simple GPM, set the first payment at $500, and then increase the payments $50 per month over the year until they reach market level.

FIGURE 14.2: Graduated Payment Mortgage

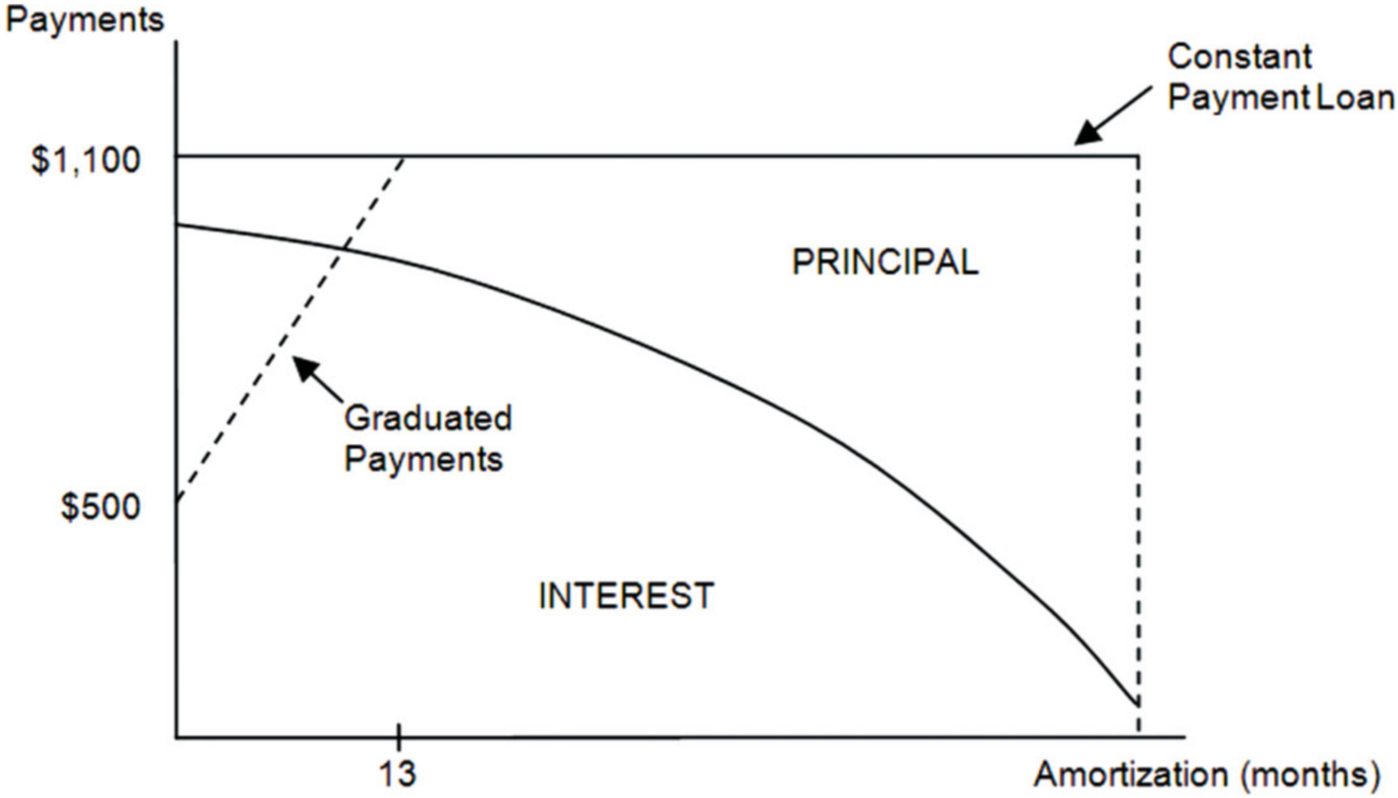


Figure 14.2 illustrates the payments to be made in this graduated payment loan, against the backdrop of what would be the regular payments on a constant payment loan. Initial payments do not even cover the interest portion of the constant payments. During the early months of this loan, this unpaid interest will accrue and be added to the loan balance. At approximately Month 10, the borrower's payment finally rises sufficiently to fully pay interest accruing each month along with some principal repayment. By the 13th payment, the graduated payments are at "market" level and the loan becomes a regular constant payment mortgage (being technical, the interest accrual over the first year will have to be accounted for, either by a longer amortization period, a slightly higher payment, or a larger outstanding balance owing if partially amortized).

The advantage of the graduated payment loan is that it can be used to help prospective purchasers buy real estate who otherwise might not be able to qualify for a conventional loan. For organizations that want to promote affordable home ownership, this loan type may be an attractive option.

Risks of Graduated Payment Mortgages

The benefit of graduated payment mortgages is the improved affordability of home ownership. While this is generally considered a positive for society, at the extreme it also is a potential negative. Consider the subprime crisis in the US, where loans were made to people with little or no income, by keeping the initial payments extremely low. When the payments increased to market level, these loans were not sustainable. With rising real estate values, perhaps some of these loans could have been saved from default by aggressive refinancing, increasing the loan balance and extending the amortization periods to keep payments affordably low. However, coupled with dropping real estate values, borrowers faced with a 300% increase in mortgage payments and “negative equity” (property value less than the mortgage value) simply handed over their keys – a massive wave of foreclosures and a global financial crisis. This is not to say that GPMs are a bad option – used judiciously, they can provide a particularly useful means of improving affordability. But like almost everything in life, “all good things in moderation” ... caution is in order, for both borrowers and lenders!

- **Risk to borrower:** low initial payments improve loan qualification but may not be affordable in the long term.
- **Risk to lender:** initial payments add to debt raising loan-to-value; protected from default losses if property values are increasing but high risk of loss if property values are dropping.

SINKING FUND ASSISTED MORTGAGES (SFAMs)

A sinking fund assisted mortgage (or SFAM) is related to a graduated payment mortgage as both are methods to reduce initial payments to improve affordability. The primary difference is that a GPM reduces payments below interest cost, and the interest effectively accrues, adding to the loan balance as payments increase. In contrast, a SFAM advances the borrower an amount that is less than the face value of the loan, setting the difference aside in an account that is used to draw down initial payments. The funds set aside are typically placed in an interest-bearing account, with an amount withdrawn each payment period as a subsidy to reduce the mortgage payments. The withdrawals continue until the account balance is zero.

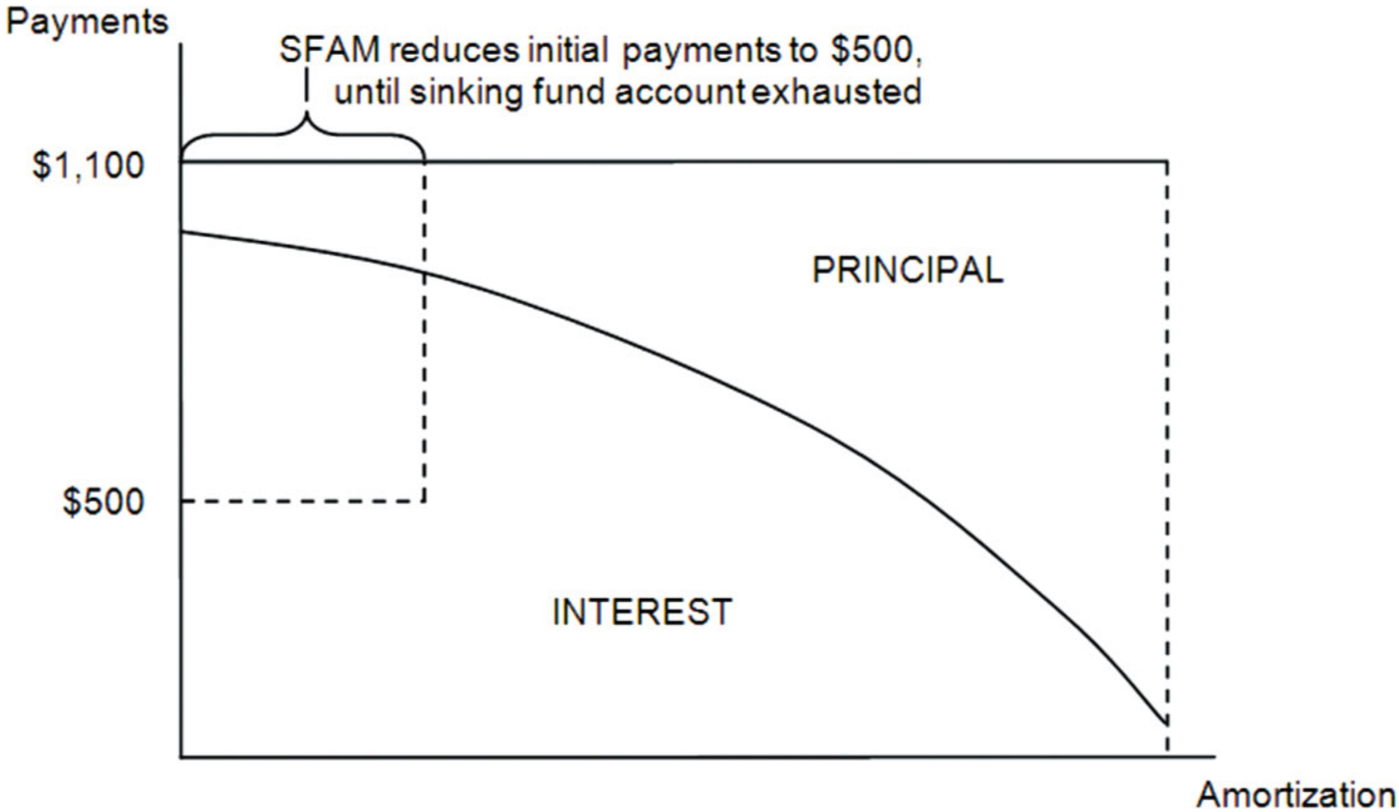
A sinking fund assisted mortgage is effectively a form of bonused loan, in that the borrower is not advanced the full amount of the loan upfront. The advantage over a GPM is that the increased indebtedness from the “bonus” is clearly established upfront, rather than the somewhat hidden impact of interest accrual in the GPM. The lender’s administration of the savings account somewhat reduces default risk, and thereby permits higher loan-to-value and gross debt service ratios, although perhaps not to the extent that would be found under a government-insured program.

Illustration 14.5 (continued)

Rather than establishing a graduated payment loan, the non-profit society is considering a sinking fund assisted mortgage to aid the prospective home owner.

- The loan funds advanced will remain at $200,000.
- The loan's face value is $205,000.
- $5,000 is put into an interest-bearing account.
- Monthly draws are made from this account to subsidize payments, lowering the market loan payment ($1,100) until the sinking fund is exhausted.
- Sinking fund withdrawals could be a set amount each month, e.g., $500 per month until the fund is exhausted, with initial payments steady and then a sudden jump to market level (illustrated in the following figure).
- Alternatively, sinking fund withdrawals could be large at first and gradually decline, such that the mortgage payments start small and gradually increase, similar to the graduated payment loan. The calculations for determining the initial payment and rate of decrease are complicated – demonstrated briefly in Illustration 14.6.

FIGURE 14.3: Sinking Fund Assisted Mortgage (Constant Withdrawals)



An advantage of using a SFAM in this example, rather than a GPM, is that the impact of artificially lowering initial payments is more apparent upfront. The loan balance includes an additional $5,000 that the borrower does not receive, and this amount is either amortized over the loan term or repaid upon expiry of the term (with a higher loan balance, assuming the mortgage is renewed). In a GPM, the interest accrued in early payments also adds to the loan balance, but the impact of this may not be as clear to borrowers – once again, see the text box outlining GPM risks.

Illustration 14.6

	Price of house	$470,000	
–	Down payment	– 47,000	
	First mortgage	$446,500	(95% loan-to-value)
–	Amount advanced to borrower	– 423,000	(90% loan-to-value)
	Amount of bonus deposit	$23,500	
	Required cash down payment	$47,000	

Assume the mortgage provides for a five-year graduation with payments, net of the deposit fund, to increase annually until standard payments are reached at the end of the fifth year. The first mortgage is at a rate of j_2 = 10.5% amortized over 25 years. Assume that the deposit will earn j_2 = 10.5%.

In analyzing the sinking fund assisted mortgage, it may be necessary to work by trial and error (iterate) to a solution. No straightforward equation can be used to calculate (simultaneously) the savings withdrawal necessary to exactly deplete the deposit account in exactly the specified time. This occurs because the interest per month on the original loan is predicated on one rate and amortization schedule (as set out in the original loan) and the deposit account grows at a different rate and schedule. However, this type of problem may be solved using "Solver" or "Goal Seek" applications in spreadsheets, used for solving simultaneous equations.

Because of these reasons, SFAMs present complicated calculations and these will not be illustrated here. Table 14.1 summarizes the SFAM results for Illustration 14.6.

Table 14.1: Comparison of GPM, SFAM, and Interest Only Options

Cash Advance of $423,000 at j_2 = 10.5% with a 25-year amortization

	1st Payment	30th Payment	60th Payment	OSB After 60th Payment	OSB as % of Loan Amount
Conventional Repayment	$3,926.90	$3,926.90	$3,926.90	$399,277.10	94.4%
Pure GPM (2% Graduation)	$3,349.68	$3,513.89	$3,692.22	$432,311.50	102.2%
SFAM (10.5% Deposit Rate)	$3,385.00	$3,670.30	$3,982.00	$421,175.20	99.6%
Interest Only Loan	$3,622.80	$3,622.80	$3,622.80	$423,000.00	100%

Note: Because of risk issues the interest rates of these mortgages would not actually be the same (for instance, the rate would be lower with a conventional mortgage).

The results show that both the GPM and SFAM alternatives reduce initial payments below those on a conventional mortgage loan, but both result in a significantly larger outstanding balance five years into the loan. The SFAM can reduce payments below the conventional mortgage, similar to the GPM's payment but with slightly less impact on the loan's outstanding balance. The SFAM reduced initial payments below an interest only loan, with a similar OSB in five years.

The following illustration introduces the symbolic notation for the future value relationship for a stream of regular payments:

$$FV = PMT \times s[\![n, j_m]\!]$$

where: FV = Future value of the stream of regular cash flows

PMT = Dollar amount of the regular payment or cash flow

$s[\![n, j_m]\!]$ = Future value of \$1 per period for n periods, discounted at a nominal rate of j_m

n = Number of compounding periods

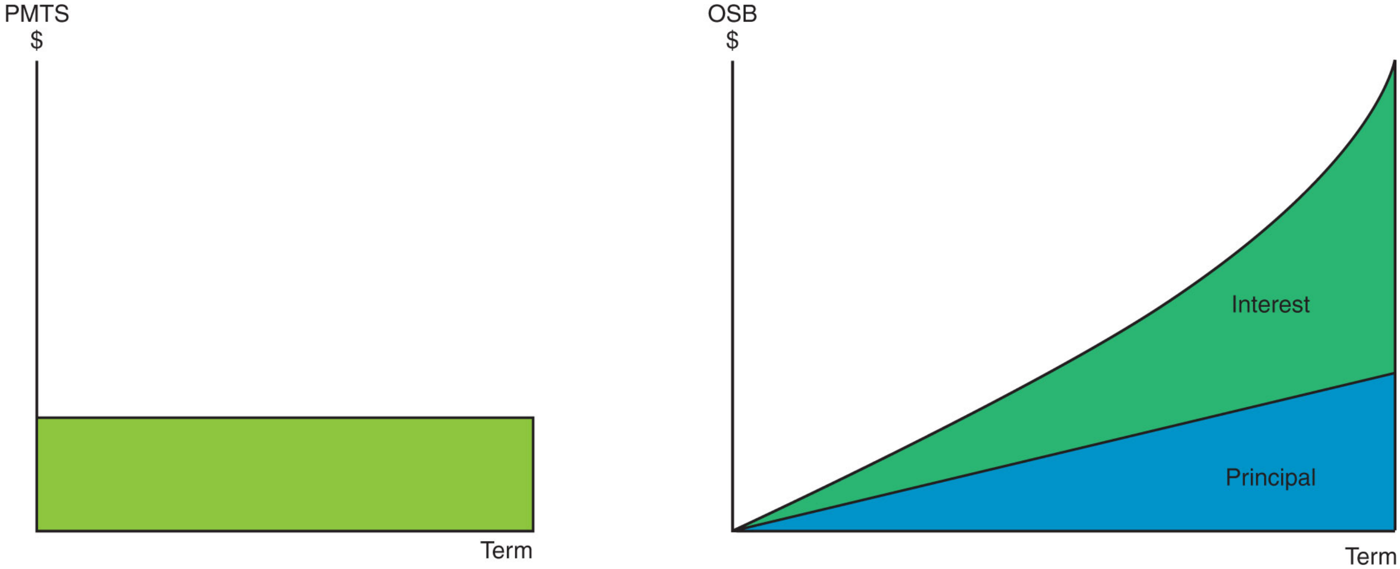
j = Interest rate per annum; requires compounding frequency

It is important to understand that $s[\![n, j_m]\!]$ is not a mathematical formula; it is simply a shorthand or symbolic way of writing something that is otherwise very cumbersome. It represents the mathematical relationship to be used in solving problems where streams of regular payments are involved. The double square brackets "$[\![$, $]\!]$" are traditionally used to separate the values for n and j.

REVERSE MORTGAGES OR REVERSE ANNUITY MORTGAGES (RAMs)

Occasionally real estate financing vehicles are created where the lender advances funds periodically rather than as a single capital sum. These arrangements are common in development financing where the funds are advanced as a series of mortgage draws as construction progresses. Another form of periodic investment stream is the reverse annuity mortgage (RAM). The reverse annuity mortgage, like its conventional counterpart, involves a regular series of payments. However, in a RAM the periodic payments are made by the lender to the borrower. At the end of the contractual term (or upon the death of the borrower), the accumulated balance of principal advances and accrued interest becomes repayable.

FIGURE 14.4: Periodic Payments and Outstanding Balance on a Reverse Annuity Mortgage (RAM)



From the borrower's perspective, a reverse mortgage allows equity-rich borrowers to postpone selling their residence when income is insufficient to cover increasing real property taxes, insurance and maintenance costs, or non-housing expenditures. Conventional mortgage financing is generally ill suited to the financial constraints of many elderly mortgage borrowers as a means of converting accumulated real estate equity to supplement income. Many seniors own their homes (i.e., they are asset rich) but have little ongoing income (i.e., they are cash poor). A reverse mortgage may provide the borrower with a source of ongoing cash without losing their home.

Illustration 14.7

A reverse mortgage has been created to improve a borrower's monthly income based on the security of a $300,000 home. The loan is written at 6.25% per annum, compounded monthly, over a 10-year term. The outstanding balance is not to exceed 25% of the market value of the home at the time the loan was written.

What is the monthly amount the borrower will receive?

Solution:

The maximum amount the loan may equal, at any time, is 25% of the value of the house:

$$FV = \$300{,}000 \times 0.25 = \$75{,}000$$

The payments, accumulated with interest, may grow to this amount:

$$FV = PMT \times s[\![n, j_{12} = 6.25\%]\!]$$
$$\$75{,}000 = PMT \times s[\![120, j_{12} = 6.25\%]\!]$$
$$PMT = \$451.48$$

Calculation

Press	Display	Comments
6.25 I/YR	6.25	Stated nominal rate
12 ■ P/YR	12	Stated and desired compounding frequency
75000 FV	75,000	Maximum future debt
120 N	120	Length of term
0 PV	0	
PMT	–451.475727	Monthly payment

Thus, the borrower will receive $451.48 per month over the 10-year term.

Alternatively, a home owner could pursue a conventional mortgage, borrowing a lump sum secured by their real property, depositing the bulk of the funds in a highly liquid vehicle (likely a savings account or GIC), and then draw on the account regularly to add to current purchasing power. This option places the borrower in an undesirable position – borrowing at the first mortgage rate and investing in a financial institution at a rate likely well below the cost of funds. The reverse mortgage provides a means by which the borrower avoids this negative rate differential and pays interest only on funds actually utilized to supplement income. In a rising market, the home owner also continues to enjoy the benefits of property appreciation.

Reverse Mortgage Examples

Reverse mortgages have been offered by HomeEquity Bank (the CHIP Reverse Mortgage, *chipmoney.chip.ca*) in Canada since 1986. In addition to CHIP, as of June 2023, Equitable Bank (*www.equitablebank.ca*) and Bloom Finance Company *(bloomfin.ca/product)* also offer reverse mortgages. Details of the reverse mortgage lenders are available on their websites. For purpose of illustration, we will refer to the details on the CHIP Reverse Mortgage.

Under CHIP's reverse mortgage programs, home owners aged 55 and older can take the loan as a lump sum or in periodic payments and the money received is not taxable. CHIP insists that potential borrowers obtain independent legal advice before signing up and guarantees that home owners can never owe more than the fair market value of the property at the time of repayment. CHIP offers fixed and variable rate loans.

Under CHIP, home owners can receive up to 55% of the value of their home with the specific amount to be based on the home owner's age, the location of the property, the type of home, and the home's current appraised value. Home owners can receive the money as one lump sum advance, periodically over time, or planned advances over a set period of time (or even a combination of a lump sum and ongoing advances over time). The money received is tax-free: it is not added to taxable income. The proceeds received can be used without restriction: to build up savings, cover unexpected expenses, renovations, or to be given to family members. The only condition is that any outstanding loans secured by the home must be retired with proceeds from the CHIP Home Income Plan. No payments are required while residing in the home. The full amount only becomes due and payable when the home is sold, or if the home owner moves out. However, home owners can choose to repay a part or all of the annual accrued interest and home owners have the option to repay the principal and interest in full at any time (may be subject to a penalty). Home owners maintain ownership and control of the home but must maintain the property and maintain current status with the payment of property taxes, insurance, and condominium or maintenance fees while the home owner resides in the home. CHIP guarantees that the amount to be repaid will never exceed the fair market value of the home at the time that it is sold.

The following illustration provides two scenarios available with a CHIP reverse mortgage.

Illustration 14.8

CHIP recently approved a $240,000 reverse mortgage for the Senior family based on their recently appraised market value of $600,000 on their single-family detached residence. The term of the mortgage is set at 5 years at a rate of

5.5% per annum, compounded semi-annually on an interest accrual loan, i.e., no payments will be made until the end of the term.

a. If the Senior family decides to purchase a $240,000 annuity with the proceeds, what amount will they receive per month from the annuity? The rate on the annuity is set at a rate of 2.5% per annum, compounded semi-annually with monthly payments over the 5-year term. Assume that the rates for the mortgage and the annuity will remain fixed for the duration of the 5-year term.

b. What annual rate of house depreciation could occur and still have adequate proceeds to successfully repay the loan from the house proceeds? (Assume zero selling costs).

c. Assume now that the Senior family will receive a lump sum today for $100,000 and monthly payments of $1,000 over the 5-year term on the CHIP reverse mortgage. Calculate the amount owing on this loan at the end of the 5-year term. Assume that the rate on the mortgage is 5.5% per annum, compounded semi-annually.

Solution:

a. $PV = PMT \times a\llbracket n, j_2 = 2.5\% \rrbracket$

$\$240{,}000 = PMT \times a\llbracket 60, j_{12} = 2.487078\% \rrbracket$

$PMT = \$4{,}258$

Calculation

Press	Display	Comments
2.5 ■ NOM%	2.5	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	2.515625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	2.487078	Equivalent j_{12} rate
240000 PV	240,000	Annuity amount
60 N	60	Amortization period
0 FV	0	
PMT	–4,257.999447	Annuity payment

The annuity payment per month that the Seniors will receive is $4,258.

b. Determine the loan amount owed at the end of the five-year term and then calculate the annual rate of depreciation on the house.

$$FV = PV \times (1+ i)^n$$

$$FV = \$240{,}000 \times (1+ i_{sa})^n$$

$$FV = \$240{,}000 \times (1+ i_{sa})^{10}$$

$$FV = \$314{,}796.25$$

where: $i_{sa} = 2.75\%\ (5.5\% \div 2)$

Calculation

Press	Display	Comments
5.5 I/YR	5.5	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
240000 PV	240,000	Reverse mortgage amount
10 N	10	Length of loan (10 semi-annual periods)
0 PMT	0	
FV	–314,796.247821	Loan amount to repay

The amount owed on the loan at the end of the 5-year term is $314,796.25.

To calculate the annual rate of house depreciation that could occur, compare the present value of the home to that of the future value amount of the loan.

$FV = PV \times (1+ i)^n$

$\$314{,}796.25 = \$600{,}000(1 + i_a)^5$

$i_a = -12.1\%$

Calculation

Press	Display	Comments
600000 PV	600,000	House value today
314796.25 +/– FV	–314,796.25	OSB loan at end of 5-year term
5 N	5	5-year term
0 PMT	0	
1 ■ P/YR	1	
I/YR	–12.102675	Annual rate of depreciation

The annual rate of house depreciation is approximately 12%. This means that the property could decline in value by approximately 12% per year over the 5-year period and there will be adequate value in the property to repay the loan amount.

c. To find the amount owed on this loan at the end of the term, find the future value of the lump sum and the future value of the annuity payment.

$$FV = PV \times (1+i)^n + PMT \times s⟦n, j_m⟧$$

$$FV = \$100{,}000 \times (1+i)^{60} + \$1{,}000 \times s⟦60, j_{12} = 5.438018\%⟧$$

$$FV = \$199{,}936.71$$

Calculation

Press	Display	Comments
5.5 ■ NOM%	5.5	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	5.575625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	5.438018	j_{12} equivalent rate
1000 PMT	1,000	Monthly payment
60 N	60	Amortization period
100000 PV	100,000	Lump sum amount
FV	–199,936.709907	OSB on loan

If the home owners receive a lump sum amount for $100,000 and monthly payments of $1,000 for the duration of the 5-year term, the outstanding balance owing on the loan is $199,936.71.

Pros and Cons of Reverse Mortgages

Reverse mortgages allow home owners the opportunity to maintain, sustain, or even improve their current lifestyle and still live in their home. Reverse mortgages may be a good option if the home owner is not able to take on a regular bank loan or line of credit secured by the value of the house and make regular loan payments. If the home owner has value tied up in the house but does not have enough cash to satisfy their needs, reverse mortgages may be an appropriate option (albeit at rates 2-3% higher than a home equity line of credit).

However, some financial planners are hesitant to recommend reverse mortgages because they are not interested in supporting a product that puts a client back into debt and are not comfortable advising clients to spend an asset. From a financial planning perspective, some might question the wisdom of postponing a real property downsizing decision by using a reverse mortgage, as the compounding accrued interest erodes the "nest egg".

Since reverse mortgages are rising debt loans, it is important that seniors' children also understand the process involved in a reverse mortgage because it is the children's inheritance that can be substantially reduced upon repayment of the debt. Many borrowers, and possibly regulatory bodies, will continue to regard the incremental erosion of real estate equity to be undesirable. Furthermore, in a dropping real estate market, depending on the proportion borrowed in the reverse mortgage, a rising loan balance coupled with declining property value could be disastrous for the borrower.

Reverse mortgages are not recommended as a short-term financing vehicle given their higher interest rates than other forms of short-term borrowing, e.g., a home equity loan. In addition, with CHIP, there are prepayment penalties in the first three years.

Finally, tax treatment of reverse mortgages should be investigated. These negatives should not be taken as a definitive conclusion that a reverse mortgage is not a wise financial decision. However, the implications of a reverse mortgage relative to other alternatives should be fully considered.

REFINANCING THE PROPERTY

We now turn our focus to an examination of property financing where transfers of title are not necessarily taking place and where some financing is already in place. In these circumstances, the term refinancing is generally adopted to place emphasis on the pre-existing financial arrangement. A number of alternative avenues may be explored in the context of refinancing, each potentially involving significant implications in terms of risk, yield (or cost), and cash flow for both lenders and borrowers. Thus, in the sections that follow, we discuss refinancing arrangements, examine the details appropriate for each, and highlight management implications for both lenders and borrowers.

Investors in real property are motivated to consider refinancing in a wide variety of circumstances. First, as virtually all mortgages initiated in Canada are partially amortized, refinancing (sometimes called "rollover") is required upon maturity of the original mortgage. In this case, the options available to the owner/investor are limited, as the outstanding principal balance at maturity must either be refinanced or paid out by the borrower. A typical term maturity (or call) clause would be presented as follows:

> The principal sum of Three Hundred Thousand Dollars ($300,000) shall be paid by monthly instalments of One Thousand Four Hundred Thirty-Six Dollars ($1,436) each commencing on the first day of January, 20x8, continuing on the first day each and every month thereafter until the first day of December 20x9, the whole of the principal sum then remaining due and interest as herein provided shall become due and be paid.

Note that the maturity clause requires the payment of all outstanding principal upon maturity of the loan. The clause is absolute, not permissive. Therefore, the borrower must either pay the balance due or refinance the balance through a mutually acceptable renewal of the mortgage. If the balance is paid out, the borrower must raise the required funds from their own sources or through the creation of a new mortgage from an alternative source. As the initiation of a new mortgage with another lender can entail significant costs to the borrower (in the form of legal, appraisal, and survey costs), the borrower must consider the initiation costs of alternatives relative to the costs involved in refinancing the loan with the original lender. Furthermore, as refinancing with the original lender introduces the possibility of alterations to terms and conditions of the contract, the borrower must carefully review all relevant facets of a renewal with the original lender and conditions available from alternative sources. Refinancing occasioned by the maturity of one or more existing loans is generally referred to as a mandatory refinancing. Situations where a property owner elects to refinance before the term expires are called voluntary refinancing.

Analysis for Refinancing

If a borrower wishes to refinance, either voluntarily or involuntarily, the appropriate analysis is to select the best alternative method. Ignoring variations in the amount borrowed, the loan term, and amortization, the best alternative is that which has the lowest cost, i.e., lowest effective interest rate.

Illustration 14.9

A residential borrower arranged for a mortgage of $180,000 almost five years ago and has made all payments as due. The outstanding balance of $163,277.98

is due soon and the borrower has been offered the following alternatives. The original lender is willing to refinance for a further 5-year term and the payments are to be based on the remaining 20 years in the original amortization period. The rate required by the lender is 5.24% per annum, compounded semi-annually. Monthly payments are to be rounded up to the next higher dollar. A nominal renewal fee of $100 is levied to contribute to the lender's legal and registration costs. An alternative has been offered by another lender. Under this option, funds are advanced at 4.9% per annum, compounded semi-annually, again to be repaid over a 5-year term and based on a 20-year amortization period. Monthly payments are to be rounded up to the next higher dollar. The "new" lender will charge the borrower for legal and appraisal fees totalling $1,000.

Based on a cost of borrowing analysis, determine which option is to the advantage of the borrower. For rate comparisons, quote effective annual interest rates. Assume that the borrower must pay the fees out of pocket in each case, i.e., the fees are not added to the face values of the loans.

Solution:

	Option A *Original Lender*	Option B *New Lender*
Face Value (Original OSB_{60})	$163,277.98	$163,277.98
− Fees	− 100.00	− 1,000.00
Net Proceeds	$163,177.98	$162,277.98

A variety of approaches may reasonably be applied to determine which option is to the advantage of the borrower. Two of the most commonly applied methods of analysis are detailed below. The first method involves determining the rate of interest paid by the borrower by making the required payments in exchange for the net proceeds available under each alternative. The second method focuses on the "margin" and involves calculating the incremental differences in up-front fees, monthly payments, and outstanding balances.

Method One: Cost of Effective Rates on Net Funds Advanced

Option A

To solve for the effective rate, first find the payment and outstanding balance at the end of the 5-year term based on the contract rate of j_2 = 5.24%. Then, calculate the effective rate paid by the borrower on the net proceeds.

Calculation

Press	Display	Comments
5.24 ■ NOM%	5.24	j_2
2 ■ P/YR	2	
■ EFF%	5.308644	j_1
12 ■ P/YR	12	
■ NOM%	5.183696	j_{12}
163277.98 PV	163,277.98	Total loan amount
0 FV	0	
240 N	240	Amortization period
PMT	–1,094.200175	Monthly payment
1095 +/– PMT	–1,095	Rounded payment
60 INPUT ■ AMORT	60–60	
= = =	136,652.751496	OSB_{60}
136652.75 +/– FV	–136,652.75	Enter OSB_{60} as FV
60 N	60	Compounding periods
163177.98 PV	163,177.98	Funds advanced
I/YR	5.198732	j_{12}
■ EFF%	5.324411	j_1

Thus, the impact of the $100 renewal fee is to increase the rate paid by the borrower from the contract rate of j_1 = 5.308644% to j_1 = 5.324411%. This analysis is repeated below for the new lender to evaluate the impact of the larger fees.

Option B

To solve for the effective rate, first find the payment and outstanding balance at the end of the 5-year term based on the contract rate of j_2 = 4.9%. Then, calculate the effective rate paid by the borrower on the net proceeds.

Calculation

Press	Display	Comments
4.9 ■ NOM%	4.9	j_2
2 ■ P/YR	2	
■ EFF%	4.960025	j_1
12 ■ P/YR	12	
■ NOM%	4.850715	j_{12}
163277.98 PV	163,277.98	Total loan amount
0 FV	0	
240 N	240	Amortization period
PMT	–1,064.142784	Monthly payment
1065 +/– PMT	–1,065	Rounded payment
60 INPUT ■ AMORT	60–60	
= = =	135,841.017155	OSB_{60}
135841.02 +/– FV	–135,841.02	Enter OSB_{60} as FV
60 N	60	Compounding periods
162277.98 PV	162,277.98	Funds advanced
I/YR	5.00078	j_{12}
■ EFF%	5.117006	j_1

The effect of the $1,000 payment for legal and appraisal fees is to increase the rate paid by the borrower for the use of funds from the contract rate of j_1 = 4.960025% to j_1 = 5.117006%. Compared to the cost of funds from the original lender (j_1 = 5.324411%), the lower interest rate offered by the new lender more than compensates for its higher fees.[2]

Method Two: Cost of Marginal Funds

Another equally valid approach to analyzing this problem involves considering the cost incurred by the borrower on the marginal dollars advanced under the first option relative to the second.

	Original Lender	–	New Lender	=	Marginal Dollars
Up-front Payments	$100.00	–	$1,000.00	=	–$900.00
Periodic Payments	$1,095.00	–	$1,065.00	=	$30.00
OSB at Term	$136,652.75	–	$135,841.02	=	$811.73

The borrower must pay $900 more in up-front fees to the new lender compared with the original lender. The benefit of this initial cost to the borrower is that, compared to the original lender, the borrower will make a series of 60 monthly payments that are $30 lower, and the outstanding balance at the end of the 5-year term is $811.73 lower. The following calculation can be used to determine the return on the initial investment that is achieved by the borrower.

Calculation

Press	Display	Comments
60 N	60	Compounding periods
900 +/– PV	–900	Marginal dollars released
30 PMT	30	Marginal payment
811.73 FV	811.73	Marginal OSB_{60} enter as FV
12 ■ P/YR	12	
I/YR	39.349217	j_{12}
■ EFF%	47.281911	j_1

Thus, the borrower effectively earns a return of 47.28% per annum, compounded annually on the initial "investment" of $900. This analysis, as with the first analysis, indicates that refinancing with the new lender is the best option. The rate shown above is essentially an internal rate of return (IRR), and any rate above the borrower's discount rate (say, j_1 = 5%) indicates it is worth pursuing the new refinancing option. By varying the cost of the investment in the calculations above (PV), we see that any fee increases up to approximately $2,310 still yields an IRR above 5%. Beyond $2,310, the initial cost of the investment would outweigh the benefits.[3]

The analysis presented above is appropriate in measuring the rate paid by the borrower on funds advanced in refinancing situations where existing financing must be terminated. The analysis is exactly analogous to that for comparison of alternatives during initial financing. The sections and analyses that are presented in the remainder of this chapter focus on refinancing situations where the borrower has the option of retaining existing financing and chooses to refinance.

Blended Rate Refinancing

When a borrower is pursuing refinancing to free up additional funds, the alternatives typically include renegotiating a new first mortgage or adding a second mortgage. However, renegotiating a first mortgage typically involves prepayment penalties, while second mortgages have higher, often prohibitive, interest rates. A third option is to negotiate a blended rate or "blend and extend" alternative. The blend and extend option takes advantage of the continued low-rate borrowing on the current first mortgage funds as well as additional financing at a rate lower than the second mortgage, while avoiding prepayment penalties. This is illustrated in the following example.

Illustration 14.10

A borrower wants to obtain additional mortgage funds by renegotiating the first mortgage, adding a second mortgage, or taking out a blend and extend mortgage. The borrower's annual income is $72,000. The current outstanding balance on the borrower's loan is $102,474.74. The borrower is currently making payments of $1,000 per month and will continue to make those payments for 12 more years. Based on the total debt service ratio, the maximum annual payment is $21,600. (For simplicity of illustration, we will ignore property taxes.) The current interest rate is 6% per annum, compounded monthly. New funds are charged at a rate of 8% per annum, compounded monthly. Any new funds are calculated based on a 20-year amortization.

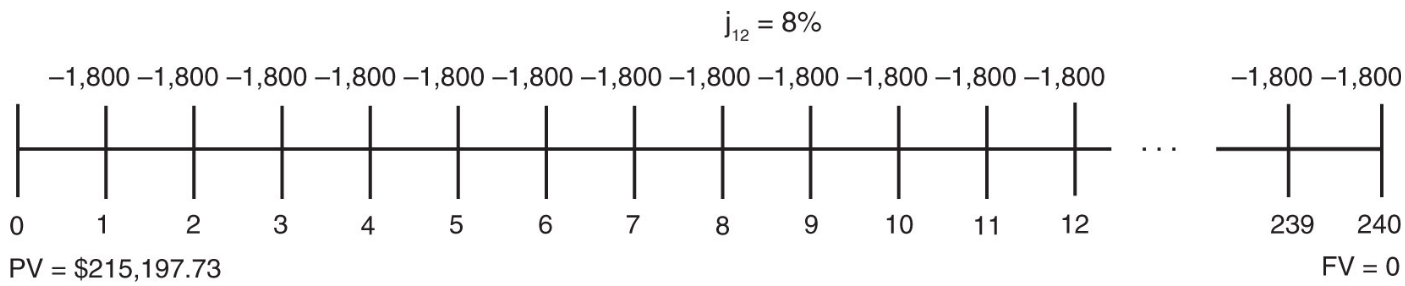
a. Calculate the maximum amount of additional funds that can be acquired with a new first mortgage where the penalty for prepayment is 3 months' interest at the contract rate.
b. Calculate the maximum amount of additional funds that can be acquired with a second mortgage where the second mortgage rate is 10% per annum, compounded monthly. Assume that the maximum second mortgage payment is the differential payment over a 20-year amortization.
c. Calculate the maximum amount of funds that can be acquired using the blend and extend option. Comment on why borrowers might choose this option.
d. Compare the three options. Which is best?

Solution:

a. If the borrower pays out the existing first mortgage (subject to a penalty), the additional funds advanced is calculated as the present value of the full maximum payment over 20 years at the current rate less the penalty and the outstanding balance on the existing first mortgage.

$PV = \$1,800 \times a[[240, j_{12} = 8\%]]$

$PV = \$215,197.73$



Calculation

Press	Display	Comments
8 I/YR	8	Stated nominal rate
12 ■ P/YR	12	Stated and desired compounding frequency
1800 +/– PMT	–1,800	Maximum payment
0 FV	0	
240 N	240	Amortization period
PV	215,197.725064	PV of payment

$\text{Penalty} = OSB_{today} \times imo \times 3$

where: $i_{mo} = 6\% \div 12 = 0.5\%$

Penalty = $102,474.74 × 0.5% × 3

Penalty = $1,537.12

Additional Funds = $215,197.73 B $1,537.12 B $102,474.74

Additional Funds = $111,185.87

b. If the borrower retains the existing first mortgage and obtains a second mortgage, the amount of additional funds is the present value of the differential payment over the 20-year amortization period.

Differential Payment = Maximum Allowable Payment (as per TDSR) – Existing First Mortgage Payment

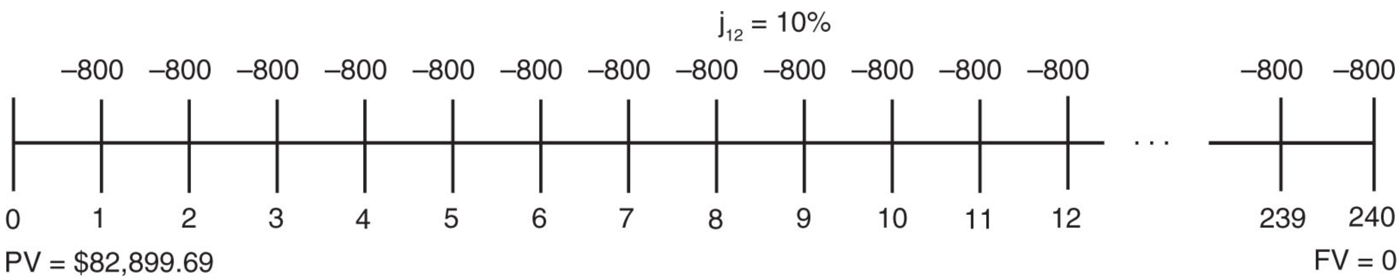
where: Maximum Allowable Payment = $21,600 per year = $1,800 per month

Differential Payment = $1,800 – $1,000

Differential Payment = $800

$PV = \$800 \times a[[240, j_{12} = 10\%]]$

PV = $82,899.69 (Additional funds)



Calculation

Press	Display	Comments
10 I/YR	10	Stated nominal rate
12 ■ P/YR	12	Stated and desired compounding frequency
800 +/– PMT	–800	Maximum payment ($1,800 – $1,000)
0 FV	0	
240 N	240	Amortization period
PV	82,899.694953	PV of payment

c. Under the blended rate option, the borrower will pay the maximum loan payment of $1,800 for the next 20 years. The amount of additional funds available will be a combination of the present value of the difference in the maximum payment for the remaining term on the existing first mortgage plus the present value of the full maximum payment for the remaining 8 years.

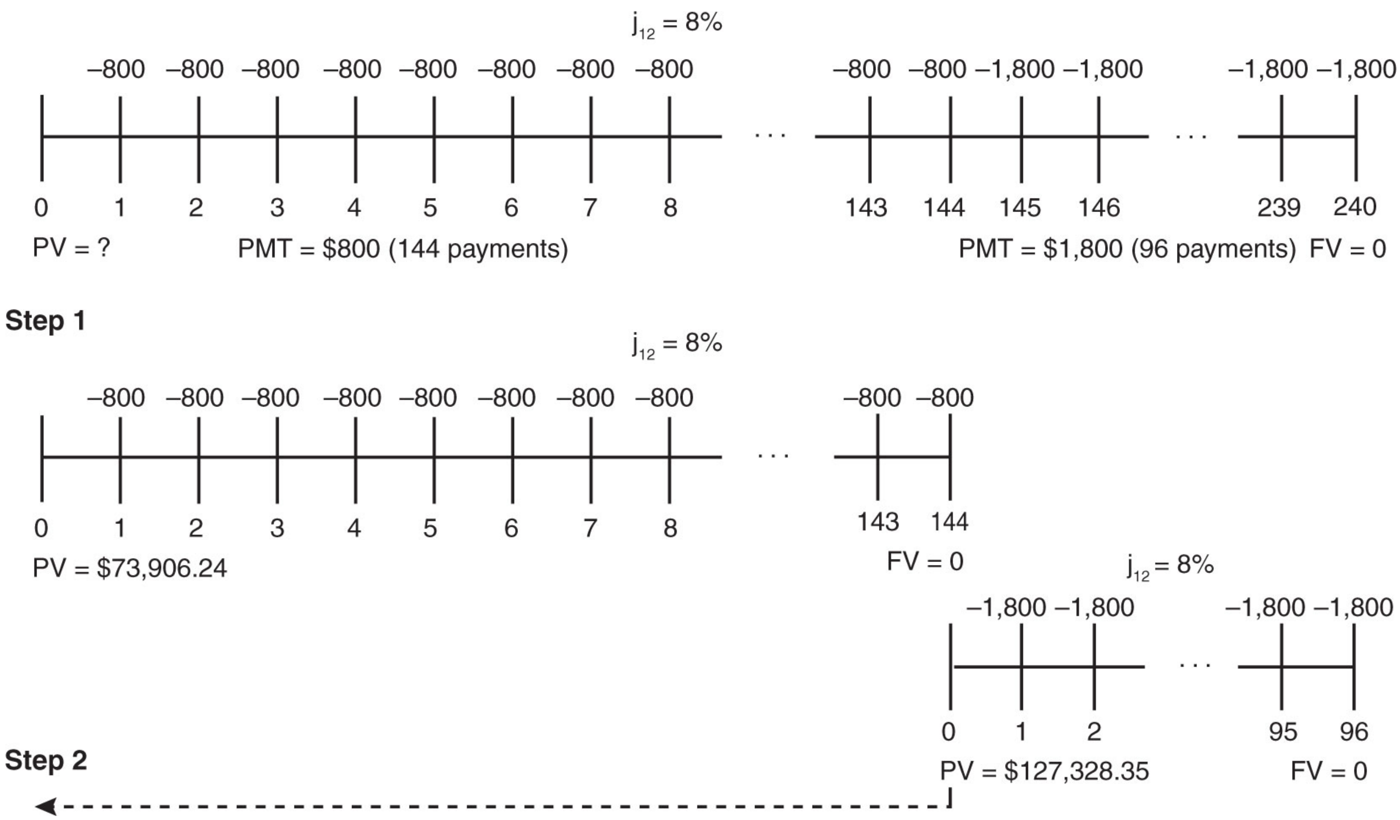
With a maximum annual payment of $21,600, the maximum monthly payment is $1,800. For the remaining 12 years of the first mortgage, the differential payment is $800 ($1,800 – $1,000). Then the payment is the full maximum of $1,800 for the remaining 8 years.

$$PV = \$800 \times a\llbracket 144, j_{12} = 8\% \rrbracket + \$1{,}800 \times a\llbracket 96, j_{12} = 8\% \rrbracket (1+i_{mo})^{-144}$$

$$PV = \$73{,}906.24 + [\$127{,}328.35(1+i_{mo})^{-144}]$$

$$PV = \$73{,}906.24 + \$48{,}908.69$$

$$PV = \$122{,}814.93$$



Calculation

Press	Display	Comments
8 I/YR	8	Stated nominal rate
12 ■ P/YR	12	Stated and desired compounding frequency
800 +/– PMT	–800	Payment for first 12 years
0 FV	0	
144 N	144	Remaining amortization period on current loan
PV	73,906.239614	PV of differential payment
→M	73,906.239614	
1800 +/– PMT	–1,800	Full maximum payment
96 N	96	Remaining amortization after 1st mortgage is repaid
PV	127,328.34689	Value at end of 144 months
127328.35 +/– FV	–127,328.35	
144 N	144	
0 PMT	0	
PV	48,908.687127	PV of payments
+ RM =	122,814.926741	Additional funds advanced

The maximum amount of additional funds is $122,814.93. Buyers might choose this option to obtain additional funds without losing the advantage of the current low interest rate on the existing first mortgage.

d. Summary:

New first mortgage: additional funds	$111,185.87
Second mortgage: additional funds	$82,899.69
Blend and extend: additional funds	$122,814.93

The blend and extend option provides the largest amount of new funds and is the best choice for the borrower because the borrower retains the low rate first mortgage and benefits from the higher payments over an extended amortization period. The second mortgage retains the lower rate first mortgage, but the second mortgage requires a higher interest rate. The new first mortgage generates more additional funds than the second mortgage because the borrower can qualify for a larger payment today; however, the borrower is subject to a penalty and must pay the higher current rate on the new first mortgage.

Nazzy Holding Case

In preliminary discussions with three lenders, Nazzy Holding has identified several alternative financing packages and seeks to determine a refinancing arrangement that will result in the largest equity release. This section of the chapter presents the alternatives, introduces the appropriate analytical techniques and, where appropriate, discusses implications regarding risk, yield (or cost), and cash flow to the lender and borrower.

Illustration 14.11(a): Fully Amortized Mortgage

Assume that Nazzy Holding is a limited company active in long-term real estate investment. The firm is wholly owned by private interests and has been active in long-term industrial and commercial real estate investments since its incorporation.

Six years ago, Nazzy Holding purchased, for $7,300,000, the 1040 Office Building, a 10-storey office building that was newly completed at the time of Nazzy's acquisition. Nazzy Holding is reviewing this investment in the context of the recent finalization of major rent reviews. While Nazzy has no desire to dispose of the building, it does wish to alter the financing. The property represents a large pool of equity, as the property's current (November 1) lending value of $15,000,000, is more than three times the current outstanding balance on the existing first mortgage. Nazzy wishes to refinance the property to obtain capital to use in other investment opportunities. Nazzy's objective, relative to 1040 Office Building, may be simply stated as a desire to refinance, using conventional sources, to release as much equity as possible.

1040 Office Building is in a Canadian metropolitan area on a major arterial approximately 15 minutes from the city centre. The 6-year old building is comprised of 10 stories. The gross area of each floor is 10,800 square feet. An entrance foyer, elevators, and stairs reduce net rentable area to 9,000 square feet on the main floor, with the remainder of the floors averaging 9,600 square feet of net rentable area. At the time of the purchase, the building was fully leased with all leases having 6-year terms (expiring on December 31 of the current year). Original rents averaged $7.25 per square foot, fully net. The original value of the improvements was $4,750,000 (of which $807,455 has been claimed for capital cost allowance on income taxes). Current annual net operating income is $900,000.

Initial financing was arranged as a first mortgage at 5.5% per annum, compounded semi-annually. Given the long-term lender's underwriting policy at the time, a $5,200,000 loan was initiated. The loan called for monthly payments, and a 25-year amortization period and contractual term. Such a loan satisfied the lender's criteria that a 75% loan-to-value ratio is not exceeded (lending value was set at $7,300,000; therefore, the initial loan-to-value ratio was 71.2%) and that a 15% safety margin existed between debt service and net operating income. The financial terms of the existing loan are summarized below:

Existing Loan[4]

Original Loan Amount:	$5,200,000
Contract Rate:	5.5% per annum, compounded semi-annually
Payments:	$31,740.36, monthly, commencing February 1, 6 years ago
Term and Amortization:	25 years/25 years commencing January 1, 6 years ago
Outstanding Balance (at date of refinancing):	$4,505,810.90 (January 1, this year – current OSB)

The first mortgage contains a prepayment prohibition; however, preliminary discussions with the first lender have determined that the lender will waive the prohibition without penalty providing the loan is fully prepaid.[5]

All leases have been recently reviewed. Rents have been renegotiated to the market rate of $14.25 per square foot (average), inclusive of base year expenses, effective January 1 of next year. In each case, leases were renegotiated with 10-year terms and further 10-year options to renew at negotiated rent levels.

Nazzy is considering three refinancing alternatives:

1. New First Mortgage
2. Traditional Second Mortgage
3. Blended Rate Mortgage

Alternative 1: New First Mortgage Refinancing

The first and most obvious alternative the owners could consider would be to pay out the existing mortgage and replace it with a new first mortgage at current rates and conditions.

Loan Constraints:	75% maximum loan-to-value
	15% minimum safety margin
Proposed Terms:	
Loan Amount:	No ceiling, subject to above constraints
Contract Rate:	9% per annum, compounded semi-annually
Payments:	Monthly
Term and Amortization:	25 years (fully amortized)

Calculate:

- The total amount of financing available to the investors under the terms of the refinancing, considering both the capital and income constraints suggested above.
- The amount of equity released because of the refinancing.

Solution:

Capital Constraint

Maximum Loan = Value × Loan-to-Value Ratio

Assuming the lending value is equal to the market value, the maximum loan is:

Maximum Loan = $15,000,000 × 0.75
Maximum Loan = $11,250,000

Income Constraint

In this case, given that the net operating income is $900,000 and the lender requires a 15% safety margin, only 85% of the net operating income is available to support the loan payments.

$$\text{Annual Debt Service} = \text{NOI} \times (1.0 - \text{Safety Margin})$$
$$\text{Annual Debt Service} = \$900{,}000 \times (1.0 - 0.15)$$
$$\text{Annual Debt Service} = \$900{,}000 \times 0.85$$
$$\text{Annual Debt Service} = \$765{,}000 \text{ (maximum allowed for annual debt service)}$$

$$\text{Maximum Monthly Payment} = \frac{\text{Annual Debt Service}}{12} = \frac{\$765{,}000}{12} = \$63{,}750 \text{ per month}$$

Find the maximum loan based on the income constraint.

$$PV = PMT \times a[\![n, j_2 = 9\%]\!]$$
$$PV = \$63{,}750 \times a[\![300, j_{12} = 8.835748\%]\!]$$
$$PV = \$7{,}699{,}485.62$$

Calculation

Press	Display	Comments
9 ■ NOM%	9	j_2
2 ■ P/YR	2	
■ EFF%	9.2025	j_1
12 ■ P/YR	12	
■ NOM%	8.835748	j_{12}
63750 +/– PMT	–63,750	Payment
300 N	300	Compounding periods
0 FV	0	
PV	7,699,485.6153	Maximum loan under income constraint

The binding constraint in this case will be the servicing income. Although the loan-to-value ratio results in a loan of $11,250,000, the servicing income will only support a loan of $7,699,485.62 (51% of lending value). Incremental funds advanced because of the refinancing equals $3,193,674.72.

Thus, the replacement first mortgage would provide total financing of $7,699,485.62 or 51% of the value of the security at the time of the refinancing. Incremental funds advanced because of the refinancing equals the new loan amount minus the balance outstanding on the original loan at the time of refinancing or $3,193,674.72 ($7,699,485.62 – $4,505,810.90).

The new first mortgage replaces the existing loan, eliminating the obvious benefits associated with the relatively low 5.5% rate on the existing financing.

Rather than retaining the outstanding balance of $4,505,810.90 at 5.5% per annum, compounded semi-annually, the borrower will replace this amount by the same amount on the new loan but will pay 9% per annum, compounded semi-annually on these funds. Clearly, the owners will want to consider refinancing alternatives that will leave the lower rate of 5.5% on the current outstanding balance and only pay the higher rate on additional funds.

Alternative 2: Traditional Second Mortgage Refinancing

The second refinancing option available to the property owners is to obtain a conventional second mortgage. This alternative will have the desired effect of leaving the existing mortgage in place. The equity dollars will be released by arranging for additional mortgage funds by further encumbering the property. The traditional approach in this context is to obtain a second mortgage, which, given the prior claim held by the existing lender and the narrower debt service margin typically allowed, involves greater lender risk. As a result, the second lender will require a higher interest rate, and likely a shorter amortization period, than would prevail on a new first mortgage. The specific terms of this alternative are as follows:

Loan Constraints:	85% loan-to-value maximum for first and second combined 8% minimum safety margin
Proposed Terms:	
Loan Amount:	No ceiling, subject to above constraints
Contract Rate:	13% per annum, compounded semi-annually
Payments:	Monthly
Term and Amortization:	19 years (to match existing first mortgage)

Calculate the maximum amount of the second mortgage and the total amount of financing available to the property owners.

Solution:

Capital Constraint

Maximum Total Financing = Value × Loan-to-Value Ratio
Maximum Total Financing = $15,000,000 × 0.85

Maximum Total Financing = $12,750,000
Maximum Additional Financing = Maximum Funds – OSB Existing First Mortgage
Maximum Additional Financing = $12,750,000 – $4,505,810.90
Maximum Additional Financing = $8,244,189.10

Income Constraint

Annual Debt Service = NOI × (1.0 – Safety Margin)
Annual Debt Service = $900,000 × (1.0 – 0.08)
Annual Debt Service = $900,000 × 0.92 = $828,000

$$\text{Maximum Monthly Payment} = \frac{\text{Annual Debt Service}}{12} = \frac{\$828{,}000}{12} = \$69{,}000$$

	Maximum amount available to monthly debt service	$69,000.00
–	Payments to existing first mortgage lender	– 31,740.36
	Maximum payments on second mortgage	$37,259.64

To calculate the maximum amount of the second mortgage, determine the present value of the available payments at the contract rate of 13% per annum, compounded semi-annually.

$PV = PMT \times a⟦n, j_2 = 13\%⟧$
$PV = \$37{,}259.64 \times a⟦228, j_{12} = 12.661289\%⟧$
$PV = \$3{,}208{,}765.49$

Calculation

Press	Display	Comments
13 ■ NOM%	13	j_2
2 ■ P/YR	2	
■ EFF%	13.4225	j_1
12 ■ P/YR	12	
■ NOM%	12.661289	j_{12}
37259.64 +/– PMT	–37,259.64	Payment
228 N	228	Compounding periods
0 FV	0	
PV	3,208,765.49248	PV of payments

Once again, the binding constraint on the size of the combined mortgages is the income limitation. While a second mortgage of $8,244,189 would be permitted under the loan-to-value ratio limit, the application of the income constraint margin results in a maximum second mortgage loan of $3,208,765.49. In summary, the traditional second mortgage provides for the following:

	First Mortgage	Second Mortgage	Total
Loan Amount	$4,505,810.90	$3,208,765.49	$7,714,576.39
Monthly Payment	$31,740.36	$37,259.64	$69,000.00
Loan-to-Value			51%
Remaining Amortization	228	228	

Thus, the second mortgage alternative will provide the property owners with additional funds of $3,208,765.49 or total financing of $7,714,576.39 for the existing first and new second mortgage. This level of debt finances the property to a total of 51% of value, well below the maximum permitted by the capital value constraint. Relative to the new first mortgage option, this option provides additional funds of $15,090.77 ($7,714,576.39 – $7,699,485.62) and leaves the existing first mortgage in place. Not only will greater equity be released, but also the attractive rate prevailing on the existing first mortgage is retained. Note that the borrowers will have less cash flow left each month because of higher total mortgage payments.

Alternative 3: Blended Rate Refinancing

The investors may also approach the existing lender and attempt to arrange a blended rate replacement first mortgage,[6] which would recognize the lower interest rate currently being charged on the outstanding $4,505,810.90. In attempting to retain the benefit of this lower rate, and yet obtain additional funds, the borrowers will have to offer some form of inducement to the lender. The lender is being asked to sacrifice an extremely secure existing mortgage for the increased risks of a standard first mortgage.

The "sweetener" offered in these cases is generally an agreement that the borrower will pay more than the prevailing first mortgage rate on the additional funds the lender will advance. Thus, the "new" mortgage will be larger than the current outstanding balance, will have a lower contract interest rate than would exist on the new standard first mortgage, and the lender will yield more than the prevailing first mortgage rate on the additional funds advanced to the borrower. It is unlikely that a prepayment penalty would be charged on such a replacement loan with the same lender.

Nazzy Holdings has approached the lender who advanced the existing first mortgage and was offered a blended rate replacement first mortgage. The lender offered new funds at a rate of 10% per annum, compounded semi-annually, on the new funds. The replacement mortgage is to be fully amortized.[7]

Loan Constraints:	75% loan-to-value maximum 15% minimum safety margin
Proposed Terms:	
Loan Amount:	No ceiling, subject to above constraints
Contract Rate:	To be determined – lender is to yield j_2 = 10% on incremental funds advanced

Calculate:

- The amount of new monies advanced.
- The total amount of financing subsequent to the refinancing.
- The appropriate contract rate on the blended rate mortgage.

Solution:

Capital Constraint

Maximum Total Financing = Value × Loan-to-Value Ratio
Maximum Total Financing = \$15,000,000 × 0.75 = \$11,250,000
Maximum Additional Funds = Maximum Funds – OSB Existing First Mortgage
Maximum Additional Funds = \$11,250,000 – \$4,505,810.90
Maximum Additional Funds = \$6,744,189.10

Income Constraint

Annual Debt Service = NOI × (1.0 – Safety Margin)
Annual Debt Service = \$900,000 × (1.0 – 0.15)
Annual Debt Service = \$900,000 × 0.85 = \$765,000

$$\text{Maximum Monthly Payment} = \frac{\text{Annual Debt Servce}}{12} = \frac{\$765{,}000}{12} = \$63{,}750 \text{ per month}$$

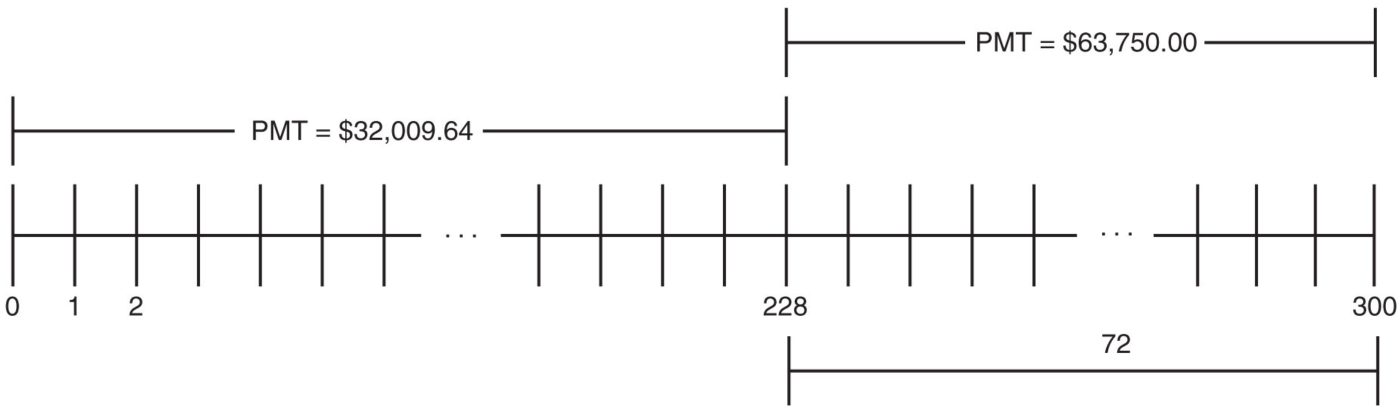
The amount of additional funds advanced would be the present value of the incremental payments made by the borrower, discounted at the lender's desired yield rate on new funds advanced (j_2 = 10%). As the existing mortgage would run for another 19 years in the absence of the refinancing, the incremental payment stream would be as follows:

	Years 1-19	Years 20-25
Available for Debt Service	\$63,750.00	\$63,750.00
– Payments on Existing First	– 31,740.36	– 0.00
Incremental Payments	\$32,009.64	\$63,750.00

The present value of this uneven cash flow, at j_2 = 10%, would be determined with the following relationship:

$$PV = \$32{,}009.64 \times a⟦228, j_2 = 10\%⟧ + \$63{,}750 \times a⟦72, j_2 = 10\%⟧(1+i)^{-228}$$

$$PV = \$3{,}848{,}341.15$$



Calculation

Press	Display	Comments
10 ■ NOM%	10	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	10.25	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	9.797815	Equivalent j_{12} rate
32009.64 +/– PMT	–32,009.64	Payment
228 N	228	Compounding periods
0 FV	0	
PV	3,306,462.66704	PV of payments in Years 1-19
→M	3,306,462.66704	Store for later use
72 N	72	Amortization period for Years 20-25
63750 +/– PMT	–63,750	Payment
PV	3,460,152.75874	PV of Years 20-25 valued at Year 20
3460152.76 +/– FV	–3,460,152.76	FV amount
228 N	228	19 years
0 PMT	0	No intervening payments
PV	541,878.485022	PV of payments in Years 20-25
+ RM =	3,848,341.15206	Maximum loan

Thus, under the fully amortized 25-year blended rate alternative, the amount of new money advanced by the lender is $3,848,341.15 that, when combined with the outstanding balance on the existing first mortgage, provides a total of $8,354,152.05 ($3,848,341.15 + $4,505,810.90). Note that once again the income constraint is the binding constraint on the maximum loan amount.

This option provides additional capital release in comparison to the traditional second mortgage (total financing with the second mortgage of $7,714,576.39 compared to $8,354,152.05 under the blended rate arrangement). In addition, the borrower has financing in place for a full 25-year period under this option.

The rate that the blended rate first mortgage would be written at would be the rate at which the total payments ($63,750 for 25 years) repay the total loan amount ($8,354,152.05) over a 25-year amortization.

$$PV = PMT \times a[\![n, j_{12} = ?]\!]$$
$$\$8{,}354{,}152.05 = \$63{,}750 \times a[\![300, j_{12} = ?]\!]$$
$$j_{12} = 7.868019\%$$
$$j_2 = 7.998122\%$$

Calculation

Press	Display	Comments
8354152.05 PV	8,354,152.05	Loan amount
63750 +/– PMT	–63750	Payment
300 N	300	Compounding periods
0 FV	0	
12 ■ P/YR	12	
I/YR	7.868019	j_{12}
■ EFF%	8.158047	j_1
2 ■ P/YR	2	
■ NOM%	7.998122	j_2

Under the conditions proposed in the fully amortized blended rate alternative, the borrowers will receive a new loan with a face value of $8,354,152.05 at a nominal rate of 7.998122% per annum, compounded semi-annually and the loan is fully amortized over 25 years. The existing first mortgage is surrendered.

Summary of Refinancing Alternatives

Table 14.2 summarizes each of the three options, showing the amount of new funds disbursed, payments to be made, and cost of borrowing (yield on investment).

Table 14.2: Summary of Refinancing Options

	Alternative Type of Loan	Net New Funds Released	Incremental Payments	Yield on Investment (or Cost of Borrowing)
1	New First	$3,193,674.72	$32,009.64 per month for 228 months + $63,750 per month for 72 months	j_2 = 12.4%
2	Traditional Second	$3,208,765.49	$37,259.64 per month for 228 months	j_2 = 13%
3	Blended Rate	$3,848,341.15	$32,009.64 per month for 228 months + $63,750 for subsequent 72 months	j_2 = 10%

Each of the alternative refinancing methods presented creates significant variances from the perspective of risk management, yield (and cost), and cash flow. The first possibility required that the borrower retire an existing loan with a significant remaining term and a reasonably low rate of interest. The rate charged on the new mortgage (9% per annum compounded semi-annually) was deemed to be the market rate for new first mortgages and, as such, would be considered to compensate the lender for their cost of capital, overhead, profit, risk, and lessened liquidity. Under this alternative, the borrower simply "lost" the benefits associated with the low rate prevailing on the original loan. In contrast, the blended rate first mortgage has the borrower and lender ultimately bargaining to "share" the impact of retiring the existing loan through a negotiated discounted payout. Regarding the yield (and/or cost) associated with each option, the analyst can readily compare the amount of the lender's (new) investment with the resulting income over the loan term.

The refinancing arrangements generate a broad range of possibilities: equity released by two plans is just over $3 million and yet under the blended rate option increases to over $3.8 million. The rate earned by the lender (or conversely, paid by the borrower) varies from a low of 10% per annum to a high of 13%. These variations in mortgage form, in dollar amount, contractual term, and rate, and the resulting implications for cash flow and default risk must be analyzed by both parties.

In this case, the blended rate option provides the best course of action, generating the largest amount of new funds released. This option retains the relatively low interest rate on the existing loan while the borrower pays a slightly higher rate on the additional funds (compared to a new first mortgage). The larger loan amount satisfies the borrower, while the higher rate that is charged on the additional loaned funds satisfies the lender, as it acts as compensation for the additional risk that the lender must undertake.

Illustration 14.11(b): Negotiated Discount

In Illustration 14.11(a)'s first mortgage alternative, the borrower is approaching the lender to prepay the existing loan and originate a new loan at a much higher interest rate and for more funds. Clearly this would be an attractive business opportunity for the lender. To encourage this possibility, it stands to reason the lender would be willing to waive any prepayment penalties. However, a savvy borrower might also attempt to negotiate with the lender to share some of the benefit from the increased interest rate and loaned funds. This might take the form of a discount on the new loan.

Solution:

Find the present value of the first mortgage payment over its existing term using the current first mortgage rate of 9% per annum, compounded semi-annually.

$PV = PMT \times a[\![n, j_2 = 9\%]\!]$

$PV = \$31{,}740.36 \times a[\![228, j_{12} = 8.835748\%]\!]$

$PV = \$3{,}501{,}380.24$

Calculation

Press	Display	Comments
9 ■ NOM%	9	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	9.2025	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	8.835748	Equivalent j_{12} rate
31740.36 +/– PMT	–31,740.36	Payment
228 N	228	Number of payments
0 FV	0	
PV	3,501,380.2381	Value at current market rates

The above analysis indicates that the lender would be indifferent either to allowing the existing loan to run to maturity or accepting a prepayment of $3,501,380.24 to fully discharge the loan and reinvesting these funds[8] at the going market rate. Providing the lender can reinvest the funds at 9% per annum, compounded semi-annually, both courses of action will result in a monthly cash flow to the lender of $31,740.36. Therefore, this arrangement would entail the lender accepting a discounted prepayment of no less than $3,501,380.24. The maximum amount of the discount is $1,004,430.66 ($4,505,810.90 – $3,501,380.24).

Some form of probability analysis could be employed by the lender to quantify the likelihood of the borrower prepaying the existing financing if the lender did not release the mortgage now. Having ascribed probabilities to the likelihood of this event at different points in time, the lender could determine a buy-out figure derived from a present value analysis weighted by discounted values assuming prepayment at a variety of points throughout the remaining term. A more intuitive approach would involve determining a mutually agreeable prepayment figure through direct negotiation with the borrower. For the purposes of this illustration, let's assume that the parties to the original contract have agreed, through negotiation, to a $550,000 discount on the prepayment of the existing mortgage. A new standard first mortgage will then be initiated, which is identical to the terms determined in Illustration 14.11(a).

In other words, the lender will accept:

	Outstanding Balance at Contract Rate		$4,505,810.90
–	Discount for Prepayment	–	550,000.00
	Net Pay-Out		$3,955,810.90

As all underwriting criteria are unchanged relative to Illustration 14.11, the total proceeds of the new first loan will once again be $7,699,485.62. However, in this instance the borrower will receive an additional benefit of $550,000 because of the prepayment discount negotiated to discharge the existing first mortgage, thereby increasing the amount of capital released to $3,743,674.72.

	New First Mortgage		$7,699,485.62
–	Cash to Discharge Existing First	–	3,955,810.90
	Net Pay-Out		$3,743,674.72

With the payment unchanged, $31,740.26 for 228 months and $63,750 for 72 further months, increasing the net new funds released from $3,193,674.71 to $3,743,674.71 decreases the cost of borrowing from 12.4% to 10.2%. This is now just slightly above the cost of borrowing for the blended rate alternative. If the negotiated discount is significant, the new first mortgage could be the more desirable option for the borrower.

Illustration 14.11(c): Partially Amortized Mortgage

The analysis presented in Illustration 14.11(a) is a practical example of how a borrower might approach the problem of raising additional funds through refinancing. However, it is not realistic, in that fully amortized loans in Canada are rare to the point of non-existent. To add more realism to this illustration, we will now repeat the analysis but with the assumption that the original loan was for a 10-year term – of which 6 years have elapsed, so there are 4 years remaining in the term. The loan's current outstanding balance is $4,505,810.90 (six years from loan origin). At the end of 4 years from now (10 years from loan origin), the original loan's outstanding balance is $3,900,273.78.

The refinancing alternatives are all presented for 5-year terms, so an additional 12 months beyond the existing term, at which time the outstanding balance is due. How does this impact Nazzy Holding's refinancing decision?

Note for the purposes of simplifying this illustration, we will ignore the discount issue in 14.11(b) and assume the interest rates are the same as in the fully amortized scenario.

Solution:

We will not show detailed calculations for each alternative but instead simply summarize them in Table 14.3.

Table 14.3: Summary of Refinancing Options: Partially Amortized

	Alternative Type of Loan	Net New Funds Released	Incremental Payments	Yield on Investment (or Cost of Borrowing)
1	New First	$3,193,674.72	$32,009.64 per month for 48 months + $63,750 per month for 12 months + $7,169,450.41 OSB in 60 months (and less $3,900,273.78 OSB that no longer must be paid in 48 months)	j_2 = 12.3%
2	Traditional Second	$3,208,765.49	$37,259.64 per month for 60 months + $2,925,805.81 OSB in 60 months	j_2 = 13%
3	Blended Rate	$3,230,861.47	$32,009.64 per month for 48 months + $63,750 for subsequent 12 months + $7,199,713.26 OSB in 60 months (and less $3,900,273.78 OSB that no longer must be paid in 48 months)	j_2 = 12.1%

Revisiting Illustration 14.11(a) by removing the simplifying assumption of full amortization yields interesting results. In Illustration 14.11(c), the blended payment option is no longer as attractive, providing similar cost of borrowing as the first mortgage. This is sensible because the benefit of a blended rate alternative is primarily to take advantage of an established low mortgage rate that extends over a long term. Here, the borrower gets the benefit of keeping the low 5.5% first mortgage rate for 48 months but pays for this with a higher rate on the remaining 12 months (the lender's 10% yield versus the 9% new first mortgage rate). Because of the shortened term of the initial loan, the advantageous low interest rate does not extend for a long enough term to overcome the impact of the blended mortgage's higher overall rate.

Illustration 14.11(d): Decrease in First Mortgage Rates

What if interest rates decrease? How does this change the scenario and analysis? Assume first mortgage interest rates have decreased over the six years since Nazzy Holdings purchased this property. Assuming Nazzy Holdings still wishes to refinance and seek additional funds, how does this decreasing interest rate scenario affect their decision?

[Note: to simplify this illustration, we will ignore the discount issue in 14.11(b) and the partially amortized calculations in 14.11(c) – this illustration continues from the original calculations in Illustration 14.11(a).]

Alternative 1: New First Mortgage Refinancing at j_2 = 5%

Calculate:

- The total amount of financing available to the investors under the terms of the refinancing, considering both the capital and income constraints suggested above.
- The amount of equity released because of the refinancing.

Solution:

Capital Constraint

Assuming the lending value is equal to the market value, the maximum loan is $11,250,000.

Income Constraint

Find the maximum loan based on the income constraint.

$$PV = PMT \times a[\![n, j_2 = 5\%]\!]$$
$$PV = \$63,750 \times a[\![300, j_{12} = 4.948699\%]\!]$$
$$PV = \$10,961,047.73$$

Calculation

Press	Display	Comments
5 ■ NOM%	5	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	5.0625	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	4.948699	Equivalent j_{12} rate
63750 +/– PMT	63,750	Payment
300 N	300	Number of payments
0 FV	0	
PV	10,961,047.7283	Maximum loan

The binding constraint in this case is still the servicing income. Although the loan-to-value ratio would result in a loan of $11,250,000, the servicing income will only support a loan of $10,961,047.73 (73% of lending value). Incremental funds advanced because of the refinancing would be $6,455,236.83.

If we continue to assume the first mortgage has no prepayment penalty, the new first mortgage option is attractive because the new mortgage has a lower rate of interest, both on the new funds borrowed and on the remaining term for the existing funds.

However, ignoring the prepayment penalty might not be realistic in this scenario, as a lender would be reluctant to refinance at a lower rate without compensation. Prepayment penalties are established in the initial mortgage contract. If the penalty is based on interest rate differential (IRD), the penalty would recover the difference in interest over the remaining 19 years. This is calculated as the difference in monthly interest rate × current OSB × 228 months remaining in the term, or $427,606.83. This large penalty for refinancing would reduce the amount of additional funds available to the borrower and potentially make this an unattractive option. However, if the lender charged a prepayment penalty of 3 months' interest, this would cost $61,256.70. With a lower penalty, there is an opportunity for interest savings for the borrower in a decreasing interest rate environment.

Alternative 2: Traditional Second Mortgage Refinancing at j_2 = 9%

Calculate the maximum amount of the second mortgage and the total amount of financing available to the property owners.

Solution:

Capital Constraint

Maximum total financing is $12,750,000 and maximum additional financing is $8,244,189.10.

Income Constraint

	Maximum amount available to monthly debt service	$69,000.00
–	Payments to existing first mortgage lender	– 31,740.36
	Maximum payments on second mortgage	$37,259.64

$$PV = PMT \times a[\![n, j_2 = 9\%]\!]$$
$$PV = \$37{,}259.64 \times a[\![228, j_{12} = 8.835748\%]\!]$$
$$PV = \$4{,}110{,}229.60$$

Calculation

Press	Display	Comments
9 ■ NOM%	9	Stated nominal rate
2 ■ P/YR	2	Stated compounding frequency
■ EFF%	9.2025	Equivalent effective annual rate
12 ■ P/YR	12	Desired compounding frequency
■ NOM%	8.835748	Equivalent j_{12} rate
37259.64 +/– PMT	37,259.64	Payment
228 N	228	Number of payments
0 FV	0	
PV	4,110,229.59962	Maximum loan

Once again, the binding constraint on the size of the combined mortgages is the income limitation. While a second mortgage of $8,244,189 would have been permitted under the loan-to-value ratio limit, the application of the income constraint margin results in a maximum second mortgage loan of $4,110,229.60. In summary, the traditional second mortgage provides for the following:

	First Mortgage	Second Mortgage	Total
Loan Amount	$4,505,810.90	$4,110,229.60	$8,616,040.50
Monthly Payment	$31,740.36	$37,259.64	$69,000.00
Loan-to-Value			57%
Remaining Amortization	228	228	

The second mortgage alternative will provide the property owners with additional funds of $4,110,229.60. Combining the first and second mortgages, total financing is $8,616,040.50. This level of debt finances the property to a total of 57% of value, well below the maximum permitted by the capital value constraint. Relative to the new first mortgage option, this option provides fewer additional funds of $2,345,007.23 ($10,961,047.73 – $8,616,040.50). While this option leaves the existing first mortgage in place during and beyond the five-year term of the second mortgage, the high cost of the second mortgage makes this option less desirable. In addition, the borrowers will have less cash flow left each month because of higher total mortgage payments.

Alternative 3: Blended Rate Refinancing at j_2 = 10%

A blended rate alternative does not make logical sense if interest rates have dropped. From the borrower's perspective, the benefit of a blended rate refinancing option is to free up additional funds at the market rate while at the same time taking advantage of the low interest rate on existing financing. If interest rates have dropped, the borrower has no incentive to maintain the existing first mortgage, so a blended alternative offers little or no benefit.

This assumes prepayment penalties on the new first mortgage option are not prohibitive. If prepayment penalties are substantial, it may be less costly for the borrower to pay the normally higher rates on a second mortgage, to avoid these penalties. In either case, it is unlikely that the blended rate alternative would provide any benefit worth pursuing.

WRAP-AROUND MORTGAGES

Wrap-around mortgages are a creative form of providing additional financing where an existing mortgage encumbers property. While wrap-around mortgages are no longer used in contemporary mortgage lending in Canada because of legal decisions limiting their function, they are still used in other jurisdictions and for that reason it may be of interest to know how they operate.

Where a first mortgage is already registered against a property and the owner requires additional financing, the owner has several alternatives:

a. if the first mortgage allows, it can be paid out and a new, larger first mortgage on the title can be arranged; or

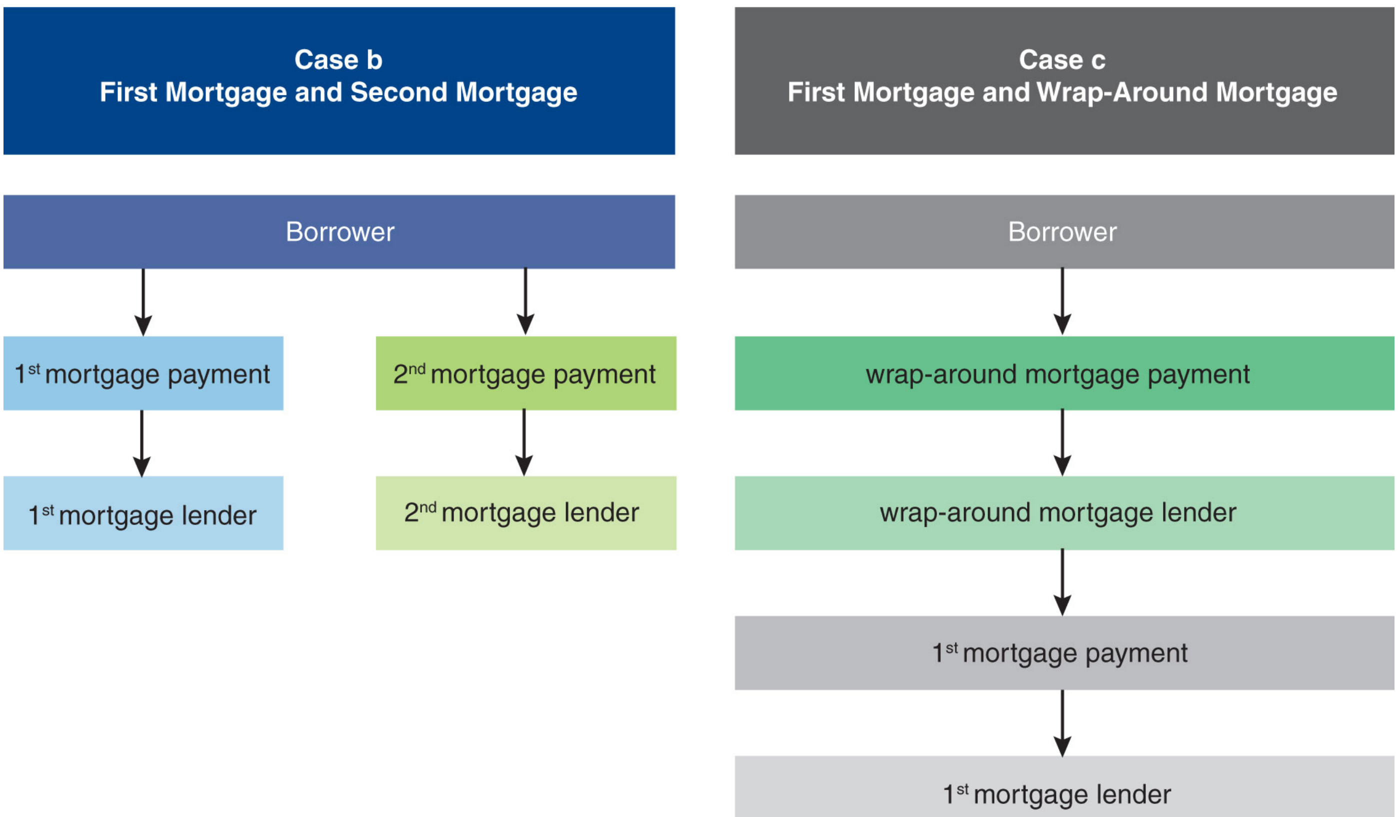
b. a conventional second mortgage can be arranged to raise the extra funds and the existing first mortgage can be kept in place; or

c. the borrower may wish to arrange a second mortgage containing a wrap-around clause.

A borrower would prefer b and c if the first mortgage has a very low interest rate or if the first mortgage has a large prepayment penalty.

In both b and c there are two mortgages registered against the property. However, the flow of payments is different. Figure 14.5 shows the way in which payments flow in each case.

FIGURE 14.5: Flow of Payments



With a wrap-around mortgage, the borrower makes only one payment to the wrap-around lender. The wrap-around lender keeps its part of the payment and forwards the first mortgage payment amount to the first mortgage lender. Since the wrap-around lender controls the first mortgage payments, there is less risk of payments being late. Because of the lower risk, the wrap-around lender charges a lower interest rate on the wrap-around mortgage than a lender would charge on a conventional second mortgage.

The mortgage contract for the wrap-around mortgage will show an amount equal to the outstanding balance on the first mortgage plus the extra funds the borrower is borrowing. Therefore, it will appear as if the borrower owes more than they do because the first mortgage contract will show the amount borrowed under that mortgage loan and the wrap-around will show the amount still owed under the first mortgage, plus the new amount being borrowed. However, a clause in the wrap-around mortgage will state that only the new amount and interest on it need to be repaid to obtain a discharge of the wrap-around. In practice, wrap-around loans have a rate of interest higher than the rate on the existing first mortgage but roughly the same as rates on new conventional first mortgages. An example of a wrap-around clause is shown below:

> …The said payments, when made, shall be applied firstly in payment of the monthly instalments due and payable under the First Mortgage, secondly in payment of interest as aforesaid, thirdly on all other amounts due hereunder and the balance, if any, upon principal monies secured hereby.
>
> AND THE MORTGAGOR covenants and agrees to pay the monthly instalments under the First Mortgage up to and including the payment due on the first day of __________, 20___.
>
> PROVIDED, however, that after redemption of the First Mortgage each of the said monthly instalments shall be applied firstly in payment of interest as aforesaid, secondly on all other amounts due hereunder and the balance, if any, upon principal monies secured hereby.
>
> It is further acknowledged by the Mortgagor that although the amount actually advanced to the Mortgagor hereunder is the principal sum less the amount owing under the First Mortgage for the purpose of computation of interest hereunder, the principal sum shall be deemed to have been fully advanced.

A wrap-around mortgage can allow a borrower to obtain additional funds at a rate below the market rate for such funds, similar to blended rate financing. Mathematically, a wrap-around mortgage is much the same as a blended rate mortgage. The key difference conceptually is a wrap-around mortgage is held by a second mortgage lender rather than offered as an extension of the first mortgage.

CONCLUSION

Mortgage brokers should be aware of the various mortgage repayment plans and refinancing options that can help bring borrowers and lenders together. In this chapter, we examined several advanced loan repayment options: variable rate mortgages, graduated payment mortgages, sinking fund assisted mortgages, and reverse annuity mortgages. Variable rate mortgages and reverse annuity mortgages are two plans that have seen a significant rise in use over the past decade.

As well, this chapter analyzed borrowers' refinancing options – a subject that mortgage brokers will encounter frequently. The ability to evaluate options such as a new first mortgage, a second mortgage, or a blended rate mortgage will enable mortgage brokers to find the best available product for their client.

1 It is important to understand that the “face amount” or “face value” of the loan does not necessarily correspond to the number of dollars that end up in the borrower’s hands. Borrowers (both residential and commercial) frequently covenant to repay more than the net amount they receive by virtue of the mortgage arrangement. This may be the result of disbursements such as insurance, taxes, approval charges, and legal fees, or it may be the result of a bonus or brokerage fee, charges that are intended either as additional compensation to the lender for advancing funds or to compensate mortgage brokers for their services in arranging financing.

2 The analysis presented on this page would be very similar if the fees had been included in the face value of the loan. For example, to determine the cost of funds advanced to the borrower from the original lender, one would enter $163,377.98 as the total loan amount (initial PV), then enter $163,277.98 for the funds advanced (subsequent PV). The remaining calculator steps and the conclusion are the same, with the result being a cost of funds of 5.3244% on Option A and 5.11603% for Option B.

3 If this problem had been restructured to have fees added to the face value, a cost of marginal funds calculation would no longer work because the initial investment is $0 (the borrower has no up-front fees under either option). Instead, an analyst could calculate the present value of the benefits and costs of the second lender’s offer: savings of $24 per month and a reduced OSB_{60} of $73.88. Discounted at, say, j_1= 5%, the present value is $1,333.10 (this is also the net present value since the initial investment cost is $0). This indicates that the savings for Option B offers a benefit of $1,333.10 in present value terms, with no initial investment required – clearly a beneficial alternative.

4 Note: Using large loan amounts creates the potential for minor differences in the results due to different approaches used.

5 If the lender allowed prepayment, but did not waive the prepayment penalty, there would be additional costs upon refinancing that would reduce the amount of additional funds available. A penalty of 3 months’ interest would cost $61,256.70. Using the interest rate differential, the penalty would be much larger for this fully amortized loan; the prepayment penalty to recover the difference in interest over the remaining 19 years is $2,974,746.56 (calculated as the difference in monthly interest rate × current OSB × 228 months remaining in the loan term). It is logical that the lender would waive the prepayment in a situation like this, if given the opportunity to refinance with more funds at a higher interest rate.

6 One new mortgage based on the rate on the existing loan and the new rate.

7 Readers should note that fully amortized loans are very rarely initiated by lenders. In the case at hand, such an arrangement may prove possible as the lender is otherwise committed to carrying the original loan for another 19 years at a very unattractive rate. Under these conditions, the lender may elect to offer a replacement mortgage that is fully amortized to improve a “bleak” situation. A more realistic partially amortized example is presented later in this illustration.

8 Readers should note that the analysis presented presumes that funds can be reinvested at 9% per annum (with semi-annual compounding) over a 19-year period. Some uncertainty must be recognized here as virtually all loans currently involve terms considerably less than 19 years. Further, the analysis could be altered to incorporate differing rates to reflect the relative risk and liquidity of the original and the replacement mortgages.