**1 -** Yes

**2 -**
**i.** CSCI - 5521 Machine Learning Fundamentals
In addition, I've been in a 2 years of research developing Q-learning algorithms.

**ii.** Mathematics Probability and Statistics for Engineers (Bilkent University)
Mechanical Eng. Advanced Mathematical Techniques in Engineering (Bilkent University)

**iii.** Mathematics Linear Algebra (Bilkent University)

**iv.** Mechanical Eng. Advanced Mathematical Techniques in Engineering (Bilkent University)
EE - 5239 Nonlinear Optimization (Currently taking)

**3 -**
The objective function $J(\omega)$:

$$J(\omega) = \frac{1}{2}\|y - X\omega\|^2 + \frac{\lambda}{2}\|\omega\|^2$$

Take the derivative of $J(\omega)$ with respect to $\omega$:

$$\nabla J(\omega) = -X^T(y - X\omega) + \lambda\omega$$

Set the derivative equal to zero to find the critical points:

$$-X^T(y - X\omega) + \lambda\omega = 0$$

Rearrange the equation to solve for $\omega^*$:

$$X^T X\omega^* + \lambda\omega^* = X^T y$$

Factor out $\omega^*$:

$$(X^T X + \lambda I)\omega^* = X^T y$$

To find $\omega^*$, multiply both sides by $(X^T X + \lambda I)^{-1}$:

$$\omega^* = (X^T X + \lambda I)^{-1} X^T y$$

Solution is valid regardless of relation between n and p. However, when n¡p, $X^T X$ may not be full rank therefore $(X^T X + \lambda I)$ wouldn't be invertible. Then, the solution wouldn't be unique. There are techniques to stabilize the solution when n¡p. These techniques basically make $X^T X$ invertible by adding small numbers to the matrix.

**4 -**

```
opt_w = np.matmul(np.matmul(np.linalg.inv(np.matmul(X.T, X) + lambda * np.identity(p)), X.T), y)
```

I initially thought using np.argmin benefiting the given equation in the homework but it wouldn't directly calculate $\omega^*$ based on the closed-form solution. It does not yield the actual $\omega^*$values

**5 -**
These types of problems are called Rayleigh quotient maximization/minimization problem. Let's say the solution to these problems is $\omega^*$. For maximization, $\omega^*$ is called the principal eigenvector or the dominant eigenvector of the positive definite matrix $A$. The principal eigenvector $\omega^*$ corresponds to the largest eigenvalue of the matrix $A$. For minimization, $\omega^*$ is the corresponding eigenvector associated with the smallest eigenvalue of $A$.