

Date

Data Structures

Scope Of a Variable

- ① Static Scoping
- ② Dynamic Scoping

int a=500, b=600;

Main()

```
{ int a,b;  
  a=10; b=20;
```

Swap();

```
{ printf("%d %d",a,b);  
 }
```

swap()

```
{ int t;
```

t=a;

a=b;

b=t;

```
printf("%d %d",a,b);  
 }
```

O/p :-

600 500

(means variable 10, 20
are not in this
fn.)

Static Scoping :-

O/p :- 600 500 10 20

when the free variables are taken from
as global variables.

Dynamic Scoping :-

O/p :- 20 10 20 10

when the free variables are taken from previous function
call till we reach main().

If we dont get variables in even in main, then we refer
dynamic/scoping global variable.

AC language follow Static Scoping, Older languages follow
dynamic scoping.

O/p in C :-

600 500 10 20

* int a=10, b=20;

main()

```
{ int a=1,b=1;  
  A();  
 }
```

A()

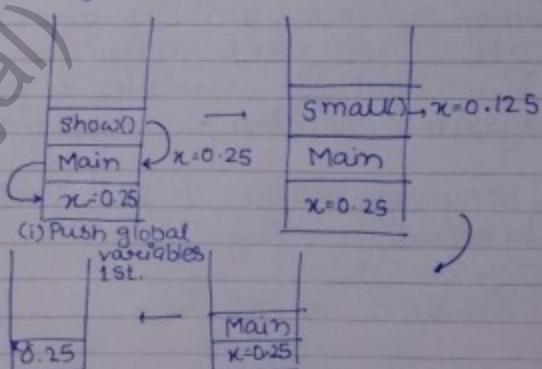
```
{ int a=2,b=2;  
  B();  
 }
```

B()

```
{ int a=3,b=3;  
  printf("%d %d",a,b);  
 }
```

Date

04.08.12

- boolean b=true;
procedure p
begin
 b
 pf(b);
end
boolean b=false
p;
end
- static scoping: true
dynamic scoping: false
- var x:real
procedure show()
begin
 pf(x);
end
procedure small()
begin
 var x:real
 x=0.125;
 show();
end
begin
 x=0.25
 Show();
 small();
end
- no name for this fn.
hence main
C: it is the main fn.)
- no concept of stack.
since at compile time, we know
only global variable.
(at compile time)
- Static scoping: 0.25 0.25
• dynamic scoping: 0.25 0.125
(at run time)
- For dynamic:-

- var x,y:integer
procedure p (var n:integer)
begin
 x=(n+2)/(n-3);
end
begin
 n=7; y=8;
- Static: 3 6
• dynamic: 6 7

```

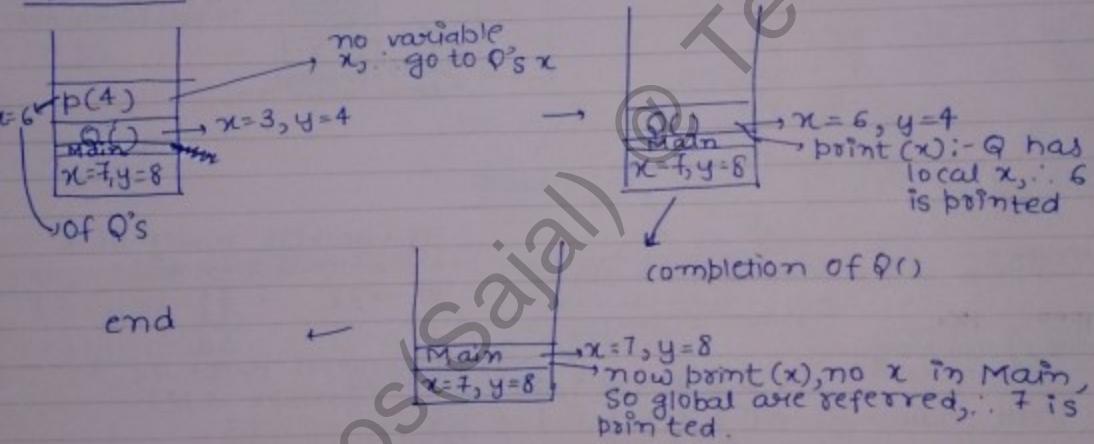
Q();
pf(x);
end
procedure Q
begin
var x,y:integer
x=3,y=4;
pf(y);
pf(x);
end

```

static:

completion of p(4)	x :- global $\rightarrow x = 6$
pf(x)	$x=3,y=4 \rightarrow \text{local}$
local available	Main $\rightarrow x=7,y=8 \rightarrow \text{global}$
3	x,y:0 global variable was modified $\therefore x = 6$

dynaminc:



```

Q. Var r:integer
procedure Two
begin
pf(r);
end
procedure One
begin
var r:integer
r=5;
two;
end

```

changed
by
main 2

```
begin
  r=2;
  two;
  one;
  two;
end
```

Static:

2 2 2

↑
printed
by bf(r) of
two (global
variable)

↑
printed by
bf(r) of two
(when called by
one)

dynamic:

two()
Main
r=0

no & here
So goes back to main
further we say go
to global

so go
back to
two()
no r
two() → r=5
one
Main
r=2

∴ O/p :- 2 5 2

two()
Main
r=2
two() completed
hence main too

one completed
no & here
so goes back to main

• int a; → big endian

• a=55; → 1000 1001
↓ little endian

1000	1001	0000 0000
0011 0111	0000 0000	

55 → nearest power of 2
(smaller is 2⁵)

$$2^5 \rightarrow 32 \rightarrow \frac{55}{32}$$

23 → nearest power of 2
(smaller is 2⁴)

$$2^4 \rightarrow 16 \rightarrow \frac{23}{16}$$

7 → nearest power of 2
(smaller is 2²)

$$2^2 \rightarrow 4 \rightarrow \frac{7}{4}$$

3 → nearest power of 2
(smaller is 2¹)

$$2^1 \rightarrow 2 \rightarrow \frac{3}{2}$$

1 → nearest power of 2
(smaller or equal)

$$2^0 \rightarrow 1 \rightarrow \frac{1}{0}$$

`pf(a) → 55`
`pf(&a) → 1000`
`pf(*(&a)) → 55`
 ↓
 55
 1000
 value at address 1000

`pf("%d", a) → 55`

`pf("%u", &a) → 1000`

unsigned
(for addresses)

`pf("%d", *(&a)); → 55`

int a, → [a]
garbage
1000

a = 55; → [a]
55
1000

a → 55

b → 1000

*b → 55

*a → error (compilation) • `printf("%u", b); → 1000`

&b → 2000

*(&b) → 1000

→ referencing operator

* → dereferencing "

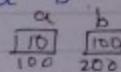
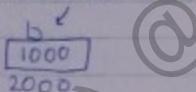
int a = 10, b = -1;

`printf("%u", a); → 10`

`printf("%u", b); → 2's complement
of -1 (if range
of int is upto
4294967295)
then the 0/b is
4294967295)`

int b = &a; // we can't write like this,
// because writing int b,
// we committed that we will
// store a value at 'b'

we write :- `int *b = &a;`



int a = 10;
 int b = (int)&a;
`printf("%d", a); → 10`
`printf("%d", &a); → 1000`
`printf("%d", b); → 100`
`printf("%d", *b); → compile time error`

Q1 int a = 10, → [a]
int *b = &a, → [b]
10
1000
2000

• single pointer :- storing address
of a value.

• double pointer :- storing address
of a pointer.

int **c = &b, → [c]
because b is
itself a pointer
2000
3000

int ***d = &c, → [d]
d is storing
an address which
is a pointer to a pointer.
3000
4000

- ★ If variable is holding a value, then to get the value \rightarrow no \ast req.
- ★ " " " " " single pointer, then to get the value \rightarrow 1 \ast , req.
- ★ " " " " " double pointers, " " " " " \rightarrow 2 \ast , req.

Parameter Passing Techniques

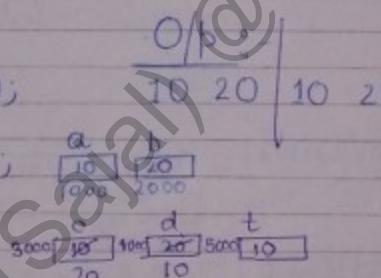
1. Call by value / Call by Copy
2. Call by reference
3. Call by copy-restore
4. Call by Name
5. Call by Need
6. Call by Text

Call by Value :-

main()

```

    { int a=10,b=20;
      printf ("%d %d",a,b);
      swap(a,b);
      printf ("%d %d",a,b);
    }
    swap(int c,int d)
    { int t;
      t=c; // c=20
      c=d; // d=10
      d=t;
    }
  
```



- ★ In C compiler, by default call by value is done,
i.e. swap(a,b) \rightarrow call by value
- ★ If any compiler supports call by reference by default
then swap(a,b) \rightarrow call by reference
(it will pass addresses of a & b, & not values of a & b)

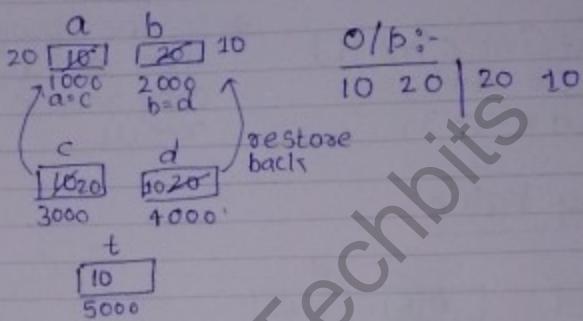
Call by Copy-Restore

```

main()
{
    int a=10, b=20;
    printf("%d %d", a, b);
    swap(a, b);
    printf("%d %d", a, b);
}
swap(int *c, int *d)
{
    int t;
    t = *c;
    c = d;
    d = t;
}

```

* executing the program in a compiler having call by copy-restore by default.



* Changes to actual parameter takes place at the end using call by copy restore.

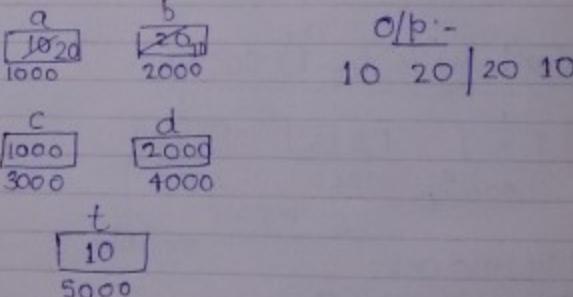
* Changes to actual parameter takes place on the fly using call by reference.

Call by Reference

```

main()
{
    int a=10, b=20;
    printf("%d %d", a, b);
    swap(&a, &b);
    printf("%d %d", a, b);
}
swap(int *c, int *d)
{
    int *t;
    t = *c;
    *c = *d;
    *d = t;
}

```



$$\frac{5}{5} \times 5 \\ 5 \times 5$$

Q. Consider the following program:-

```
main()
{
    int a=5;
    Test(a,a);
    printf(a);
}

Test(int c,int d)
{
    c+=d*2;
    d*=c-3;
}
```

- ★ Call by reference ~~or~~ call by copy restore gives diff. answer if the program contains aliasing.
- ★ If the program contains aliasing then call by reference & call by copy restore may ~~not~~ generate same result

Call by Value:-

a	c	d
5	815	860
1000	2000	3000

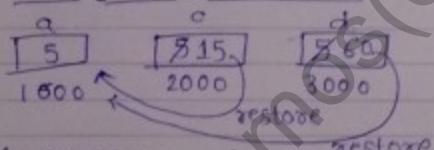
$$\begin{aligned} \text{C} &= \text{C} + \text{d} * 2 \rightarrow \text{C} = \text{C} + \text{d} * 2 \\ &= 5 + 10 \\ &= 15 \\ \text{d} * &= \text{c} - 3 \rightarrow \text{d} = \text{d} * (\text{c} - 3) \\ &= 15 * (12) \\ &= 15 * 12 \\ &= 180 \end{aligned}$$

in this case, always R.H.S. is calculated first irrespective of the priorities.

O/p:-
5

pink(assista)

Call by Copy-Restore:-



My answer:-

O/p:- 60 15/60

whichever is restored last is the output.

Call by Reference:-

a	c	d
5	180	1000
1000	2000	3000

$$\begin{aligned} (*\text{d}) &= (*\text{d}) * 2 \\ (*\text{c}) &= (*\text{c}) + (*\text{d}) * 2 \\ (*\text{c}) &= 5 + (5 * 2) \\ &= 15 \\ (*\text{d}) &= (*\text{d}) * ((*\text{c}) - 3) \\ &= 15 * (15 - 3) \\ &= 15 * 12 \\ &= 180 \end{aligned}$$

O/p:-
180

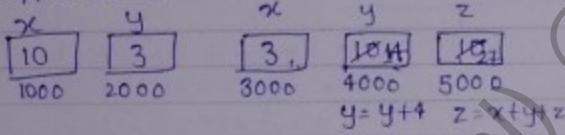
Q. Consider the following programs:-

```
main()
{ int x=10,y=3;
  fun(y,x,x);
  printf("%d,%d");
}
```

```
fun(int x,int y,int z)
{
  y = y+4;
  z = x+y+z;
}
```

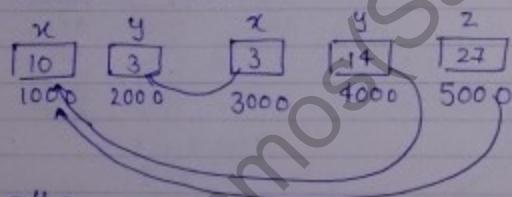
Call by Value :-

O/p:- 10 3



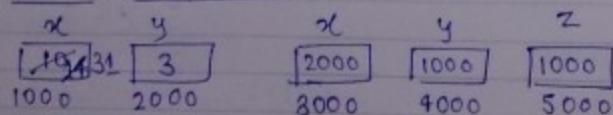
Call by Copy restore :-

Same as above



O/p:-
14/27 3

Call by reference



$$*y = *y + 4$$

$$= 10 + 4$$

$$= 14$$

$$*z = (*x) + (*y) + (*z)$$

$$= 10 + 14$$

$$= 3 + 14 + 14$$

$$*z = 31$$

O/p:-
31 3

Aliasing :- $y \rightarrow$

int $x;$ \rightarrow 

int $b = x;$ means y address is same as x address.
★ C doesn't support aliasing.

Q. [Imp.]

main()

{ int a, b; }

a = 2; b = 3;

P(a+b, a);

printf(a, b); } this is done
in temporary
variable & its
address is passed.

P(int x, int y, int z)

{ y = y+1;

z = z+x;

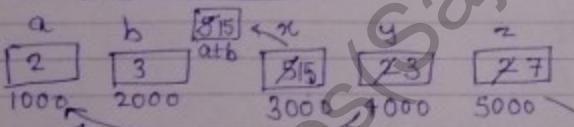
x = x+y+z;

}

Call by Value :-

O/p:- 2, 3

Call by Copy restore :-



$$y = y + 1 = 3$$

$$z = z + x = 2 + 5 = 7$$

$$x = 5 + 3 + 7 = 15$$

$$a = 3 / 7$$

$$at\ b = 15$$

if $a = 3$, then $b = 12$
if $a = 7$, then $b = 8$

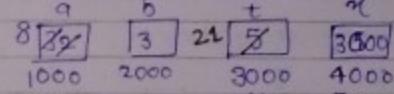
∴ O/p:-

3/7 12/8] → My answer

but answer:-

3/7 3]

Call by reference :-



$$\begin{aligned} *y &= *y + 1 \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} *z &= (*z) + (*x) \\ &= 3 + 5 = 8 \end{aligned}$$

$$\begin{aligned} *x &= (*x) + (*y) + (*z) \\ &= 5 + 8 + 8 \\ &= 21 \end{aligned}$$

O/p:-
8 3

Q. main()

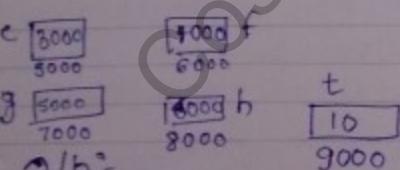
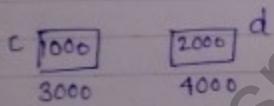
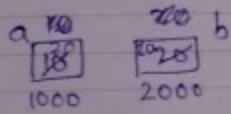
```
{ int a=10, b=20;  
    swap(a,b);  
    printf("%d",b);
```

```
swap(int *c, int *d)  
{ interchange(c,d);  
    printf("%d %d", *c, *d);
```

```
interchange(int *e, int *f)  
{ A(*e, *f);
```

```
A(int **g, int **h)  
{ int t;  
    t = *g; // t = ***g;  
    *g = *h; // ***g = ***h;  
    *h = t; // ***h = t;
```

Call by reference :-

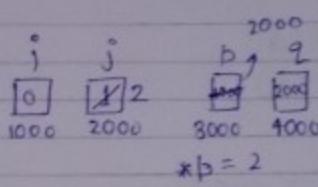


O/p:-
20 10

Q. Consider following C program:-

```
#include <stdio.h>
void f(int *p, int *q)
{
    p = q;
    *p = 2;
}
int i = 0, j = 1;
main()
{
    f(&i, &j);
    printf("%d %d", i, j);
    return(0);
}
```

∴ Output :- 0 2

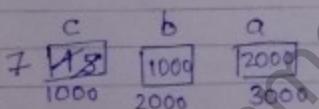


Ans: $\boxed{1000}$
 $\boxed{2000}$

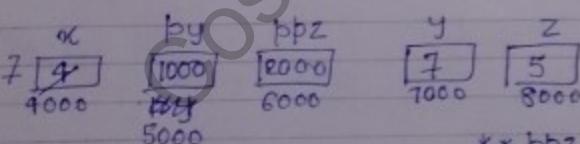
point f

Q. What is the o/p pointed by the following C program:-

```
main()
{
    int c, *b, **a;
    c = 4;
    b = &c;
    a = &b;
    printf("%d", f(c, b, a));
}
```



```
f(int x, int *py, int **pz)
{
    int y, z;
    **pz = 1;
    z = **pz;
    *py = 2;
    y = *py;
    x = 3;
    return (x+y+z);
}
```



$$\begin{aligned} **pz &= **pz + 1; & y &= *py; \\ &= 2000 + 1 = 2001 & y &= 2 \\ z &= **pz & x &= x + 3 \\ &= 2001 & &= 3 + 3 = 6 \\ *py &= *py + 2 & & \\ &= 1000 + 2 = 1002 \end{aligned}$$

O/p:-
 $\boxed{19}$

Q. Consider the following C program:- (Static Scoping by default as its a C program)

```
#include <stdio.h>
int x;
void Q(int z)
{ z+=x;
    printf(z);
}
```

```
void P(int *y)
{ int x = *y + 2;
    Q(x);
    *y = x - 1;
    printf(x);
}
```

```
main()
{x=5;
P(&x);
printf(x);}
```

x	y	x(of P)	z
6 [8]	1000	7	7 [12]
1000	2000	3000	4000

$x = *y + 2$
 $= 5 + 2$
 $= 7$
 $xy = 7 - 1 = 6$
 $z = 7 + 5 \text{ (global)}$
 $= 12$

O/p:
[12, 7, 6]

Q. int a[5] = {10, 20, 30, 40, 50};
 \rightarrow 1000 1002 1004 1006 1008
 \rightarrow 10 | 20 | 30 | 40 | 50
1000 to 1009

| array name indicates starting element address
| variable name indicate inside value.

a = 1000

*a = 10
 $a+1 = 1000 + 1 \times 2 = 1002$ [as integer is of 2 bytes]

$*a+1 = *(1002) = 20 = a[1]$

$\star + a$

$*a+4 = 50 = a[4]$

★ at time of preprocessing $a[4]$ is converted to $*(a+4)$.

$a[50] = *(a+50) = *(base address + 50 \times 2)$

random access is possible due to pointers.

$a+0 \rightarrow$ skipping 0 elements $\rightarrow a[0]$

$a+1 \rightarrow$ " 1 " $\rightarrow a[1]$

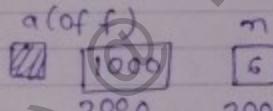
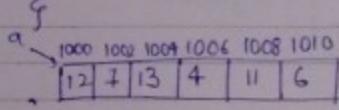
$a+2 \rightarrow$ " 2 " $\rightarrow a[2]$

Q. #include <stdio.h>

```
int f(int *a, int n)
{ if (n <= 0)
    return 0;
else
    if (*a % 2 == 0)
        return *a + f(a+1, n-1);
    else
        return *a - f(a+1, n-1);
```

main()

```
{ int a[] = {12, 7, 13, 4, 11, 6};
printf ("%d", f(a, 6));
return 0;
```



O/p :-
15

$$\begin{aligned} & 12 + (7 - (13 - (4 + (11 - (6 + 0))))) \\ &= 12 + (7 - (13 - (4 + 5))) \\ &= 12 + (7 - 9) \\ &= 12 + 3 = 15 \end{aligned}$$

$$\begin{aligned} & *a \% 2 = 12 \% 2 = 0 \\ & \therefore *a + f(a+1, n-1) = 12 + 3 = 15 \\ & 12 + f(1002, 5) \\ & 1 * 7 + f(1004, 4) \\ & 13 + f(1006, 3) \\ & 4 + f(1008, 2) \\ & 11 - f(1010, 1) \\ & 6 + f(1012, 0) \\ & 0 \end{aligned}$$

Q. What is the O/p of the following C-program?

```
#include<stdio.h>
int f(int n, int k)
{if (n == 0)
    return 0;
else
    if (n % 2)
        return f(n / 2, 2 * k) + k;
    else
        return f(n / 2, 2 * k) - k;
```



```
main()
{printf("%d", f(20, 1));
}
```

O/P :-
[]

f(20, 1) ← 9
f(10, 2) - 1 ← 10 - 1 = 9
f(5, 4) - 2 ← 10 - 2 = 10
* f(2, 8) + 4 ← 8 + 4 = 12
f(1, 16) - 8 ← 16 - 8 = 8
f(0, 32) + 16 ← 16

```
Q.
Main()
{int a[] = {10, 20, 30, 40, 50};
int *p[] = {a, a+1, a+2, a+3, a+4};
int **ptr = p;
ptr++;
printf("%d/%d/%d", ptr - p, *ptr - a, **ptr);
*(++ptr);
printf("%d/%d/%d", ptr - p, *ptr - a, **ptr);
++(*ptr);
printf("%d/%d/%d", ptr - p, *ptr - a, **ptr);
*ptr++; ← unary operator, execution from right to left
printf("%d/%d/%d", ptr - p, *ptr - a, **ptr);
```

ptr++ = p
& after semicolon
is encountered,
then only p is incremented

10	20	30	40	50
1000	1002	1004	1006	1008

P	1000	1002	1004	1006	1008
	2000	2002	2004	2006	2008

~~ptr~~ ~~ptr~~
 2000 3000
 $\rightarrow \text{ptr} + 1, \rightarrow \text{ptr} = \text{ptr} + 1 \rightarrow$ 2002
 3000

$$\begin{array}{ccccc} 0 & / & p & . & 7 \\ \cancel{2} & & \cancel{2} & & \cancel{20} \\ \cancel{4} & & \cancel{4} & & \cancel{30} \\ \cancel{4} & & \cancel{6} & & \cancel{40} \end{array}$$

It gives no. of elements

1st printf → $\text{ptr} - \text{p} = 2002 - 2000 = 2 \rightarrow$ 2 means only 1 element
 $*\text{ptr} - \text{a} = 1002 - 1000 = 2 \rightarrow$ 2 means only 1 element
 $**\text{ptr} = 22$

$$**pt\vartheta = 20$$

$\rightarrow * (+ + \text{ptr}) \rightarrow \text{ptr}$

2nd point f → $\text{ptr} - p = 2004 - 1000 = 1004$ → 4 means '2 no. of elements' * $\text{ptr} - a = 1004 - 1000 = 4$ → 4 means '2 no. of elements' ** $\text{ptr} = 30$

$\rightarrow ++(*\text{ptr}) = \text{*ptr} + 1$, $++(1004) = 1006$

	2000	2002	2004	2006	2008
Year	1000	1002	1006	1006	1008

pt^o [2004]
3000

\rightarrow 3rd pointf \rightarrow $\text{ptr} - \text{p} = 2004 - 2000 = 4 \rightarrow 4$ means '2' no. of elements.
 $*\text{ptr} - \text{a} = 1006 - 1000 = 6 \rightarrow 6$ means '3' no. of elements.
 $**\text{ptr} = 40$

$\rightarrow *ptr++$; $\rightarrow *ptr$ will take first $\rightarrow 1006$.

pt2 will not
take place till
; is encountered
pt2 takes place 1st, & after execution pt2 is incremented

	2000	2002	2004	2006	2008	pt2
	100	1002	1008	1006	1008	2006
						3000

(*pt2), (*pt2)+X;
after this statement ends, pt2 will take place

Note:- '*' and '++' having same priority, so we will go to associativity, & associativity is right to left.

→ 4th printf statement → $\text{ptr} - \text{p} = 2006 - 2000 = 6 \rightarrow 6$ means 3 elements
 $*\text{ptr} - a = 1005 - 1000 = 5 \rightarrow 5$ means '4' no. of elements
 $**\text{ptr} = 5/0 4 0$

Note:- If two addresses undergoes arithmetic opn.

O/p

1	1	20
2	2	30
2	3	40
24/150 33 40		

Q. #include <stdio.h>
main()
{ int a[] = {0, 1, 2, 3, 4};
int *p[] = {a, a+1, a+2, a+3, a+4};
int **ptr = p;
**ptr++;
printf("%d.%d.%d", ptr - p, *ptr - a, **ptr);
++*ptr;
printf("%d.%d.%d", ptr - p, *ptr - a, **ptr);
++**ptr;
printf("%d.%d.%d", ptr - p, *ptr - a, **ptr);
}

a

0	1	2	3	4
1000	1002	1004	1006	1008

b

1000	1002	1004	1006	1008
2000	2002	2004	2006	2008

ptr → **ptr++;
↳ from right to left
ptr++ will take place 1st
∴ ptr++, $\Rightarrow \text{ptr} = 2000$
& $*\text{ptr} = 1000$
& $**\text{ptr} = 0$
& after ; , ptr++, ∴ $\text{ptr} = 2002$

O/p:-

1	1	1
1	2	2
1	2	3

1st printf :- $\text{ptr} - p = 2002 - 2000 = 2 \rightarrow 2$ means '1' element
 $*\text{ptr} - a = 1002 - 1000 = 2 \rightarrow " "$
 $**\text{ptr} = 1$

$\rightarrow *++*\text{ptr} \rightarrow$ from right to left

$*\text{ptr} = 1002$

$\downarrow + * \text{ptr} \rightarrow (*\text{ptr}) = (**\text{ptr}) + 1$

$\therefore p \rightarrow$

1000	1004	1004	1006	1008
2000	2002	2004	2006	2008

$*1004 = 2$

2nd printf :- $\text{ptr} - p = 2002 - 2000 = 2 \rightarrow 2$ means '1' element
 $*\text{ptr} - a = 1004 - 1000 = 4 \rightarrow 4$ '2' elements
 $**\text{ptr} = 2$

$\rightarrow ++**\text{ptr} \rightarrow$ from right to left

$*\text{ptr} = 1004$

$**\text{ptr} = 2$

$\leftarrow \text{---} \quad ++**\text{ptr} = \text{---} \quad **\text{ptr} = **\text{ptr} + 1$

$\therefore a$

1000	1002	1004	1006	1008
0	1	3	5	4

\therefore 3rd printf $\rightarrow \text{ptr} - p = 2002 - 2000 = 2 \rightarrow 2$ means '1' element
 $*\text{ptr} - a = 1004 - 1000 = 4 \rightarrow 4$ '2' elements
 $**\text{ptr} = 3$ [$*\text{ptr} = 1004 \rightarrow **\text{ptr} = 3$]

★ Subtracting two pointers means $*(\text{ptr} - p)$
gives the total size of the elements $*(*\text{ptr} + 1)$;
that can reside in b/w them.
if the elements are integers, then
total no. of int = total size b/w them

Note 1 :- For the pointer variable, we can add integer constants.
e.g. $p_1 + 5$ (skipping 5 elements in forward direction).

Note 2 :- We can subtract integer constant from pointer variable
e.g. $p_1 - 1 = 1014 - 1 = 1013$

e.g. $p_1 - 2$ (skipping 2 elements in backward direction)
e.g. $p_1 - q$ given no. of elements in
 $q - p$

Note 3 :- Subtraction b/w 2 pointers is possible ($p_1 - p_2$) iff
 $p_1 - p_2$ [p_1 & p_2 should be pointers of the same array]

where ' $p_1 - p_2$ ' indicates no. of elements b/w two pointers including 1st one & excluding last one.

Note 4 :- Multiplication, division & addition is not possible b/w two pointers because it has no meaning.

Q. main() , Array of characters
{ char a[] = "gotoshell";
char *b = "gotoshell";
a[3] = printf("%s", a);
printf("%s", b); }
a[3] = 'w';
b[3] = 'w';
printf("%s", a);
printf("%s", b); }
a , take 9 bytes
String constant
b , take 9 bytes
b , 12000 , 3000 , 11 Bytes

* $a = a + 5 \rightarrow$ error
(we can't change the base address of the array.)

* but $b = b + 3 \rightarrow$ no error
because b is character pointer & can be changed.

★ `printf("%s", 1000)` → means it will print characters starting from 1000 address till null character (null character is not printed.)

★ `printf("%s", *1000)` → it will print only one character at 1000 address.

∴ `printf("%s", a) → gotohell`
`printf("%s", a+4) → hell`
`printf("%s", *(a+1)) → l`
`printf("%s", b) → gotohell`
`printf("%s", b+5) → ell`
`printf("%s", *(b+5)) → e`
`printf("%s", *(a+3)) → o`

- $a[3] = *(a+3)$.
 $* (a+3) = 'o'$ ∴ $a[3] \text{ o t } \text{ h e l l } \text{ \n}$
- $b[3] = *(b+3)$

`printf("%s", a); → gotohell`
`printf("%s", b); → gotohell`

Note 1:- Base address of the array can't be changed by string constant can be changed.
Contents of the array can be changed, but string constant content can't be changed,
if contents of string constant are changed, result is undefined.

Date

05.08.12

main()

```
char p[] = "gate2012";
printf("%s", p + p[8] - p[1]);
```

{

O/P:

te2012 2012

1001	1003	1005	1007
1000	1002	1004	1006
g	a	t	e
at	te	2	0
ate	te2	01	2012

$$1000 + 1003 - 1001 \\ = 1000 + 2 \\ = 1002$$

$$p[3] = *(p+3) = e \\ e-a = 1004 \quad 4$$

$$p[1] = *(p+1) = a$$

Warning in compilation takes place (program)
When there is no loss of data but it doesn't follows semantic rules.

e.g. int a;

char b;

$a=b;$ //only warning, implicitly it is converting $a=(int)b$
// coercion (implicit type conversion)

// $b=a;$ → this generate error message, because loss of data
// we are storing 2 byte data into 1 byte storage, & hence
// error.

$b=(char)a;$ //correction, explicit conversion.

★ $\text{printf}("c", 'a');$ → 'a'
★ $\text{printf}("d", 'a');$ → 97

★ Lower Address → Higher Byte] → Big endian
Higher " → Lower Byte

★ Lower Address → Lower Byte] → Little endian
Higher " → Higher "]

★ char s = 'a';
 $s = s + 1;$ // s now contains 'b'

★ char s = 255;
 $s = s + 1;$ // s will be 0.

★ char s;
 $s = 'a' + 'b'; \rightarrow s = 97 + 98 = 195$
 $s = 'b' - 'a'; \rightarrow s = 98 - 97 = 1$

Semantic rule
Lower bit
Variable using
lower memory
can be stored in
a variable using
more memory
but not vice versa.

★ $*(\text{p}+i) = \text{p}[i]$ > These two are same.
 $*(\text{i}+\text{p}) = i[\text{p}]$

- ★ `printf("%s", p);` → date2012
- ★ `printf("%s", p+5);` → 012
- ★ `printf("%c", *(p+5));` → 0
- ★ `printf("%c", p);` → error message, because we are using address with %c
- ★ `printf("%s", *p);` → error message, as *p means j.
- ★ `printf("%c", *p);` → g

Q. main()

{ char p[20]; }

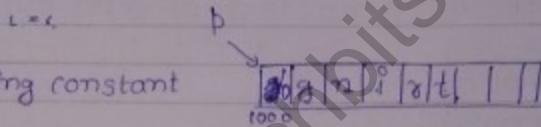
char *s = "String"; //String constant
int length = strlen(s);

for(int i=0; i<length; i++)

{ p[i] = s[length-i]; }

printf("%s", p);

Q/p :- Nothing
(because first character is '\0' itself)



S 2001 2002 2004 2006
2000 t 2003 2005
r i n g \0

length
6

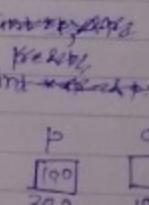
- ★ `sizeof("abcd");` → 5 (including /0) sizeof means the size of "abcd" in the memory.
- ★ `strlen("abcd");` → 4

Q. main() → Let Address to be of 4 Bytes.

{ char *p;

printf("%d%d", sizeof(p), sizeof(*p));

Q/p :- *p means p is pointing to a character, which takes 1 B.
4 1 → sizeof(p) means p store address, address takes 4 B.



Q. main()

{ char *p = "good";

char a[] = "good";

printf("%d%d%d", sizeof(p), sizeof(*p), strlen(p));
printf("%d%d", sizeof(a), strlen(a));

★ if $k = 0$
then O/p :- 0 0 1 0 3 1 is false
 \downarrow
 $(i++ \& j++ \& k++)$
∴ we need to execute $i++$

★ If $i = 0, j = 2, k = 3$ & $l = 2$
∴ O/p :- 1 2 3 3 1

Tmp.

★ int a[10] = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}

1000 1002										
10 20 30 40 50 60 70 80 90 100	↓									

0 1 2 3 4 5 6 7 8 9

int *b, = (int *) a; a [1000] b [1000]
char *c; // c waiting for an address of character.

float *d;

void *e;

c = (char *) a; → [1000] c [3000]

d = (float *) a; → [1000] d [4000]

printf(d) → 1000

printf(*d) → 10.20 (because d is a float pointer & will
print 4 Bytes which is the size of float)
0000 0000 0000 0100, 0000 0000 0000 1010,
0000 0000 0000 0100, 0000 0000 0000 1010,
0000 1010 0000 0000 0001 0100 0000 0000
1000 1001 1002 1003

∴ O/p :- 20 10 it is in little endian &
hence has to be reversed
if it is in big endian, then
O/p will be :- 10 20

printf(c) → 1000

printf(*c) → only 1 Byte will be retrieved; ∴ only 1000
address value is pointed, ∴ O/p :- 10

printf(e) → 1000

printf(*e) → error.

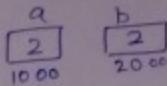
★ if c++, then c = 1001

★ if d++, then d = 1004

★ if b++, then b = 1002

★ if e++, then error.

* No compilation errors, but windows stopped the program execution.



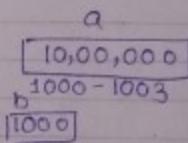
Q.

int a=2;
int *b=(int *)a; → b → 2 is an integer pointer now, which
*if Cb) printf(b) → 2 → 2000 is integer value before
printf(*b) → garbage (value at address 2)
++b → 4 (because 2 is now integer pointer & hence
++a → ++b will increment it to 4).
3 (because value in a is integer value.)

Q. int a=2;
float *b=(float *)a; → b → integer value '2' is converted to
pointf(b) → 2 → floating pointer
* pointf(*b) → garbage (value at address 2,3,4,5)
b → 6 (because floating pointer increments it by 4).
+ta → 3

Q. int a=2;
char *b=(char *)a; → b → integer value '2' is converted to
pointf(b) → 2 → character pointer
* pointf(*b) → garbage (value at address 2)
++b → 3 (increment by 1 only because character pointer)
+ta → 3

Q. float a=10,00,000; → takes 4 Bytes
int *b=(int *)a;
pointf(b) → 1000
* pointf(*b) → since b is an integer pointer,
& hence will print 2 Bytes out
of 4 Bytes (hence data loss)



* if address lines are of 2 Bytes

then float a=10,00,000;
int *b=(int *)a

We are storing a (4 Bytes data) into 2 Bytes
integer pointer & hence the 4 Byte a (converted
to integer pointer) is stored in 2 Byte 'b' &
hence address loss.

Static Variable

```
for(int i=5; i>0; i--)
    {
        for(int j=5; j>i; j--)
            cout<<j;
        cout<<endl;
    }
```

main()

```
{ Static int var=5;
    printf("%d", var-);
    if (var)
        main();
}
```

O/p:-

5 4 3 2 1

g

* Writing 'static' means at the time of compilation time memory is allotted. (Static binding) [compile time memory is allocated]

* if 'static' isn't there then the memory is allotted at run time. (Dynamic binding) [memory is allotted at run time]

- ① For the static variable memory will be created only once during its life time (compile time only)
(Recompiling the program will allocate different memory location to the static variable.)
- ② For the static variable initialization will be done once.
(By default)
- ③ By default Static & Global variable are initialized to zero:

Q. void main()

```
{ Static int i=5;
    if (--i)
        { main();
            printf("%d", i);
        }
}
```

g

O/p:
0 0 0 0

main()
i=5

main()
4

main()
3

main()
2

main()
1

main()
0

main()
-1

main()
-2

main()
-3

```

Q main()
{
    int f(int n)
    {
        static int x=0;
        if(n<=0)
            return 1;
        if(n>3)
            {x=n;
            return f(n-2)+2;
        }
        return f(n-1)+x;
    }
}

```

What is the value of $f(5)$?

```

f(5)
  ↗ (n>3)
  ↗ return f(3)+2 = 16+2 = 18
  ↗ f(3)
  ↗ return f(2)+5 = 11+5 = 16
  ↗ f(2)
  ↗ return f(1)+5 = 6+5 = 11
  ↗ f(1)
  ↗ return f(0)+5 = 6
  ↗ f(0)
  ↗ return 1

```

x 5
1000

O/p:-
18

Note :- If we use register instead of static, then x is stored in register for faster access.

```

#include <stdio.h>
main()
{
    struct a
    {
        char c[7];
        char *Str;
    };
    struct b
}

```

```

f char *c;
struct a ss1;

```

O/p:-
Raipur Jaipur
airpur airpur

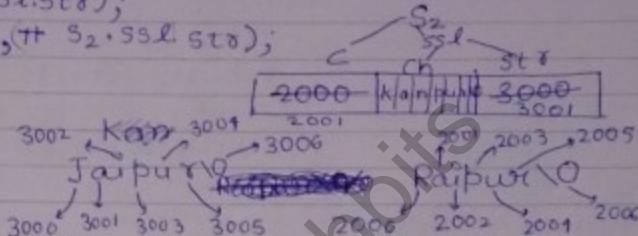
```

};  
Struct b S2={"Raipur", "Kanpur", "Jaipur"};  
printf("%s\n", S2.C, S2.ss1.str);
printf("%s\n", ++S2.C, ++S2.ss1.str);
}

```

★ int a=10;
int *b=a; //no error

a	b
10	10
1000	2000



Structures & Union

```

Struct s1;
{ int a;
  float b;
  char c;
}

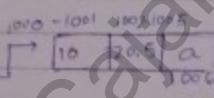
```

NOTE:- Whenever we reference a struct, many variables are referenced, hence selection is req. to select the intended variable using .

```

Struct s2, x;
x = {10, 20.5, 'a'};

```



★ printf(x) → means nothing. first element of the struct will be printed.

★ printf(x.a) → 10
★ printf(x.b) → 20.5
★ printf(x.c) → a

★ if struct s1

```

{ int a;
}

```

then s1 and not required s2 will also point a.

Both are possible.

```

Struct s1;
{ int a;
  float b;
  char c;
}

```

Struct s2;

```

{ char d;
}

```

```

Struct s1 e;

```

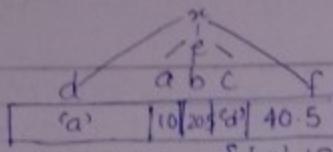
```

{ float f;
}

```

5. 123

← we can declare struct within struct.



struct $s_2 = \{ 'a', 10, 20.5, 'd', 40.5 \}$

struct $x = \{ 'a', 10, 20.5, 'd', 40.5 \}$

$x.d = 'a'$

$x.e.a = 10$

$x.e.b = 20.5$

$x.e.c = 'd'$

$x.f = 40.5$

Q. struct s

int a;

float b;

};

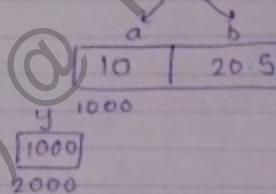
1. struct $s, x = \{ 10, 20.5 \} ;$

2. struct $s, *y = &x;$

3. printf(y) $\rightarrow 1000$

printf(*y).a) $\rightarrow 10$

or printf(y->a); $\rightarrow 10$



Q. main()

struct s

char *z;

int;

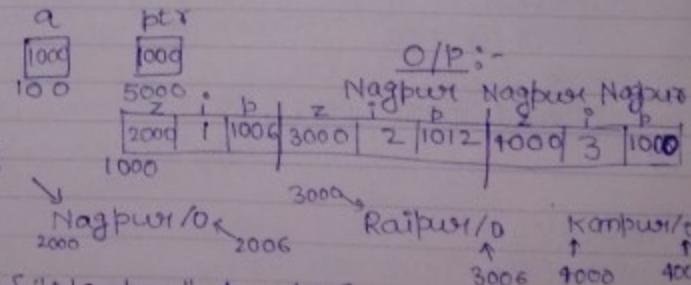
struct $s, *b;$

};

struct $s, a[] = \{ \{ "Nagpur", 1, a+1 \},$
 $\{ "Raipur", 2, a+2 \},$
 $\{ "Kanpur", 3, a+3 \} \};$

struct $s, *ptr = a;$

printf("%s/%s/%s/s", a[0].z, ptr->z, a[2].b->z);
 \downarrow
 $(*ptr).z \quad a[2].(*p).z$



After processor will change $b \rightarrow c$ into $(*b).c$

main()

{ struct s;

char *z;

int i;

struct s, *b;

};

Struct s, a[] = { "Nagpur", 1, a[1] }, { "Raipur", 2, a[2] },
{ "Kapur", 3, a[3] };

Struct s, * Ptr = a; $\rightarrow (*b).z$

printf ("%s", ++(Ptr->z)); \downarrow 3600

printf ("%s", a[(++b)->i].z);

printf ("%s", a[--(b->b->i)].z);

printf ("%d", -a[2], i);

O/p:-

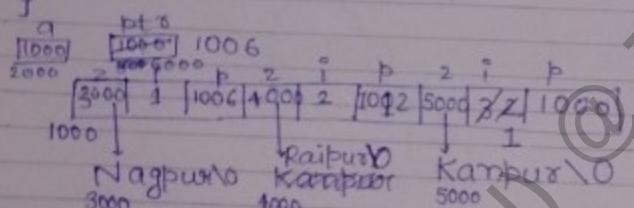
agpur

Kapur

Kapur

Kapur

21



3000 better/o 5000 ancestor/o

Q. #include <stdio.h>

struct Test

int i;

char *c;

};

Struct Test st[] = { 5, "become", 4, "better", 6, "jungle",
8, "ancestor", 7, "brother" };

main()

{ Struct Test *p = st;

b = b+1; \rightarrow (*b).c

printf ("%s", ++(b->c)));

printf ("%s", *++b->c);

printf ("%d", P[0].i);

printf ("%s", b->c);

*(*b).c ++

O/p:-

etter, u, c, ungle

P[0] = *(P+0)

Q. void main()
 { char S[] = "hello";
 char R[] = "hello";
 if (S == R)
 printf("equal");
 else
 printf("not equal");
 }

* if $*S == *R$ ($'h' == 'h'$)
 O/p:- equal

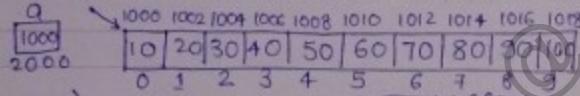
O/p:-
 Not equal

because S & R both stores diff. base addresses.

Arrays

1-D (1-Dimensional)

int a[10] = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}



$$a+5 = 1000 + 2*5$$

$$L(a[5]) = 1000 + 2*(5-0) \quad \text{offset}$$

in C compiler it is like
 $1000 + 2*5$
 (so C compiler saves 1 subtraction.)

$$L(a[9]) = 1000 + 2*(9-0)$$

This means before 5th element, how many elements are there.

$$\text{if array starts with index '1'}$$

$$L(a[5]) = 1000 + 2*(5-1) \quad \text{offset}$$

$$L(a[9]) = 1000 + 2*(9-1)$$

Q. Array A is defined as follows:-

- A[-5 ... +5]
- Base address (BA) = 999
- Size of element = 100B

Total no. of elements :-
 Last index - 1st index + 1

Location of $a[-1], a[+5]$

$$L(a[-1]) = 999 + 100(-1 - (-5)) = 999 + 100 \times 4 = 1399$$

$$L(a[5]) = 999 + 100 \times (5 - (-5)) = 999 + 100 \times 10 = 1999$$

In General :-

- $A[lb..ub]$
- BA \rightarrow Base Address
- C \rightarrow Size of 1 element

then Location of i^{th} element :-
$$\text{Loc}(a[i]) = \text{B.A.} + C \times (i - lb)$$

Two-Dimensional Arrays

int a[4][4];

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

* Every multidimensional array is stored in the form of multidimensional or 1-dimensional array.

Row-Major Order :-

1000	1002	1004	1006	08	10	12	14	16	18	20	22	24	26	28	30
a ₁₁	a ₁₂	a ₁₃	a ₁₄	a ₂₁	a ₂₂	a ₂₃	a ₂₄	a ₃₁	a ₃₂	a ₃₃	a ₃₄	a ₄₁	a ₄₂	a ₄₃	a ₄₄
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$$L(a[i][j]) = \text{B.A.} + C * (j - lb_2) * (i - lb_1)$$

$$L(a[4][3]) = 1000 + (4(4-1) + (3-1)) * 2 \\ = 1000 + 28 \\ = 1028$$

$$L(a[2][4]) = 1000 + (4(2-1) + (4-1)) * 2 \\ = 1000 + 7 * 2 \\ = 1014$$

Q. Consider the following 2-D array :-

Total Rows

Last Row No. -
1st Row No. + 1

Total Columns

Last Column No. -
1st Column No. + 1

array starting
from A[1][1]

$a[5..25, 50..75]$

$B.A = 0$

$C = 5B$

$$L(a[20][70]) = 0 + ((70-50)(20-5) \times (75-50+1) + (70-50)) \times 5 \\ = (15 \times 25 + 20) \times 5 \\ = \cancel{375} \times 5 (390 + 20) \times 5 \\ = \cancel{1975} \quad 410 \times 5 = 2050$$

$$Loc(a[25][75]) = 0 + ((25-5) \times (75-50+1) + (75-50)) \times 5 \\ = (20 \times 26 + 25) \times 5 \\ = \cancel{445} \times 5 \quad 545 \times 5 \\ = \cancel{2225} \quad 2725$$

In General (Row Major Order)

$A[lb_1..ub_1, lb_2..ub_2]$
 BA, C

$$Loc(a[i][j]) = BA + ((i-lb_1) \times (ub_2 - lb_2 + 1) + (j - lb_2)) \times C$$

Date

11.08.12

- int a[4][4];

$a_{11} \quad a_{12} \quad a_{13} \quad a_{14}$
 $a_{21} \quad a_{22} \quad a_{23} \quad a_{24}$
 $a_{31} \quad a_{32} \quad a_{33} \quad a_{34}$
 $a_{41} \quad a_{42} \quad a_{43} \quad a_{44}$

$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline & 1000 & 02 & 04 & 06 & 08 & 10 & 12 & 014 & 10 & 18 & 20 & 22 & 24 & 26 & 28 & 30 \\ \hline a_{11} & a_{21} & a_{31} & a_{41} & a_{12} & a_{22} & a_{32} & a_{42} & a_{13} & a_{23} & a_{33} & a_{43} & a_{14} & a_{24} & a_{34} & a_{44} \\ \hline \end{array}$

$a[15][ub_1][lb_2] \dots ub_2]$

$$\text{Loc}(a[i][j]) = B.A. + ((j - lb_2) \times (ub_1 - lb_1 + 1) + i - lb_1) \times C$$

$$\begin{aligned} \cdot \text{Loc}(a[3][4]) &= 1000 + ((4-10) \times 4 + (3-1)) \times 2 \\ &= 1000 + (12+2) \times 2 \\ &= 1000 + 28 \\ &= 1028 \end{aligned}$$

$$\begin{aligned} \cdot \text{Loc}(a[4][3]) &= 1000 + ((3-1) \times 4 + (4-1)) \times 2 \\ &= 1000 + (8+3) \times 2 \\ &= 1000 + 22 \\ &= 1022 \end{aligned}$$

$$\cdot \text{Loc}(a[4][1]) = 1000 + ((1-1) \times 4 + (4-1)) \times 2 \\ = 1000 + 6 = 1006$$

Q. Consider the array (using Column Major Order)

$$\cdot A[-5..+20][25..75]$$

$$\cdot BA = 999$$

$$\cdot C = 10B$$

$$\begin{aligned} \text{Loc}(A[0][74]) &= 999 + ((74-25) \times (20-(-5)+1) + (0-(-5))) \times 10 \\ &= 999 + (49 \times 26 + 5) \times 10 \\ &= 999 + 12790 \\ &= 13789 \end{aligned}$$

$$\begin{aligned} \text{Loc}(A[-2][30]) &= 999 + ((30-25) \times 26 + (-2+5)) \times 10 \\ &= 999 + (130+3) \times 10 \\ &= 999 + 1330 = 2329 \end{aligned}$$

Lower Triangular Matrix

int a[4][4]

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

* it is valid only for Square matrices.

$$[A[i][j] = 0 \text{ if } i < j]$$

Storing using Row Major :-

a	1000	02	04	06	08	10	12	14	16	18
	a ₁₁	a ₁₂	a ₁₃	a ₁₄	a ₂₂	a ₂₃	a ₂₄	a ₃₃	a ₃₄	a ₄₄
1st	2nd	3rd	4th							

$$\begin{aligned} \text{LOC}(a[4][3]) &= 1000 + ((3-1) \times ((1+2+3) \times (4-1) + (3-1)) \times 2 \\ &= 1000 + 18 \\ &= 1000 + ((4-1)(1+2+3) + (3-1)) \times 2 \\ &= 1000 + (8) \times 2 \\ &= 1016 \end{aligned}$$

$$\text{Loc}(a[i][j]) = [B.A. + ((i-lb_1)NNS + (j-lb_2)) \times c]$$

(i-lb₁) → Natural No. Sums upto from 1 to (i-lb₁)

→ (4-1) sum of natural nos. (i.e. sum of natural nos. upto 3.)

$$\begin{aligned} \text{LOC}(a[4][2]) &= 1000 + ((1+2+3) + (2-1)) \times 2 \\ &= 1000 + (6+1) \times 2 = 1014 \end{aligned}$$

Q. Consider matrix defined as follows:-

$$A[25, -+25, -25, \dots, +25]$$

$$BA = 0$$

Row Major Order, Lower Triangular Matrix

$$C = [100 \ 0]$$

$$\begin{aligned} \text{Loc}(A[-20][-21]) &= 0 + ((-20+25)NNS + (-21+25)) \times 100 \\ &= 0 + ((1+2+3+4+5) + 4) \times 100 \\ &= 0 + (19) \times 100 \\ &= 1900 \end{aligned}$$

$$\frac{5 \times 5 \times 3}{2} =$$

$$\begin{aligned} \text{Loc}(A[0][0]) &= 0 + ((0+25)NNS + (0+25)) \times 100 \\ &= 0 + \left(\frac{25 \times 2^13}{2} + 25 \right) \times 100 = 35,000 \end{aligned}$$

Column Major Order :-

00	02	04	06	08	10	12	14	16	18
a ₁₁	a ₂₁	a ₃₁	a ₄₁	a ₂₂	a ₃₂	a ₄₂	a ₃₃	a ₄₃	a ₄₄

My answer

$$\text{Loc}(a[i][j]) = \text{B.A.} + \left(\frac{i-1}{2} \text{ to } j \right) \text{ reverse order sum} + \left(\frac{j-1}{2} \text{ to } i \right) \times c$$

$$\begin{aligned} \text{Loc}(a[4][3]) &= 1000 + ((4+3) + (4-3)) \times 2 \\ &= 1000 + (7 + 1) \times 2 \\ &= 1016 \end{aligned}$$

★ Or $\text{Loc}(a[4][3]) = 1000 + \left(\left(\frac{4 \times 5}{2} \right) - (4-3+1)NNS + (4-3) \right) \times 2$

skip all columns we actually want to go to 3rd column

a ₁₁	a ₁₀	0	0	0
a ₂₁	a ₂₂	0	0	0
a ₃₁	a ₃₂	a ₃₃	0	0
a ₄₁	a ₄₂	a ₄₃	a ₄₄	

we are at here (at the end)

we are at here (actually after a₄₂) (after subtracting)

$$\begin{aligned} &= 1000 + (10 - (24) + 1) \times 2 \\ &= (1000 + 8) \times 2 \\ &= 1016 \end{aligned}$$

$$\begin{aligned} \text{Loc}(a[4][1]) &= 1000 + \left(\frac{\frac{2}{2} \times 5}{2} - (4-2+1)NNS + (4-1) \right) \times 2 \\ &= 1000 + (10 - 10 + 3) \times 2 \\ &= 1000 + 6 = 1006 \end{aligned}$$

int * a = 2;
pf(*a); [A[ub₁, ub₁, lb₂ ub₂]]

↓
skip all columns

go to the starting
of the desired column

NNS
↑
go to the desired row

$$\text{LOC}(A[i][j]) = \text{B.A} + ((ub_2 - lb_2)^{\text{NNS}} - (ub_2 - j + 1)^{\text{NNS}} + (i - j)) \times c$$

Q. A[-5...+5, -5...+5]

$$BA = 1000, C = 10$$

Lower Triangular Matrix
Column Major Order

$$66 - 21 \\ = 45$$

$$\frac{7 \times 8}{2} = 28$$

$$\begin{aligned} \cdot \text{LOC}(A[0][0]) &= 1000 + (11^{\text{NNS}}(5-0+1)^{\text{NNS}} + 1) \\ &= 1000 + (\frac{11 \times 12^6}{2} - \frac{36 \times 7}{2} + (0-0)) \times 10 \\ &= 1000 + 45 \times 10 \\ &= 1450 \end{aligned}$$

$$\begin{aligned} \cdot \text{LOC}(A[2][-1]) &= 1000 + (11^{\text{NNS}} - (5-(-1)+1)^{\text{NNS}} + (2+1)) \times 10 \\ &= 1000 + (66 - 28 + 3) \times 10 \\ &= 1000 + (41) \times 10 \\ &= 1410 \end{aligned}$$

To i-diagonal Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

$$N-1 + N + N-1 = 3N-2 \quad \text{memory space.}$$

* In Tridiagonal Matrix, apart from 1st & last row (which has 2 elements), rest all other rows have 3 elements.

a. Using Row Major :-

a_{11}	a_{12}	a_{21}	a_{22}	a_{23}	a_{32}	a_{33}	a_{34}	a_{43}	a_{44}
1000	02	04	06	08	10	12	14	16	18

$$\text{Loc}(a[3][4]) = 1000 + (3(2+3) + (4-2)) \times 2 \\ = 1000 + (5+2) \times 2 \\ = 1014$$

assuming, skipping desired no. of rows

but 1st row contains only 2 elements, subtracting 1 element

$$[\text{Loc}(a[i][j]) = B.A. + (3 \times (j - lb_1) - 1 + (j - i + 1)) \times c]$$

$$\text{Loc}(a[4][4]) = 1000 + (3 \times (4-1) - 1 + (4-4+1)) \times 2 \\ = 1000 + 3 \times 2 = 1018$$

going to the desired column.

Using Column Major

$$[\text{Loc}(A[i][j]) = B.A. + (3 \times (j - lb_2) - 1 + (i - j + 1)) \times c]$$

$$\text{Loc}(A[3][4]) = 1000 + (3 \times (4-1) - 1 + (3-4+1)) \times 2 \\ = 1000 + (-8) \times 2 \\ = 1016$$

Using Diagonal by Diagonal :- (Principal diagonal, Lower, Upper)

a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	a_{24}	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}
a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	a_{24}	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}

Principal				Lower			Upper								
a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	a_{24}	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}

$\text{Loc}(A[i][j]) \rightarrow$ if ($i = j$) (Principal diagonal)

$$= B.A. + (j - lb_1) \times c \quad [\text{if } i = j]$$

$$= B.A. + ((ub_2 - lb_2 + 1) + j - lb_2) \times c \quad [\text{if } i > j]$$

$\downarrow N (ub_2 - lb_2 + 1 = N)$

$$= [B.A + (N + (N-1) + (i - lb_1)) \times c \quad (\text{if } i \leq j)]$$

Z-Matrix

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	a_{23}	a_{24}
a_{31}	a_{32}	a_{33}	a_{34}
a_{41}	a_{42}	a_{43}	a_{44}

Storage :-

① Row Major

② Column Major

③ Store 1st row, last row & remaining elements in diagonal.

Row-Maj

Using 3rd approach:-

a_{11}	a_{12}	a_{13}	a_{14}	a_{41}	a_{42}	a_{43}	a_{44}	a_{23}	a_{32}
$\downarrow N$	$\downarrow N$	$\downarrow N$	$\downarrow N-2$						

∴ total space = $3N-2$

$$\text{Loc}(A[i][j]) = \begin{cases} B.A + ((i - lb_2)) \times c & [\text{if } i = lb_1] \\ B.A + (N + (j - lb_2)) \times c & [\text{if } i = ub_1] \\ B.A + (2N + (i - lb_1 - 1)) \times c & [\text{if } i \neq lb_1 \text{ &} \\ & j \neq ub_1 \text{ &} \\ & (i+j) = N+1] \end{cases}$$

Toepplitz Matrix

if $A[i][j] = A[i-1][j-1] \forall i, j$ where $i \geq 1 \& j \geq 1$

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	a_{23}	a_{24}
a_{31}	a_{32}	a_{33}	a_{34}
a_{41}	a_{42}	a_{43}	a_{44}

10	20	30	40
50	10	20	30
20	50	10	20
70	60	50	10

Store only $(2N-1)$ elements

Store only these

Storage:-

① Row-Order

② Column-Order

③ Store 1st row, remaining element in 1st column

1000	a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{24}
	10	20	30	40	50	60	70
	00	02	04	06	08	10	12

$$\text{Loc}(a[i][j]) = \begin{cases} \text{B.A} + (j - lb_2) \times c, & (\text{if } i \leq f) \\ \text{B.A} + (N + (i - lb_2 - 1)) \times c & (\text{if } i > j) \\ (lb_2 - lb_2 + 1) & \end{cases}$$

[Elements belong to upper part.]
[Elements belong to lower part.]

Three-Dimensional Matrix

int a[2][3][4] → 2 arrays of size 3x4

BA = 1000

Row Major Order

$$\begin{aligned} \text{Loc}(A[2][3][4]) &= 1000 + (2-1) \times 12 + (3-1) \times 4 + (4-1) \times 2 \\ &= 1000 + (12 + 8 + 3) \times 10 \\ &= 1000 + 46 \times 10 = 1230 \end{aligned}$$

Loc(a)

int a[-5...+5][-10...+50][-25...+25]

• BA = 0

• Column Major Order

• C = 5

$$\begin{aligned} \text{Loc}(A[0][0][0]) &= 0 + (5 \times 61 \times 51 + (0+25) \times (61) + (0+10)) \times 5 \\ &= (3115 + 1525 + 10) \times 5 \\ &= 85450 \end{aligned}$$

$$\begin{array}{r} 3115 \\ 1525 \\ 10 \\ \hline 5 \\ \hline 15555 \\ 1535 \\ \hline 17090 \end{array} \quad \begin{array}{r} 2 \\ x25 \\ \hline 50 \\ 1525 \\ 1525 \\ \hline 3050 \\ 311 \\ \hline 311 \end{array} \quad \begin{array}{r} 61 \\ x51 \\ \hline 305 \\ 311 \\ \hline 311 \end{array}$$

$A[lb_1 \dots ub_1][lb_2 \dots ub_2][lb_3 \dots ub_3]$

Row Major Order

$$\text{Loc}(a[i][j][k]) = B.A + ((i-lb_1) \times (ub_2-lb_2+1) \times (ub_3-lb_3+1) + (j-lb_2) \times (ub_3-lb_3+1) + (k-lb_3)) \times C$$

Column Major Order

$$\text{Loc}(a[i][j][k]) = B.A + ((i-lb_1) \times (ub_2-lb_2+1) \times (ub_3-lb_3+1) + (k-lb_3) \times (ub_2-lb_2+1) + (j-lb_2)) \times C$$

specifying this is optional

Q. int a[5] = {10, 20, 30, 40, 50};
↑
mandatory

int a[][3] = {10, 20, 30, 40, 50, 60};
↑
Optional

int a[][][3]
↑
Optional mandatory

int a[5]; → garbage values

int a[5] = { };

4th & 5th elements are 0.

int a[5] = { }; → all are 0.

int a[5] = {10, 20, 30, 40, 50, 60};

↓
no compilation error, but a[5] is not accessible

Note:-

In multidimensional Array, M1 indexes are mandatory.

Q. main()

{ int a[3][3];
printf("%d", ((a= *a) + (*a = a[0])));

a[0][0] = 1000;
a[0][1] = 1000;
a[0][2] = 1000;
a[1][0] = 1006;
a[1][1] = 1006;
a[1][2] = 1012;
a[2][0] = 1012;
a[2][1] = 1012;
a[2][2] = 1012;

a=1000
*a=1000
*a=1000
a[0]=*(a+0)
=1000
[0/b]-1

a[i][j]-
↓
b
↓
b[j]
↓
*(b+j)
↓
*(a[i]+j)
↓
(*(*a+i)+j)

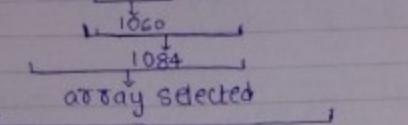
row is not selected
row is selected.
 $*a+1 = a[1]$

row is not selected
(a+1) = 1006
& hence
we can change
the row.

$\boxed{a=1000}$
 $\boxed{a+1=1006}$
 $*a+1 = 1006 \rightarrow \text{row is selected}$
 $*a+1+1 = 1008 \quad (\text{we can't change the row now.})$

$$12 \times 7 \\ = 84$$

Q. $((* ((a+5)+2) + 2) + 1)$



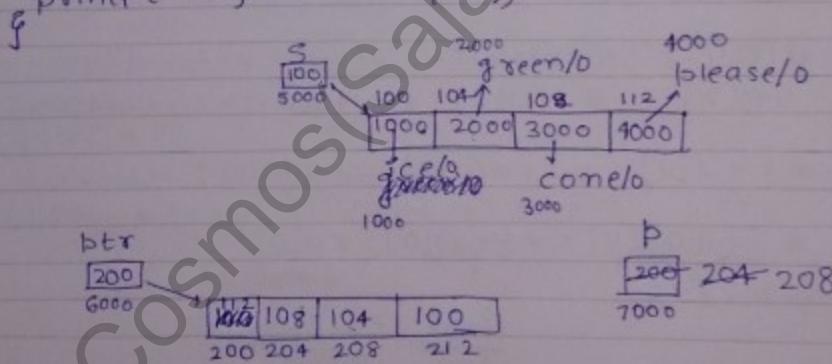
$$\text{skip 2} \\ \text{move 800} \\ 1084 + 2 \times 4 \\ = 1084 + 8 \\ = 1092$$

$$\text{skip 1} \\ \text{move 800} \\ 1092 + 1 \times 4 \\ = 1096$$

#include <stdio.h>

Q. main()

```
{ char *s[] = {"ice", "green", "cone", "please"};
  char **ptr[] = {s+3, s+2, s+1, s};
  char ***p = ptr;
  printf("%s", ***++p);
  printf("%s", ***--*++(p+1));
```



O/p:-

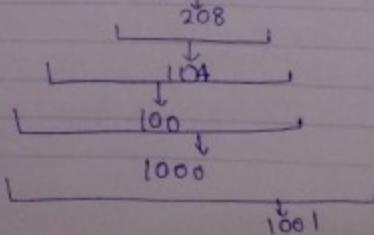
$$b = 200$$

$$++p = 204$$

$$*++p = 3000$$

$$**++p = \text{cone}$$

$$* - - * + + p + 1$$



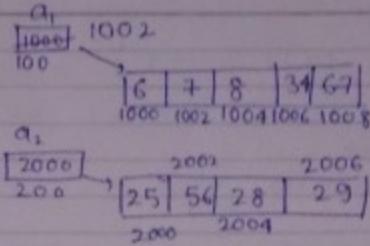
O/p:-

cone

ce

Q #include <stdio.h>

int a[] = {6, 7, 8, 34, 67};
int a2[] = {25, 56, 28, 29};
int a3[] = {-12, 27, -31};
int *x[] = {a1, a2, a3};



O/p :-
8
-12
7
25

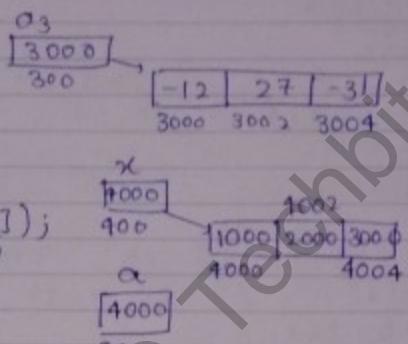
main()

{ B(x);

}

B(int **a)

{ printf("%d", a[0][2]);
printf("%d", *a[2]);
printf("%d", *++a[0]);
printf("%d", *(++a)[0]);



O/p :-

1st printf :- a[0][2] = *(*(a+0)+2) = *(1000 + 2*2) = *(1004)

1st printf :- a[0][2] = *(*(a+0)+2) = *(1000 + 2*2) = *(1004)
2nd printf :- *a[2] = *(*(a+2)) = *(*(4000 + 2*2)) = *(4004) = *(3000)

3rd printf :- *++a[0] = *++*(a+0) = *(4000) = *(1000) = *(1002)

4th printf :- *(++a)[0] = *(a+1)[0] = *a[0] = *(4000) = *(1000) = *(1002)
 ↓
 ((a+0)) = *(4000) = *(2000) = 25

even though
x is 1-D pointer array.

x[1][2] = *(*(x+1)+2) = 28

★ using one dimensional array of pointer, we can get two-dimensional array.

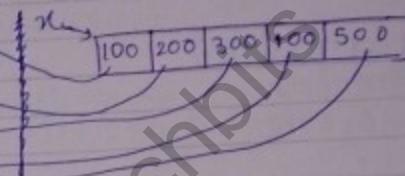
`int/a[5]`

★ `int a[][][5]` → 2-D array, where every row contain uniform no. of columns.

★ `int *x[]` → 2-D array, but it will create variable columns in every row.

★ `int a[5] = {6, 7, 8, 34, 67};`
`int a[5] = {25, 56, 28, 29};`
`int a[5] = {-12, 27, -31};`
`int a[5] = {10, 20};`
`int a[5] = {20};`

`int *x[5] = {a1, a2, a3, a4, a5};`



Q. consider following C-declarations:-

(1) `int A[10][10];`

(2) `int *B[10];`

Which is true?

(a) `A[5][6]=10;` T

(b) `A[5]=1000` F (we can't change the base address of 5th row of an array.)

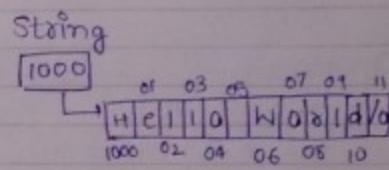
(c) `B[5][6]=10;` T

(d) `B[5]=1000` T

Q. `main()`

```
{ Char String[] = "Hello World"
    display(String); }
```

```
void display(Char *String)
{ printf("%s", String); }
```



★ In the above program, we are using `display` fn. w/o declaration so compiler assumes `display` fn. by default integer value, but above program is returning void, so above program will give error of type mismatch.

Q. double foo(double); /* line 1 */

int main()

{ double da, db;
db = foo(da);
return 0;

double foo(double a)

{ return a;
}

* If we delete line 1 then
conflicting datatype for fn. foo.
type mismatch

Q. void main()

{ int i=0;
for(int i=0; i<20; i++)
{

switch(i)

{ Case 0: i+=5; // We can
case 1: i+=2; but default
case 5: i+=5; anywhere,
default: i+=4; he will have
break; same result.

if default is
at *

printf("%d", i);

* In switch statement, we
can write default wherever
we want, but it will be
executed whenever there is
no match.

O/p:-

i = i + 5

5

16

21

36

39

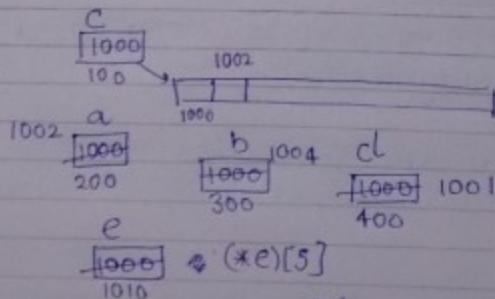
44

47

52

Q. main()

{ int c[100];
int *a=(int *)c;
float *b=(float *)c;
char *d=(char *)c;
int (*e)[5]=(int *)c;
for(i=0 to 5)
{ printf(c+i,a,b,d,e);
a++;
b++;
d++;
e++;
}



O/p:-

0	1000	1000	1000	1000	1000
1	1002	1002	1004	1001	1000
2	1004	1004	1008	1002	1020
3	1006	1006	1012	1003	1030
4	1008	1008	1016	1004	1040
5	1010	1010	1020	1005	1050

★ $\text{int } (*a)[5] \rightarrow a$ is a pointer to array which contains 5 elements,
 \therefore when we do $a++$, then a is incremented to 10B.

Function returning pointer :-

```
int fun();
main()
{ int x=10;           x [10]
  int *b;             b [300]
  b=fun();            200
  printf("%d",x+(*b));
}
O/p:- 10+20=30
```

```
int *fun()
{ int y=20;
  return(&y);
}
```

★ function will get the memory when the fn is called, & will be deallocated when the fn. returns.

extern int a;
 static
 int a;

★ fun is a function which returns pointer.

Functional Pointer

```
main()
{ int display(); // creating memory
  int (*funptr)(); // for display
  funptr= display; // in those
  (*funptr)(); // address is allocated
}

```

Value at 1000 (at 1000, it is the fn. display)

funptr is a pointer, which internally contain function's address.

O/p:-
 Hi

funptr	display
1000	1000
2000	3000

★ function name indicates base address
 : display indicate its base address.

★ $\text{int } (*p)()$ → p is pointer pointing to a function.
 $\text{int } *p[5]$ → p is an array of pointers pointing to
 $\text{int } *p[5]()$ → p is an array of pointers pointing to 5 fn.

Q. main()

{ $\text{int } (*ptr[3])();$
 $\text{ptr}[0] = \text{aaa};$
 $\text{ptr}[1] = \text{bbb};$

[1000] int aaa() ↗
 $\text{ptr}[0] = \text{bf}("aaa");$ ↗
 $\text{return } 0;$ ↗
 $\text{ptr}[1] = \text{bbb}();$ ↗
 int bbb() ↗
 $\text{return } 1;$ ↗

$\text{ptr}[2] = \text{ccc};$

$\text{ptr}[2]();$ or $(*\text{ptr}[2])(())$ ↗
 $\text{int ccc}()$ ↗
 $\text{bf}("ccc");$ ↗

ptr

4000	4002	4004
1000	2000	3000

0 1 2

$*ptr[0] \rightarrow **(ptr + 0)$ $ptr[0] = *(ptr + 0)$
 $= **(1000)$

$= *4000$
• value at address 4000

$(*\text{ptr}[2])(())$

\downarrow
 $(* *(\text{ptr} + 2))()$

\downarrow
 $(* *4004))()$

\downarrow
 $(*3000))()$

at ccc fn.

Q. What does the following C-statement indicate

$\text{int } (*f)(\text{int}, \text{int});$

★ f contains address of a function which is taking two int variables as argument & returning an integer.

• $\text{int } * (*f)(\text{int } *, \text{int } *);$

f is a pointer containing address of a function which takes 2 integer pointers as arguments & returns integer pointer.

② `void (*abc)(int, void (*def)());`

abc is a pointer containing address of a function which takes 2 argument as i/p, out of which one is integer & the other is def is a pointer pointing to a fn. which returns void.

& returns void.

③ `char * (*(*a[N]))();`

array of N pointers
pointing to function(s)
[doesn't take any input]

these function(s) are
returning pointer(s). (pointer type
, is not specified)

these returned pointers
are pointing to a function

this function is returning
character pointer.

* Array of N pointers pointing to functions which will return a pointer which is pointing to a function which is returning character pointer.

④ `*(char * (*)()) (*ptr[N])();`

(ii) a pointer
to a function

(ii) the function returns
a character pointer.

(iii) character pointer
is pointing to an
array of pointers
which are pointing
to functions which
are returning
a pointer.

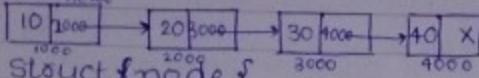
brackets are
having associativity
from left to right

Date

12.08.12

Linked List :-

data next



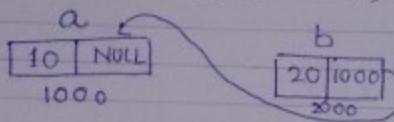
Struct node {

int data;

struct node *next;

};

struct node a = {10, NULL};



Struct node b = {20, &a};

Struct node *ptr = &b;

Self referential data structure

• float type pointer is pointing to float.

• int type pointer is pointing to int.

ptr = 2000

*ptr → inside 2000 (i.e. Selected the address 2000).

(*ptr).next = 1000

(*(*ptr).next) :- Selected the address 1000

(*(*ptr).next).data = 10

(*ptr->next).data = 10

(ptr->next)->data = 10

Q. Write a C program to print all data fields parts in every node of the linked list.

Ans.

```
#include <stdio.h>
```

```
struct node { int data; struct node *next;};
```

struct

main()

```
{ struct node *start; // starting address of linked list
```

while(1)

ptr = start;

while(ptr != NULL)

```
{ printf("%d", ptr->data);
```

```
    ptr = (*ptr).next;
```

}

```
int a  
int *ptr=&a;  
ptr=(int)a;
```

* If s is NULL, then *s is not allowed (it gives segmentation error)

Q. Write a C program to return data of the last node of linked list.

Ans. $\text{ptr} = \text{start};$

```
while( $\text{ptr} \neq \text{NULL}$ )  
{ if( $(\text{ptr}).\text{next} == \text{NULL}$ )  
    {  $\text{printf}(\text{"%d"}, (\text{ptr}).\text{data})$ ;  
    }  
     $\text{ptr} = (\text{ptr}).\text{next}$ ;  
}
```

// Linked list contain atleast 1 node.

* Struct node {

```
    int data;  
    struct node *next;  
};
```

int Display(struct node *s)

```
{ struct node *p=s;
```

```
if ( $p == \text{NULL}$ )
```

```
{  $\text{printf}(\text{"Link list is empty"})$ ;
```

```
exit(0);
```

```
}
```

while ($p \rightarrow \text{next} \neq \text{NULL}$) // Linked list contain atleast 2 elements

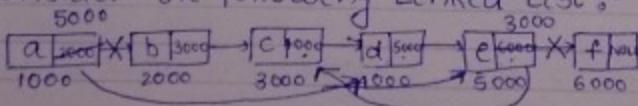
```
{ p = p  $\rightarrow$  next;
```

```
}
```

```
 $\text{printf}(\text{"%d"}, p \rightarrow \text{data})$ ;
```

```
}
```

Q. Consider the following Linked List :-



What would be the O/P after executing following sequence of statements :-

```

int a=10;
int b=20;
printf("%d", a+b);

```

p
3000

first
1000

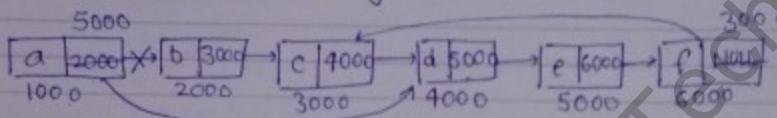
```

struct node *p;
p=first->next->next;
first->next = p->next->next;
p->next->next->next = p;
printf(first->next->next->next->data);

```

O/p:-
d

Q. consider the following linked list -



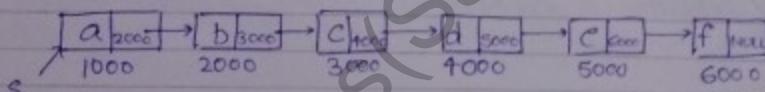
- (i) Struct node *p
- (ii) p=first->next->next
- (iii) first->next = p->next->next;
- (iv) p->next->next->next = p;
- (v) printf(first->next->next->next->data);

3000
2000

p
3000
1000

O/p:-
c

Adding Node in Linked List



(i) Starting of the linked list -

Add at Start (struct node *s, int x)

```

{ struct node *ptr=(node *)malloc(sizeof(node));
(*ptr).next=s;
(*ptr).data='g';
s=ptr;
}

```

(ii) Write a C program to add a node with data x at the end of the linked list.

```

Addatend (node * start)
{ node * ptr = (node *) malloc (sizeof (node));
(*ptr).data = x;
node * temp = Start; if (start == NULL)
while (ptr->next { start = ptr; ptr->next = NULL; exit(0);
while (temp->next != NULL)
{ temp = temp->next;
}
temp->next = ptr;
ptr->next = NULL;
}

```

(iii) Write a C program to add a node with data x, after a node with data y.

```

→ Addinbtw (node * start)
{ node * ptr = (node *) malloc (sizeof (node));
(*ptr).data = x;
node * temp = Start;
while (temp->data != y) { temp = temp->next;
{ temp = temp->next; if (temp == NULL) if (temp == NULL)
if (temp == NULL) { exit(0); }
temp = temp->next; if (temp == NULL) { exit(0); }
ptr->next = temp->next;
temp->next = ptr;
}

```

(iv) Write a C program to add a node with data x before a node with data y.

```

Addbefore (node * start)
{ node * ptr = (node *) malloc (sizeof (node));
(*ptr).data = x;
node * temp = Start;
node * tempbef = Start;
while (temp->data != y)
{ tempbef = start; temp;
temp = temp->next;
if (temp == NULL) exit(0);
}

```

```

if (tempbef == Start)
{
    if (ptr->next == Start)
        Start = ptr; exit(0);
    temp->next = ptr;
    ptr->next = tempbef->next;
    tempbef->next = ptr;
}

```

Or

```

Addbef (struct node *start)
{
    node *p = (node *) malloc (sizeof (node));
    p->data = x;
    if (node * q = start;
        if (q == NULL)
            return (1);
        q
        if (q->data == 'y')
            if (p->next = q;
                Start = p;
            q
    while (q->next != NULL && (q->next)->data != 'y')
        q = q->next;
    if (q->next == NULL)
        exit(1);
    q
    p->next = q->next;
    q->next = p;
}

```

Q. Which one of the following statement, adding an element x after current position

- current = Newnode (x, current)
- current = Newnode (x, current->next);
- current->next = Newnode (x, current);
- current ->next = Newnode (x, current->next);

Q Consider the fn. f defined below:-

```
Struct item  
{int data;  
Struct item *next;  
};
```

```
int f(Struct item *b)  
{  
return ((b==NULL)|| (b->next==NULL)|| ((b->data <= b->next->data)  
&& f(b->next)));  
}
```

for a given LL ,the fn. f returns 1 iff

- (a) if LL is empty or exactly one element
- (b) elements in the LL are sorted in non-increasing order.
- (c) " " " " " " " -increasing "
- (d) none of the above .

indirectly means
if LL has 0 or 1 element
then it is also in non-
increasing order.

Q Write a C program to delete a node at the beginning of the linked list.

```
#include<stdio.h>  
#include<stdlib.h>  
  
Struct node  
{ int data;  
Struct node *next;  
};  
  
void del(Struct node *start)  
{ node *ptr = Start;  
Start = (*Start).next;  
free(ptr); //the memory is deallocated pointed by  
//ptr ,now *ptr is dangling pointer, so to avoid  
ptr=NULL; //any segmentation errors, we perform ptr=NULL.  
}
```

* if we write int a, then we can't deallocate the memory using free()

Q Write a C program to delete a node at the end of LL.

```
delatEnd(Struct node *start)  
{ node *ptr = start, *prev = start;  
if (ptr == NULL)  
{ exit(1);  
}
```

at start

```
while(ptr->next!=NULL)
{ prev=ptr;
  ptr=ptr->next;
  if (prev==start)
  { free(prev);
    start=prev=ptr=NULL;
  }
  else
  { prev->next=NULL;
    free(ptr);
    ptr=NULL;
  }
}
```

Q. Write a C program to delete a node before which contains data x.

```
del (Struct node * start)
{ node * ptr = start, * prev = start;
  if (ptr == NULL)
  { exit(1);
  }
  while (ptr->data != 'x' && ptr->next != NULL)
  { prev = ptr;
    ptr = ptr->next;
    if (ptr == NULL) { exit(1); }
  }
  if (prev == ptr == start)
  { start = start->next;
    free(ptr);
    ptr = NULL;
  }
  else
  { prev->next = ptr->next;
    free(ptr);
    ptr = NULL;
  }
}
```

Q Write a C program to delete a node before x.
Using 3 pointers

```
del(node * start)
{ node * ptr = start, * pprev = start, * prevtwobef = start;
  if (ptr == NULL)
    {exit(1);
     }
  while (
    if (start->data == 'x')
      {exit(0);
       }
    if (start->next->data == 'x')
      { start = start->next;
        free(ptr);
        ptr = pprev = prevtwobef = NULL;
      }
  while (
    ptr = (start->next)->next;
    pprev = start->next;
    prevtwobef = start;
    while (ptr->data != 'x' && ptr->next != NULL)
      { prevtwobef = pprev;
        pprev = ptr;
        ptr = ptr->next;
      }
    if (ptr->next == NULL || ptr->data != 'x')
      {exit(1);
       }
    prevtwobef->next = pprev->next;
    free(pprev);
    prevtwobef = pprev = ptr = NULL;
  }
```

Or

Using 2 pointers

```
del(node * start)
{ node * ptr = start, * prev = start;
  if (start == NULL)
    {exit(1);
     }
  if (start->data == 'x')
    {exit(1);
     }
  if (start->next->data == 'x')
    { start = start->next;
      prev = ptr = NULL;
      exit(0);
      }
  while (ptr->next
        ptr = start->next->next;
        prev = start->next;
        while ((ptr->next)->data != 'x' && (ptr->next) != NULL)
          { prev = ptr;
            ptr = ptr->next;
            }
        if ((ptr->next) == NULL || ptr->data != 'x')
          {exit(1);
           }
        if (& prev->next = ptr->next;
            free(ptr);
            ptr = prev = NULL;
            }
      }
```

Using 1 pointer

```
del(node * start)
{ node * ptr = start;
  if (start == NULL)
    {exit(1);
     }
  if (start->data == 'x')
    {exit(1);
     }
```

Note :- In order to delete any node in the given linked list apart from last node two pointers (minimum 2 pointers) are required.

Q. Write a program to find middle of the linked list

```
find(node *start)
{ node *ptr = start;
  int i=0;
  while(ptr->next!=NULL)
  {
    ptr = ptr->next;
    i++;
  }
  if(ptr==NULL)
  {
    exit(1);
  }
  if(ptr == start)
  {
    return(ptr);
  }
  if(i%2==0)
  {
    i=i/2;
  }
  else
  {
    i=i/2+1;
    ptr = start;
    for(int j=1;j<i;j++)
    {
      ptr = ptr->next;
    }
  }
}
```

Using 2 loops

$O(n)$

Using 1 loop

```
struct node*
find(node *start)
{
  node *ptr = start, *bomid = start;
  while(ptr->next != NULL && (ptr->next)->next != NULL)
  {
    bomid = bomid->next;
    ptr = (ptr->next)->next;
  }
  return(bomid);
}
```

$\Theta(n)$

$\frac{n}{2}$ times

* Binary Search can be applied on Linked List but it is not efficient because it will take $\Theta(n)$ times + $\log_2 n$ extra space.

Q Write a C program to perform Binary search on LL.

```
node* BS(node * start, int x)
{
    node * ptr = start;
    node * ptomid = start; node * m;
    while (ptrmid->data)
        if (start->next == NULL)
            if (start->data == x)
                return (start);
            else
                return (NULL);
        else
            if (m = findmid(start)); ————— O(n)
                if (m->data == x) ]
                    { return (m); } ————— O(1)
                if
                    else if (m->data > x) ] ————— O(1)
                    { m->next = NULL; }
                    BS (start);
                if
                    else
                        { Start = m->next; }
                        BS (start);
    }
}
```

Let $T(n)$ be the time complexity of Binary Search algo on a linked list which contains n elements, then

$$T(n) = \Theta(1), \text{ if } n=1$$

$$n + T\left(\frac{n}{2}\right), \text{ if } n \geq 1$$

\downarrow
 $\boxed{\Theta(n)}$

$n^{\log_2 2} = n^1 = n^{0+1}$

$\left[\begin{matrix} \log_2 n \\ \text{Stack Space} \end{matrix} \right]$

Q Write a C program to sort the given linked list.

BubbleSort (node * start)

```
{ node * ptr = start;  
  node * prev = start;
```

for (int i

int i = 1;

while (ptr->next != NULL)

```
{ ptr = ptr->next;
```

i++; // length of linked list

```
{ ptr = start->next;
```

for (int j=0; j < i; j++)

```
{ while (ptr->next != NULL)
```

```
{ if (prev->data > ptr->data)
```

```
{ prev->int temp = prev->data;
```

prev->data = ptr->data;

ptr->data = temp;

}

prev = ptr;

ptr = ptr->next;

```
{ ptr = start;
```

}

08

Sort (node * start)

```
{ node * ptr = start; null
```

int i = 1;

while (ptr->next != NULL)

```
{ ptr = ptr->next;
```

i++;

ptr = start;

for (int j=0; j < i; j++)

while (ptr->next != NULL)

Or

Sort (struct node * s)

```

{ struct node *p,*q;
for(Cp=S; p!=NULL; p=p->next)
{ if((p->data)
    for(q=p->next; q!=NULL; q=q->next)
{ if (p->data > q->data)
{ swap(p->data, q->data);
}
}
}
}

```

$$= O(n^2)$$

Q. Write a C-program to reverse the given LL.

```

Reverse(node *start)
{ node *ptr = start, node *temp1, node *temp2;
int i=1; int temp1 = start;
while (ptr < N) temp2 = start->next;
while (ptr < N)
{ ptr->temp1 = ptr->next;
temp2 = (ptr->next)->next;
ptr->next = temp2;
(ptr->next)->next = temp1;
temp1 = temp1->next;
temp2 = temp2->next;
}

```

node

```
Struct node1  
{ Struct node * addr; struct node* next;  
};
```

Reverse (Struct node* start)

```
{ node *ptr = start; node1 *ptr1;  
while (ptr != NULL)  
{ ptr1 = (node1 *) malloc(sizeof(node1));  
ptr1->addr = ptr;  
ptr = ptr->next;  
}
```

ptr : start;

Or

Reverse (Struct node **s)

```
{ struct node *b, *q, *r;  
r = s; q = p = NULL;  
while (r != NULL)  
{ p = q;  
q = r;  
r = r->next;  
q->next = p;  
s = q;  
}
```

→ 3 pointers
compulsory.

$O(n)$

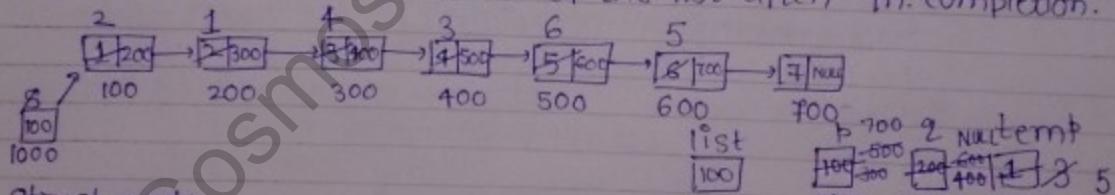
Q. The following C func. takes a single LL as i/p argument. It modified the list by moving the last node to the front of the list & returns the list. Some part of the code is left blank.

```
Type#definef Struct node{  
    int value;  
    Struct node *next;  
};node;
```

```
node* modifiedList(node* head)  
{node *p,*q;  
 if((head==NULL)|| (head->next==NULL))  
     return(head);  
 q=NULL;  
 p=head;  
 while(p->next!=NULL)  
 {q=p;  
 p=p->next;  
 q->next=head;  
 q->next=NULL;}  
 head=p;  
 return(head);}
```

Q. The following C fn. takes a singly linked list of integers as parameter & rearranges the elements of list. The fn. is called with the list containing the integers:-
 1,2,3,4,5,6,7

What will be the contents of the list after fn. completion.



```
Struct node  
{ int value;  
    Struct node *next;  
};
```

```
void arrange (struct node *list)
{node *p,*q;
 int temp;
 if (!list || !list->next)
    return;
 p=list;
 q=list->next;
 while(q)
 {temp=p->value; p->value=q->value; q->value=temp;
  p=q->next;
  q=p->next;
 }
}
```

list now :- 2, 1, 4, 3, 6, 5, 7

Q. Write a C program to find a cycle (loop) in the given linked list.

```
Struct node1
{node * addr;
 node1 * next;
};
```

```
findcycle (node * start)
{node1 * start1= (node1 *)malloc (sizeof(node1));
 no start1->addr= start;
 start1->next = NULL;
 node1 * pprev= start1;
 node1 * ptot= start;
 while (ptot!=NULL)
 {node1 * ptot1= (node1 *)malloc (sizeof(node1));
  pprev->next= ptot1;
  pprev= ptot1;
  ptot1->addr= ptot;
  ptot1->next= ptot->next;
 }
```

Or

Algorithm

- ① Take two pointers p_1 & p_2 .
- ② Increment p_1 by 1 & p_2 by 2.
- ③ Max. n loops p_1 & p_2 are same comes out to be same.
When p_1 & p_2 comes out to be equal, then there's a cycle.

Algorithm :-

Store every address in Array if that is not found in array
continue until match is found.

- * If we allocate memory using Malloc, then elements inside struct, values have garbage value.
- * If we allocate memory using Calloc, then elements inside struct will be initialized to 0.

Q. Write a C program to implement Stack using LL.

Q. Write a C " " " Queue " "

Q. " " " " " LL " arrays.

Q. " " " " " perform following opn on LL:-

(i) Union of 2 LL.

(ii) Intersection of 2 LL.

(iii) Polynomial addition of 2 LL.

(iv) " multiplication of 2 LL.

Date

18.08.12

Stacks

```
struct stack { int data;
                struct stack *next;
}; struct stack *top; int n;
```

Push(struct stack *top, int n);

```
struct stack *p = (stack *) malloc(sizeof(stack));
if (p == NULL)
    { printf("Memory is full");
        exit(1);
}
else
{ p->data = n;
    p->next = top;
    top = p;
}
```

}

→ [O(1)]

```
Pop(int n, struct stack *top)
{ struct stack *temp = top;
if (top == NULL)
    { printf("Stack is empty");
        exit(1);
}
top = temp;
temp = top->next;
n = temp->data;
free(temp);
temp = NULL;
}
```

→ [O(1)]

Queue

★ If To implement Queue property, we need atleast 2 pointers (Front & Rear).

Add

```
struct queue {
    int data;
    queue *next;
};
```

```
struct queue *front = NULL, *rear = NULL;
```

```
Add(int x)
```

```
{ queue *temp = (queue *)malloc(sizeof(queue));  
temp->data = x;  
if (front == NULL && rear == NULL)  
{ front = temp;  
rear = front;  
front->next = NULL; } } → [O(1)]
```

```
else if (front == rear)  
{ if (temp->next == front) NULL; rear->next = temp;  
rear = front = temp; } }
```

```
Del(int x)
```

```
{ if (queue *temp = front;  
if (front == rear == NULL)  
{ printf("Queue is empty");  
exit(1); } } // no element } } → [O(1)]
```

```
if (front == rear)  
{ front = front NULL;  
rear = NULL; free(temp); } // only one-element  
temp = NULL; }
```

```
else  
{ front = front->next;  
free(temp); } } ← [O(1)]
```

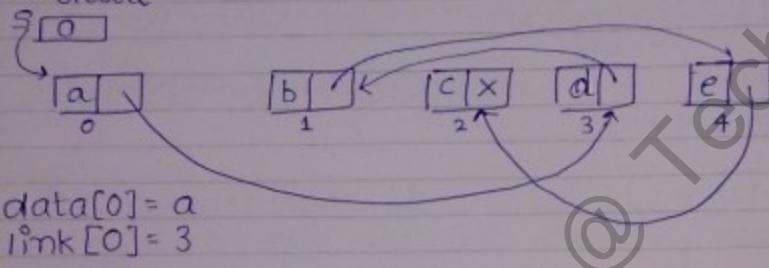
Implementing Linked List Using Arrays

0	a
1	b
2	c
3	d
4	e

data

0	3
1	4
2	-1
3	1
4	2

link



Q. Write a C program to display data part of the linked list using array representation.

```
display(int *d, int *l, int *start)
{ int *temp = start;
  while (*temp != -1)
    { printf("%d", data[temp]);
      temp = link[temp];
    }
}
```

```
display(data, link, s)
{ int temp = s;
  while (temp != -1)
    { printf("%d", data[temp]);
      temp = link[temp];
    }
}
```

Q. Write a C program to delete a ^{last} node from linked list

```

del(data, link, s)
{ temp = s;
  pprev = s;
  while (link[temp] != -1)
    { prev = temp;
      temp = link[temp];
      link[pprev] = -1;
    }
}

```

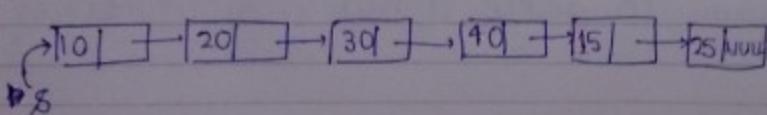
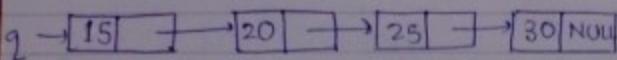
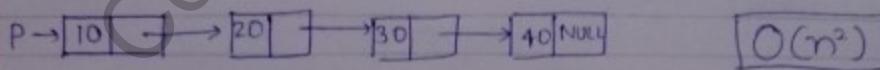
or

```

deletelast(data, link, s)
{ temp = pprev = s;
  if(link[s] == -1)
    { s = -1;
      exit(1);
    }
  else
    { while(link[temp] != -1)
      { pprev = temp;
        temp = link[temp];
        link[pprev] = -1;
      }
    }
}

```

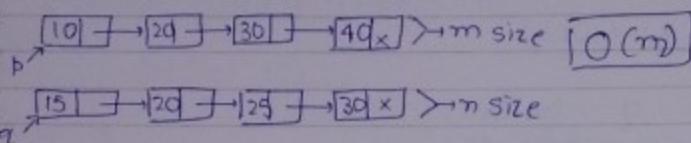
Union Of 2 Linked List :-



Algo:-

- 1 Add first linked list to the result. $\rightarrow O(n)$
- 2 Take every element of 2nd LL & check if it is available in first Linked List or not using Linear Search.
 - if it is available, don't add it in LL.
 - otherwise add it to LL. $\rightarrow O(n^2)$

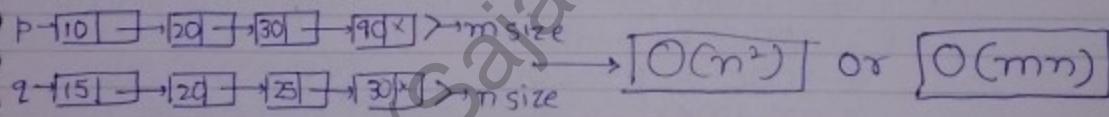
Concatenation of 2 LL



Algo:-

- 1 Go to the last node of the 1st LL. $\rightarrow O(n)$
- 2 & link that node with the starting node of the 2nd LL. $\rightarrow O(1)$.

Intersection of 2 LL:-



- 1 Take the 1st node of 1st Linked List & check whether the node's data (of 1st LL) matches with any node's data of 2nd Linked List.
 - if it matches with α then add it to result LL.
 - otherwise don't add.
- 2 Repeat Step 1 with all the other nodes of 1st LL.

Polynomial Addition

$$p: 10x^3 + 5x^2 + 2x + 5$$

$$q: 20x^3 + 15x^2 + 5x + 20$$

$$30x^3 + 20x^2 + 7x + 25$$

$$p: 10x^5 + 20x^5 + 3x^2 + 16$$

$$q: 5x^5 + 77x + 10$$

$$10x^5 + 20x^5 + 25x^5 + 3x^2 + 77x + 26$$

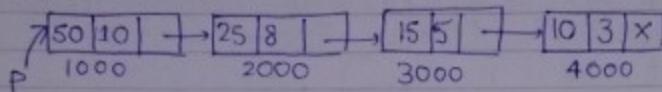
struct poly{

int power;

int coe;

poly * next;};

Polynomial addition



Time complexity:

$O(m+n)$

while($p \neq \text{NULL}$ && $q \neq \text{NULL}$)

{

 if ($p \rightarrow \text{power} == q \rightarrow \text{power}$)
 { $r \rightarrow \text{power} = q \rightarrow \text{power};$
 $r \rightarrow \text{coe} = p \rightarrow \text{coe} + q \rightarrow \text{coe};$
 $p = p \rightarrow \text{next};$
 $q = q \rightarrow \text{next};$
 $r = r \rightarrow \text{next};$ }
 }

 else

 { if ($p \rightarrow \text{power} > q \rightarrow \text{power}$)
 { $r \rightarrow \text{power} = p \rightarrow \text{power};$
 $r \rightarrow \text{coe} = p \rightarrow \text{coe};$
 $r = r \rightarrow \text{next};$
 $q = q \rightarrow \text{next};$
 $p = p \rightarrow \text{next};$ }
 }
 else
 { $r \rightarrow \text{power} = q \rightarrow \text{power};$
 $r \rightarrow \text{coe} = q \rightarrow \text{coe};$
 $r = r \rightarrow \text{next};$
 $q = q \rightarrow \text{next};$ }
 }

 if ($p == \text{NULL}$ && $q == \text{NULL}$)
 return (r);

 else

 { if ($p == \text{NULL}$)
 { add q to r ; }
 else
 { add p to r ; } }
 }

Polynomial Multiplication :-

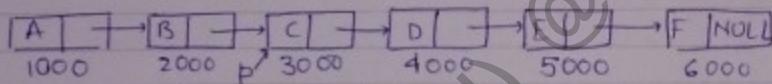
```

while (p != NULL)
  {
    while (q != NULL)
      {
        r->power = p->power + q->power;
        r->coe = p->coe * q->coe;
        r = r->next;
        q = q->next;
      }
    p = p->next;
    q = s; // starting of q
  }

```

$\rightarrow O(n^2)$

Consider the following LL:-



from the above linked list delete a node pointed by p if only p is given. (starting is not given.)

Ans.

```

del(node * p)
{
  node * temp = start;
  node * node = start;
  while (temp != p)
  {

```

copy the content

```

  p->data = (p->next)->data; // but if p is last node,
  p->next = (p->next)->next; // then this algo fails
  free(p->next);
}

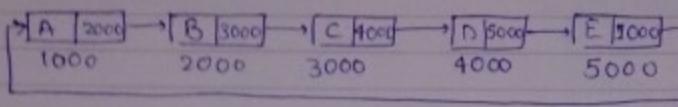
```

p

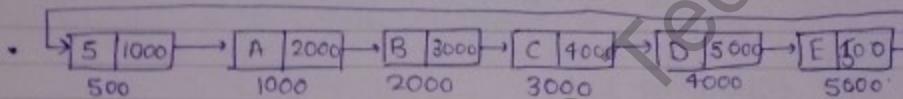
Drawbacks of Singly LL :-

- ① We can move only in one direction. (we can't go back)
- ② In singly LL last node's next part is always NULL w/o any use (we can keep starting node address, so that we can go back.)

Circular SLL :-



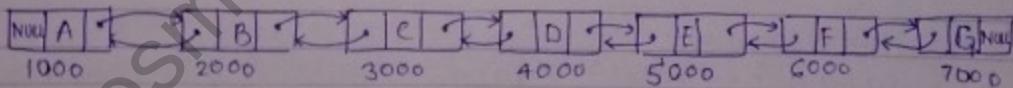
1. We can go back.
2. The drawback with circular SLL it may enter into the infinite loop.
3. In order to eliminate the infinite loop in circular SLL, we are going to head node.
4. Head node data part will contain valuable info. like total no. of nodes in circular SLL.



if

Doubly Linked List

```
struct DLL  
{ struct DLL *left;  
    int data;  
    struct DLL *right);  
};
```



Q. Write a C program to delete a node which contain data as x in the given double linked list.

Ans. del(int x)

```
{ struct DLL *ptr = start;  
if (start->data == x)  
{ Start-> = start->right;  
    start->left = NULL;  
    free(ptr);  
    ptr = NULL;
```

```

* if (ptr!=NULL)
    {exit(1);
    }

else
{ while (ptr->data!=x && ptr!=NULL)
    { ptr=ptr->right;
        if (ptr->right==NULL) { (ptr->left)->right = ptr->right;
                                free(ptr); ptr=NULL; }
        else { (ptr->left)->right = ptr->right;
               (ptr->right)->left = ptr->left;
               free(ptr);
               ptr=NULL;
        }
    }
}

```

Q Write a C program to insert a node with data x before a node with data y.

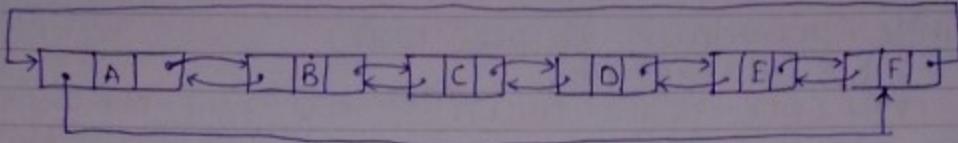
Ans. add (int y, dll * start)

```

dll * ptr= start; dll * temp= (dll *)malloc(sizeof(dll));
while (ptr->data!=y && ptr!=NULL)
{ ptr=ptr->right;
    if (ptr==NULL) {exit(1); }
    if (ptr==start)
    { temp->data=x;
      temp->left=NULL;
      start->left=temp;
      temp->right=start;
      start=temp;
    }
    else if (start==NULL)
    { temp->data=x;
      exit(1);
    }
    else
    { temp->data=x;
      temp->left=ptr->left;
      temp->right=ptr;
      (ptr->left)->right=temp;
      ptr->left=temp;
    }
}

```

Circular Doubly LL



Q

- (i) → →
- (ii) →

mark all nodes with flag 0, & make the flag as 1 as the node is visited (do this for (i)st LL & then for (ii)nd & check if the flag is 0 or 1, if it is already 1 from that node onwards, count the no. of nodes till NULL) → O(n)
 $\Omega(n)$

★ Application of Doubly LL :- Binary Tree

Binary Tree

Q. Write a C program to find no. of leaf nodes in the given binary tree.

Ans. totleafnodes (tree * p)

{ if (p->left == NULL & p->right == NULL)

{ if t++; return;

{ else

totleafnodes (p->left);

totleafnodes (p->right);

}

void main()

{ tree * start; // assume it is assigned to root address.

if (start != NULL)

{ totleafnodes (start); }

```
struct tree
{
    tree *lc;
    int data;
    tree *rc;
};
```

Or

```
NLN (struct tree *s)
{if (s==NULL) return 0;
else{if (s->lC==NULL && s->rC==NULL)
    {return(1);}
else{
    xL=NLN(s->lC); → T(n/2)
    xR=NLN(s->rC); → T(n/2)
    return(xL+xR); → c
    }
}
}
```

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + c \\ = 2T\left(\frac{n}{2}\right) + c \\ \therefore T(n)$$

$$\text{or } T(n) = T(a) + T(n-(a+1)) + c \\ \text{if } a > 0 \\ T(n) = T(n)$$

Q. Write a C program to find no. of non-leaf nodes in a given binary tree.

```
int i=0;
totnonleafnodes (tree *b)
{if (b->left!=NULL || b->right!=NULL)
{if (i++)
    return;
}
}
```

else
{totnonleafnodes(b->left);
 totnonleafnodes(b->right);
}
}

```
void main()
{if (root!=NULL)
{totnonleafnodes(root);
}
}
```

Or

```
NLN (tree *s)
{if (s==NULL) return 0;
else
{if (s->lC==NULL && s->rC==NULL)
    return(0);
}
}
```

```
if (b==NULL)
return;
if (b->left==NULL && b->right==NULL)
{return;
}
}
```

```

else
    x_L = NLN(S -> lC);
    x_R = NLN(S -> rC);
    return(L + x_L + x_R);
}
?
```

Q Write a C program to find total no. of nodes in the given binary tree.

```
int i=0;
totnodes(tree *p)
{ if(p==NULL)
    return;
    if(p->left == NULL & p->right == NULL)
        i++;
    return;
}
else
{ i++;
    totnodes(p->left);
    totnodes(p->right);
}
```

```

Or
NLN(tree + s)
  if (s == NULL) return 0;
  else
    if (s->lc == NULL &&
        return (1);
    else
      xL = NLN(s->lc);
      xR = NLN(s->rc);
      return (1 + xL + xR);
}

```

Q. Write a C program to find height of a given binary tree.

Ans. height (tree * s)

```
{if (s == NULL) return 0;
else
{if (s->lC == NULL && s->rC == NULL)
    return(0);
else
{xL = height(s->lC);
xR = height(s->rC);
return(1 + max(xL, xR));}}
```

Or

height (tree * p)
fn: no. of nodes

Q. Write a C program to check given binary tree is strict binary tree or not.

bool flag = true;

strictOrNot (tree * p)

```
{if (p->left == NULL && p->right == NULL)
    {return;
     }
else if ((p->left == NULL && p->right != NULL) ||
         (p->left != NULL && p->right == NULL))
    {flag = false;
     return;
     }
else
{strictOrNot(p->left);
 strictOrNot(p->right);}}
```

g
Or

Strict_BT (tree * s)

```
{if (s == NULL) return (1);
else
{if (s->lC == NULL && s->rC == NULL)
    return 1;
else}
```

```

if(s->lc==NULL && s->rc==NULL)
    return (strict_BT(s->lc) & strict_BT(s->rc));
else
    { return 0;
}

```

Q Consider the following C program:-

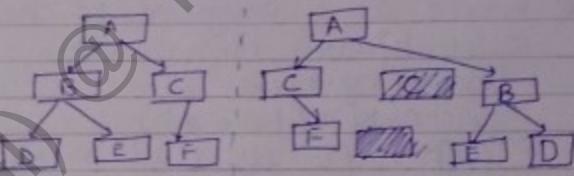
Tree (struct BtNode *s)

```

if (s==NULL)
    { t=(struct noBtNode *)malloc(sizeof(struct BtNode));
      t->data = s->data;
      t->lc = s->rc; tree(s->rc);
      t->rc = s->lc; tree(s->lc);
      return(t);
}

```

Mirror Image



Q Write a C program to find min. element in the given binary search tree.

Ans. int min= root->data;
findmin(tree *p)
{ if(p->left==NULL) ~~return p->right->data;~~
 return (p->data);
 if(p->data<
 p=findmin(p->left));
}

or

```

find_min(tree *s)
{ if(s==NULL) return;
else
    { while(s->lc!=NULL)
        s=s->lc;
    return(s->data);
}
}

```

Q. A data structure comprises of nodes, each of which has exactly 2 pointers to other nodes with no Null pointers, the following C program is to be used to count the no. of nodes in given binary tree. It uses a mark field assumed to be '0' initially for all nodes. Find the missing statement in the code :-

→ Struct test

```
struct test { int info, mark; }
```

```
struct test *p, *q;
```

```
{ };
```

```
int nodecount (struct test *a)
```

```
{ if (a->mark)
```

```
    return (0);
```

```
else
```

```
{ a->mark=1; }
```

```
return (nodecount (a->p)+nodecount (a->q)+1);
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

```
{ };
```

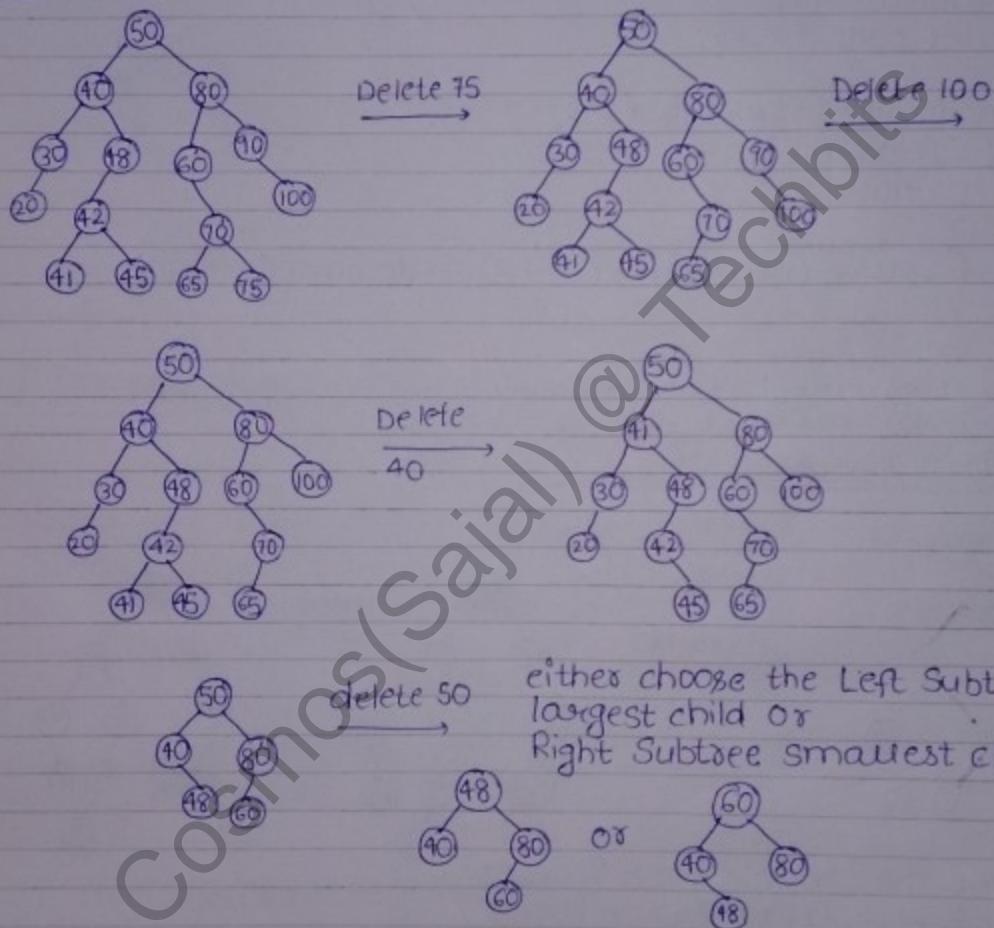
```
{ };
```

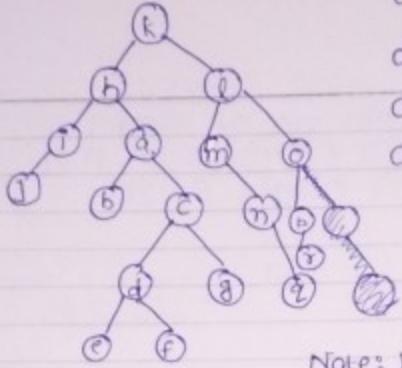
```
{ };
```

Algo

- ① Find the place of $x \rightarrow O(n)$
- ② Insert element $x \rightarrow \frac{O(1)}{O(n)}$

Deletion





$\text{delete}(a)$: inorder predecessor or
inorder successor $\rightarrow \text{lo}(a)$

$\text{delete}(c) = e$ f (o) g

$\text{delete}(d) = e$ (o) f

$\text{delete}(o) = \text{connect } l \& p$

$\text{delete}(q) = \text{no need to do anything}$

$$n + O(1) = O(n)$$

\downarrow
finding for place adjustment

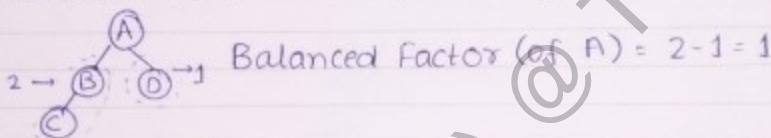
Note: If we delete any node from BST replacement will be done by inorder successor or inorder predecessor.

AVL Tree

(Adelson, Valski, Landis)

Or Balanced Binary Search tree [Size $\rightarrow O(\log n)$]

Balanced Factor = $H(RST) - H(LST)$ or $H(RST) - H(LST)$



Note: A BST in which every node's Balance factor is -1, 0 or (max. diff.) is called AVL tree.

Height = 2 \rightarrow

min. no. of nodes for a tree to be AVL of height 2 = 4

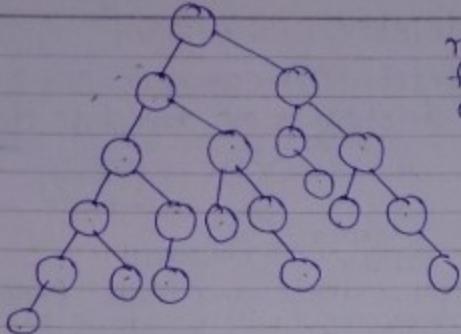
Height = 3 \rightarrow

min. no. of nodes for a tree to be AVL of height 3 = 7

Height = 4

min. no. of nodes for a tree to be AVL of height 4 = 12

height = 5



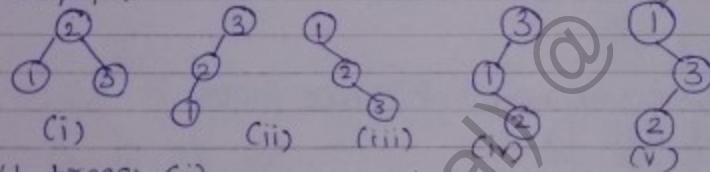
min. no. of nodes
for a tree to be AVL
of height 5 = 20

Min. no. of nodes in a height H AVL tree:

$$MNN(H) = MNN(H-1) + MNN(H-2) + 1$$

$$\boxed{MNN(H) = MNN(H-1) + MNN(H-2) + 1}$$

Q. n = 3(1, 2, 3) → BST



AVL trees - (i)

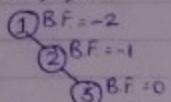
BST - (i), (ii), (iii), (iv), (v)

(ii), (iii), (iv), (v) are converted to (i) using rotations, & this rotation should take constant time.

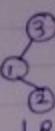
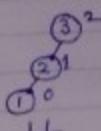
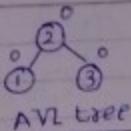
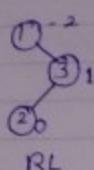
Date

21.08.12 AVL Trees

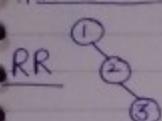
$$n=3 \Rightarrow (1, 2, 3)$$



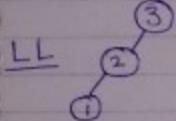
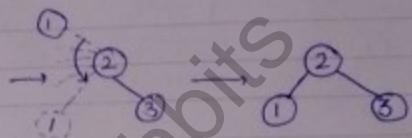
In this case ① has problem from R (because of ②) & ② has problem from its right, i.e. RR problem.



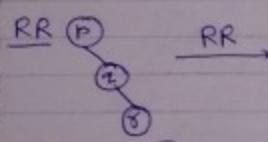
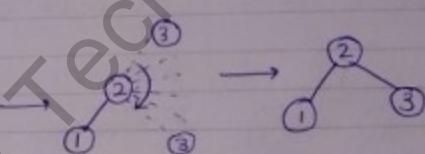
Rotations :-



→ 1. keep the nodes as it is which has $B.F = 0, 1, +$ & rotate the node which has $B.F \neq 0, 1, -1$ to the anticlockwise (one left rotation)

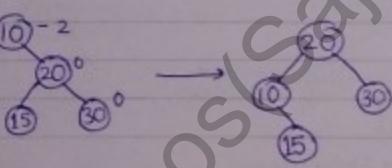


→ 1. keep the nodes as it is which has $B.F = 0, 1, -1$ & rotate the node which has $B.F \neq 0, 1, -1$ clockwise (one right rotation)

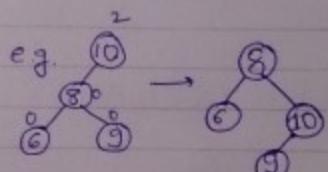
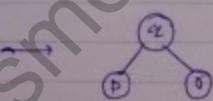
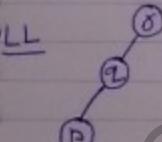


→ $q \rightarrow lc = p$
 ~~$p \rightarrow rc = NULL$~~

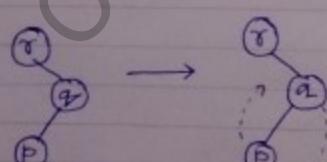
e.g.



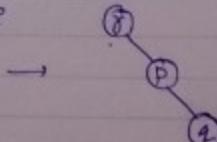
$q \rightarrow lc = r$
 $r \rightarrow lc: NULL$



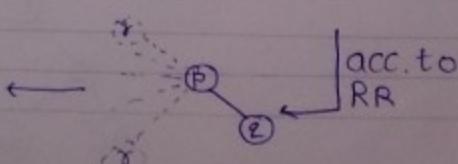
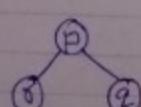
RL



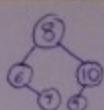
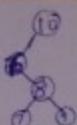
I do one clockwise rotation



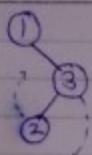
now this has RR problem.



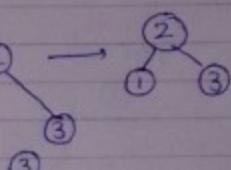
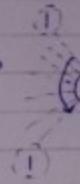
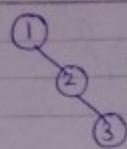
acc. to RR



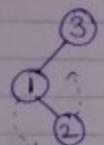
e.g.



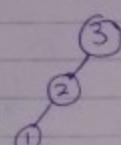
1 clockwise
rotation



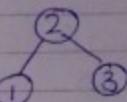
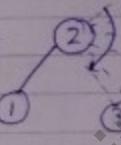
LR



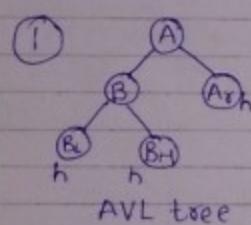
1 anti-clockwise
rotation



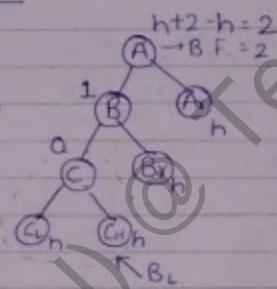
now its



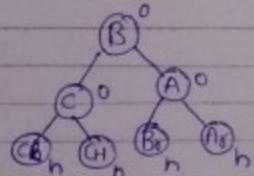
Insertion into AVL tree:



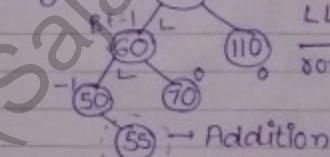
Insert an
element into
 B_L



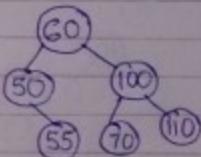
L-L rotation
(problem arises because of insertion in B_L which is Left to B (immediate left of A))



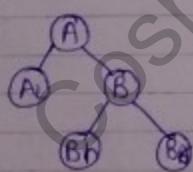
e.g.



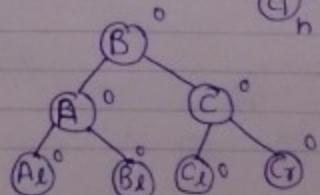
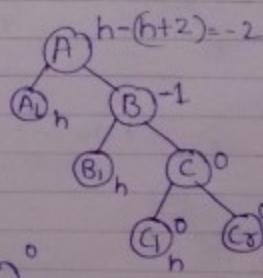
LL
rotation

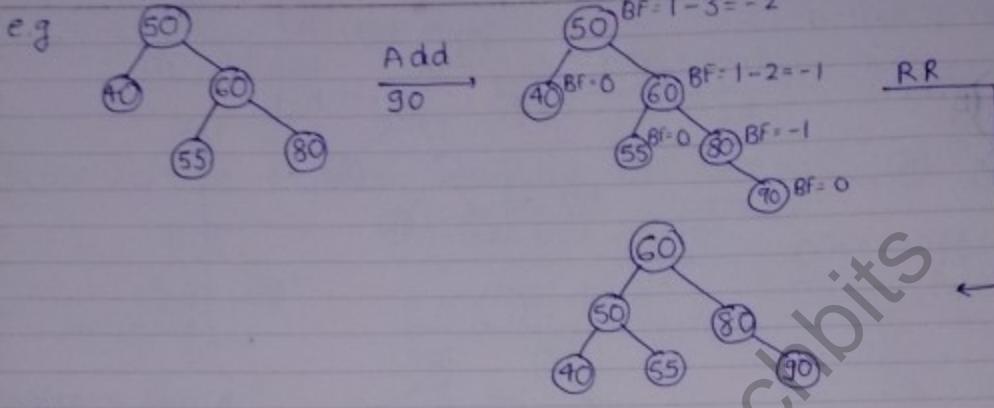


② RR problem

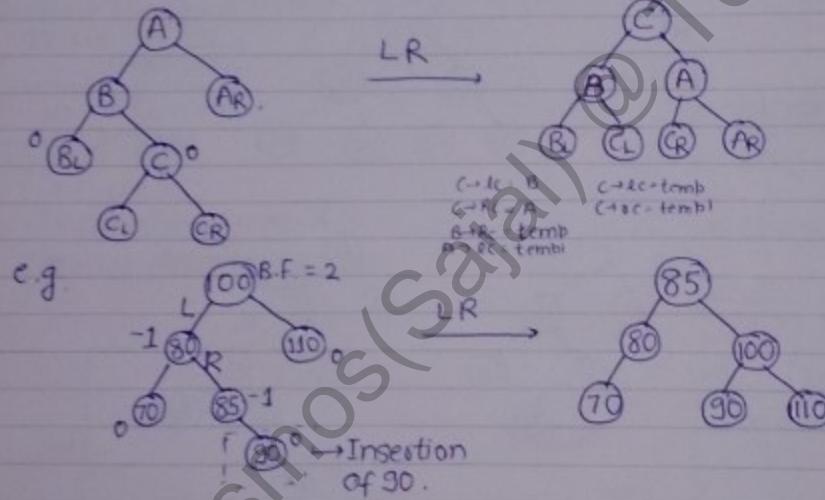


Insertion
into B_R



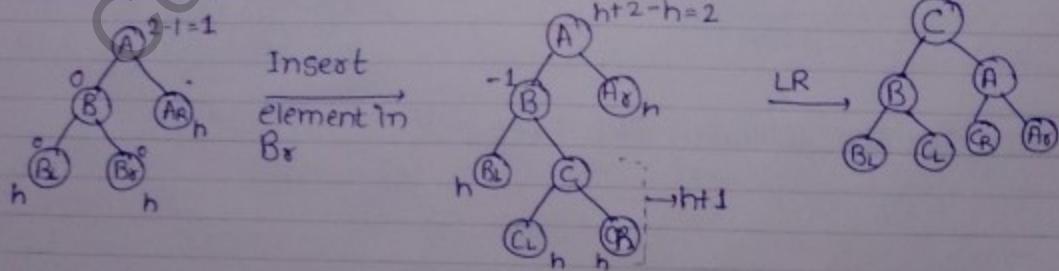


LR

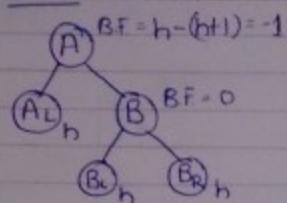


- * In LR problem 2 rotations are compulsory.
- * After doing one rotation we will get LL. After getting LL problem, apply LL rotation.

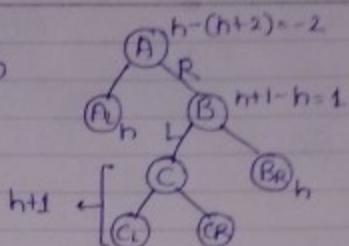
Sir's explanation:-



RL

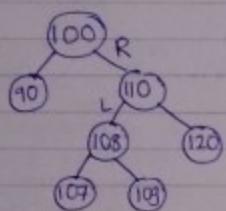


Insert into
 B_L

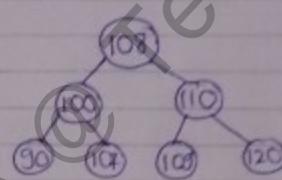


RL
rotation

e.g.

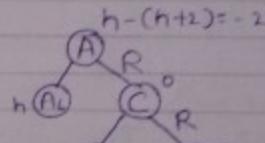
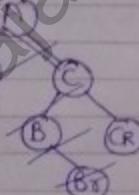
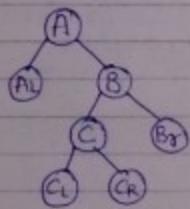


RL
problem

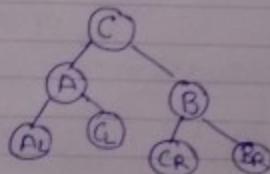


- * In RL rotation 2 severe rotations are compulsory.
- * After 1 rotation we will get RR problem.
- * Apply 1 more RPL rotation to get AVL tree.

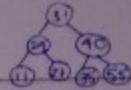
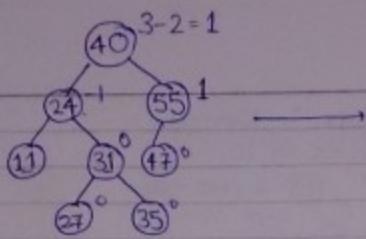
Or



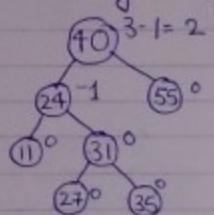
RR



Q. Consider the following AVL tree:-



After deleting 47, show resultant AVL tree.



★ To delete an element,
Search that element,
Searching complexity :-
& then perform
the deletion algorithm.

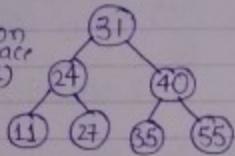
Search that element, Searching complexity :- $O(n)$ [BST] & not AVL
 & then perform $O(log n)$ [AVL & BST]

then perform $O(\log n)$ [AVL & BST] the deletion algorithm.

the deletion algorithm

~~O(n) [BST] & not AVL
O(logn) [AVL & BST]~~

LR
Left
rotat



Complexity :-

① Finding place of deletion

② Deletion of that element → constant

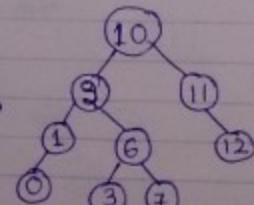
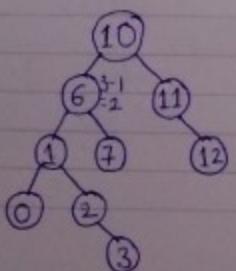
③ Traversing the complete path from site of deletion to the root to check the balancing factors = logn

$$2\log n = \Theta(\log n)$$

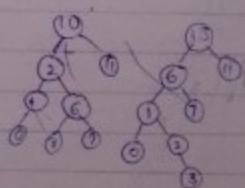
Q. What will be the resulting AVL tree if we insert following elements into AVL tree:-

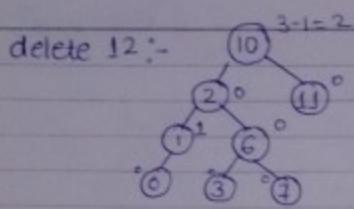
10,6,11,12,1,7,0,2,3

Addition $\rightarrow O(\log n)$



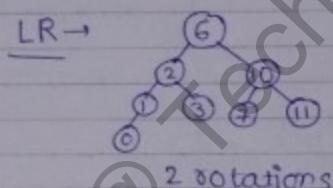
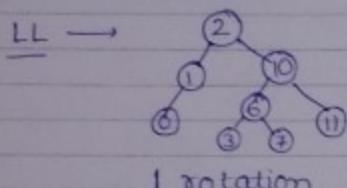
AVL Tree





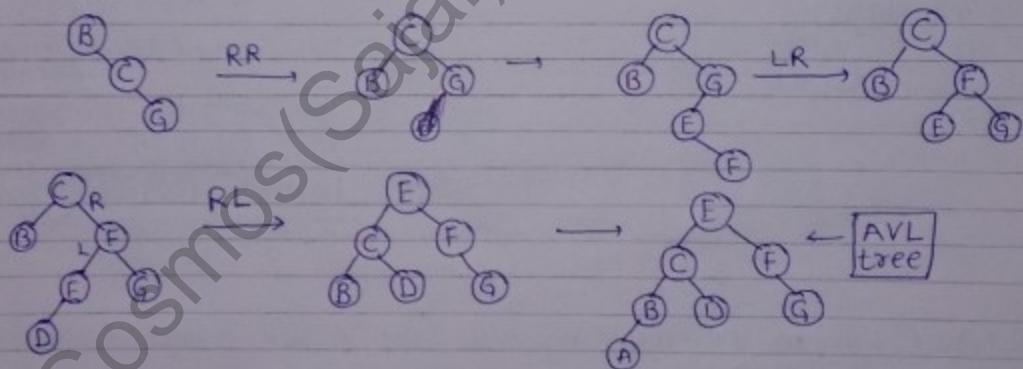
Now problem is because of Left subtree of 10, i.e., L, now, we are at 2, we can 2 has B.F.=0, so we can go both left & right, going left results in LL, " " right " " LR

What will be the resulting AVL tree if we delete 12.

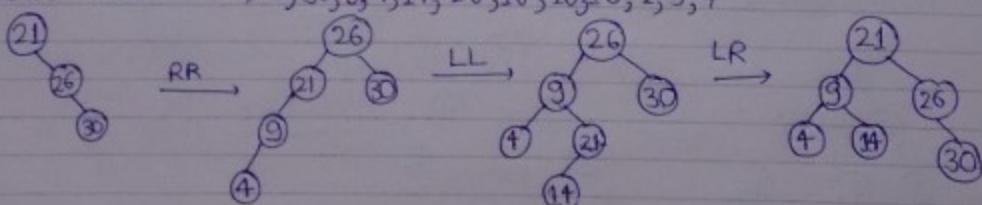


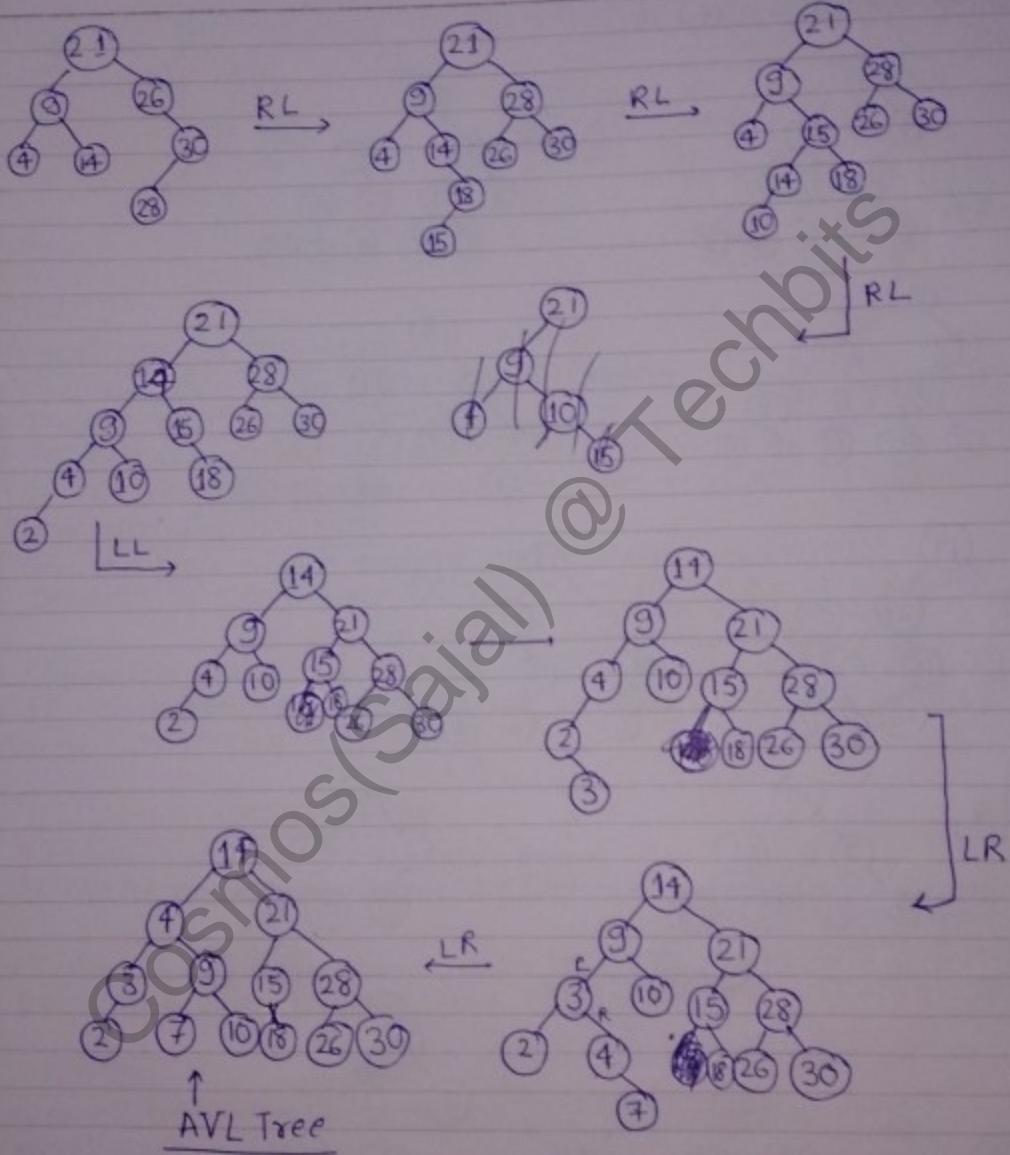
We could apply any of these.
(Either LL or LR.)

Q. Construct AVL trees from following keys.
B, C, G, E, F, D, A

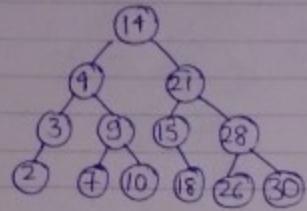


Q. Create AVL - 21, 26, 30, 9, 4, 14, 28, 18, 15, 10, 2, 3, 7

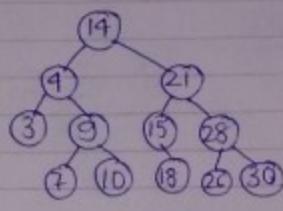




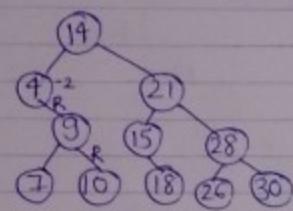
Q. Delete 2, 3, 10, 18, 4, 9, 14, 7, 15



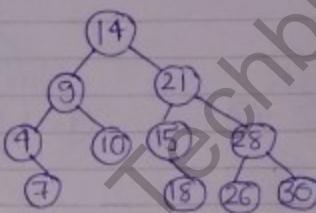
2 →



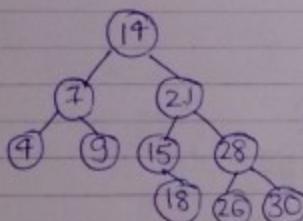
3 →



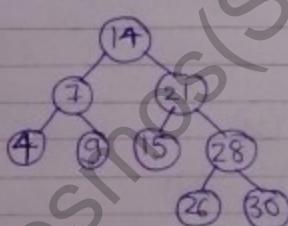
→



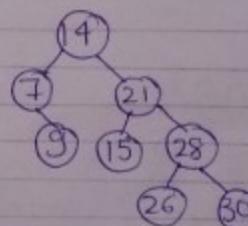
↓ 10



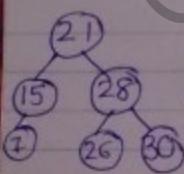
↓ 18



4 →

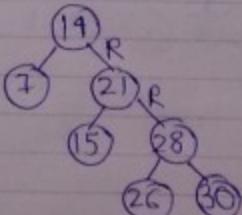


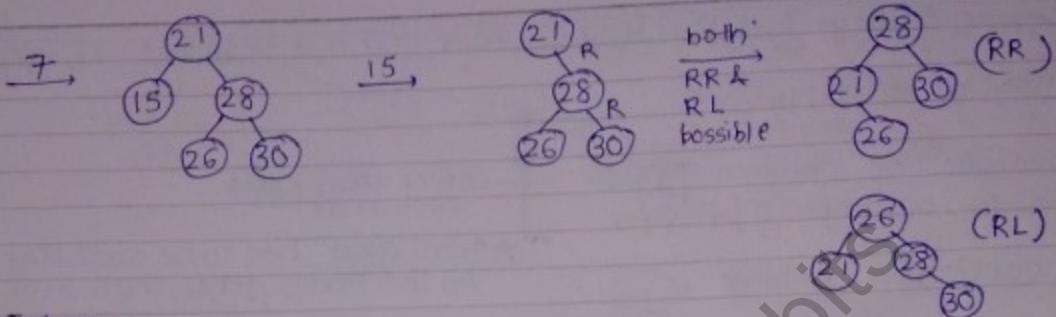
↓ 9



14 ←

RR ←





Note:-

Algorithm for insertion

1. Find the place of the element $\rightarrow O(\log n)$.

2. Insert that element $\rightarrow O(1)$.

3. Traverse the same path to find BF $\rightarrow O(\log n)$

4. If not balanced at some node, perform rotation $\rightarrow O(1)$

Total complexity: $O(\log n)$

Algorithm for deletion

1. Find the place of the element to be deleted $\rightarrow O(\log n)$

2. Deletion of element $\rightarrow O(\log n)$ [when there are 2 children of a node to be deleted.]

$\rightarrow O(1)$ [When children are 0 or 1.]

3. Check the balancing factors $\rightarrow O(\log n)$ by traversing

4. If not balanced at some node, perform rotation $\rightarrow O(1)$

Total Complexity: $O(\log n)$

Multiway Search Tree

1. Search Tree (Ordered Tree)

2. At every node minimum 2 children allowed (order of multiway search tree(m)).

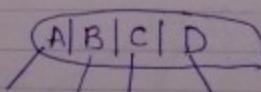
3. Height of Multiway search tree is $O(\log_m n)$. [Balanced M-way search tree.]

4. Deletion & Insertion will take $O(\log_m n)$.

B-Tree:-

Order of B-Tree = m (assume) [means each node can have max. ' m ' children.]

∴ maximum $(m-1)$ keys
maximum m -children



min. $\lceil \frac{m}{2} \rceil$ children

min. $\lceil \frac{m}{2} \rceil - 1$ keys.

Degree = 5 | Keys: 5, 10, 12, 13, 14, 1, 2, 3, 4, 20, 18, 19, 17, 16, 15, 25, 23, 24, 22, 21, 30, 28, 29

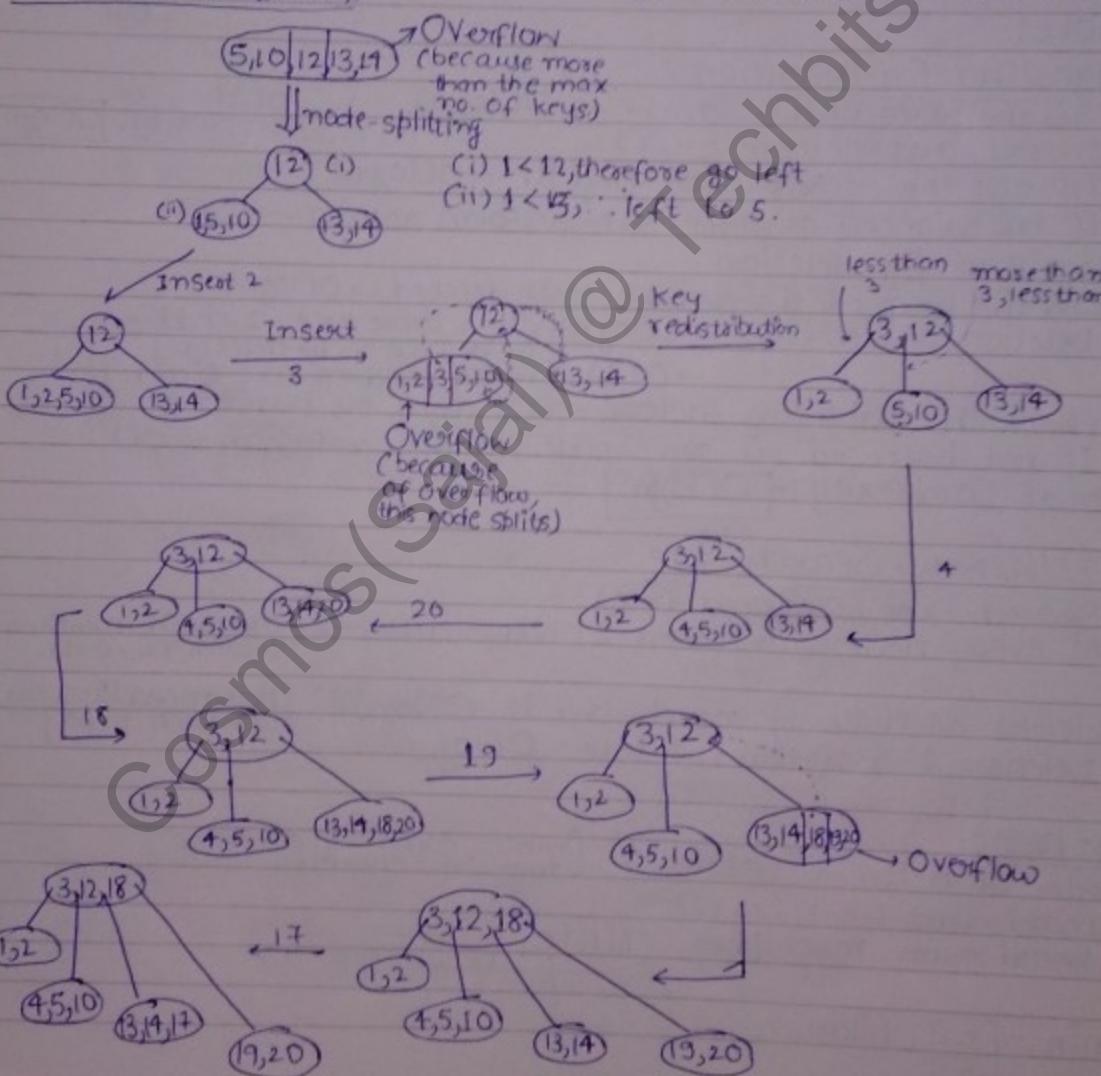
max. no. of children = 5

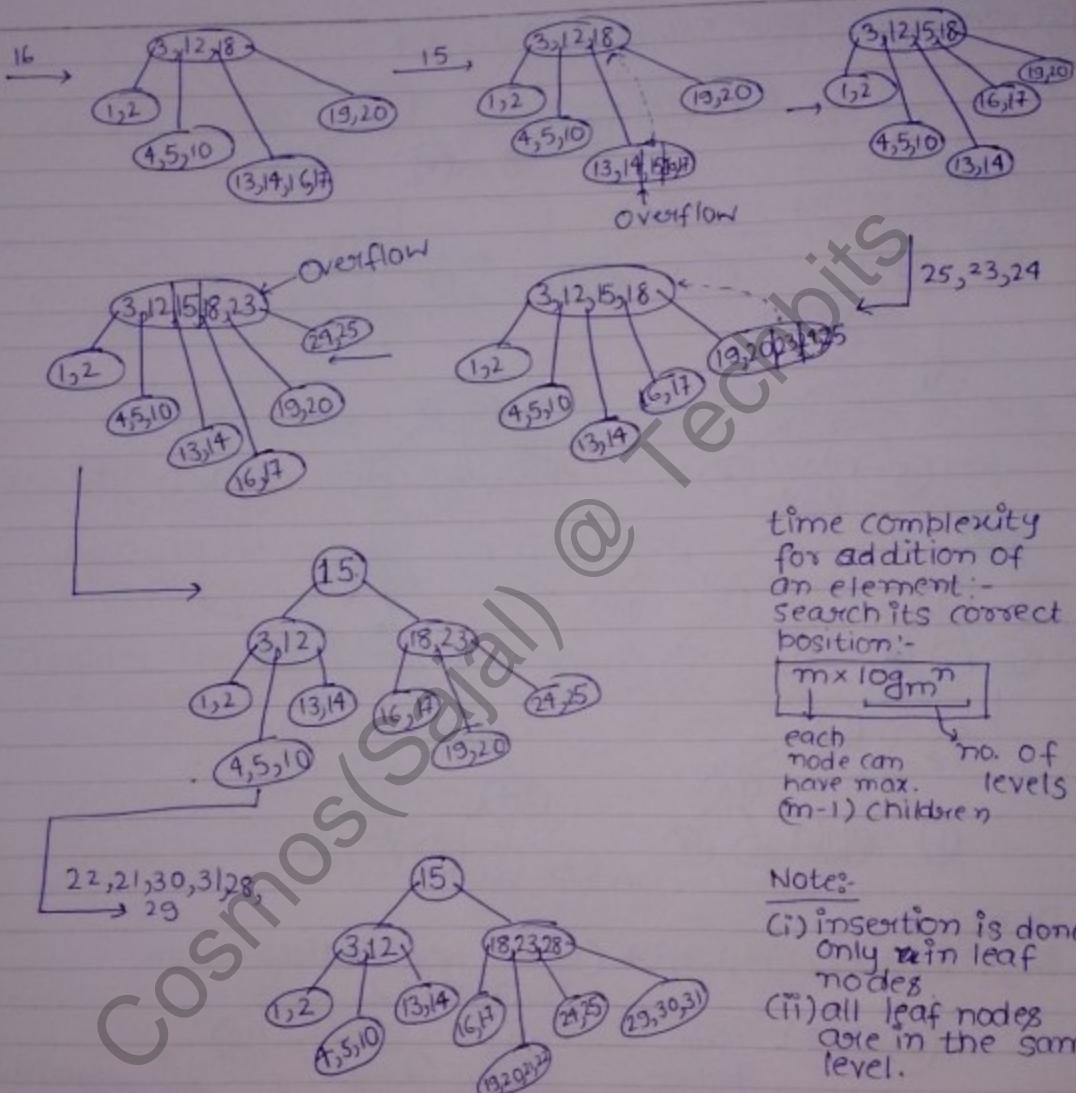
" " keys = 4

min. no. of children = $\lceil \frac{5}{2} \rceil = 3$

min. no. of keys = $\lceil \frac{5}{2} \rceil - 1 = 3 - 1 = 2$

Insertion in B-Trees





time complexity
for addition of
an element:-
Search its correct
position:-

$$m \times \log_m n$$

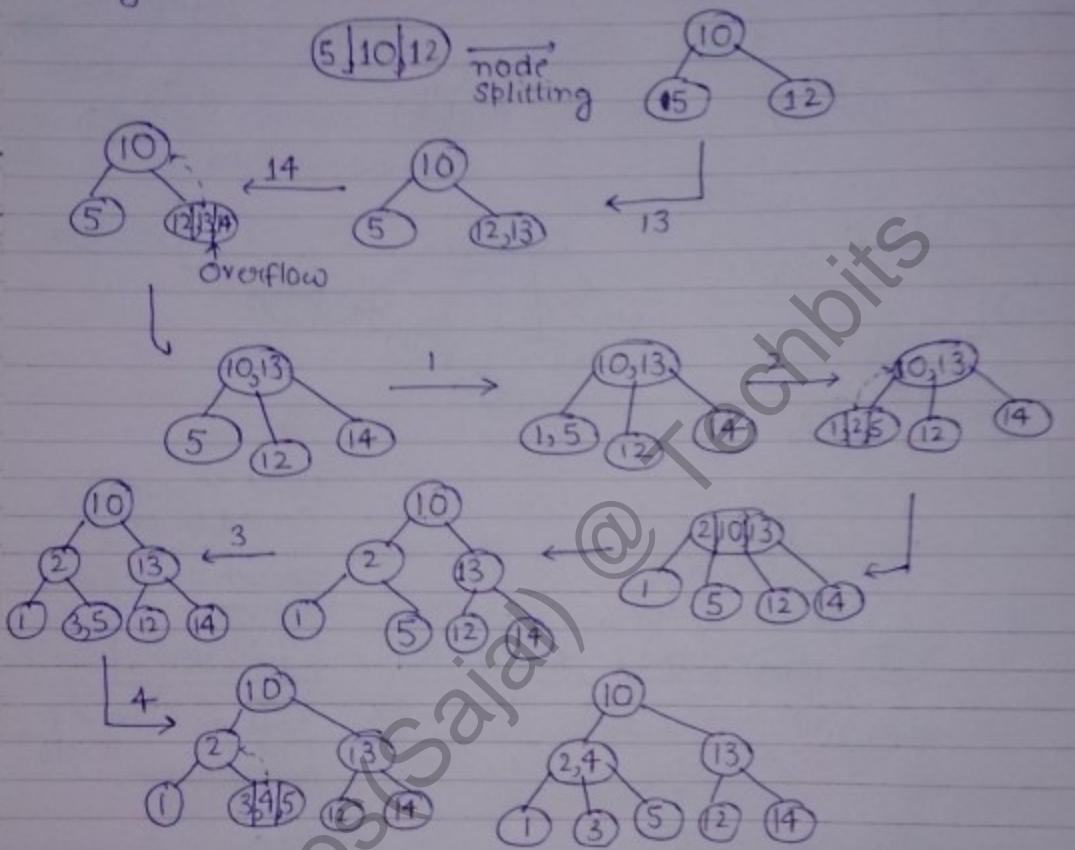
each node can have max. no. of levels
(m-1) children

Note:-

- (i) insertion is done only in leaf nodes
- (ii) all leaf nodes are in the same level.

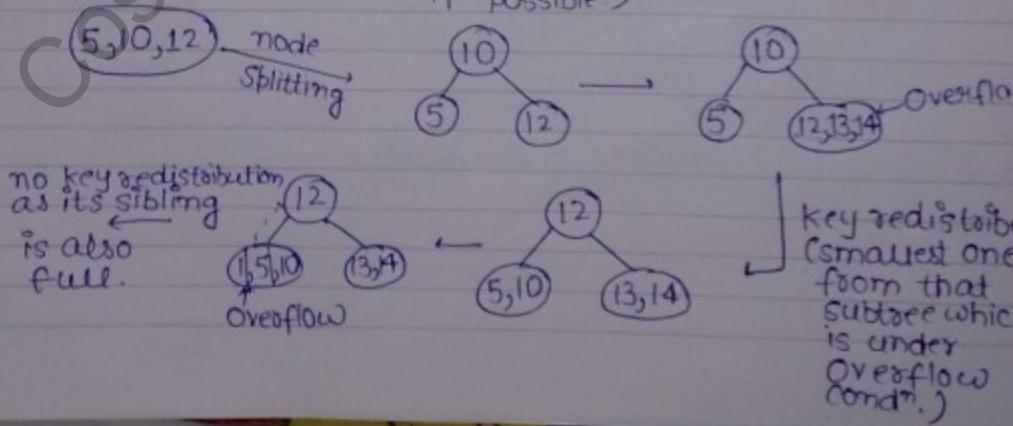
★ A 2-3 tree means max. 3 children

Q. degree = 3 | keys: 5, 10, 12, 13, 14, 1, 2, 3, 4

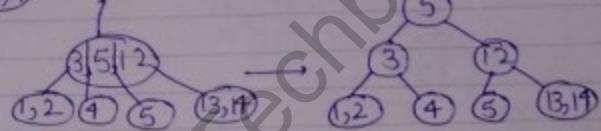
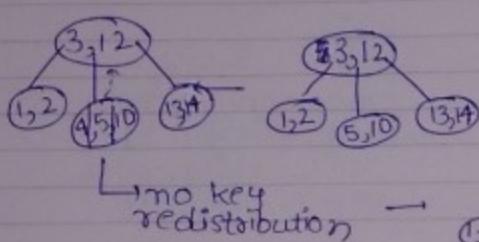
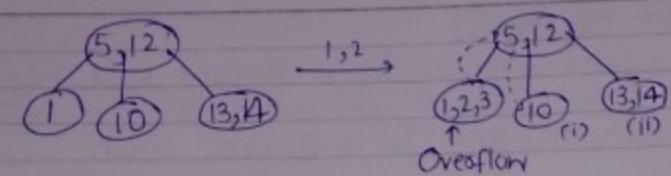


Q. Degree = 3

Keys: 5, 10, 12, 13, 14, 1, 2, 3, 4 (using key redistribution if possible)



if both the
 siblings (left &
 right) have
 space, carry
 on redistribution (left or
 right)
 for key
 redistribution,
 ask only the
 immediate siblings
 so ask for
 key redistribution
 with i_1 only.

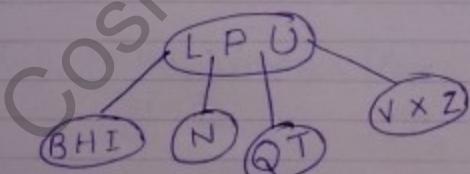


Note:- While constructing B-Tree for the given trees & keys, key-redistribution is always better than node splitting, because we are not wasting space & height of B-Tree also decreases, so time complexity also decreases

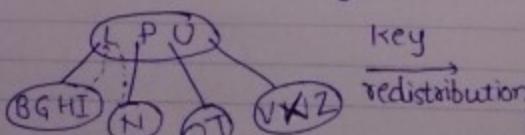
Q. Consider the following B-Tree:

2-3-4 Tree (A B-Tree with min. degree of 2, i.e., max
 min. children = 2
 min. keys = 1
 max. children = 4
 max. keys = 3)

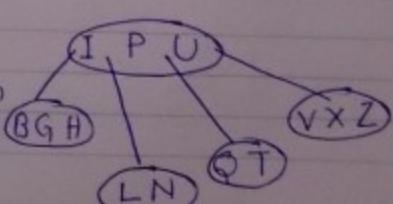
in which each data item is a letter



What is the result of inserting G in the above tree:-



Key
redistribution



Q. Consider a B-Tree with order = 32

(i) find max. no. of nodes present in a B-Tree with height 3.

(ii) find min. no. of nodes present in a B-Tree with height 3.

Ans. (i) height = $\log_{32} n \approx 3$

$$\begin{aligned} 3+1 &= \log_{32} n \\ 4 &= \log_{32} n \\ 32^4 &= 32^{\log_{32} n} \\ 32^4 &= n \end{aligned}$$

$$\begin{aligned} \text{max. children} &= 32 \\ \text{max. keys} &= 31 \\ \text{min. children} &= 16 \\ \text{min. keys} &= 15 \end{aligned}$$

for max.

Level	nodes	keys	children
1	1	31	32
2	32	32×31	32×32
3	32×32	$32 \times 32 \times 31$	$32 \times 32 \times 32$
4	$32 \times 32 \times 32$	$32 \times 32 \times 32 \times 31$	$32 \times 32 \times 32 \times 32$

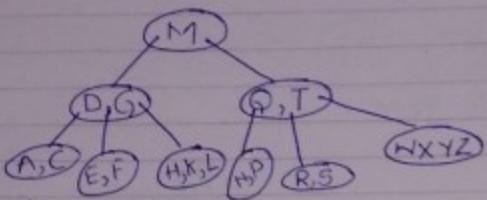
$$\begin{aligned} \text{Total nodes} &= 32^3 + 32^2 + 32^1 + 32^0 \\ &= 32 \left[1 [32^4 - 1] \right] \\ &\approx 32^3 \end{aligned}$$

max. no. of nodes in a height h, order m B-Tree:-

$$\begin{aligned} &= m^0 + m^1 + \dots + m^h \\ &= \frac{1(m^{h+1} - 1)}{m-1} = \frac{m^{h+1} - 1}{m-1} \end{aligned}$$

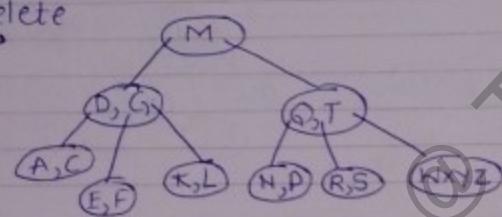
$$\text{max. keys: } (m-1)[m^0 + m^1 + \dots + m^h] = (m-1) \frac{(m^{h+1} - 1)}{(m-1)} = (m^{h+1} - 1)$$

Q. Consider the following B-Tree



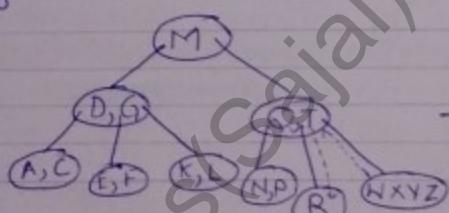
max. no. of keys = 4
Order = 5
min. no. of children = $\lceil \frac{5}{2} \rceil = 3$
min. no. of keys = $3 - 1 = 2$

Delete H



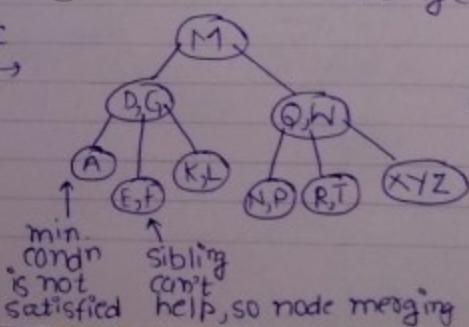
* If the element H is present in leaf node, delete that element. Check for the min. condn. If it is satisfied, then do nothing, otherwise

Delete G

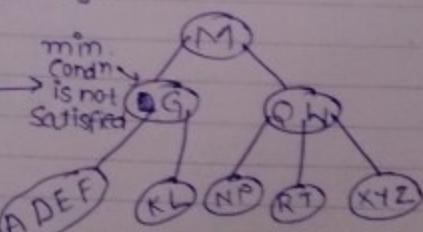


min. condn not satisfied
ask left sibling → can't give
ask right → can [if min. condn is not satisfied,
distribute the keys from immediate
sibling (if possible)].

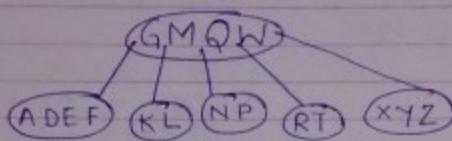
Delete C



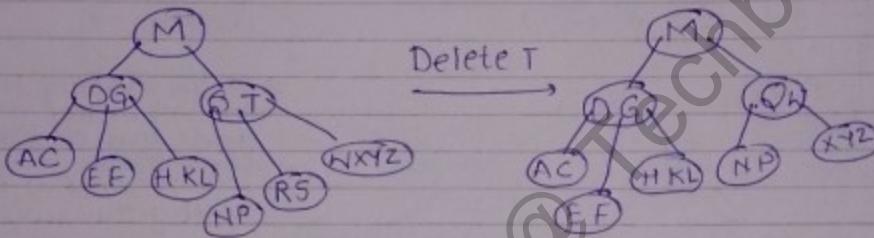
min. condn is not satisfied
sibling can't help, so node merging



G asks the sibling, but it can't help, \therefore node merging

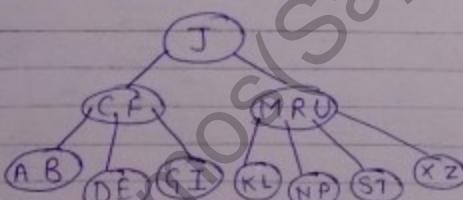


Q. Consider the following B-Tree.

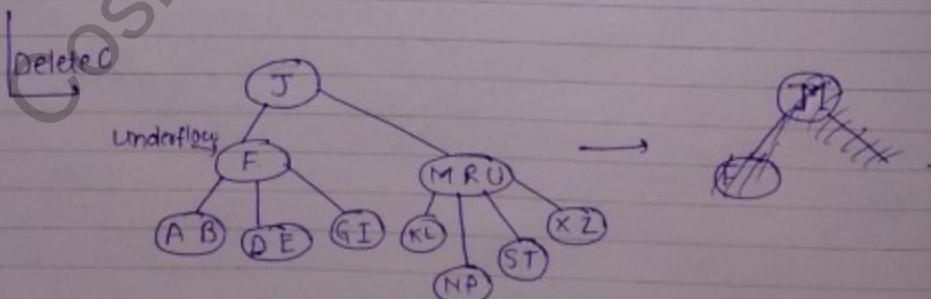


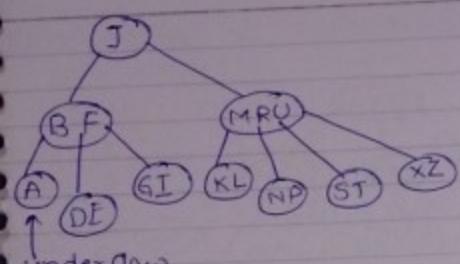
* If we delete any element which is present in non-leaf node, replacement will be done by children, then if not by children, then it is done by sibling, without satisfying merge with parent.

Q.

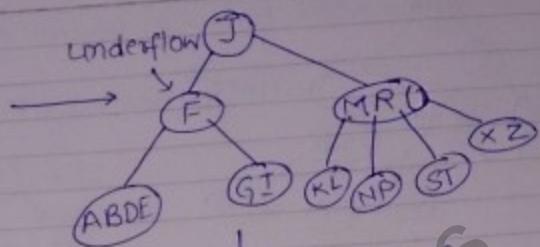


min. keys = 2
min. children = 3
max. keys = 4
max. children = 5

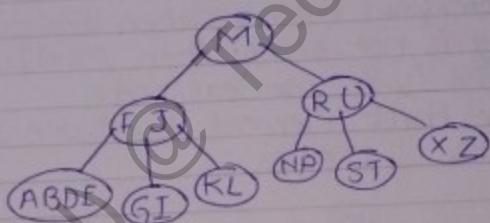




underflow
Crowd will ask its sibling, if not satisfied by underflow, ask then merge with parent
[in case of underflow, ask its sibling, if not satisfied by underflow, ask then merge with parent]
in case of deletion, replace blindly with children & then perform other opⁿ



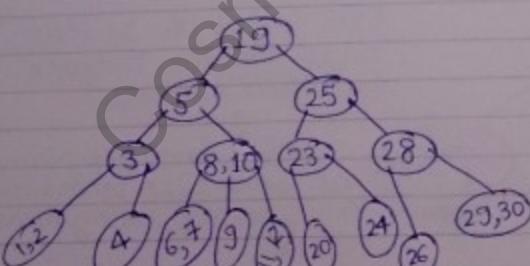
in case of underflow, ask the sibling 1st, (MRU) can give



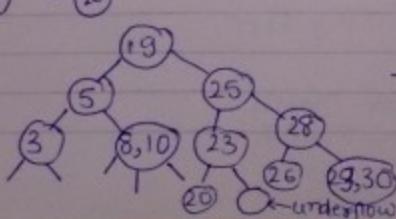
★ In case of element deleted from non-leaf node, replace the element (or take the element) from children & then perform opⁿ of underflow (if there is some underflow in children).

In case of underflow, ask help from siblings, if not then merge with parents.

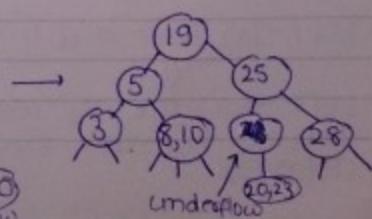
Q.

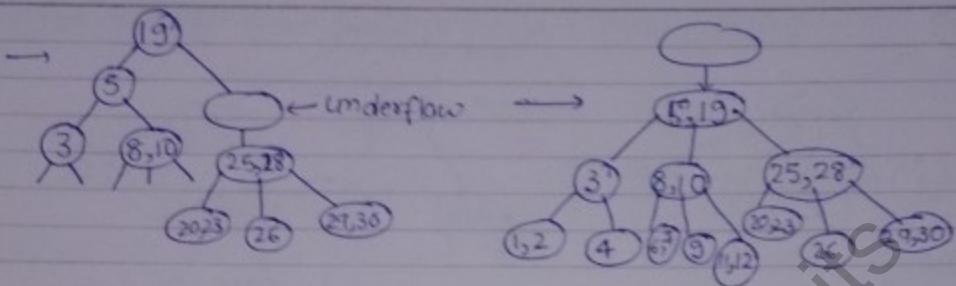


Delete 24



min-keys = 1
min-children = 2
max-children = 4
max-keys = 3





Deletion Algorithm:-

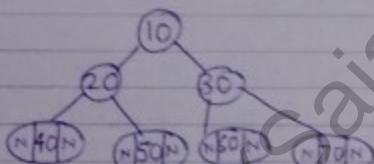
1. Element is in Leaf node

 delete element & check for underflow
 if underflow happened then borrow keys from siblings. If
 Sibling is not ready then combine with the parent.

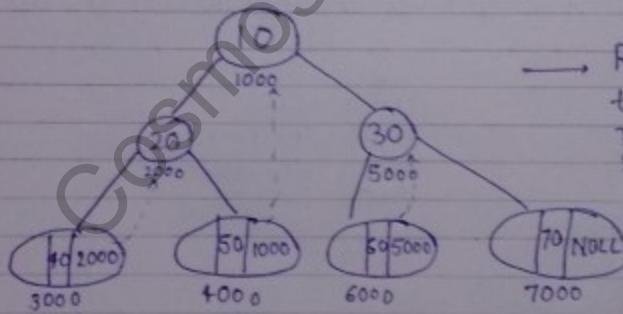
2. Element is in non-leaf node

 delete that element & replace by inorder predecessor or
 inorder successor.

Q.



In Binary Tree, all leafnodes
leftnodes Left pointer &
right pointer are null.



→ Right In threaded binary
tree (right pt. of leaf
node contain inorder
successor & address)

To find inorder no. need
of recursive procedure
(no stack req.) using Threaded
BT.

→ Left In Threaded Binary Tree

↓
left pointer of leaf nodes
contain inorder successor
address.

32 → children

Q In order 32 B-Tree with height 3, find min. keys & min. nodes.

level	nodes	keys	children	[min.]
1	1	1	2	
2	2	15×2	2×16	
3	$32(2 \times 16)$	32×15	$2 \times 16 \times 16$	
4	$2 \times 16 \times 16$	$2 \times 16 \times 16 \times 15$	0	

Order m B-Tree with height h contain min. no. of nodes:

$$1 + 2 \times \left(\frac{m}{2}\right)^0 + 2 \times \left(\frac{m}{2}\right)^1 + 2 \times \left(\frac{m}{2}\right)^2 + \dots + 2 \times \left(\frac{m}{2}\right)^{h-1}$$
$$= 1 + 2 \left\{ \frac{1 \left(\left(\frac{m}{2}\right)^h - 1 \right)}{\left(\frac{m}{2} - 1\right)} \right\} = 1 + 2 \left\{ \frac{\left(\frac{m}{2}\right)^h - 1}{\left(\frac{m}{2} - 1\right)} \right\}$$

Stack

one side is open another side is closed.

LIFO OR FILO.

Top is variable, which contain position of top most element.

ADT of Stack (what op's perform on data structures)

1. push (stack, e) = stack (return)

2. pop (stack) = element at top.

3. IS empty (stack) = Boolean.

4. FULL (stack) = Boolean.

Implementing Stack using array

int a[s] = {---};

a[10] = 0;

push(a[10]) [a[10] is not allocated to you it give garbage]

Segmentation error = Mem is not available.

Implementation (push)

push(g,n,top)



g: Name of stack or memory

top: at stack

n: size of stack, array

DS

push(S, N, TOP, X)

{
 if (TOP == N - 1)

}
 printf("Stack is overflow");

 exit(1) \Rightarrow abnormal termination

else

{
 TOP++;
 S[TOP] = X
}
}

pop(S, n, TOP)

{

 if (TOP == -1)

}
 printf("Stack is underflow"); \Rightarrow O(1)
 exit(1)

else

{
 int y = S[TOP];
 printf("%d");
 TOP--;
}

}

DS

TOP = 2 (insertion)

a	b	c	x
o	1	2	x
x	x	x	

deletion
TOP:
--TOP = 2
--TOP = 1
--TOP = 0
--TOP = 1

[
 S[TOP+1] = x [leads to data loss]
 S[++TOP] = x]

5	4
4	3
3	2
2	1
1	0

++ = 'y'

Date _____

Let S be a empty stack, starting from empty stack n natural no are pushed into stack. When they then perform 'n' pop opn. Assume push and pop opn will take 'n' second each and y second elapsed b/w end of such stack and before the start of next opn.

→ Stack life of element 'm' defined as the time elapsed from push(m) to before the pop(m)

find average stack life of an element in stack
 a) $n \times (n+1)y$ b) $By + 2x$ c) $(n+1)y - n$ d) None

		T ₁	T ₂	
a)		$n=5$	$y=1$	$n=2$ $y=4$
b)		$b=3, 5, 3, 5, 3$		
c)		$c=5, 2, 4, 1, 2, 4, 3, 5, 1$		
d)		$d=3, 5, 3, 5, 3$		
pop				
e)		$e=3, 5, 3, 5, 3$		
f)		$f=3, 5, 3, 5, 3$		
g)		$g=3, 5, 3, 5, 3$		
h)		$h=3, 5, 3, 5, 3$		
i)		$i=3, 5, 3, 5, 3$		
j)		$j=3, 5, 3, 5, 3$		

$$\text{lit time of } d = 4 = 4 \quad (n=2, y=4)$$

$$\text{if } n=1, \text{ lit time of } c = 24, 4, 2, 4 = 26$$

$$\text{if } n=1, \text{ lit time of } b = 1, 2, 4, 2, 4, 4, 1, 2, 4 = 28$$

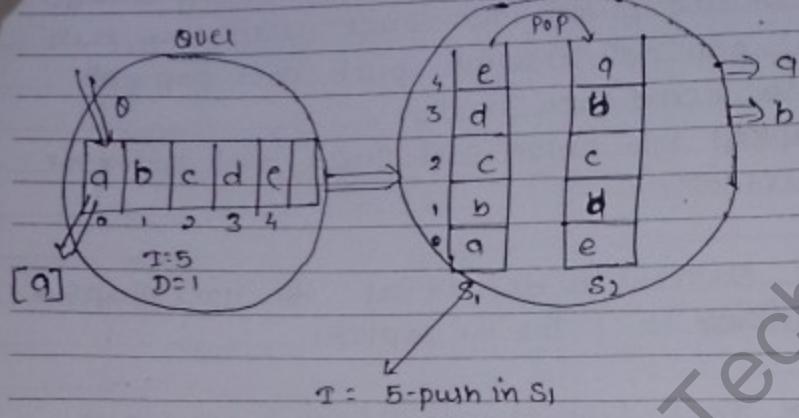
$$\text{if } n=1, \text{ lit time of } q = 8, 2, 4, 2, 4, 1, 2, 4, 4, 1, 2, 4, 1, 2, 4 = 40$$

$$\frac{4+26+28+40}{4} = \frac{98}{4} = 24.5$$

$$\frac{4+16+28+40}{4}$$

4

Q) write a c program to implement overwriting stack
overwriting stack

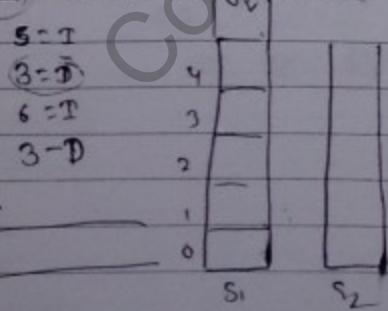


2-D:
 5 pop from S1
 5 push into S2
 1 pop from S2

\Rightarrow q (give 'o' as deleted element)

- 1 pop from $S_2 \Rightarrow a$
 - 2 pop from $S_2 \Rightarrow b$
 - 3 pop from $S_2 \Rightarrow c$
 - 4 pop from $S_2 \Rightarrow d$
 - 5 pop from $S_2 \Rightarrow e$

③ over with static



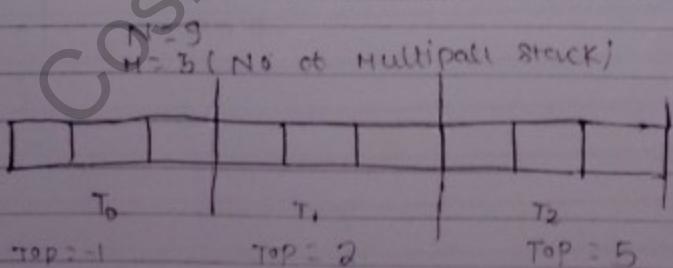
~~S = push - S₁~~
~~S = pop = S₁~~
~~S = push = S₂~~ $pop = 3 + 8 + \underline{2} + \underline{4} + 3$
~~S = pop = S₂~~ = 15 pop
~~S = push = S₁~~
~~S = pop = S₂~~ DR: preth
~~S = push = S₁~~
~~S = push = S₂~~ 17 pop
~~S = pop = S₂~~

program

Desquelo)

```
if (s2 is empty)
    if (s1 is empty)
        ps1 o (s underflow)
    else
        while (s1 is not empty)
            x = pop(s1);
            push(s2, x);
```

Implementing multiple stack in single



Time top = 1st stack: (How many stack) $\frac{N-1}{N}$

$$\frac{N-1}{N} \times 9 = -1$$

$$1^{\text{st}} \text{ stack} = \left(1 \times \frac{9}{3}\right) - 1 = 3 - 1 = 2$$

$$2^{\text{nd}} \text{ stack} = \left(2 \times \frac{9}{3}\right) - 1 = 2 \times 3 - 1 = 5$$

$$50^{\text{th}} \text{ stack} = 50 \left(\frac{9}{3}\right) - 1 =$$

$$i^{\text{th}} \text{ stack} = \boxed{i \left(\frac{9}{3}\right) - 1}$$

Top

† When we say the To is full.

Any) When top of Stack or Top is moved to Top of 1st stack.

When 50th stack is full

$$\text{Top}_{50^{\text{th}}} = 50 \left(\frac{9}{3}\right) - 1 \quad (\text{next stack top})$$

push opⁿ.

Date _____

push (S, N, H, Ti, x) (T_i : Top of i^{th} stack)

{

[if ($T_i == (i+1) * N_H - 1$)]

{

 if stack is full;
 exit(1);

}

else

{

T_i++ ;
 $S[T_i] = x$;

}

}

pop (S, Ti) N, H

{

[if ($T_i == i * N_H - 1$)]

{

 if stack is underflow
 exit(1);

}

else

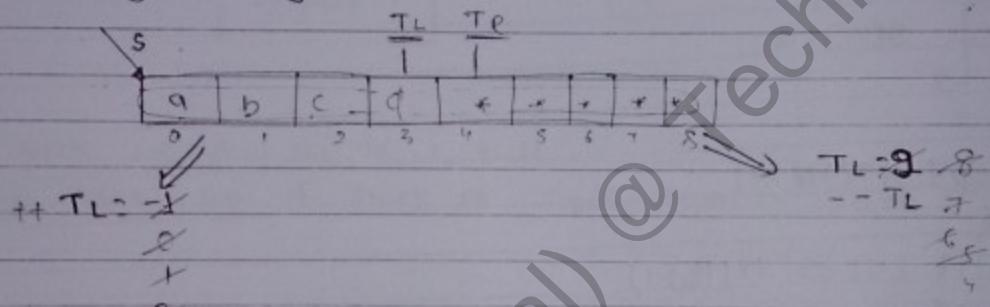
{

 y = $S[T_i]$;
 T_i-- ;
 return y;

}

1) Using above C program we can implement multiple stack in single array but it is not efficient if '1' stack is all other remainig stacks are empty it is giving stack overflow.

2) Efficient way of implementing multiple stack in single array.



3) If TL and TR are side by side then stack will give full condition for efficient use.

$$TL = TR - 1$$

$$TL = TR - 1$$

both should side by side
 $TL = 5$ $TR = 6$

Application of Stack

1) Recursion

(a) Total recursion

(b)

- 1) Recursion
 - i) Tail recursion
 - ii) Non-tail recursion
 - iii) Nested recursion
 - iv) Indirect recursion

2) Prefix to Postfix

3) postfix evaluation

4) Tower - Hanoi

5) Fibonacci series

Q1 write a c program to find minm element in the given array

Ans)

```
void min(a)
{
    int a[10];
    a[0] = min;
    for (int i = 1; i < n; i++)
        if (a[i] < a[0])
            a[0] = a[i];
}
```

Recursing

```
#include <stdio.h>
int min(a, i, j)
{
    if (i == j)
        return a[i];
    else
```

time complexity = $O(n)$

Space complexity = $O(1)$

```
? if (a[i] > a[j]) Recursion
    min(a, i+1);
else
```

min(a, i+1)

$$T = 1 + T(n-1)$$

$$\Rightarrow 1 + T(n-1) = O(n)$$

}

Q1. Write a C-program to print the given array in the reverse order.

A. pr(a,i,j)

```
    {  
        if (i==j)  
            return a[i];  
        else  
        {  
            printf("%d ", a[j]);  
            pr(a,i,j-1);  
        }  
    }
```

$$T(n) = 1 + T(n-1)$$

$$= O(n)$$

Space complexity: $O(n)$;

Time complexity: $O(n)$;

Tail recursion

$$T(n=5)$$

#1 TR(n)

```
- if (n <= 0) return;
```

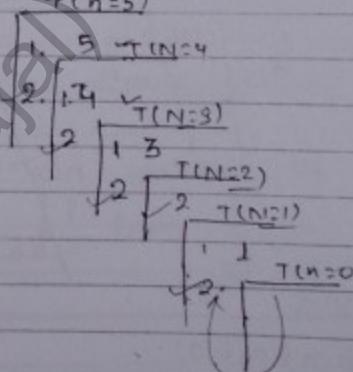
else

```
{
```

```
    printf("%d",
```

```
    TR(n-1));
```

```
}
```



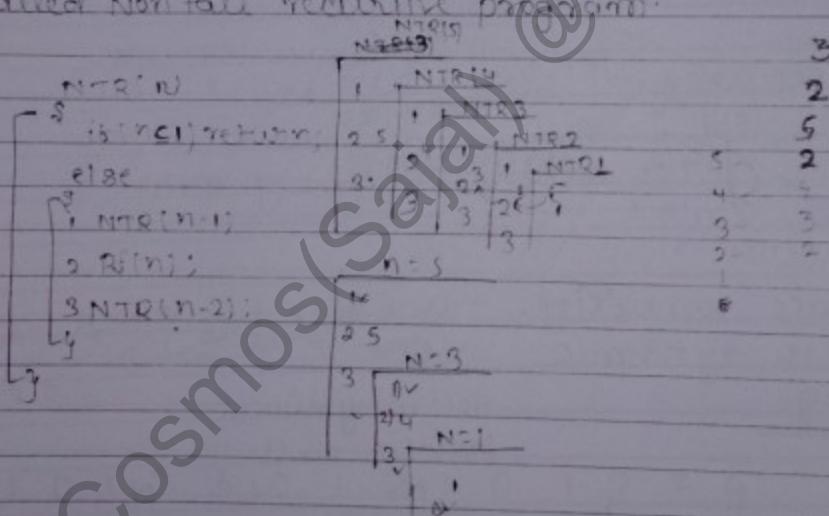
1. In the given recursive program, at the recursive function call is there then the function is called tail-recursive program.

2. The draw back with tail recursive program is - we are wasting lot of stack space.

3. The advantage with the tail recursive program is, it is very easier to write, equivalent non recursive program with the help of for loop.
4. In given recursive program only one recursive program only one recursive ic is also at the end then called tail recursive program.

Nontail recursive program

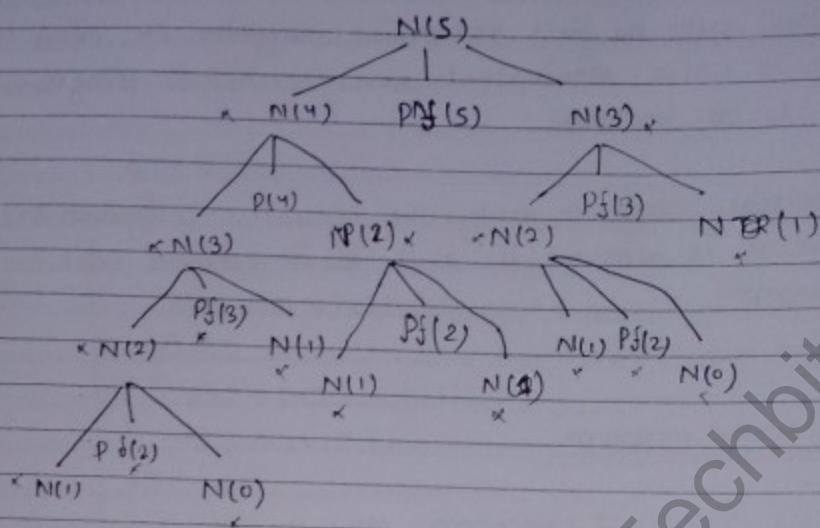
In the given recursive program, After the recursive call there are something to do then recursive program called Non tail recursive program.



2 3 4 2 5 2 3 A

Tree Method

Date: _____



n level
binary
tree.
 \downarrow
 2^n
 \downarrow
 $O(2^n)$

2 3 4 2 5 2 3 A.

$T(n)$ recurrence: $T(n) = T(n-1) + n \cdot 2$

TD C(2^n) [with dynamic]

Space complexity: Table size + Stack
 with dynamic \Downarrow

prog How many distinct
 function call

$$\boxed{S = O(2^N)}$$

without dynamic $T = O(2^n)$
 Space: $O(n)$

with dynamic Space: $O(n)$

Date _____

a) consider the recursive program

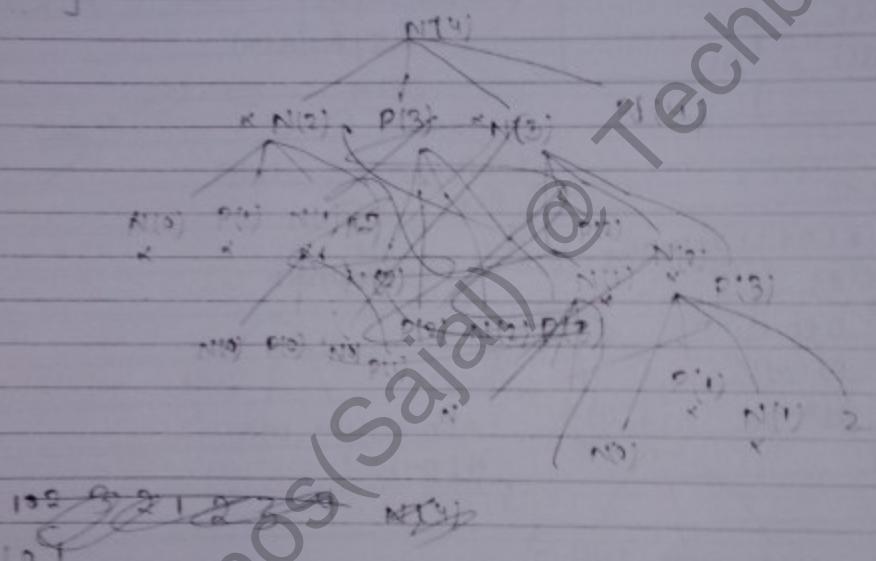
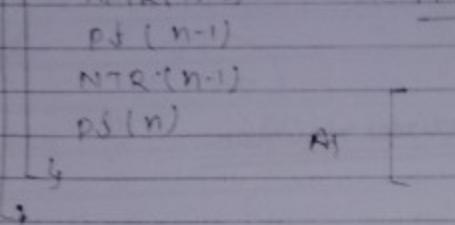
$NTR(n)$

It is in bottom up

else

$\left\{ \begin{array}{l} NTR(n-2) \\ P_1(n-1) \\ NTR(n-1) \\ DS(n) \end{array} \right.$

$NTR(4)$



1 0 1 3 0 1 2 1 0 1 2 3 4 .

Time complexity $\Rightarrow O(n)$ for dynamic programming
Space complexity $\Rightarrow O(n)$

When we store function value in table then it's dynamic programming

NOTE! we are not wasting stack space necessarily

⇒ It is very difficult to find non-recursive program with the help of for loop.

⇒

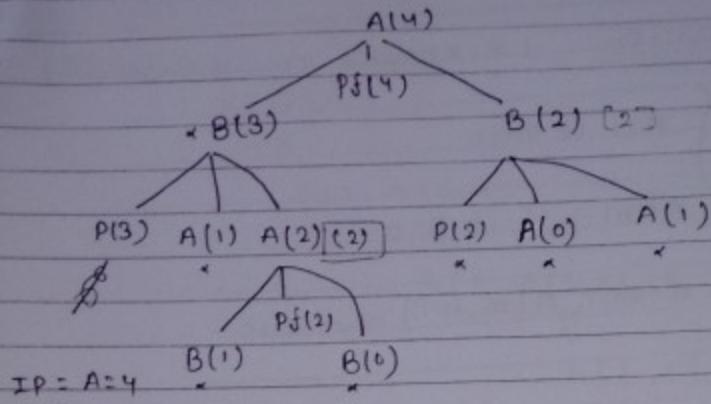
Indirect recursion

```
#1 A()      B()  
{  
    B()  
    {  
        A()  
    }  
}
```

$$T/P = A(n)$$

```
A(n)  
{  
    if (n ≤ 1) return  
    else  
        {  
            B(n-1);  
            Pf(n);  
            B(n-2);  
        }  
}
```

```
B(n)  
{  
    if (n ≤ 1) return  
    else  
        {  
            Pf(n)  
            A(n-2);  
            A(n-1);  
        }  
}
```

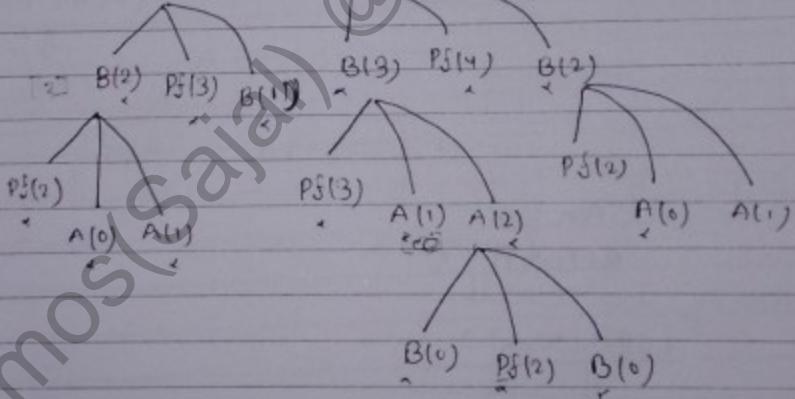
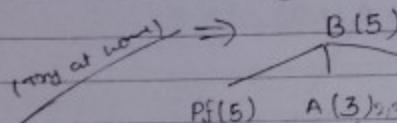


$$IP = A = 4$$

↓

3 2 4 2 phys.

$$\left[\begin{array}{c} IP = B = 5 \\ \downarrow \end{array} \right]$$



5 2 3 3 2 4 2

If T is minimum then difference between function = n^2 difference

Mixed recursion (Ackermann's recursion relation)

$$A(m, n) = \begin{cases} n+1 & \text{if } m=0 \\ A(m-1, 1) & \text{if } n=0 \\ A(m-1, A(m, n-1)) & \text{otherwise.} \end{cases}$$

$A(1, 3)$

$A(1, 3)$

0

Return

$A(0, A(1, 2))$

$A(0, A(1, 1))$

$A(0, A(1, 0))$

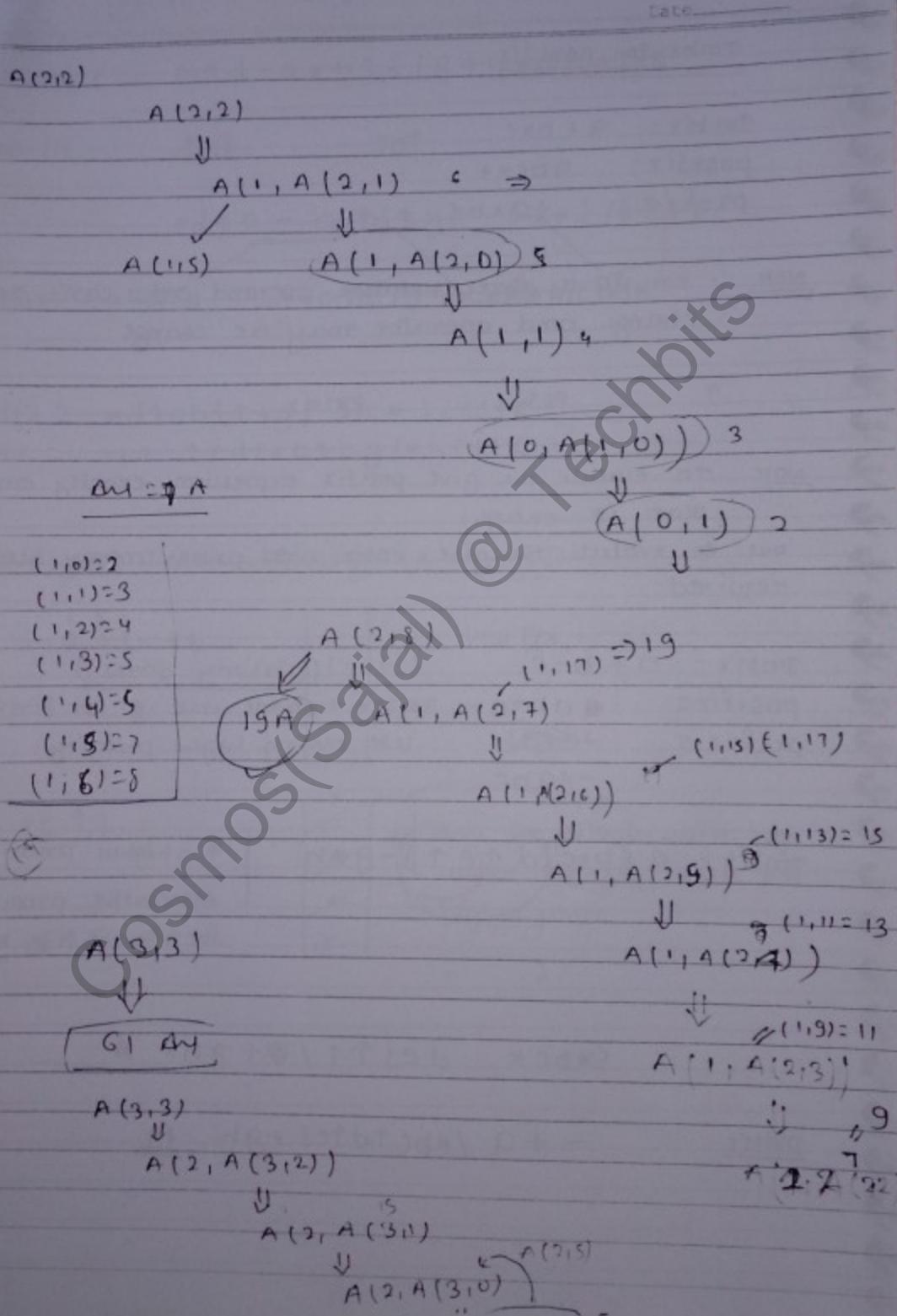
~~After API~~

$A_m = 5$

Return $= 5$

$A(0, A(0, 1))$

↓ 2



Infix to postfixInfix: $a + b * c$ Postfix: $abc*+$ Prefix: $+a*b*c$

Note :- In all the three notation operand order can't be change and operator may be change.

↑	pos	prefix
---	-----	--------

Note To evaluate the give postfix expression exactly one scan is enough.

but to evaluate the give infix and prefix many scans are required.

Infix: $a + b - c$ ($+, -$) ((priority same))

Postfix: $a b + c -$ when same who is in first will having high priority.

Infix: $(a + b * c) / d \uparrow e \uparrow f - g * h$

\uparrow = higher priority

Associative property
for \uparrow is right to left

a+b*c* def $\uparrow\uparrow$ / @+gh*x Aprefix- + a / * b c \uparrow d \uparrow e f * g h A

② infix = $e \uparrow d - a * b \uparrow f / g + h * c / i + j - k$

postfix

$\uparrow e d$

$\uparrow b f$

prefix = $- + - \uparrow e d$

$+ a \uparrow b f / g$

$* h * i / + j + k -$

postfix = $ed \uparrow abf \uparrow * g / - hc * i / + j + k -$

prefix = $- + + - \uparrow ed / * a \uparrow bg / * hc ijk$

Bracket associativity is $L \rightarrow R$

$a \stackrel{1}{+} \stackrel{2}{b} \stackrel{3}{*} c$

$a b c + + A$

+	3	pop
+	2	

postfix

$a + b * c \uparrow d - e$

$a b c d \uparrow + + e -$

*
*
+

[when we occur with top:
lower priority then pop
higher priority]

a ↑ b ↑ c ↑ d ↑ e

↑
↑
↑
↑
↑

q b c d e ↑ ↑ ↑ ↑ *

Lower priority i is coming \Rightarrow push

High to low

\Rightarrow pop

Left to right

\Rightarrow pop

Right to left

\Rightarrow push

Infix e ↑ d - a * b ↑ f / g + h * c / i + j - k

ed↑qb�↑*g/-h c*i / @+j+o K -

while converting infix to postfix we are using operator stack (only operators are pushed into stack)

*
xx
xx
xx

Max^m stack space is = '3'.

a + b * (c + (d + e) f) g

abcedf*g++

*
xx
xx
xx
+

从 prefix 到 postfix

	prefix		postfix
1)	a	—	a
2)	+qb	—	qb+
3)	*a+b+c	—	abc+*

4) + * / - abc d↑e/-efgh

$$ab - c/d * ef - g/h \uparrow +$$

* a/b - c↑↑↑gfh + e*f,g, (try at noon);

Another way start from right.

$$+ a/b - c \uparrow \uparrow \uparrow \uparrow b^f h + e * j, g,$$

Date _____

Q) $a \circ + (c \circ d) \rightarrow ?$

Find out infix (Postfix \rightarrow infix)

Ans $(a \circ b) * (c \circ d) = a \circ b$

Q) $a \circ b + (c \circ d) \circ +$

(~~operator~~; $a \circ b + c \circ d$ Ans.

Postfix evaluation

Infix $= 2 + 3 \times 4 + 5 \uparrow 2 - 3 \times 6$

Postfix

2 3 4 × 5 1 ↑ 2 / 3 6

Max^m Stack Size = 2.

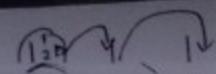
X	/	-	X
+			

Total string length = 'N'

$\lceil \frac{N}{2} \rceil$ = operand $\lfloor \frac{N}{2} \rfloor$ = operator.

Evaluation $\Rightarrow 2 3 4 \times + 5 1 \uparrow 2 / + 3 6 + -$

	OP ₂	POP(1)= ^T 4 = 12	2nd Step	3rd
4	OP ₁	POP(1)= ^{2nd} 3	12 = 14	$1 = S = 5$
3	12 5 S		2	
2	14			



#		$14 + 2 = 16$
2	6	* pop = OP ₂ = 6 OP ₁ = 3 = 18
14	18	- pop = OP ₂ = 18 OP ₁ = 14

$16 - 18 = -2 \leftarrow A$

```

# int eval();
n = postfix[i]
while ( n != 10 )
{
    if ( n is operand )
        push(n);
    i++
}
else
{
    OP2 = pop();
    OP1 = pop();
    Y = OP2 x OP1;
    push(Y);
}

```

$\Theta(n)$ = Time complexity.

operand stack

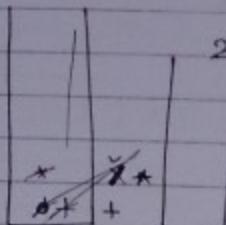
Max^m Size = $\lceil \frac{n}{2} \rceil$

NOTE while evaluating the given postfix expression we are using operand stack only operand are pushed into stack.

Date _____

infix = $2 + 3 * 4 - 5 + 6 / 7 * 4 \uparrow 5$

postfix



$234* - 567 / 4 * \uparrow \uparrow 5$

postfix = $234* + 5 - 67 / 45 \uparrow * +$

evaluation =

	postfix
3	12
2	$* i *$
	$+ +$

[Maxm Stack size = '2']

Fibonacci Series

n	0	1	2	3	4	5	6	7	8
sib(n)	0	1	1	2	3	5	8	13	21

$$sib(n) = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ sib(n-1) + sib(n-2) & \text{otherwise.} \end{cases}$$

Combining = n if $n=0$ and $n=1$

def fib(n)

21

$i\pi (n=0 \text{ or } n=1)$

return n;

else

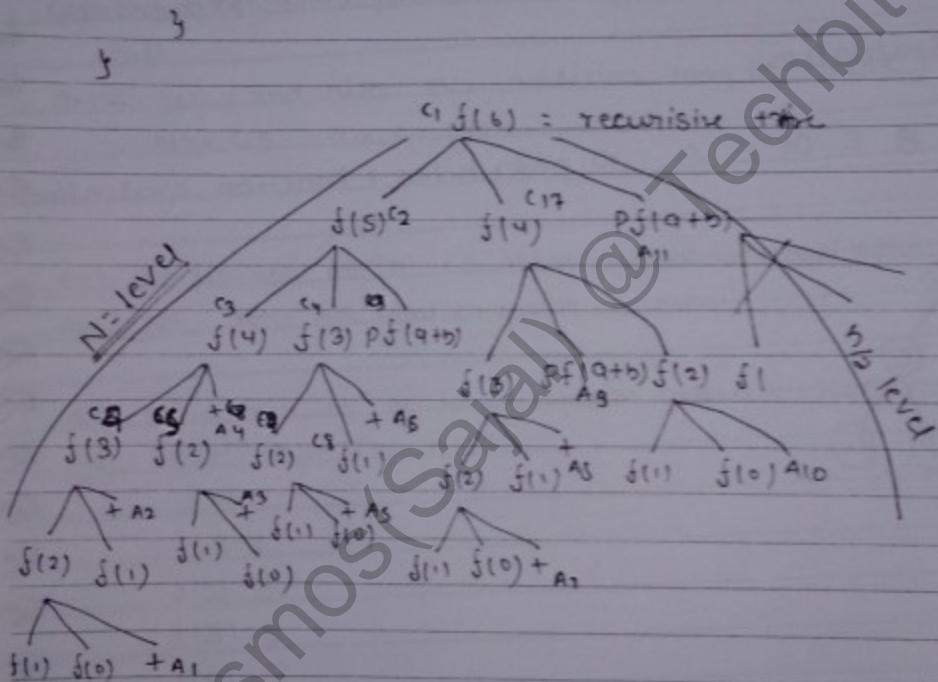
$$q = \text{fib}(n-1);$$

b := fib(n-2);

return a+b)

3

5

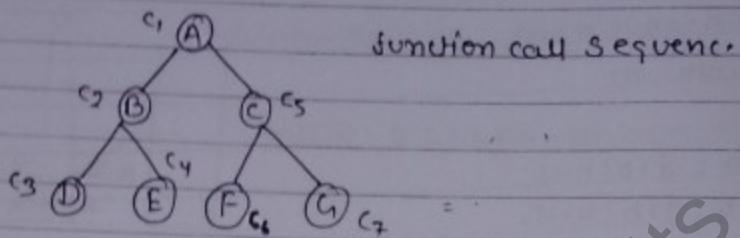


Make tree and give function nbe

max^m Stack size No of table.

$\therefore \text{O} \Rightarrow \text{WTF}$

Function called in programming language = preorder.

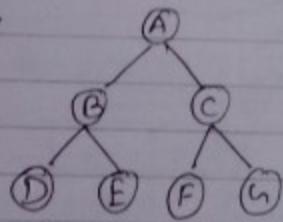


function call sequence.

A B D E C F G = function call sequence \Rightarrow preorder
function evaluation

D E B F C G A \Rightarrow post order (function evaluation)

6.



Q1) In Fibonacci of 'N' after how many function first addition will perform = $N+1$ function call.

Ans) Draw recursive tree it may be for any recursive program and give func call order and count.

Q2) In Fibonacci of 'n' is there any addition after last function call.

Ans) Yes (and also $n/2$ addition may be there)
After calls = A₁₀, A₁₁, A₁₂.

Q3) In Fibonacci of 'n' is there any function call after last addition.

Ans) No after A₁₂ total program is over.

Q4) In Fibonacci of 'N' How many function call are there.

Ans) $\text{Max } n \text{ function } O(2^n)$

$$f(0) = 1$$

$$f(1) = 2$$

$$f(2) = 3$$

$$f(3) = 5$$

$$f(4) = 9$$

$$f(5) = 15$$

$$f(6) = 25$$

$$f(7) = 41$$

⋮

$$f(n) = f(n-1) + f(n-2) + 1$$

In Fibonacci of 'n' how many addition will be there.

No. of addition

$$\Rightarrow f(0) = 0;$$

$$\Rightarrow f(1) = 0;$$

$$\Rightarrow f(2) = 1$$

$$f(3) = 2$$

$$f(4) = 4$$

$$f(5) = 7$$

$$f(6) = 12$$

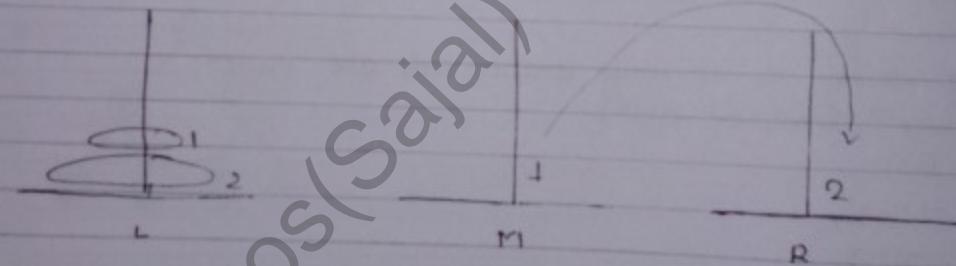
$$f(7) = 20$$

⋮

$$f(n) = f(n-1) + f(n-2) + 1.$$

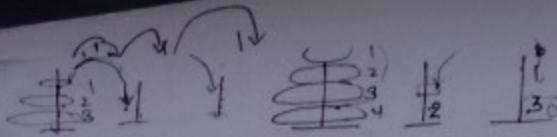
$$T(n) = 1 + T(n-1) + T(n-2)$$

Tower of Hanoi

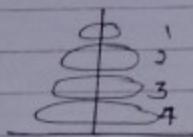


Moves:

- 1) L - M
- 2) L - R
- 3) M - R



$N=3$	$N=4$
$L - R$	$L - R$
$L - M$	$L - M$
$R - H$	$R - M$
$L - R$	
$M - L$	
$H - R$	
$L - R$	S



$$N=4 \Rightarrow$$

Top 3 have to send Middle = ie take 7 Step

The more L-R (4 No block)

then 3 block at middle take 1 step Max total step '7'

- | | | |
|--------|---------|---------|
| 1) L-H | 6) R-H | 11) R-L |
| 2) L-R | 7) L-M | 12) M-R |
| 3) H-R | 8) L-R | 13) L-H |
| 4) L-H | 9) H-R | 14) L-R |
| 5) R-L | 10) H-L | 15) M-R |

N=5

- 1) MOV C Top 4 from L to M = 15
 2) MOV Top 4 from L to R = 1 } = 31 Step.
 3) MOVE Top 4 from M to R = 15



$\text{TOH}(n, \underbrace{\text{L}, \text{R}}_{\text{Source}}, \text{M})$ destination

$\left\{ \begin{array}{l} \text{if } n=0 \text{ return} \\ \text{else} \end{array} \right.$

$\text{TOH}(n-1, \underbrace{\text{L}, \text{R}}_{\text{Source}}, \text{M})$

TOFF:

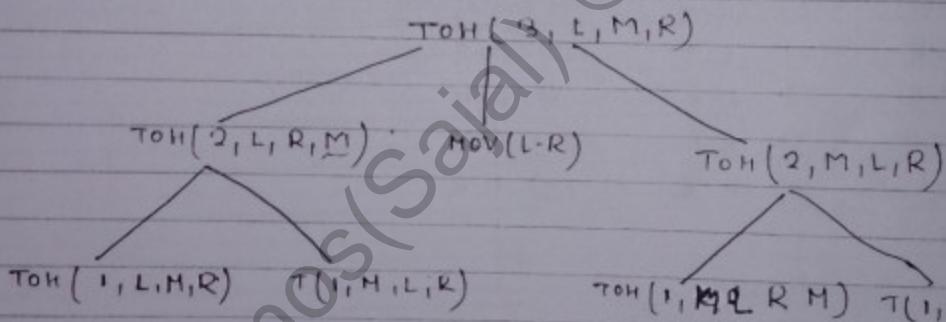
Move ($\text{L} \rightarrow \text{R}$)

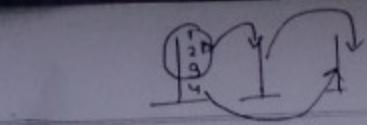
$\text{TOH}(n-1, \text{M}, \text{L}, \text{R})$

$$T(n) = T(n-1) + 1 + T(n-1)$$

$$T(n) = 2T(n-1) + 1 \quad (\text{No of moves})$$

$$T(n) = 2T(n-1) + C = \text{time complexity}$$





Recursive tree for TOH

Date _____

C1 TOH(3, L, M, R)

C2 TOH(2, L, R, M)

M(L-R)

C3 TOH(2, M, L, R)

C4 TOH(1, L, M, R)

C5 TOH(1, R, L, M)

C6 TOH(1, M, R, L)

M(H-R)

C7 TOH(1, L, H, R)

C8 TOH(0, L, R, H)

C9 TOH(0, M, L, R)

C10 TOH(0, R, M, L)

C11 TOH(0, L, R, M)

C12 TOH(0, M, L, R)

C13 TOH(0, R, M, R)

C14 TOH(0, M, R, L)

C15 TOH(0, M, L, H)

Q1 In TOH(n) after how many function call move is take place

A1 N+1 function [After C1 there is M]

Q2 TOH m(n) is true any move after last function call

A1 NO MOVE (last)

Q3 In TOH is there any function call after last move

\Rightarrow after H there is [C15 (last function call)]

Q4 In TOH of '3' after how many function there is move in

M \Rightarrow R

A1 It is 6th move. (before it C12 function call)

Q5 In TOH of 'N' how many function call are there

No. of function calls

$$TOH(0) = 1$$

$$TOH(1) = 3$$

$$TOH(2) = 7$$

$$TOH(3) = 15$$

$$TOH(4) = 31$$

$$TOH(5) = 63$$

$$T(n) = 2T(n-1) + 1$$

Q1 If $T(n)$ is the time taken to solve 'N' problems, how many more are there.

$$TOH(0) = 0$$

$$TOH(1) = 1$$

$$TOH(2) = 3$$

$$TOH(3) = 7$$

$$TOH(4) = 15$$

$$TOH(5) = 31$$

⋮

$$T(n) = (2T(n-1) + 1)$$

$$T(n) = 2^n - 1$$

$$\text{Time complexity } T(n) = O(2^n)$$

Space: No. of levels $O(n)$

Dynamic programming

$$T(n) = \begin{cases} 1 & n=1 \\ T(n-1) + T(2) + T(1) & n > 1 \end{cases} = O(n!)$$

$$\boxed{T = O(n)}$$

Space complexity = stacksize + distinct function calls
 $n + 6n = 7n$

B chapter

Q5) C - ROW-MAJOR ORDER

$\text{char } a[100][100]$

Base address = '0'

$$\begin{aligned} a[40][50] &= 0 + (40)100 + (50-0) \\ &= 4000 + 50 \\ &= 4050 \text{ A} \end{aligned}$$

Q6) a 7) b 8) c 9) d 10) b 11) a 12) c 13) d 14) e 15) d

$$\begin{array}{l} \text{Q6)} \\ \text{A} = \begin{bmatrix} \text{a} & \text{b} & \text{c} & \text{d} \\ \text{e} & \text{f} & \text{g} & \text{h} \\ \text{i} & \text{j} & \text{k} & \text{l} \\ \text{m} & \text{n} & \text{o} & \text{p} \end{bmatrix} \quad \text{B} = \begin{bmatrix} \text{a} & \text{b} & \text{c} & \text{d} \\ \text{e} & \text{f} & \text{g} & \text{h} \\ \text{i} & \text{j} & \text{k} & \text{l} \\ \text{m} & \text{n} & \text{o} & \text{p} \end{bmatrix} \end{array}$$

11) $\text{int a=10};$
 int b=9

$$\begin{bmatrix} 1 & 5 & 9 & 13 \\ 0 & 0 & 10 & 14 \\ 0 & 0 & 11 & 15 \\ 0 & 0 & 0 & 16 \end{bmatrix}$$