CS 3430: S19: SciComp with Py
Practice Problems for Exam 3

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April 18, 2019

3:00–5:00

1/2 problems conceptual
edge detection
line detection
hough transform
problems in slide

1 Introduction

The final exam will be comprehensive. Some questions will be conceptual. A conceptual question will be either multiple choice or will ask you to write 2-3 (no more!) sentences. Other questions will require you to write code.

Below are sample problems on the topic themes we studied in this class: Differentiation, Theory of the Firm, Rates of Change and 1D Function Optimization, Growth and Decay Models, Partial Differential Equations, Antidifferentiation, Curve Fitting, Newton Raphson Algorithm, Pell Equations, Linear Programming in 2 Variables, Linear Systems, Computer Vision and Image Processing.

2 Differentiation

Problem 1

Write a Python function `fun(expr)` that takes a function representation of $f(x) = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}$, differentiates it, and plots $f(x)$ and $f'(x)$ on the interval $[x_1, x_2]$. This problem generalizes to an arbitrary differentiable function that we have learned to differentiate in Lectures 1 - 10.

Problem 2

Write a Python function `fun(expr)` that takes a function representation of $f(x) = t^2 \ln t$, computes $\frac{d^2}{dt^2} f(x)$, and evaluates it at t_1. This problem generalizes to an arbitrary differentiable function that we have learned to differentiate in Lectures 1 - 10.

Problem 3

Write a Python function `fun(expr)` that takes a function representation of $f(x) = e^x$, computes a function representation of the tangent line to $f(x)$ when $x = -1$, converts the tangent line representation to a Python function and returns both the tangent line representation and the function. This problem generalizes to the problem of finding a tangent line equation to a differentiable function at a given point. find HW

Problem 4

Write a Python function `fun` that computes how fast the y-coordinate of a point is changing at $(5, -2)$ if the point is moving along the curve $x^2 - 4y^2 = 9$ and its x-coordinate is changing at the rate of 3 units per second at $(5, -2)$. This problem generalizes to an arbitrary elipse curve and an arbitrary point on that curve. rate of change

3 Theory of The Firm - max/min revenue

Problem 1

The demand equation for a certain product is $p = 85 - 0.003x$. Write a Python function `fun` that takes a function representation of the demand equation, the number of units x of the product to be produced and determines the rate of change of the revenue if the company increases production by 150 units when the level of production is at 5000 units. This problem generalizes to arbirary demand equations, values of x, and values of the current production level. derivative of revenue function

Problem 2

The demand equation for a certain commodity is $p = 2 - 0.001x$. Write a Python function `fun` that returns the value of x and the corresponding price p that maximize the revenue. This function generalizes to arbirary demand equations.

Problem 3

Implement the function `demand_elasticity(p)` that computes the elasticity of demand at the price p. Use your implementation to solve the following problem.

The demand equation for a certain metal is $f(p) = 100 - 2p$, where p is the price per pound and $f(p)$ is the quantity in millions of pounds. What is the elasticity of demand at $p = 10$, $p = 20$, $p = 30$?

Problem 4

Write the function `net_change(mrexpr, pl1, pl2)` that takes a function expression of a marginal revenue function `mrexpr` and two production levels `pl1` and `pl2` and returns the net change in revenue if the production level is raised from `pl1` and `pl2`. Use your implementation to solve the following problem.

Integration problem

Problem 5

Write the function `consumer_surplus(dexpr, a)` that takes a function expression of a demand curve and a sales level a (i.e., quantity demanded) and returns the consumer's surplus at a. Use your implementation to solve the following problem.

Find consumer surplus formula

Let the demand curve be $p = 50 - 0.06x^2$ and the current sales level $a = 20$. What is the consumer's surplus at this level of sales?

4 Rates of Change and 1D Function Optimization

- HW3/HW1?

local max/min on interval

Problem 1

Health officials in a Utah county use the following model to predict the size of the mosquito population, P, at time t, where t is the number of months from March 1 ($t = 0$): $P(t) = -t^3 + 8.5t^2 + 100$, $0 \leq t \leq 9$. Write the function `fun` that takes a function representation of this model and returns the time value where the mosquito population is growing at the fastest rate.

Inflection point

Problem 2

A patient is receiving radiation treatment for a tumor in his neck. The tumor has a spherical shape. His doctor determined that at a point in time when when radius r of the tumor is r', the radius is decreasing at a rate of r_d mm per week. The volume of the tumor is $V = C\pi r^3$. Write the function `fun` that takes a function representation of the volume of the tumor and $\frac{dr}{dt}$ and returns the rate of chage of the volume of the tumor at the time of the observation. This problem generalizes to volumes of arbitrary objects, i.e., oil spills, cylinders, etc.

rate of change

Problem 3

Understand and be ready to answer conceptual questions or write code to solve problems that use the 1st and 2nd derivative tests and local maxima/minima.

Partial Differential - know the one type

5 Growth, Decay, Terminal Velocity, Partial Differential Equations

know how to use which ~~problem~~ model for a give promblem/examp

Problem 1

A sample of 8 grams of radioactive material is placed in a vault. Let $P(t)$ be the amount remaining after t years. It has been determined that $P(t)$ satisfies $P'(t) = -0.021P(t)$.

1. Write a function `fun1(expr, P0)` that takes a function representation of $P(t)$ and P_0 and computes the decay constant;
2. Write a function `fun2(expr, n)` that takes a function representation of $P(t)$ and computes the amount of material that will remain after n years?
3. Write a function `fun3(expr, half_life, n)` that takes a function representation of $P(t)$, the half-life value of this material (an integer constant), and a number of years n (another integer constant), and computes how much of this material will remain after n years.

Problem 2

Write a function `fun(k)` where k is an arbitrary constant in $y' = ky$ and finds and graphs 3 arbirary solutions to $y' = ky$.

Problem 3

Write a function `fun(k, y0)` where k is an arbitrary constant in $y' = ky$ and $y(0) = y0$, and computes and graphs the unique solution to $y' = ky$.

Problem 4

Be ready to write code or answer conceptual questions to solve problems that use the terminal velocity, logistic growth, plant growth, spread of epidemics/news models and plot the resulting curves.

6 Antidifferentiation

approximation
trapezoid
simpson
riemann sum

Problem 1

Be ready to use your code from `antideriv.py` to compute indefinite intergrals such as the ones given below.

1. $\int x^2 dx$; **answer:** $\frac{1}{3}x^3 + C$, C is real;
2. $\int e^{-2x} dx$; **answer:** $-\frac{1}{2}e^{-2k} + C$, C is real;
3. $\int \sqrt{x} dx$; **answer:** $\frac{2}{3}x^{\frac{3}{2}} + C$, C is real;

4. $\int \frac{1}{x^2}dx$; **answer:** $-\frac{1}{x} + C$, C is real;

5. $\int \frac{1}{x}dx$; **answer:** $ln|x| + C$, C is real;

6. $\int \left(\frac{1}{x^3} + 7e^{5x} + \frac{4}{x}\right) dx$; **answer:** $-\frac{1}{2x^2} + \frac{7}{5}e^{5x} + 4ln|x| + C$, C is real;

7. $\int 4x^3dx$; **answer:** $x^4 + C$, C is real;

8. $\int (5x-7)^{-2}dx$; **answer:** $-\frac{1}{5}(5x-7)^{-1} + C$, C is real.

9. $\int \frac{3}{2+x}dx$; **answer:** $3ln|2+x| + C$, C is real.

10. $\int (3x+2)^4\, dx$; **answer:** $\frac{1}{15}(3x+2)^5 + C$, C is real.

Problem 2

Be ready to write a function that takes a function representation of a function $f(x)$ and the upper and lower bounds of an interval $[a, b]$ and approximates the definite integral of $f(x)$ on $[a, b]$ with 1) a Riemann sum; 2) the Midpoint Rule; 3) the Trapezoidal Rule; and 4) Simpson's Rule.

7 Curve Fitting

- linear regression no numpy
- review method of least squares

Problem 1

exam2 ★ Write a function `bell_curve_iq_approx(a, b)` that uses the Bell Curve IQ model to return the proportion of the population whose IQs are in $[a, b]$.

Problem 2

Be ready to write a function that takes a file of data points $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ and uses the method of least squares to compute the line that best fits the data and compare the error (sum of the squared differences between the fit line predictions and the real data) of your line with the error found with `scipy.polyfit()` and `scipy.poly1d()`.

Problem 3

Write the function `taylor_poly(fexpr, a, n)` that returns a Python function for the n-th Taylor polynomial of the function $f(x)$ represented by the function expression `fexpr` at $x = a$. HW problem

Problem 4

Write the function `taylor_poly(fexpr, a, n)` that returns a Python function for the n-th Taylor polynomial of the function $f(x)$ represented by the function expression `fexpr` at $x = a$.

8 Newton-Raphson and Pell Equations

Problem 1

Be ready to use the Newton Raphson algorithm to find zeros of polynomials encoded as function expressions (e.g., $f(x) = x^3 - x - 2$).

Problem 2

Be ready to use the Newton Raphson algorithm to compute $\sqrt{n}$, where n is a positive integer.

Problem 3

Be ready to approximate a zero of a given polynomial with a finite number of iterations of the Newton Raphson algorithm.

Problem 3

Be ready to use the Newton-Raphson algorithm to approximate solutions to equations (e.g., $e^x - 4 = x$). similar to problem

Problem 4

review

Be ready to solve or use Pell Equations to find x and y that satisfy $x^2 - ky^2 = 1$ or $x^2 - ky^2 = -1$ and to approximate square roots.

similar to HW12 + lecture examples

9 Linear Programming in 2 Variables

Problem 1

Be ready to write code (using numpy) that solves problems like the one below.

```
Teds Toys makes toy cars and toy trucks using plastic and steel. Each
car requires 4 ounces of plastic and 3 ounces of steel, while each truck
requires 3 ounces of plastic and 6 ounces of steel. Each day Ted has 30
pounds of plastic and 45 pounds of steel to use in making toy cars and
trucks, and he can sell all the cars and trucks he makes with these
materials. His profit is $5 per car and $4 per truck. Ted would like to
know how many cars and trucks he should make in order to maximize the
total profit from the sale of these toys.
```

Problem 2

Write code that solves the following problem. Minimize/maximize $4x + 5y$ subject to the following constraints:

1. $x \geq 0$;

2. $y \geq 0$;
3. $0.8x + 0.2y \geq 90$;
4. $3x + 6y \geq 600$.

Problem 3

Understand how the LP algorithm works in 2D and be ready to answer conceptual questions about it.

10 Linear Systems

Gauss-Jordan numpy allowed

Problem 1

Know how to use `numpy` to solve linear systems of equations of the form $\mathbf{Ax} = \mathbf{b}$ and compute inverse and transpose matrices.

Problem 2

Understand how to compute determinants and be ready to answer conceptual questions about determinants or write code to solve problems that require the use of determinants.

> lect 26

Problem 3

Understand Cramer's rule and be ready to use to solve $\mathbf{Ax} = \mathbf{b}$.

11 CVIP

Problem 1

Understand the RGB and HSV color spaces and be ready to answer conceptual questions about them or to write code that converts an RGB image to an HSV image.

Problem 2

Understand the basic edge detection algorithm and be ready to answer conceptual questions about it or to use to detect edges in a given image.

Problem 3

Understand the Hough Transform algorithm and be ready to answer conceptual questions about it or to use it to detection lines in a given image.

Understand edge detection
line detection - understand rho & theta & how they're computed

Problem 4

Understand image similarity functions such as cosine similarity, euclidean distance, and jaccard and be ready to answer conceptual questions about them or to use them to compare two images.

Problem 5

Understand how to compute image histograms and their similarity coefficients. Be ready to answer conceptual questions about image histograms or write code that computes image histograms and compares them to other histograms.

similarity methods

Use determinants to

How to use Cramer's rule for linear systems