BAMS 506 - Homework Assignment 1
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1. Problem 3.5-4. This is Problem 3.6-3 in the older version available on the web. Answer questions (a) to (e).

(a) Formulate a linear programming model for this problem.

Decision Variables: let
A1 – denote the number of units of Activity 1 conducted. [units]
A2 – denote the number of units of Activity 2 conducted. [units]

Objective: To minimize the cost of conducting two nonnegative activities as to achieve three benefits that do not fall below their minimum values.
min { (A1*60) + (A2*50) } [$]

Constraints:

1. **Non-negativity:** A1, A2 ≥ 0 [units]
   -The number of units of Activity 1 and 2 conducted cannot be negative.
   [this constraint is denoted as C1 for questions (b, c)]

2. **Benefit 1 Acceptance Level:** (A1*5) + (A2*3) ≥ 60 [type 1 benefits]
   -The level of type 1 benefits achieved by conducting Activities 1 and 2 must meet or exceed 60.
   [this constraint is denoted as C2 for question (b, c)]

3. **Benefit 2 Acceptance Level:** (A1*2) + (A2*2) ≥ 30 [type 2 benefits]
   -The level of type 2 benefits achieved by conducting Activities 1 and 2 must meet or exceed 30.
   [this constraint is denoted as C3 for question (b, c)]

4. **Benefit 3 Acceptance Level:** (A1*7) + (A2*9) ≥ 126 [type 3 benefits]
   -The level of type 3 benefits achieved by conducting Activities 1 and 2 must meet or exceed 126.
   [this constraint is denoted as C4 for question (b, c)]

(b) Use the graphical method to solve this problem.

Geometric Representation in (A1, A2) space:

$$\begin{cases} \min \{(60 * A1) + (50 * A2)\} \quad [\$] \\ s.t. \\ (C1A, C1B)\ A1, A2\ \geq 0\ (units) \\ (C2)\ 5\ A1 + 3\ A2\ \geq 60\ (level\ of\ Benefit\ type\ 1) \\ (C3)\ 2\ A1 + 2\ A2\ \geq 30\ (level\ of\ Benefit\ type\ 2) \\ (C4)\ 7\ A1 + 9\ A2\ \geq 126\ (level\ of\ Benefit\ type\ 3) \end{cases}$$

The graphical model for this linear model is shown in Figure 1 below:

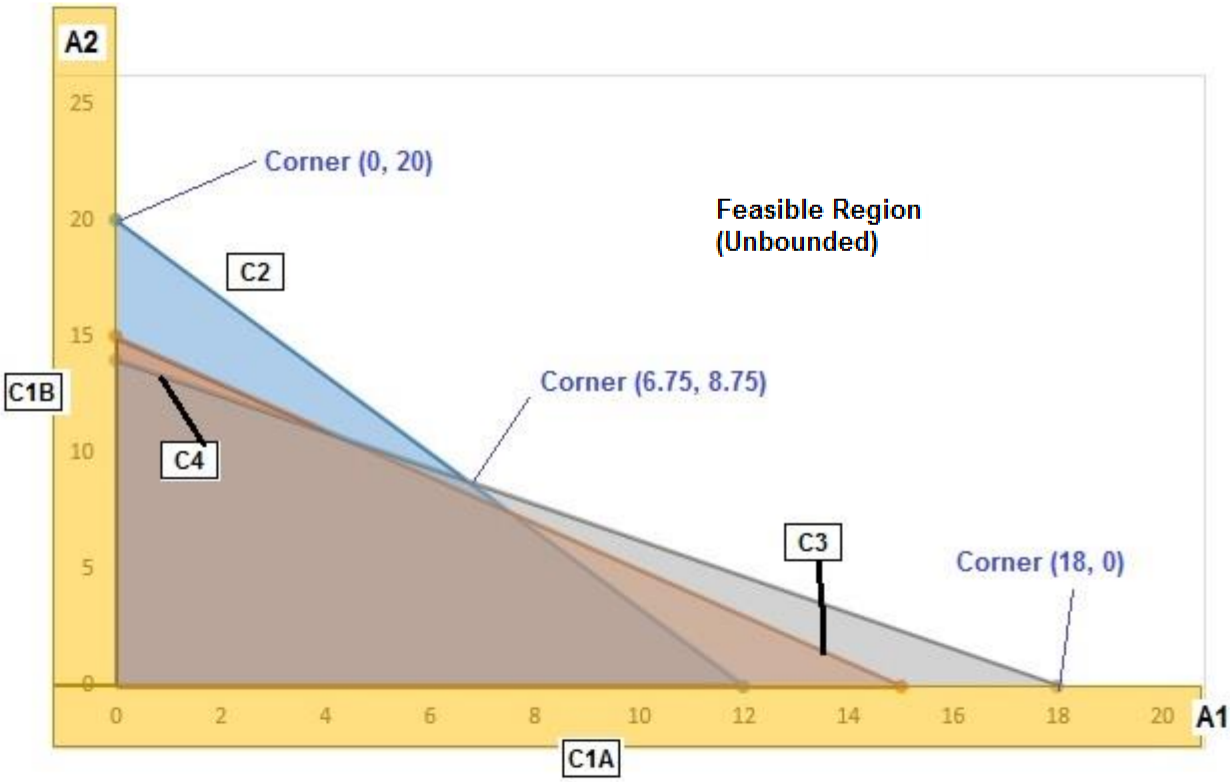

Figure 1. Set of solutions for units of Activity 2 plotted against units of Activity 1. The feasible set of solutions is left blank.

The feasible space is unbounded (to the right). There are 3 corners which could provide solutions: (0, 20), (18, 0) and (6.75, 8.75), the intersection of C2 and C4, calculated as follows:

(C2) 5A1 + 3A2 = 60
(C2) 3A2 = 60 – 5A1
(C2) A2 = 20 – 5/3 A1

And
(C4) 7A1 + 9 A2 = 126

Using C2 in C4,

7A1 + 9(20 – 5/3A1)	= 126
7A1 + 180 – 15A1	= 126
8A1	= 54
A1	= 6.75

Using A1 in (C2),

A2	= 20 – 5/3A1
A2	= 20 – 5/3 (6.75)
A2	= 8.75

Using the objective function, we calculated the cost of each solution:

1. **A1=0, A2=20 :** 60 (0) + 50 (20) = 1000 [$]
2. **A1=18, A2=0 :** 60 (18) + 50 (0) = 1080 [$]
3. **A1=6.75, A2=8.75:** 60 (6.75) + 50 (8.75) = 842.50 [$]

The third solution minimizes the cost. Therefore, **the minimum benefit requirements for Benefit 1, 2 and 3 can be met at minimal cost of $842.50 by conducting 6.75 units of Activity A and 8.75 units of Activity B.**

(c) Display the model in an Excel spreadsheet.

The constructed linear model on an Excel spreadsheet is as follows:

	A	B	C	D	E	F	G	H	I	J	K
1	BAMS 506 - Question 1 - #3.6-3 from 9th Ed										
2	LP Model										
3	Students: Gurpal Bisra and Sonya Sabourin										
4											
5	**DATA:**			**Benefits:**					**Constraints:**		
6		**Unit Cost**		**Benefit**	**Activity 1**	**Activity 2**	**Minimum Acceptance Level**	**Units**	**1. Non-negativity:**	A1, A2 $\geq$ 0	[units]
7									**2. Benefit 1 Acceptance Level:**	**62**	[type 1 benefits]
8	Activity 1	60	$/unit	1	5	3	60	Type 1 Benefits	**3. Benefit 2 Acceptance Level:**	**32**	[type 2 benefits]
9	Activity 2	50	$/unit	2	2	2	30	Type 2 Benefits	**4. Benefit 3 Acceptance Level:**	**130**	[type 3 benefits]
10				3	7	9	126	Type 3 Benefits			
11											
12											
13	**MODEL:**										
14	**Decision Variables**		**A1**	**A2**							
15	Activities Conducted		**7**	**9**	**Units**						
16											
17	**Objective Function:**		min { (A1*60) + (A2*50) }								
18	Minimize total net shipping cost				*870.00*	[$]					

(d) Use the spreadsheet to check the following solutions: (x_1, x_2) = (7, 7), (7, 8), (8, 7), (8, 8), (8, 9), (9, 8). Which of these solutions is feasible? Which of these feasible solutions has the best value of the objective function?

Using the spreadsheet of the model as shown in (c), we checked the indicated solutions by varying values of A1 and A2 in the Decision Variables box. The cost resulting from each solution (displayed in the green Objective Function box) was recorded. The results are displayed in the following spreadsheet:

			Constraints:				
Activity 1 [Units]	**Activity 2 [Units]**	**Objective Function [$]**	**Non-Negativity (C1)**	**Type 1 Benefit Acceptance Level [type 1 benefits]**	**Type 2 Benefit Acceptance Level [type 2 benefits]**	**Type 3 Benefit Acceptance Level [type 3 benefits]**	
				≥ 60	≥ 30	≥ 126	**Requirement**
7	7	770		56	28	112	
7	8	820		59	30	121	
8	7	830		61	30	119	
8	8	880		64	32	128	
8	9	930		67	34	137	
9	8	940		69	34	135	

(blue)	means feasible
(pink)	means infeasible

The feasible solutions include (A1, A2) = (8, 8), (8, 9) and (9, 8). The solution which produces the smallest (i.e. best) value for the objective function is (A1, A2) = (8, 8), with a resulting cost of $880.

(e) Use the Excel Solver to solve this model by the simplex method.

Using the Excel Solver add-on to solve the linear model yields the solution below:

	A	B	C	D	E	F	G	H	I	J	K
1	BAMS 506 - Question 1 - #3.6-3 from 9th Ed										
2	LP Model										
3	Students: Gurpal Bisra and Sonya Sabourin										
4											
5	**DATA:**			**Benefits:**					**Constraints:**		
6		**Unit Cost**		**Benefit**	**Activity 1**	**Activity 2**	**Minimum Acceptance Level**	**Units**	**1. Non-negativity:**	$A1, A2 \geq 0$	[units]
7									**2. Benefit 1 Acceptance Level:**	**60**	[type 1 benefits]
8	Activity 1	60	$/unit	1	5	3	60	Type 1 Benefits	**3. Benefit 2 Acceptance Level:**	**31**	[type 2 benefits]
9	Activity 2	50	$/unit	2	2	2	30	Type 2 Benefits	**4. Benefit 3 Acceptance Level:**	**126**	[type 3 benefits]
10				3	7	9	126	Type 3 Benefits			
11											
12											
13	**MODEL:**										
14	**Decision Variables**		**A1**	**A2**							
15	Activities Conducted		**6.75**	**8.75**	**Units**						
16											
17	**Objective Function:**	min { (A1*60) + (A2*50) }									
18	Minimize total net shipping cost				***842.50***	[$]					

As in part (b), **we find that conducting 6.75 units of Activity 1 and 8.75 units of Activity 2 ensures the objective function is minimized; the minimum benefits of type 1, 2, and 3 are met for a total price of $842.50**.

2. Problem 3.4-11 (The Medequip Company). This is Problem 3.4-11 in the older version on the web. Answer question (a) and then use ~~GAMS and its default~~ Excel's Solver to solve your model.

(a) Formulate a linear programming model for this problem.

Decision Variables: let
F1C1 – denote the number of units shipped from Factory 1 to Customer 1 per month. [units]
F1C2 – denote the number of units shipped from Factory 1 to Customer 2 per month. [units]
F1C3 – denote the number of units shipped from Factory 1 to Customer 3 per month. [units]
F2C1 – denote the number of units shipped from Factory 2 to Customer 1 per month. [units]
F2C2 – denote the number of units shipped from Factory 2 to Customer 2 per month. [units]
F2C3 – denote the number of units shipped from Factory 2 to Customer 3 per month. [units]

Objective: To minimize this month's shipping cost of units from Factory 1 and 2 to Customers 1, 2 and 3.
min { (F1C1*600) + (F1C2*800) + (F1C3*700) + (F2C1*400) + (F2C2*900) + (F2C3*600) } [$]

Constraints:

1. **Non-negativity:** F1C1, F1C2, F1C3, F2C1, F2C2, F2C3 ≥ 0 [units]
   -The number of units shipped, from any factor to any customer, per month cannot be negative.

2. **Factory 1 Output:** F1C1 + F1C2 + F1C3 = 400 [units]
   -The total number of units shipped to Customers 1, 2, and 3 from Factory 1 must equal 400.

3. **Factory 2 Output:** F2C1 + F2C2 + F2C3 = 500 [units]
   -The total number of units shipped to Customers 1, 2, and 3 from Factory 2 must equal 500.

4. **Customer 1 Order:** F1C1 + F2C1 = 300 [units]
   -The total number of units shipped to Customer 1 from Factory 1 and 2 must equal 300.

5. **Customer 2 Order:** F1C2 + F2C2 = 200 [units]
   -The total number of units shipped to Customer 2 from Factory 1 and 2 must equal 200.

6. **Customer 3 Order:** F1C3 + F2C3 = 400 [units]
   -The total number of units shipped to Customer 3 from Factory 1 and 2 must equal 400.

We solved the linear model by the simplex method using Excel Solver add-on (see below):

	A	B	C	D	E	F	G	H	I	J
1	**BAMS 506 - Assignment 1 - Question #2 [3.4-11 9th Ed.]**									
2	Medequip Company									
3	LP Model									
4	Students: Gurpal Bisra and Sonya Sabourin									
5										
6	**DATA:**			**Per Month**			**Constraints:**			
7		**Shipping Cost**	Units	**Amount Available**		Units	**1. Non-negativity:**	F1C1, F1C2, F1C3, F2C1, F2C2, F2C3 ≥ 0	[units]	
8							**2. Factory 1 Output:**	F1C1 + F1C2 + F1C3 = 400	[units]	400
9	F1C1	600	$/unit	Factory 1	400	units	**3. Factory 2 Output:**	F2C1 + F2C2 + F2C3 = 500	[units]	500
10	F1C2	800	$/unit	Factory 2	500	units	**4. Customer 1 Order:**	F1C1 + F2C1 = 300	[units]	300
11	F1C3	700	$/unit				**5. Customer 2 Order:**	F1C2 + F2C2 = 200	[units]	200
12	F2C1	400	$/unit	Customer 1 Order	300		**6. Customer 3 Order:**	F1C3 + F2C3 = 400	[units]	400
13	F2C2	900	$/unit	Customer 3 Order	200					
14	F2C3	600	$/unit	Customer 3 Order	400					
15										
16										
17										
18	**MODEL:**									
19	**Decision Variables**		**F1C1**	**F1C2**	**F1C3**	**F2C1**	**F2C2**	**F2C3**		
20	Shipped		**0**	**200**	200	300	0	200	Units	
21										
22	**Objective Function**	min { (F1C1*600) + (F1C2*800) + (F1C3*700) + (F2C1*400) + (F2C2*900) + (F2C3*600) }						[$]		
23	Minimize total net shipping cost					*540000*				

Therefore, to meet each customer's orders, Mediquip should ship, from Factory 1, 0 units to Customer 1, and 200 units to both Customers 2 and 3. From Factory 2, 300 cases should be shipped to Customer 1, 0 cases to Customer 2 and 200 Customer 3. Hence, the minimum shipping cost for this month to meet all customers' demands is $540,000.