

Case Report: Assignment Art

Prepared for

**Mr. Ash Briggs and Celeste McKenzie
San Francisco Museum of Modern Art**

Date: October 28, 2016

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Centre for Operations Excellence**

1 Executive Summary

Mr. Briggs, a previously struggling artist living in San Francisco, recently won the lottery and became a millionaire. After careful consideration, he has decided to use his fortune to donate a million dollars to and fund an exhibit, of up-and-coming modern artists, the San Francisco Museum of Modern Art. Mr. Briggs met with Celeste McKenzie, the museum's director, and both individuals determined their preferences for which pieces of art could be displayed from local artists' work. Unfortunately, not all pieces can be showcased due to budgetary and preference limitations. Our services were requested to maximize the number of pieces displayed in the exhibit without exceeding the budget.

This report discusses the binary integer programming models we formulated under various scenarios and our recommendations for San Francisco Museum of Modern Art to optimize their art exhibit strategy. We first considered how many, and which types, of pieces could be displayed in the exhibit using \$4 million. Next, we determined the total cost and which pieces should be included in the exhibit of at least 20 pieces need to be shown. Moreover, we then determined which pieces should be displayed and how much the patron of Rita Losky would be required to pay to ensure that exactly 20 pieces are displayed in the exhibit given all of Rita's pieces are displayed.

The key objective here for the consultation was to maximize the number of pieces displayed in the exhibit and, under various scenarios, minimizing the total cost of the exhibit. Given the aforementioned goals, we determined that 15 paintings can be displayed by adhering to the \$4 million budget. The list of pieces can be found in Table 2, in Section 3.2 of the report. In particular, only \$3.95 million is required to actually display 15 paintings.

To ensure that the exhibit draws the attention of the public, we determined a cost of \$5.45 million is required to display at least 20 pieces to meet the requirements set by Mr. Briggs and Celeste. Unfortunately, this is \$1.45 million more than Mr. Brigg's ideal budget of \$4 million. The list of pieces meeting these criteria can be found in Table 3, in Section 3.3 of the report.

Per request of the Patron, we estimated that 20 pieces, which include all of Rita Lowsky's works, can be displayed for a total cost of \$5.8 million. This means the patron would pay \$1.8 million in addition to Mr. Brigg's donation of \$4 million. The list of pieces meeting the aforementioned criteria can be found in Table 4, in Section 3.4 of the report.

Our analysis was conducted while meeting the following requirements:

1. **Total Cost:** The total cost of all selected pieces cannot exceed \$4 million.
2. **Collage:** Mr. Briggs wants to include only 1 collage of the four available ones.
3. **Wire-mesh Sculpture:** Mr. Briggs wants to include at least one wire mesh sculpture if a computer-generated drawing is displayed, and vice-versa.
4. **Photo-realistic Painting:** Mr. Briggs wants to include at least 1 photo-realistic painting displayed of the three which are available.
5. **Cubic Painting:** Mr. Briggs wants to include at least 1 cubic painting of the three available ones.
6. **Expressionist Painting:** Mr. Briggs wants to include at least 1 expressionist painting of the one available. Therefore, it must be displayed.
7. **Watercolor Paintings:** Mr. Briggs wants to include at least 1 watercolor painting of the six available ones.
8. **Oil Paintings:** Mr. Briggs wants to include at least 1 oil painting of the five available ones.
9. **Painting Ratio:** Mr. Briggs wants the number of paintings to be no greater than twice the other art forms. There are 18 paintings and 16 other art forms.
10. **Mr. Briggs Briggs's Paintings:** Mr. Briggs wants all of his own paintings included in the exhibit since his home is the site of the exhibit.
11. **Candy Tate:** Mr. Briggs wants both of Candy Tate's paintings displayed since they are lovers.
12. **David Lyman:** Mr. Briggs wants to include at least 1 piece from David Lyman of the two available ones.
13. **Rick Rawls:** Mr. Briggs wants to include at least 1 piece from Rick Rawls of the three available ones.
14. **Best Friends:** Mr. Briggs wants to include the same number of pieces from both Rick Rawls as David Lyman as he considers them both his best friends and doesn't want to show favoritism.
15. **Ziggy Lite:** Since Mr. Briggs considers Ziggy's art distasteful, he will only accept at most one piece of Ziggy's art.
16. **Advancing Feminism:** Celeste insists the exhibit should include one female piece for every two pieces included from a male artist to advance feminism.
17. **Advancing Environmentalism:** Celeste believes at least one or both of "Aging Earth" and "Wasted Resources" should be included to advance environmentalism.
18. **Native American Rights:** Celeste at least one piece by Bear Canton should be displayed to advance Native American rights.
19. **Advancing Science:** Celeste believes at least one pieces should be included to advance science.
20. **Narcissism** Celeste decides that if "Narcissism" is displayed, "Reflection" should also be displayed since "Reflection" also suggests narcissism.
21. **Floor Space:** Celeste is aware the museum only has enough floor space for four sculptures.
22. **Wall Space:** Celeste is aware the museum only has enough wall space for 20 paintings, collages, and drawings.

Furthermore, we made the following assumptions to conduct our analysis:

1. Mr. Briggs does not require personal compensation.
2. The cost for moving Mr. Briggs's pieces to the museum from his home in San Francisco is minimal. The cost of displaying his pieces therefore only includes the cost of constructing the display and insuring the pieces.
3. Ziggy Lite's "My Namesake" cartoon is not considered a drawing.

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Table 4: List of artists, and their pieces, to be displayed in the San Francisco Museum of Modern Art given 20 pieces are displayed at a minimum cost of \$5.8 million when all of Rita Losky's pieces must be displayed. 19

2 Introduction

Mr. Briggs, a previously struggling artist living in San Francisco, recently won the lottery and became a millionaire. After careful consideration, he has decided to use his fortune to donate a million dollars to and fund an exhibit, of up-and-coming modern artists, the San Francisco Museum of Modern Art. Mr. Briggs met with Celeste McKenzie, the museum's director, and both individuals determined their preferences for which pieces of art could be displayed from local artists' work. Unfortunately, not all pieces can be showcased due to budgetary and preference limitations. Our services were requested to maximize the number of pieces displayed in the exhibit without exceeding the budget.

This report discusses the binary integer programming models we formulated under various scenarios and our recommendations for San Francisco Museum of Modern Art to optimize their art exhibit strategy. We first considered how many, and which types, of pieces could be displayed in the exhibit using \$4 million. Next, we determined the total cost and which pieces should be included in the exhibit of at least 20 pieces need to be shown. Moreover, we then determined which pieces should be displayed and how much the patron of Rita Losky would be required to pay to ensure that exactly 20 pieces are displayed in the exhibit given all of Rita's pieces are displayed.

Our analysis was conducted while meeting the following requirements:

1. **Total Cost:** The total cost of all selected pieces cannot exceed \$4 million.
2. **Collage:** Mr. Briggs wants to include only 1 collage of the four available ones.
3. **Wire-mesh Sculpture:** Mr. Briggs wants to include at least one wire mesh sculpture if a computer-generated drawing is displayed, and vice-versa.
4. **Photo-realistic Painting:** Mr. Briggs wants to include at least 1 photo-realistic painting displayed of the three which are available.
5. **Cubic Painting:** Mr. Briggs wants to include at least 1 cubic painting of the three available ones.
6. **Expressionist Painting:** Mr. Briggs wants to include at least 1 expressionist painting of the one available. Therefore, it must be displayed.
7. **Watercolor Paintings:** Mr. Briggs wants to include at least 1 watercolor painting of the six available ones.
8. **Oil Paintings:** Mr. Briggs wants to include at least 1 oil painting of the five available ones.
9. **Painting Ratio:** Mr. Briggs wants the number of paintings to be no greater than twice the other art forms. There are 18 paintings and 16 other art forms.
10. **Mr. Briggs Brigg's Paintings:** Mr. Briggs wants all of his own paintings included in the exhibit since his home is the site of the exhibit.
11. **Candy Tate:** Mr. Briggs wants both of Candy Tate's paintings displayed since they are lovers.

- 12. David Lyman:** Mr. Briggs wants to include at least 1 piece from David Lyman of the two available ones.
- 13. Rick Rawls:** Mr. Briggs wants to include at least 1 piece from Rick Rawls of the three available ones.
- 14. Best Friends:** Mr. Briggs wants to include the same number of pieces from both Rick Rawls as David Lyman as he considers them both his best friends and doesn't want to show favoritism.
- 15. Ziggy Lite:** Since Mr. Briggs considers Ziggy's art distasteful, he will only accept at most one piece of Ziggy's art.
- 16. Advancing Feminism:** Celeste insists the exhibit should include one female piece for every two pieces included from a male artist to advance feminism.
- 17. Advancing Environmentalism:** Celeste believes at least one or both of "Aging Earth" and "Wasted Resources" should be included to advance environmentalism.
- 18. Native American Rights:** Celeste at least one piece by Bear Canton should be displayed to advance Native American rights.
- 19. Advancing Science:** Celeste believes at least one pieces should be included to advance science.
- 20. Narcissism** Celeste decides that if "Narcissism" is displayed, "Reflection" should also be displayed since "Reflection" also suggests narcissism.
- 21. Floor Space:** Celeste is aware the museum only has enough floor space for four sculptures.
- 22. Wall Space:** Celeste is aware the museum only has enough wall space for 20 paintings, collages, and drawings.

Furthermore, we made the following assumptions to conduct our analysis:

1. Mr. Briggs does not require personal compensation.
 2. The cost for moving Mr. Briggs's pieces to the museum from his home in San Francisco is minimal. The cost of displaying his pieces therefore only includes the cost of constructing the display and insuring the pieces.
 3. Ziggy Lite's "My Namesake" cartoon is not considered a drawing.
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2.1 Information Provided

List of Artists and their Pieces

The list of the local artists and their pieces, and the price of displaying each piece to Celeste, is depicted in Table 1 below. There are a total of 34 pieces. Upon receiving this list, both Mr. Brigg's and Celeste McKenzie formulated their individual preferences for the exhibit.

TABLE 1: List of artists, their pieces, and the price of displaying each piece to Celeste. The display price includes the cost of paying the artist for loaning the piece to the museum, transporting the piece to San Francisco, constructing the display for the piece, insuring the piece while it is on display, and transporting the piece back to its origin.

Artist	Piece Name and Number	Description of Piece	Price (\$)
Colin Zweibell	1. "Perfection"	A wire mesh sculpture of the human body	300,000
	2. "Burden"	A wire mesh sculpture of a male	250,000
	3. "The Great Equalizer"	A wire mesh sculpture of a gun	125,000
Rita Losky	4. "Chaos Reigns"	A series of computer-generated drawings	400,000
	5. "Who Has Control?"	A computer-generated drawing of intermeshed with lines of computer code	500,000
	6. "Domestication"	A pen-and-ink drawing of a house	400,000
	7. "Innocence"	A pen-and-ink drawing of a child	550,000
Norm Marson	8. "Aging Earth"	A sculpture of trMr. Briggs covering a larger globe	700,000
	9. "Wasted Resources"	A collage of various packaging materials	575,000
Candy Tate	10. "Serenity"	An all blue watercolor painting	200,000
	11. "Calm Before the Storm"	A painting with an all blue watercolor background and black watercolour centre	225,000
Robert Bayer	12. "Void"	An all black oil painting	150,000
	13. "Sun"	An all yellow oil painting	150,000
David Lyman	14. "Storefront Window"	A photo-realistic painting of a jewellery store display window	850,000
Angie Oldman	15. "Harley"	A photo-realistic painting of a Harley-Davidson motorcycle	750,000
	16. "Consumerism"	A collage of magazine advertisements	400,000
	17. "Reflection"	A mirror (considered a sculpture)	175,000

	18. "Trojan Victory"	A wooden sculpture of a condom	450,000
	19. "Rick"	A photo-realistic self-portrait (painting)	500,000
Rick Rawls	20. "Rick II"	A cubist self-portrait (painting)	500,000
	21. "Rick III"	An expressionist self-portrait (painting)	500,000
Bill Reynolds	22. "Beyond"	A science fiction oil painting depicting Mars colonization	650,000
	23. "Pioneers"	An oil painting of three astronauts aboard the space shuttle	650,000
Bear Canton	24. "Wisdom"	A pen-and-ink drawing of an Apache chieftain	250,000
	25. "Superior Powers"	A pen-and-ink drawing of a traditional Native American rain dance	350,000
	26. "Living Land"	An oil painting of the Grand Canyon	450,000
Helen Row	27. "Study of a Violin"	A cubist painting of a violin	400,000
	28. "Study of a Fruit Bowl"	A cubist painting of a bowl of fruit	400,000
Ziggy Lite	29. "My Namesake"	A collage of Ziggy cartoons	300,000
	30. "Narcissism"	A collage of photographs of Ziggy Lite	300,000
Mr. Briggs	31. "All That Glitters"	A watercolor painting of the Golden Gate Bridge	50,000
	32. "The Rock"	A watercolor painting of Alcatraz	50,000
	33. "Winding Road"	A watercolor painting of Lombard Street	50,000
	34. "Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art	50,000

Mr. Briggs's Requirements for the Exhibit

Mr. Briggs' goal is to educate Americans about art and artistic styles. Foremost, Mr. Briggs wants all of his own paintings included in the exhibit since his home is the site of the exhibit. Next, Mr. Briggs wants to include only one collage since he believes it requires limited skill to produce. In addition, Mr. Briggs wants to include at least one wire mesh sculpture if a computer-generated drawing is displayed and vice-versa. He feels this would allow views to compare three- and two-dimensional images more effectively. Furthermore, Mr. Briggs wants to include at least one painting of types: (1) photo-realistic; (2) cubist; (3) expressionist; (4) watercolor; and (5) oil painting. More specifically, he wants the number of paintings to be no greater than twice the other art forms.

On the other hand, Mr. Briggs possesses personal biases for and against some artists. For instance, Mr. Briggs wants both of Candy Tate's paintings displayed since they are lovers. Moreover, Mr. Briggs will display at least one, and the same number, of paintings from David Lyman and Rick Rawls since Mr. Briggs considers them both his best friends. Since Mr. Briggs considers Ziggy's art distasteful, he will only accept at most one piece of Ziggy's art.

Celeste McKenzie's Requirements for the Exhibit

As a museum director, Celeste wants to represent diverse artistic expressions to appeal to a variety of audience members while meeting political correctness. First, she insists the exhibit should include one female piece for every two pieces included from a male artist to advance feminism. Next, at least one or both of "Aging Earth" and "Wasted Resources" should be included to advance environmentalism. Additionally, at least one piece by Bear Canton should be displayed to advance Native American rights. Likewise, at least one of the following pieces should be included to advance science: "Chaos Reigns," "Who Has Control," "Beyond," and "Pioneers." Finally, Celeste decides that if "Narcissism" is displayed, "Reflection" should also be displayed since "Reflection" also suggests narcissism.

Limited Space at the Museum

The museum only has enough floor space for four sculptures and enough wall space for 20 paintings, collages, and drawings.

3 Analysis

3.1 Binary Integer Programming Model

Considering the aforementioned considerations in the Introduction of this report, we formulated the following binary integer programming model to maximize the number of pieces displayed in the exhibit without exceeding the budget.

Binary Decision Variables: Whether or not each of the 34 pieces (i.e. refer to “Piece Name and Number” in Table 1), x_i where $i = 1, 2, \dots, 34$, of art are displayed by letting x_i equal to:

- | | | |
|---|---|--------|
| 1 | denote the piece of art is displayed. | [none] |
| 0 | denote the piece of art is not displayed. | [none] |

Constraints: The following constraints define the binary integer programming model to find the optimal solution:

Constraint Imposed by Cost

1. **Total Cost:** The total cost of all selected pieces cannot exceed \$4 million.

$$\sum_{i=1}^{34} x_i * c_i \leq 4,000,000 \quad [\$]$$

where c_i = corresponding cost of piece of art i

Constraints Imposed by Mr. Briggs

2. **Collage:** Mr. Briggs wants to include only 1 collage of the four available ones.

$$x_9 + x_{16} + x_{29} + x_{30} = 1 \quad [none]$$

3. **Wire-mesh Sculpture:** Mr. Briggs wants to include at least one wire mesh sculpture if a computer-generated drawing is displayed, and vice-versa.

$$\{x_1 + x_2 + x_3\} \geq 0.5 * \{x_4 + x_5\} \quad [none]$$

$$(1/3) * \{x_1 + x_2 + x_3\} \leq \{x_4 + x_5\} \quad [none]$$

4. **Photo-realistic Painting:** Mr. Briggs wants to include at least 1 photo-realistic painting displayed of the three which are available.

$$x_{14} + x_{15} + x_{19} \geq 1 \quad [none]$$

5. **Cubic Painting:** Mr. Briggs wants to include at least 1 cubic painting of the three available ones.

$$x_{20} + x_{27} + x_{28} \geq 1 \quad [none]$$

- 6. Expressionist Painting:** Mr. Briggs wants to include at least 1 expressionist painting of the one available. Therefore, it must be displayed.

$$x_{21} = 1 \quad \text{[none]}$$

$$\text{[none]}$$

- 7. Watercolor Paintings:** Mr. Briggs wants to include at least 1 watercolor painting of the six available ones.

$$x_{10} + x_{11} + x_{31} + x_{32} + x_{33} + x_{34} \geq 1 \quad \text{[none]}$$

- 8. Oil Paintings:** Mr. Briggs wants to include at least 1 oil painting of the five available ones.

$$x_{12} + x_{13} + x_{22} + x_{23} + x_{26} \geq 1 \quad \text{[none]}$$

- 9. Painting Ratio:** Mr. Briggs wants the number of paintings to be no greater than twice the other art forms. There are 18 paintings and 16 other art forms.

$$\sum_{i=1}^{18} x_{i-\text{paintings}} \leq 2 * (\sum_{i=1}^{16} x_{i-\text{other}}) \quad \text{[none]}$$

$$\{ x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{26} + x_{27} + x_{28} + x_{31} + x_{32} + x_{33} + x_{34} \} - 2 * \{ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{16} + x_{17} + x_{18} + x_{24} + x_{25} + x_{29} + x_{30} \} \leq 0$$

- 10. Mr. Briggs Brigg's Paintings:** Mr. Briggs wants all of his own paintings included in the exhibit since his home is the site of the exhibit.

$$x_{31} = 1 \quad \text{[none]}$$

$$x_{32} = 1 \quad \text{[none]}$$

$$x_{33} = 1 \quad \text{[none]}$$

$$x_{34} = 1 \quad \text{[none]}$$

- 11. Candy Tate:** Mr. Briggs wants both of Candy Tate's paintings displayed since they are lovers.

$$x_{10} = 1 \quad \text{[none]}$$

$$x_{11} = 1 \quad \text{[none]}$$

- 12. David Lyman:** Mr. Briggs wants to include at least 1 piece from David Lyman of the two available ones.

$$x_{14} + x_{15} \geq 1 \quad \text{[none]}$$

- 13. Rick Rawls:** Mr. Briggs wants to include at least 1 piece from Rick Rawls of the three available ones.

$$x_{19} + x_{20} + x_{21} \geq 1 \quad \text{[none]}$$

- 14. Best Friends:** Mr. Briggs wants to include the same number of pieces from both Rick Rawls as David Lyman as he considers them both his best friends and doesn't want to show favoritism.

$$\{ x_{14} + x_{15} \} - \{ x_{19} + x_{20} + x_{21} \} = 0 \quad \text{[none]}$$

- 15. Ziggy Lite:** Since Mr. Briggs considers Ziggy's art distasteful, he will only accept at most one piece of Ziggy's art.

$$x_{29} + x_{30} \leq 1$$

[none]

Constraints Imposed by Celeste McKenzie

- 16. Advancing Feminism:** Celeste insists the exhibit should include one female piece for every two pieces included from a male artist to advance feminism.

$$2 * \{x_4 + x_5 + x_6 + x_7 + x_{10} + x_{11} + x_{16} + x_{17} + x_{18} + x_{27} + x_{28}\} - \{x_1 + x_2 + x_3 + x_8 + x_9 + x_{12} + x_{13} + x_{14} + x_{15} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} + x_{34}\} \geq 0$$

[none]

- 17. Advancing Environmentalism:** Celeste believes at least one or both of "Aging Earth" and "Wasted Resources" should be included to advance environmentalism.

$$x_8 + x_9 \geq 1$$

[none]

- 18. Native American Rights:** Celeste at least one piece by Bear Canton should be displayed to advance Native American rights.

$$x_{24} + x_{25} + x_{26} \geq 1$$

[none]

- 19. Advancing Science:** Celeste believes at least one pieces should be included to advance science.

$$x_4 + x_5 + x_{22} + x_{23} \geq 1$$

[none]

- 20. Narcissism** Celeste decides that if "Narcissism" is displayed, "Reflection" should also be displayed since "Reflection" also suggests narcissism.

$$x_{30} - x_{17} \leq 0$$

[none]

- 21. Floor Space:** Celeste is aware the museum only has enough floor space for four sculptures.

$$x_1 + x_2 + x_3 + x_8 + x_{17} + x_{18} \leq 4$$

[none]

- 22. Wall Space:** Celeste is aware the museum only has enough wall space for 20 paintings, collages, and drawings.

$$\{x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{26} + x_{27} + x_{28} + x_{31} + x_{32} + x_{33} + x_{34}\} + \{x_9 + x_{16} + x_{29} + x_{30}\} + \{x_4 + x_5 + x_6 + x_7 + x_{24} + x_{25}\} \leq 20$$

[none]

Objective Function:

The objective is to maximize the number of pieces displayed in the exhibit without exceeding the budget. Therefore, we seek to:

$$\max \{ \sum_{i=1}^{34} x_i \}$$

[none]

3.2 Optimal Solution

Question (a): Optimal Solution to Maximize the Number of Pieces Displayed Without Exceeding \$4 Million.

Using the Simplex method built-in to Excel's Solver add-on, we find that all aforementioned requirements can be met to showcase **15 paintings** for a total cost of \$3.95 million. This can be achieved by displaying the paintings as found in Table 2 below. The constructed binary integer programming model can be found as an Excel spreadsheet is found in Figure 1 in Appendix A.

TABLE 2: List of artists, and their pieces, to be displayed in the San Francisco Museum of Modern Art using a budget of \$4 million.

Artist	Piece Name and Number	Description of Piece
Colin Zweibell	1. "The Great Equalizer"	A wire mesh sculpture of a gun
Rita Losky	2. "Chaos Reigns"	A series of computer-generated drawings
Norm Marson	3. "Wasted Resources"	A collage of various packaging materials
Candy Tate	4. "Serenity"	An all blue watercolor painting
	5. "Calm Before the Storm"	A painting with an all blue watercolor background and black watercolour centre
Robert Bayer	6. "Sun"	An all yellow oil painting
David Lyman	7. "Harley"	A photo-realistic painting of a Harley-Davidson motorcycle
Angie Oldman	8. "Reflection"	A mirror (considered a sculpture)
Rick Rawls	9. "Rick III"	An expressionist self-portrait (painting)
Bear Canton	10. "Wisdom"	A pen-and-ink drawing of an Apache chieftain
Helen Row	11. "Study of a Fruit Bowl"	A cubist painting of a fruit bowl
Mr. Briggs Brigs	12. "All That Glitters"	A watercolor painting of the Golden Gate Bridge
	13. "The Rock"	A watercolor painting of Alcatraz
	14. "Winding Road"	A watercolor painting of Lombard Street
	15. "Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art

3.3 Optimal Solution for Various Conditions

Question (b): Optimal Solution to Minimize the Cost of Displaying at Least 20 Paintings.

To ensure that the exhibit draws the attention of the public, we considered how to minimize the cost of the exhibit while displaying at least 20 pieces and meeting the requirements set by Mr. Briggs and Celeste. We modified the binary integer programming model as described in Section 3.1 by removing constraint 1 (i.e. the “Total Cost” constraint) and introducing the following additional constraint:

23. Attention of the Public: Celeste wants to draw the attention of the public by displaying at least 20 pieces of art.

$$\sum_{i=1}^{34} x_i \geq 20 \quad [\text{none}]$$

In addition, we modified the objective function to:

$$\max \{ \sum_{i=1}^{34} x_i * c_i \} \quad [\text{none}]$$

Per request of the client, we found an optimal solution using the Simplex method built-in to Excel’s Solver add-on. We find that all aforementioned requirements can be met to showcase 20 paintings at a cost of **\$5.45 million**. Unfortunately, this is \$1.45 million higher than the Mr. Briggs Brigg’s ideal \$4 million budget. This can be achieved by displaying the paintings as found in Table 3 below. The constructed binary integer programming model can be found as an Excel spreadsheet is found in Figure 2 in Appendix B.

TABLE 3: List of artists, and their pieces, to be displayed in the San Francisco Museum of Modern Art given 20 pieces are displayed at a minimum cost of \$5.45 million.

Artist	Piece Name and Number	Description of Piece
Colin Zweibell	1. "Perfection"	A wire mesh sculpture of the human body
	2. "The Great Equalizer"	A wire mesh sculpture of a gun
	3. "Burden"	A wire mesh sculpture of a male
Rita Losky	4. "Chaos Reigns"	A series of computer-generated drawings
	5. "Domestication"	A pen-and-ink drawing of a house
Norm Marson	6. "Wasted Resources"	A collage of various packaging materials
Candy Tate	7. "Serenity"	An all blue watercolor painting
	8. "Calm Before the Storm"	A painting with an all blue watercolor background and black watercolour centre
Robert Bayer	9. "Sun"	An all yellow oil painting
	10. "Void"	An all black oil painting
David Lyman	11. "Harley"	A photo-realistic painting of a Harley-Davidson motorcycle
Angie Oldman	12. "Reflection"	A mirror (considered a sculpture)
Rick Rawls	13. "Rick III"	An expressionist self-portrait (painting)
Bear Canton	14. "Wisdom"	A pen-and-ink drawing of an Apache chieftain
Helen Row	15. "Study of a Violin"	A cubist painting of a violin
	16. "Study of a Fruit Bowl"	A cubist painting of a bowl of fruit
Mr. Briggs	17. "All That Glitters"	A watercolor painting of the Golden Gate Bridge
	18. "The Rock"	A watercolor painting of Alcatraz
	19. "Winding Road"	A watercolor painting of Lombard Street
	20. "Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art

Question (c): Optimal Solution Minimize the Patron's Expenses for Displaying 20 Paintings which Include All of Rita's Pieces.

Per request of the patron, who is an influential patron of Rita Losky's work who chairs the Museum Board of Directors, we considered how to minimize the cost of the exhibit while displaying at 20 pieces of work which includes all four of Rita's pieces. We modified the binary integer programming model as described in Section 3.3 by modifying the "Attention of the Public" constraint to be equal to 20 (i.e. instead of equal to or greater than), and introducing the following additional constraint:

24. Rita Losky: The influential patron of Rita Losky, who chairs the Museum Board of Directors, offers to pay the minimum amount to ensure all of her pieces are displayed.

$x_4 = 1$	[none]
$x_5 = 1$	[none]
$x_6 = 1$	[none]
$x_7 = 1$	[none]

Pre request of the client, we found an optimal solution using the Simplex method built-in to Excel's Solver add-on. We find that all aforementioned requirements can be met to showcase 20 paintings at a cost of **\$5.8 million**. Unfortunately, this is \$1.8 million higher than the Mr. Briggs Brigg's ideal \$4 million budget. This can be achieved by displaying the paintings as found in Table 3 below. The constructed binary integer programming model can be found as an Excel spreadsheet is found in Figure 3 in Appendix B.

TABLE 4: List of artists, and their pieces, to be displayed in the San Francisco Museum of Modern Art given 20 pieces are displayed at a minimum cost of \$5.8 million when all of Rita Losky's pieces must be displayed.

Artist	Piece Name and Number	Description of Piece
Colin Zweibell	1. "The Great Equalizer"	A wire mesh sculpture of a gun
	2. "Burden"	A wire mesh sculpture of a male
Rita Losky	3. "Chaos Reigns"	A series of computer-generated drawings
	4. "Who Has Control?"	A computer-generated drawing intermeshed with lines of computer code
	5. "Domestication"	A pen-and-ink drawing of a house
	6. "Innocence"	A pen-and-ink drawing of a child
Norm Marson	7. "Wasted Resources"	A collage of various packaging materials
Candy Tate	8. "Serenity"	An all blue watercolor painting
	9. "Calm Before the Storm"	A painting with an all blue watercolor background and black watercolour centre
Robert Bayer	10. "Sun"	An all yellow oil painting
	11. "Void"	An all black oil painting
David Lyman	12. "Harley"	A photo-realistic painting of a Harley-Davidson motorcycle
Angie Oldman	13. "Reflection"	A mirror (considered a sculpture)
Rick Rawls	14. "Rick III"	An expressionist self-portrait (painting)
Bear Canton	15. "Wisdom"	A pen-and-ink drawing of an Apache chieftain
Helen Row	16. "Study of a Violin"	A cubist painting of a violin
Mr. Briggs	17. "All That Glitters"	A watercolor painting of the Golden Gate Bridge
	18. "The Rock"	A watercolor painting of Alcatraz
	19. "Winding Road"	A watercolor painting of Lombard Street
	20. "Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art

4 Appendices

Appendix A. Representation of the Binary Integer Programming Model in Excel with the Optimal Solution

FIGURE 1: Optimum solution yields 15 pieces to be displayed in the San Francisco Museum of Modern Art using a budget of \$3.95 million. This means the \$4 million budgeted was not exceeded.

Decision variables						Constraints	Value		
Artist	Piece	Description of Piece	binary	value	Price				
Colin Zweibell	"Perfection"	A wire mesh sculpture of the human body	x1	0	\$300,000	1 total cost	3950000	<=	4000000
	"Burden"	A wire mesh sculpture of a mule	x2	0	\$250,000	2 collage	1	=	1
	"The Great Equalizer"	A wire mesh sculpture of a gun	x3	1	\$125,000	3 wire mesh and computer generated drawings	1	>=	0.5
Rita Losky	"Chaos Reigns"	A series of computer-generated drawings	x4	1	\$400,000		1	>=	0.33333
	"Who Has Control?"	A computer-generated drawing intermeshed with lines of computer code	x5	0	\$500,000	4 photo-realistic painting	1	>=	1
	"Domestication"	A pen-and-ink drawing of a house	x6	0	\$400,000	5 cubic painting	1	>=	1
Norm Marson	"Innocence"	A pen-and-ink drawing of a child	x7	0	\$550,000	6 expressionist painting	1	>=	1
	"Aging Earth"	A sculpture of trash covering a larger globe	x8	0	\$700,000	7 water color painting	6	>=	1
	"Wasted Resources"	A collage of various packaging materials	x9	1	\$575,000	8 oil painting	1	>=	1
Candy Tate	"Serenity"	An all blue watercolor painting	x10	1	\$200,000	9 painting ratio	10	<=	10
	"Calm Before the Storm"	A painting with an all blue watercolor background and a black watercolor center	x11	1	\$225,000	10 ash's paintings	1	=	1
Robert Bayer	"Void"	An all black oil painting	x12	0	\$150,000		1	=	1
	"Sun"	An all yellow oil painting	x13	1	\$150,000		1	=	1
David Lyman	"Storefront Window"	A photo-realistic painting of a jewelry store display window	x14	0	\$850,000	11 candy's paintings	1	=	1
	"Harley"	A photo-realistic painting of a Harley-Davidson motorcycle	x15	1	\$750,000		1	=	1
Angie Oldman	"Consumerism"	A collage of magazine advertisements	x16	0	\$400,000	12 david	1	>=	1
	"Reflection"	A mirror (considered a sculpture)	x17	1	\$175,000	13 rick	1	>=	1
Rick Rawls	"Trojan Victory"	A wooden sculpture of a condom	x18	0	\$450,000	14 best friend	1	=	1
	"Rick"	A photo-realistic self-portrait (painting)	x19	0	\$500,000	15 ziggy	0	<=	1
	"Rick II"	A cubist self-portrait (painting)	x20	0	\$500,000	16 feminism	10	>=	10
Bill Reynolds	"Rick III"	An expressionist self-portrait (painting)	x21	1	\$500,000	17 environment	1	>=	1
	"Beyond"	A science fiction oil painting depicting Mars colonization	x22	0	\$650,000	18 native american	1	>=	1
	"Pioneers"	An oil painting of three astronauts aboard the space shuttle	x23	0	\$650,000	19 science	1	>=	1
Bear Canton	"Wisdom"	A pen-and-ink drawing of an Apache chieftain	x24	1	\$250,000	20 narcissism	-1	<=	0
	"Superior Powers"	A pen-and-ink drawing of a traditional Native American rain dance	x25	0	\$350,000		2	<=	4
	"Living Land"	An oil painting of the Grand Canyon	x26	0	\$450,000	21 floor space			
Helen Row	"Study of a Violin"	A cubist painting of a violin	x27	0	\$400,000	22 wall space	13	<=	20
	"Study of a Fruit Bowl"	A cubist painting of a bowl of fruit	x28	1	\$400,000	obje max total number of paintings	15		
Ziggy Lite	"My Namesake"	A collage of Ziggy cartoons	x29	0	\$300,000				
	"Narcissism"	A collage of photographs of Ziggy Lite	x30	0	\$300,000		3950000		
Ash Briggs	"All That Glitters"	A watercolor painting of the Golden Gate Bridge	x31	1	\$50,000				
	"The Rock"	A watercolor painting of Alcatraz	x32	1	\$50,000				
	"Winding Road"	A watercolor painting of Lombard Street	x33	1	\$50,000				
	"Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art	x34	1	\$50,000				

Appendix B. Variants of the Binary Integer Programming Model in Excel with the Optimal Solutions

FIGURE 2: Optimal solution yields 20 paintings displayed in the San Francisco Museum of Modern Art at a minimum cost of \$5.45 million.

Decision variables						Constraints			
Artist	Piece	Description of Piece	binary xi	value	Price (\$)		Value		
Colin Zweibell	"Perfection"	A wire mesh sculpture of the human body	x1	1	\$300,000	1 total cost	0		0
	"Burden"	A wire mesh sculpture of a mule	x2	1	\$250,000	2 collage	1	=	1
	"The Great Equalizer"	A wire mesh sculpture of a gun	x3	1	\$125,000	3 wire mesh and computer generated	3	>=	0.5
Rita Losky	"Chaos Reigns"	A series of computer-generated drawings	x4	1	\$400,000		1	>=	1
	"Who Has Control?"	A computer-generated drawing intermeshed with lines of computer code	x5	0	\$500,000	4 photo-realistic painting	1	>=	1
	"Domestication"	A pen-and-ink drawing of a house	x6	1	\$400,000	5 cubic painting	2	>=	1
Norm Marson	"Innocence"	A pen-and-ink drawing of a child	x7	0	\$550,000	6 expressionist painting	1	>=	1
	"Aging Earth"	A sculpture of trash covering a larger globe	x8	0	\$700,000	7 water color painting	6	>=	1
	"Wasted Resources"	A collage of various packaging materials	x9	1	\$575,000	8 oil painting	2	>=	1
Candy Tate	"Serenity"	An all blue watercolor painting	x10	1	\$200,000	9 painting ratio	12	<=	16
	"Calm Before the Storm"	A painting with an all blue watercolor background and a black watercolor center	x11	1	\$225,000	10 ash's paintings	1	=	1
Robert Bayer	"Void"	An all black oil painting	x12	1	\$150,000		1	=	1
	"Sun"	An all yellow oil painting	x13	1	\$150,000		1	=	1
David Lyman	"Storefront Window"	A photo-realistic painting of a jewelry store display window	x14	0	\$850,000	11 candy's paintings	1	=	1
	"Harley"	A photo-realistic painting of a Harley-Davidson motorcycle	x15	1	\$750,000		1	=	1
Angie Oldman	"Consumerism"	A collage of magazine advertisements	x16	0	\$400,000	12 david	1	>=	1
	"Reflection"	A mirror (considered a sculpture)	x17	1	\$175,000	13 rick	1	>=	1
Rick Rawls	"Trojan Victory"	A wooden sculpture of a condom	x18	0	\$450,000	14 best friend	1	=	1
	"Rick"	A photo-realistic self-portrait (painting)	x19	0	\$500,000	15 ziggy	0	<=	1
	"Rick II"	A cubist self-portrait (painting)	x20	0	\$500,000	16 feminism	14	>=	13
	"Rick III"	An expressionist self-portrait (painting)	x21	1	\$500,000	17 environment	1	>=	1
Bill Reynolds	"Beyond"	A science fiction oil painting depicting Mars colonization	x22	0	\$650,000	18 native american	1	>=	1
	"Pioneers"	An oil painting of three astronauts aboard the space shuttle	x23	0	\$650,000	19 science	1	>=	1
Bear Canton	"Wisdom"	A pen-and-ink drawing of an Apache chieftain	x24	1	\$250,000	20 narcissism	-1	<=	0
	"Superior Powers"	A pen-and-ink drawing of a traditional Native American rain dance	x25	0	\$350,000	21 floor space	4	<=	4
Helen Row	"Living Land"	An oil painting of the Grand Canyon	x26	0	\$450,000	22 wall space	16	<=	20
	"Study of a Violin"	A cubist painting of a violin	x27	1	\$400,000	23 Attention of Public	20	>=	20
Ziggy Lite	"Study of a Fruit Bowl"	A cubist painting of a bowl of fruit	x28	1	\$400,000				
	"My Namesake"	A collage of Ziggy cartoons	x29	0	\$300,000	objective			
Ash Briggs	"Narcissism"	A collage of photographs of Ziggy Lite	x30	0	\$300,000	Minimize the Total Cost	5450000		
	"All That Glitters"	A watercolor painting of the Golden Gate Bridge	x31	1	\$50,000				
	"The Rock"	A watercolor painting of Alcatraz	x32	1	\$50,000				
	"Winding Road"	A watercolor painting of Lombard Street	x33	1	\$50,000				
	"Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art	x34	1	\$50,000				
					20				

FIGURE 3: Optimal solution yields 20 paintings displayed in the San Francisco Museum of Modern Art at a minimum cost of \$5.8 million when all of Rita Losky's pieces must be displayed.

Decision variables						Constraints			
Artist	Piece	Description of Piece	binary xi	value	Price (\$)		Value		
Colin Zweibell	"Perfection"	A wire mesh sculpture of the human body	x1	0	\$300,000	1 total cost	0		0
	"Burden"	A wire mesh sculpture of a mule	x2	1	\$250,000	2 collage	1	=	1
	"The Great Equalizer"	A wire mesh sculpture of a gun	x3	1	\$125,000	3 wire mesh and computer generated	2	>=	1
Rita Losky	"Chaos Reigns"	A series of computer-generated drawings	x4	1	\$400,000		2	>=	0.66667
	"Who Has Control?"	A computer-generated drawing intermeshed with lines of computer code	x5	1	\$500,000	4 photo-realistic painting	1	>=	1
	"Domestication"	A pen-and-ink drawing of a house	x6	1	\$400,000	5 cubic painting	1	>=	1
Norm Marson	"Innocence"	A pen-and-ink drawing of a child	x7	1	\$550,000	6 expressionist painting	1	>=	1
	"Aging Earth"	A sculpture of trash covering a larger globe	x8	0	\$700,000	7 water color painting	6	>=	1
	"Wasted Resources"	A collage of various packaging materials	x9	1	\$575,000	8 oil painting	2	>=	1
Candy Tate	"Serenity"	An all blue watercolor painting	x10	1	\$200,000	9 painting ratio	11	<=	18
	"Calm Before the Storm"	A painting with an all blue watercolor background and a black watercolor center	x11	1	\$225,000	10 ash's paintings	1	=	1
							1	=	1
Robert Bayer	"Void"	An all black oil painting	x12	1	\$150,000		1	=	1
	"Sun"	An all yellow oil painting	x13	1	\$150,000		1	=	1
David Lyman	"Storefront Window"	A photo-realistic painting of a jewelry store display window	x14	0	\$850,000	11 candy's paintings	1	=	1
	"Harley"	A photo-realistic painting of a Harley-Davidson motorcycle	x15	1	\$750,000		1	=	1
Angie Oldman	"Consumerism"	A collage of magazine advertisements	x16	0	\$400,000	12 david	1	>=	1
	"Reflection"	A mirror (considered a sculpture)	x17	1	\$175,000	13 rick	1	>=	1
	"Trojan Victory"	A wooden sculpture of a condom	x18	0	\$450,000	14 best friend	1	=	1
Rick Rawls	"Rick"	A photo-realistic self-portrait (painting)	x19	0	\$500,000	15 ziggy	0	<=	1
	"Rick II"	A cubist self-portrait (painting)	x20	0	\$500,000	16 feminism	16	>=	12
	"Rick III"	An expressionist self-portrait (painting)	x21	1	\$500,000	17 environment	1	>=	1
Bill Reynolds	"Beyond"	A science fiction oil painting depicting Mars colonization	x22	0	\$650,000	18 native american	1	>=	1
	"Pioneers"	An oil painting of three astronauts aboard the space shuttle	x23	0	\$650,000	19 science	2	>=	1
						20 narcissism	-1	<=	0
Bear Canton	"Wisdom"	A pen-and-ink drawing of an Apache chieftain	x24	1	\$250,000		3	<=	4
	"Superior Powers"	A pen-and-ink drawing of a traditional Native American rain dance	x25	0	\$350,000	21 floor space			
	"Living Land"	An oil painting of the Grand Canyon	x26	0	\$450,000	22 wall space	17	<=	20
Helen Row	"Study of a Violin"	A cubist painting of a violin	x27	1	\$400,000	23 Attention of Public	20	=	20
	"Study of a Fruit Bowl"	A cubist painting of a bowl of fruit	x28	0	\$400,000	24 Rita Losky	1	=	1
	"My Namesake"	A collage of Ziggy cartoons	x29	0	\$300,000		1	=	1
Ziggy Lite	"Narcissism"	A collage of photographs of Ziggy Lite	x30	0	\$300,000		1	=	1
Ash Briggs	"All That Glitters"	A watercolor painting of the Golden Gate Bridge	x31	1	\$50,000		1	=	1
	"The Rock"	A watercolor painting of Alcatraz	x32	1	\$50,000				
	"Winding Road"	A watercolor painting of Lombard Street	x33	1	\$50,000	objective			
	"Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art	x34	1	\$50,000	Minimize the Total Cost	5800000		