Homework Assignment 4

Due Wednesday November 23, 9:00 am

Do all 5 problems below. *Please refer to the instructions for Homework in the Course Outline. A spreadsheet containing the data for Problem 1 is posted on the course web site. All integer programming models in this homework must have linear constraints and objective. Make sure to produce and explain all relevant computer printouts. Unless specifically instructed otherwise, please solve each integer programming model to optimality (i.e., with Integer Optimality tolerance 0% if you use the Excel Solver, and similarly with other solvers). Interrupt a computer solution run if it exceeds 300 seconds.*

1. Stocking Sets. Write a Case Report for the *Stocking Sets* case (Case 11.3 in the Hillier & Lieberman text; copy of this case attached. Please note the following typo in the textbook version, 5th line below the first table: kitchen set 1 uses countertop O4 [as correctly indicated in the next data table] and not O2). Include brief answers to all Questions (a) to (e) in your one-page Executive Summary.

2. Projection. Let $P = \{ (x_1, x_2, w) \in \mathbf{R}^3 :$

$$\begin{array}{lrcl} (1) & x_1 + w & \le & 3 \\ (2) & -x_1 + w & \le & 1 \\ (3) & -w & \le & -1 \\ (4) & -x_2 - w & \le & -2 \\ (5) & x_2 - w & \le & 2 \ \}. \end{array}$$

(a) Construct a graphical representation of P in the (x_1, x_2, w)-space, with w as the "vertical" dimension. (Hint: the first three constraints, which do not involve x_2, define a "horizontal", unbounded triangular prism; the other two constraints cut this prism sideways.) List all 6 extreme points of P.

(b) Construct the projection $Q = proj_x P$ onto the subspace of the (x_1, x_2) coordinates. List the 6 constraints that define Q. Show how each of these 6 inequalities can be obtained as a nonnegative combination of the 5 inequalities defining P.

3. Valid inequalities and convex hulls. For each of the three sets below, find a missing valid inequality and verify graphically that its addition to the formulation gives the convex hull of X

(i) $X = \{x \in \mathbf{B}^2 : 0 \le x \le 1; \ \ 3 x_1 - 4 x_2 \le 1\}$

(ii) $X = \{(x, y) \in \mathbf{R} \times \mathbf{B} : 0 \le x \le 7; \ \ 0 \le y \le 1; \ \ x \le 20 y\}$

(iii) $X = \{(x, y) \in \mathbf{R} \times \mathbf{Z} : 0 \le x \le 16; \ y \ge 0; \ x \le 6 y\}$

4. Cutting planes vs. Branch-and-Bound. Consider the pure integer programming problem

$$\begin{array}{ll} \text{maximize} & 3 x_1 - 3 x_2 \\ \text{subject to} & 2 x_1 - 2 x_2 \le 1 \\ & x = (x_1, x_2) \ge 0 \ \text{ and integer.} \end{array}$$

(a) Try to solve this problem directly using the *Excel Solver* (standard version, with the default 1% Integer Optimality tolerance). What do you observe?

(b) Repeat question (a) with the *OpenSolver*.

(c) Find a Chvátal-Gomory inequality for this problem, such that its addition to the given initial formulation makes it trivial to solve, even by the *Excel Solver*.

5. Cutting planes. In each of the examples below, a set X and a point x or (x, y) are given. Find a valid inequality for X cutting off the point, and present an *algebraic proof* (in contrast with a geometric argument based on drawing a picture) of its validity for the corresponding set X (by deriving it as a Chvátal-Gomory inequality, or using any other method, at your choice).

(i) $X = \{(x, y) \in \mathbf{R}^2 \times \mathbf{B} : 0 \le x \le 1;\ \ x_1 + x_2 \le 2\, y\};\ (x_1, x_2, y) = (1,\ 0,\ 0.5)$

(ii) $X = \{x \in \mathbf{Z}^4 : x \ge 0;\ 4\, x_1 + 8\, x_2 + 7\, x_3 + 5\, x_4 \le 33\};\ \ x = (0, 0, 33/7, 0)$

(iii) $X = \{x \in \mathbf{Z}^5 : x \ge 0;\ 9\, x_1 + 12\, x_2 + 8\, x_3 + 17\, x_4 + 13\, x_5 \ge 50\};$
$x = (0, 25/6, 0, 0, 0)$

(iv) $X = \{(x, y) \in \mathbf{R} \times \mathbf{Z} : 0 \le x \le 9;\ y \ge 0;\ x \le 4\, y\};\quad (x, y) = (9,\ 9/4)$

(v) $X = \{(x, y) \in \mathbf{R}^2 \times \mathbf{Z} : (x, y) \ge 0;\ x_1 + x_2 \le 25;\ x_1 + x_2 \le 8\, y\};$
$(x_1, x_2, y) = (20,\ 5,\ 25/8)$

CASE 11.3 STOCKING SETS

Daniel Holbrook, an expeditor at the local warehouse for Furniture City, sighed as he moved boxes and boxes of inventory to the side in order to reach the shelf where the particular item he needed was located. He dropped to his hands and knees and squinted at the inventory numbers lining the bottom row of the shelf. He did not find the number he needed. He worked his way up the shelf until he found the number matching the number on the order slip. Just his luck! The item was on the top row of the shelf! Daniel walked back through the warehouse to find a ladder, stumbling over boxes of inventory littering his path. When he finally climbed the ladder to reach the top shelf, his face crinkled in frustration. Not again! The item he needed was not in stock! All he saw above the inventory number was an empty space covered with dust!

Daniel trudged back through the warehouse to make the dreadful phone call. He dialed the number of Brenda Sims, the saleswoman on the kitchen showroom floor of Furniture City, and informed her that the particular light fixture the customer had requested was not in stock. He then asked her if she wanted him to look for the rest of the items in the kitchen set. Brenda told him that she would talk to the customer and call him back.

Brenda hung up the phone and frowned. Mr. Davidson, her customer, would not be happy. Ordering and receiving the correct light fixture from the regional warehouse would take at least two weeks.

Brenda then paused to reflect upon business during the last month and realized that over 80 percent of the orders for kitchen sets could not be filled because items needed to complete the sets were not in stock at the local warehouse. She also realized that Furniture City was losing customer goodwill and business because of stock-outs. The Furniture megastore was gaining a reputation for slow service and delayed deliveries, causing customers to turn to small competitors that sold furniture directly from the showroom floor.

Brenda decided to investigate the inventory situation at the local warehouse. She walked the short distance to the building next door and gasped when she stepped inside the warehouse. What she saw could only be described as chaos. Spaces allocated for some items were overflowing into the aisles of the warehouse while other spaces were completely bare. She walked over to one of the spaces overflowing with inventory to discover the item that was overstocked. She could not believe her eyes! The warehouse had at least 30 rolls of pea-green wallpaper! No customer had ordered pea-green wallpaper since 1973!

Brenda marched over to Daniel demanding an explanation. Daniel said that the warehouse had been in such a chaotic state since his arrival one year ago. He said the inventory problems occurred because management had a policy of stocking every furniture item on the showroom floor in the local warehouse. Management only replenished inventory every three months, and when inventory was replenished, management ordered every item regardless of if it had been sold. Daniel also said that he had tried to make management aware of the problems with overstocking unpopular items and understocking popular items, but that management would not listen to him because he was simply an expeditor.

Brenda understood that Furniture City required a new inventory policy. Not only was the megastore losing money by making customers unhappy with delivery delays, but it was also

losing money by wasting warehouse space. By changing the inventory policy to stock only popular items and replenish them immediately when they are sold, Furniture City could ensure that the majority of customers receive their furniture immediately and that the valuable warehouse space was utilized effectively.

Brenda needed to sell her inventory policy to management. Using her extensive sales experience, she decided that the most effective sales strategy would be to use her kitchen department as a model for the new inventory policy. She would identify all kitchen sets comprising 85 percent of customers orders. Given the fixed amount of warehouse space allocated to the kitchen department, she would identify the items Furniture City should stock in order to satisfy the greatest number of customer orders. She would then calculate the revenue from satisfying customer orders under the new inventory policy, using the bottom line to persuade management to accept her policy.

Brenda analyzed her records over the past three years and determined that 20 kitchen sets were responsible for 85 percent of the customer orders. These 20 kitchen sets were composed of up to eight features in a variety of styles. Brenda listed each feature and its popular styles:

Floor Tile	Wallpaper	Light Fixtures	Cabinets
(T1) White textured tile	(W1) Plain ivory paper	(L1) One large rectangular frosted fixture	(C1) Light solid wood cabinets
(T2) Ivory textured tile	(W2) Ivory paper with dark brown pinstripes	(L2) Three small square frosted fixtures	(C2) Dark solid wood cabinets
(T3) White checkered tile with blue trim	(W3) Blue paper with marble texture	(L3) One large oval frosted fixture	(C3) Light wood cabinets with glass doors
(T4) White checkered tile with light yellow trim	(W4) Light yellow paper with marble texture	(L4) Three small frosted globe fixtures	(C4) Dark wood cabinets with glass doors

Countertops	Dishwashers	Sinks	Ranges
(O1) Plain light wood countertops	(D1) White energy-saving dishwasher	(S1) Sink with separate hot and cold water taps	(R1) White electric oven
(O2) Stained light wood countertops	(D2) Ivory energy-saving dishwasher	(S2) Divided sink with separate hot and cold water taps and garbage disposal	(R2) Ivory electric oven
(O3) White lacquer-coated countertops		(S3) Sink with one hot and cold water tap	(R3) White gas oven
(O4) Ivory lacquer-coated countertops		(S4) Divided sink with one hot and cold water tap and garbage disposal	(R4) Ivory gas oven

Brenda then created a table showing the 20 kitchen sets and the particular features composing each set. To simplify the table, she used the codes shown in parentheses above to represent the particular feature and style. The table is given below. For example, kitchen set 1 consists of floor tile T2, wallpaper W2, light fixture L4, cabinet C2, countertop O4, dishwasher D2, sink S2, and range R2. Notice that sets 14 through 20 do not contain dishwashers.

Brenda knew she had only a limited amount of warehouse space allocated to the kitchen department. The warehouse could hold 50 square feet of tile and 12 rolls of wallpaper in the inventory bins. The inventory shelves could hold two light fixtures, two cabinets, three

countertops, and two sinks. Dishwashers and ranges are similar in size, so Furniture City stored them in similar locations. The warehouse floor could hold a total of four dishwashers and ranges.

	T1	T2	T3	T4	W1	W2	W3	W4	L1	L2	L3	L4	C1	C2	C3	C4	O1	O2	O3	O4	D1	D2	S1	S2	S3	S4	R1	R2	R3	R4
Set 1		1				1						1		1						1		1		1				1		
Set 2		1			1				1							1				1		1				1		1		
Set 3	1						1			1			1				1				1				1				1	
Set 4			1				1				1				1				1		1		1				1			
Set 5				1				1	1				1					1			1			1			1			
Set 6		1				1				1						1				1		1			1					1
Set 7	1						1					1			1			1			1		1				1			
Set 8		1			1						1		1				1					1			1					1
Set 9		1			1					1					1			1				1		1				1		
Set 10	1				1				1				1						1		1					1			1	
Set 11			1		1						1				1		1				1		1						1	
Set 12		1				1			1					1				1				1				1		1		
Set 13				1				1			1				1		1				1			1					1	
Set 14				1				1				1	1						1				1				1			
Set 15			1				1		1				1				1								1				1	
Set 16			1				1					1	1						1					1			1			
Set 17	1							1		1					1				1							1			1	
Set 18		1					1				1			1						1			1					1		
Set 19		1						1				1				1				1				1						1
Set 20		1					1		1				1					1							1					1

Every kitchen set always includes exactly 20 square feet of tile and exactly five rolls of wallpaper. Therefore, 20 square feet of a particular style of tile and five rolls of a particular style of wallpaper are required for the styles to be in stock.

(a) Formulate and solve a BIP model to maximize the total number of kitchen sets (and thus the number of customer orders) Furniture City stocks in the local warehouse. Assume that when a customer orders a kitchen set, all the particular items composing that kitchen set are replenished at the local warehouse immediately.

(b) How many of each feature and style should Furniture City stock in the local warehouse? How many different kitchen sets are in stock?

(c) Furniture City decides to discontinue carrying nursery sets, and the warehouse space previously allocated to the nursery department is divided between the existing departments at Furniture City. The kitchen department receives enough additional space to allow it to stock both styles of dishwashers and three of the four styles of ranges. How does the optimal inventory policy for the kitchen department change with this additional warehouse space?

(d) Brenda convinces management that the kitchen department should serve as a testing ground for future inventory policies. To provide adequate space for testing, management decides to allocate all the space freed by the nursery department to the kitchen department. The extra space means that the kitchen department can store not only the dishwashers and ranges from part (c), but also all sinks, all countertops, three of the four light fixtures, and three of the four cabinets. How much does the additional space help?

(e) How would the inventory policy be affected if the items composing a kitchen set could not be replenished immediately? Under what conditions is the assumption of immediate replenishment nevertheless justified?