Homework Assignment 2

Due Wednesday November 9, 9:00 am

Do all problems below. *No Case Report is required in this homework set. Please refer to the instructions for Homework in the Course Outline. A spreadsheet containing the data for this homework set is posted on the course web site. All integer programming models in this homework must have linear constraints and objective. Make sure to produce and explain all relevant computer printouts. Interrupt a computer solution run if it exceeds 300 seconds.*

1. Solving a TSP instance. Consider the instance of the Symmetric Traveling Salesman Problem (TSP) with twelve cities named A, B, …, L and city-to-city distances given below. Choose one of the formulations given in class and estimate the number of variables and constraints if you were to write the full formulation explicitly. Solve this 12-city instance using the Relaxation & Separation (or Cutting Planes) method, starting with the degree constraints only. You may choose any strategy to solve each successive Relaxation problem and Separation problem, and you may use the computer to solve the relaxations. Detail and explain your calculations.

	A	B	C	D	E	F	G	H	I	J	K	L
A		24	26	20	23	21	25	18	22	17	29	20
B			23	28	10	17	11	27	7	25	26	17
C				30	11	10	24	29	21	16	17	8
D					27	25	29	22	26	22	32	24
E						17	10	26	8	12	25	16
F							19	24	16	22	12	2
G								28	8	26	27	18
H									25	21	32	23
I										23	24	15
J											30	21
K												11
L												

City-to-city distances (km)

2. Portfolio partitioning. An investment dealer receives orders for an asset from his customers throughout the day. He also purchases units of the asset during the day at different prices. At the end of the day, the dealer wishes to allocate the assets to the customers in an "equitable way". Specifically, customer i (i = 1,…m) has placed orders for a total of d_i units of the asset today. The dealer has purchased the total amount $T = \Sigma_i d_i$ today in n lots, where lot j (j=1,…n) consists of s_j units at price p_j (dollars per unit). Therefore we have $\Sigma_j s_j = T$ and the total purchase cost $C = \Sigma_j p_j s_j$ is to be allocated "equitably" to the m customers. Ideally, the dealer would like to charge each customer i the average price C/T. However, current regulations require the dealer to allocate to each customer specific units of the asset at their purchase price.

One possible interpretation of "equitably" is to minimize the maximum, among all customers, of the resulting average price per unit charged to the customer. (As an illustrative example using the data below, if customer A gets 14 units from lot 1, 35 units from lot 3, and 1 unit from lot 4, she is charged a total of 14×995 + 35×1001 + 1×1006 = $49,971 and thus an average price of 49,971/50 = $999.42 per unit.)

(a) Show that this problem would have a trivial solution if fractional allocations of asset units were allowed. Find this "trivial solution" for the following instance.

Customer i	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Order d_i **(units)**	50	220	160	80	65	70	90	100	40	125

Lot j	**1**	**2**	**3**	**4**	**5**
Amount s_j **(units)**	155	195	175	370	105
Purchase price p_j **($/unit)**	995	989	1001	1006	999

In fact, due to the current regulations, each unit of the asset is indivisible and must be assigned to one customer. (For example, the dealer may *not* allocate 14.5 units from lot 1 to a customer.)

(b) Formulate a linear integer programming model for this problem. Show that the "trivial solution" found in (a) above solves the LP relaxation of this problem. Explain why this implies that this integer programming problem can be expected to be very difficult to solve using LP-based branch-and-bound.

(c) Using the *Excel Solver*, try and find a feasible integer solution with objective value within *half a cent* of the optimum. (What value of the percent Tolerance option should you use?) What happens?

(d) Repeat question (c) using the *OpenSolver*. What happens?

(e) Repeat question (c) using a mathematical programming system with an algebraic modeling language. Compare with your findings in (c) and (d) above.

(f) An alternate interpretation of "equitably" is to minimize the *sum*, over all T units, of the absolute deviations of the resulting prices per unit from the "target price" C/T. (In the earlier illustrative example, the absolute deviation from the target price was |999.42 – 999.37| = $0.05 per unit for each of the 50 units allocated to customer A.) Note that summing over all units treats each unit of asset "equitably", and thus reflects the relative importance of the different customers, as indicated by their order quantities. Repeat questions (a) to (e) above for this new objective (for questions (c) to (e): within half a cent of the optimum on a per unit basis) and compare the results.

3. Ranking Objects. We want to rank n objects (e.g., investment proposals, or sports teams) in a total order, from most to least preferred. We are given an $n \times n$ cost matrix C where the entry c_{ij} is the cost of ranking object i before j ($i \neq j$). For example, c_{ij} is the number of managers who prefer investment j to i, or the number of games that team i has lost against team j. Thus we seek a total order of the n objects which minimizes the sum of the costs c_{ij} over all pairs (i, j) where i is ranked before j.

(a) Formulate a binary integer programming model to determine a total order with minimum cost. Recall that a total order is irreflexive, antisymmetric and transitive. How many variables and constraints does your model contain? What are these model sizes for n = 10? (Using symmetry, can you reduce the number of transitivity constraints by a factor of 3? Explain.)

(b) Without spending too much time on this question, try and define a spreadsheet model for your model of question (a). Ignoring any limitation on the number of variables, explain clearly, yet concisely, what main difficulty you would encounter with an instance size n = 10 or 20, and what you *could* do about it. You are *not* being asked to complete a spreadsheet implementation of your model.

(c) Formulate your integer programming model using an algebraic modeling language. Make sure to use such language constructs as indexed variables and constraints; summations; and data tables or data files. Briefly comment on the relative ease of implementation of a model of this type using an algebraic modeling language compared with a spreadsheet implementation.

(d) Using your algebraic model of question (c) and a corresponding mathematical programming system, solve both the LP relaxation and the integer program for the 10-object instance with cost matrix shown below. (For example, the cost of ranking object **A** before **B** is 4. This cost matrix is also available on the course web site.) Comment on the differences (if any) between the LP and integer solutions. Make sure to produce and explain all relevant computer printouts.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
A	.	4	2	0	1	2	2	3	0	0
B	0	.	4	2	0	4	4	2	2	1
C	2	0	.	4	1	2	1	2	0	2
D	4	2	0	.	1	4	3	4	2	1
E	3	4	3	3	.	1	4	2	1	0
F	2	0	2	0	3	.	2	2	4	0
G	2	0	3	1	0	2	.	1	4	2
H	1	2	2	0	2	2	3	.	1	1
I	4	2	4	2	3	0	0	3	.	2
J	4	3	2	3	4	4	2	3	2	.