BAMS 506 - Homework Assignment 2

September 14th, 2016

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1. Concrete Blocks Corporation (CBC) Questions (a) – (g)

*Given: Do the following **Exercise**. (Please refer to the instructions on Homework Assignments in the course outline.) The Concrete Blocks Corporation (CBC) can produce 4 types of concrete blocks, B1 to B4. Each block is subjected to several processes, of which the following 3 have limited capacity: batch mixing, mold vibrating, and inspection. The table below indicates the number of hours of each process required per pallet of blocks of each type; the number of hours available next month for each process; and the unit profit per pallet for of each type of blocks. The plant manager desires to maximize profits during the next month.*

	Process time (hours per pallet)				Hours available
Block type:	B1	B2	B3	B4	next month
Batch mixing	1	2	10	16	800
Mold vibrating	1.5	2	4	5	1,000
Inspection	0.5	0.6	1	2	340
Profit ($/batch)	8	14	30	50	

Question 1(a) Formulate a linear programming model for this problem.

<u>Decision Variables</u>:

Let B1 denotes the total number of pallets produced of block-type 1 *[pallets of block 1]*
Let B2 denotes the total number of pallets produced of block-type 2 *[pallets of block 2]*
Let B3 denotes the total number of pallets produced of block-type 3 *[pallets of block 3]*
Let B4 denotes the total number of pallets produced of block-type 4 *[pallets of block 4]*

<u>Objective</u>: To maximize next month's profit by producing an optimal mix of four block-types (i.e. B1, B2, B3, B4), while meeting all the production constraints.

Maximize : { (B1*8) + (B2*14) + (B3*30) + (B4*50)} = Z *[$]*

<u>Constraints</u>:

1. **Non-negativity:** B1, B2, B3, B4 ≥ 0 *[pallets of blocks]*
   A negative number of blocks cannot be produced. Therefore, the total number of pallets of each block-type (1, 2, 3, or 4) cannot be negative.

2. **Batch Mixing:** (B1*1) + (B2*2) + (B3*10) + (B4*16) ≤ 800 *[hours]*
   The total number of hours in a month dedicated to batch mixing all the concrete for block-types 1 to 4 must be less than or equal to 800 hours.

3. **Mold Vibrating:** (B1*1.5) + (B2*2) + (B3*4) + (B4*5) ≤ 1000 *[hours]*
   The total number of hours in a month dedicated to vibrating and molding block-types 1 to 4 must be less than or equal to 1,000 hours.

4. **Inspection:** (B1*0.5) + (B2*0.6) + (B3*1) + (B4*2) ≤ 340 *[hours]*
   The total number of hours dedicated to inspecting block-types 1 to 4 must be less than or equal to 340.

Question 1(b) Solve using the computer.

Figure 1 (below) shows an optimal solution that was found using the model described in Question 1(a). This optimal solution indicates that we expect CBC to generate a maximum profit of $6,000 if they produce 400 pallets of block-type 1 and 200 pallets of block-type 2.

Figure 1

	A	B	C	D	E	F	G	H	I	J
1	BAMS 506 - Assignment 2 - Question 1									
2	LP Model									
3	Students: Gurpal Bisra and Barend Lotter									
4										
5	DATA:	Benefits:						Constraints:		
6		B1	B2	B3	B3	Time Available Next Month	Units		Total Benefit	
7								Nonnegativity		B1, B2, B3, B4 ≥ 0
8	Batching Mixing	1	2	10	16	800	hours	Batching Mixing	800	
9	Mold Vibrating	1.5	2	4	5	1000	hours	Mold Vibrating	1000	
10	Inspection	0.5	0.6	1	2	340	hours	Inspection	320	
11										
12	Profit	8	14	30	50		$/pallet			
13										
14	MODEL:									
15	Decision Variables	B1	B2	B3	B4					
16	Pallets Produced	400	200	0	0		Pallets			
17										
18	Objective Function:		max { (B1*8) + (B2*14) + (B3*30) + (B4*50) }							
19	Maximize Profit				6000.00	[$]				

Given: Answer the following questions using, as much as possible, sensitivity analysis. When sensitivity analysis only gives you lower and/or upper bounds for a numerical answer, obtain an exact answer by solving an appropriately modified LP model. Note that the changes considered in each question below are non-cumulative, i.e., independent: each change only applies to the question where it is considered.

Question 1(c) By how much must the profit per pallet on Type 3 blocks be increased before it would be profitable to produce them?

According to the sensitivity report (Figure 2) provided by Excel, the profit for block-type 3 must increase by at least $28 / pallet for it to become profitable. The total profit for block-type 3 must therefore be equal to or more than $58 / pallet. Figure 3 (below) shows an optimal solution if block 3's profit increases to $58 / pallet. In this solution the object function will change to [(B1*8) + (B2*14) + (B3***58**) + (B4*50)] = Z and the solution indicates that CBC should produce 618.18 pallets of block 1 and 18.18 pallets of block 3 to generate a profit of $6,000. If the profit for each pallet of block-type 3 increases by more than $28, the total profit will also increase above $6,000.

Figure 2

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [A2 - Q1.xls]Better LP Model Layout
Report Created: 9/15/2016 6:01:53 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B16	Pallets Produced B1	400	0	8	1.818181818	1
C16	Pallets Produced B2	200	0	14	2	2.105263158
D16	Pallets Produced B3	0	-28	30	28	1E+30
E16	Pallets Produced B4	0	-40	50	40	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I8	Batching Mixing Total Benefit	800	5	800	200	133.3333333
I9	Mold Vibrating Total Benefit	1000	2	1000	50	200
I10	Inspection Total Benefit	320	0	340	1E+30	20

Figure 3

BAMS 506 - Assignment 2 - Question 1
LP Model
Students: Gurpal Bisra and Barend Lotter

DATA:	Benefits: B1	B2	B3	B3	Time Available Next Month	Units
Batching Mixing	1	2	10	16	800	hours
Mold Vibrating	1.5	2	4	5	1000	hours
Inspection	0.5	0.6	1	2	340	hours
Profit	8	14	58	50		$/pallet

Constraints:	Total Benefit	
Nonnegativity		B1, B2, B3, B4 ≥ 0
Batching Mixing	800	
Mold Vibrating	1000	
Inspection	327.2727273	

MODEL:					
Decision Variables	B1	B2	B3	B4	
Pallets Produced	618.18182	0	18.1818	0	Pallets

Objective Function:	max { (B1*8) + (B2*14) + (B3*30) + (B4*50) }	
Maximize Profit	6000.00	[$]

Question 1(d) What minimum profit per pallet on Type 2 blocks must be realized so that they remain in an optimum production schedule?

The minimum profit per pallet on Type 2 blocks, so that they remain in the optimum production schedule, is **$11.90.** The sensitivity analysis (Figure 4) indicated that the objective coefficient for the total number pallets of block-type 2 can be decreased by 2.105263158. Since the objective function is stated in dollars, we will round to the second decimal. If block 2's profit decreases by $2.1 / pallet (Figure 5) then its production remains profitable. If however, its profit decreases by more than that, say $2.11 / pallet, the block becomes unprofitable (Figure 6).

Figure 4

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [A2 - Q1.xls]Better LP Model Layout
Report Created: 9/15/2016 6:01:53 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B16	Pallets Produced B1	400	0	8	1.818181818	1
C16	Pallets Produced B2	200	0	14	2	2.105263158
D16	Pallets Produced B3	0	-28	30	28	1E+30
E16	Pallets Produced B4	0	-40	50	40	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I8	Batching Mixing Total Benefit	800	5	800	200	133.3333333
I9	Mold Vibrating Total Benefit	1000	2	1000	50	200
I10	Inspection Total Benefit	320	0	340	1E+30	20

Figure 5

BAMS 506 - Assignment 2 - Question 1
LP Model
Students: Gurpal Bisra and Barend Lotter

DATA:	**Benefits:**						**Constraints:**		
	B1	**B2**	**B3**	**B3**	**Time Available Next Month**	**Units**		**Total Benefit**	
							Nonnegativity		**B1, B2, B3, B4 ≥ 0**
Batching Mixing	1	2	10	16	800	hours	**Batching Mixing**	**800**	
Mold Vibrating	1.5	2	4	5	1000	hours	**Mold Vibrating**	**1000**	
Inspection	0.5	0.6	1	2	340	hours	**Inspection**	**320**	
Profit	8	**11.9**	30	50		$/pallet			

MODEL:						
Decision Variables	**B1**	**B2**	**B3**	**B4**		
Pallets Produced	400	**200**	**0**	**0**		**Pallets**

Objective Function:	max { (B1*8) + (B2*14) + (B3*30) + (B4*50) }	
Maximize Profit	*5580.00*	[$]

Figure 6

	A	B	C	D	E	F	G	H	I	J
1	BAMS 506 - Assignment 2 - Question 1									
2	LP Model									
3	Students: Gurpal Bisra and Barend Lotter									
4										
5	**DATA:**	**Benefits:**						**Constraints:**		
6		**B1**	**B2**	**B3**	**B3**	**Time Available Next Month**	**Units**		**Total Benefit**	
7								**Nonnegativity**		**B1, B2, B3, B4 ≥ 0**
8	Batching Mixing	1	2	10	16	800	hours	**Batching Mixing**	**800**	
9	Mold Vibrating	1.5	2	4	5	1000	hours	**Mold Vibrating**	**1000**	
10	Inspection	0.5	0.6	1	2	340	hours	**Inspection**	**336.8421053**	
11										
12	Profit	8	**11.89**	30	50		$/pallet			
13										
14	**MODEL:**									
15	**Decision Variables**	**B1**	**B2**	**B3**	**B4**					
16	Pallets Produced	631.578947	**0**	**0**	**10.5263158**		**Pallets**			
17										
18	**Objective Function:**		max { (B1*8) + (B2*14) + (B3*30) + (B4*50) }							
19	Maximize Profit				**5578.95**	[$]				

Question 1(e) If the 800 machine-hours capacity on the batch mixer is uncertain, for what range of machine-hours will it remain optimal to produce Type 1 and Type 2 blocks?

If the 800 machine-hours capacity on the batch mixer is uncertain, the range of machine-hours which will remain optimal to produce Type 1 and Type 2 blocks is $666.\overline{6}$ to 1000 hours. This can be calculated from the sensitivity analysis (Figure 7), which indicates that these two block types (1 and 2) will remain profitable if the current batch mixing hours increases by up to 200 or decrease by up to 133.333 hours. The model in Figure 8. shows that if the available batch mixing hours increases to 1,000, then number of pallets of block-type 1 that should be produced is still a positive number even though tends towards zero. Similarly, the model in Figure 9 shows that if the available batch mixing hours decreases to $666.\overline{6}$, then the number of pallets of block-type 2 that should be produced is still a positive number that is only slightly larger than zero.

Figure 7

5							
6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	**Cell**	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
9	B16	Pallets Produced B1	400	0	8	1.818181818	1
10	C16	Pallets Produced B2	200	0	14	2	2.105263158
11	D16	Pallets Produced B3	0	-28	30	28	1E+30
12	E16	Pallets Produced B4	0	-40	50	40	1E+30
13							
14	Constraints						
15			Final	Shadow	Constraint	Allowable	Allowable
16	**Cell**	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
17	I8	Batching Mixing Total Benefit	800	5	800	200	133.3333333
18	I9	Mold Vibrating Total Benefit	1000	2	1000	50	200
19	I10	Inspection Total Benefit	320	0	340	1E+30	20

Figure 8

	A	B	C	D	E	F	G	H	I	J
1	BAMS 506 - Assignment 2 - Question 1									
2	LP Model									
3	Students: Gurpal Bisra and Barend Lotter									
4										
5	DATA:	Benefits:						Constraints:		
6		B1	B2	B3	B3	Time Available Next Month	Units		Total Benefit	
7								Nonnegativity		B1, B2, B3, B4 ≥ 0
8	Batching Mixing	1	2	10	16	1000	hours	Batching Mixing	1000	
9	Mold Vibrating	1.5	2	4	5	1000	hours	Mold Vibrating	1000	
10	Inspection	0.5	0.6	1	2	340	hours	Inspection	300	
11										
12	Profit	8	14	30	50		$/pallet			
13										
14	MODEL:									
15	Decision Variables	B1	B2	B3	B4					
16	Pallets Produced	2.274E-13	500	0	0		Pallets			
17										
18	Objective Function:		max { (B1*8) + (B2*14) + (B3*30) + (B4*50) }							
19	Maximize Profit				7000.00	[$]				

Figure 9

	A	B	C	D	E	F	G	H	I	J
1	BAMS 506 - Assignment 2 - Question 1									
2	LP Model									
3	Students: Gurpal Bisra and Barend Lotter									
4										
5	DATA:	Benefits:						Constraints:		
6		B1	B2	B3	B3	Time Available Next Month	Units		Total Benefit	
7								Nonnegativity		B1, B2, B3, B4 ≥ 0
8	Batching Mixing	1	2	10	16	666.6666667	hours	Batching Mixing	666.6666667	
9	Mold Vibrating	1.5	2	4	5	1000	hours	Mold Vibrating	1000	
10	Inspection	0.5	0.6	1	2	340	hours	Inspection	333.3333333	
11										
12	Profit	8	14	30	50		$/pallet			
13										
14	MODEL:									
15	Decision Variables	B1	B2	B3	B4					
16	Pallets Produced	666.66667	5.4E-13	0	0		Pallets			
17										
18	Objective Function:		max { (B1*8) + (B2*14) + (B3*30) + (B4*50) }							
19	Maximize Profit				5333.33	[$]				

Question 1 (f) A competitor located next door has offered CBC additional batch mixing time at a rate of $4.00 per hour. Should CBC accept this offer? If yes, for how many hours, and what will be the net impact on total profit? If no, what is the maximum price CBC should be willing to accept for this additional time?

CBC should accept this offer up to an additional 200 batch mixing hours at a rate of $4.00 per hour. In particular, this would yield an additional profit of $200. However, once CBC accepts over an addition 200 hours of batch mixing hours, then the rate of net profit will start to decrease. This can be seen in two ways. The easiest is to look at the original sensitivity analysis (Figure 10), which shows that the shadow price for batch mixing hours is $5/hour and that the allowable increase for batch mixing hours is 200 hours. This means that we can make a profit of $1/hour for the first 200 hours of batch mixing that we outsource at $4/hour.

The second way to evaluate (and confirm) this is to remodel the original problem with changes to the objective function and the batch mixing constraint. This can be seen below.

Figure 10

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [A2 - Q1.xls]Better LP Model Layout
Report Created: 9/15/2016 6:01:53 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B16	Pallets Produced B1	400	0	8	1.818181818	1
C16	Pallets Produced B2	200	0	14	2	2.105263158
D16	Pallets Produced B3	0	-28	30	28	1E+30
E16	Pallets Produced B4	0	-40	50	40	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I8	Batching Mixing Total Benefit	800	5	800	200	133.3333333
I9	Mold Vibrating Total Benefit	1000	2	1000	50	200
I10	Inspection Total Benefit	320	0	340	1E+30	20

We remodel the original problem with changes to the objective function and the batch mixing constraint.

Objective: To maximize next month's profit by producing an optimal mix of four block-types (i.e. B1, B2, B3, B4), while meeting all the production constraints.

Maximize: { (B1*8) + (B2*14) + (B3*30) + (B4*50) **– 4*200** } = Z [$]

Constraints:

1. **Non-negativity:** B1, B2, B3, B4 ≥ 0 [pallets of block-types]
   -The total number of pallets produced of block-types 1 through 4 cannot be negative.

2. **Batch Mixing:** (B1*1) + (B2*2) + (B3*10) + (B4*16) $\leq$ 800 **+ 200** [hours]
   -The total number of hours dedicated to batch mixing for producing block-types 1 through 4 must be less than or equal to 800.

3. **Mold Vibrating:** (B1*1.5) + (B2*2) + (B3*4) + (B4*5) $\leq$ 1000 [hours]
   -The total number of hours dedicated to mold vibrating for producing block-types 1 through 4 must be less than or equal to 1000.

4. **Inspection:** (B1*0.5) + (B2*0.6) + (B3*1) + (B4*2) $\leq$ 340 [hours]
   -The total number of hours dedicated to inspection for producing block-types 1 through 4 must be less than or equal to 340.

Figure 11 shows (and confirms) that our profit will increase by $200 if we outsource 200 hours of batch mixing we at $4/hour. Figure 12 that outsourcing more than 200 hours will have a negative effect on the net profit.

Figure 11

	A	B	C	D	E	F	G	H	I	J
1	BAMS 506 - Assignment 2 - Question 1									
2	LP Model									
3	Students: Gurpal Bisra and Barend Lotter									
4										
5	DATA:	Benefits:						Constraints:		
6		B1	B2	B3	B3	Time Available Next Month	Units		Total Benefit	
7								Nonnegativity		B1, B2, B3, B4 ≥ 0
8	Batching Mixing	1	2	10	16	1000	hours	Batching Mixing	1000	
9	Mold Vibrating	1.5	2	4	5	1000	hours	Mold Vibrating	1000	
10	Inspection	0.5	0.6	1	2	340	hours	Inspection	300	
11										
12	Profit	8	14	30	50		$/pallet			
13										
14	MODEL:									
15	Decision Variables	B1	B2	B3	B4					
16	Pallets Produced	2.274E-13	500	0	0		Pallets			
17										
18	Objective Function:		max { (B1*8) + (B2*14) + (B3*30) + (B4*50) - 4*200 }							
19	Maximize Profit				6200.00	[$]				

Figure 12

	A	B	C	D	E	F	G	H	I	J
1	BAMS 506 - Assignment 2 - Question 1									
2	LP Model									
3	Students: Gurpal Bisra and Barend Lotter									
4										
5	**DATA:**	**Benefits:**						**Constraints:**		
6		**B1**	**B2**	**B3**	**B3**	**Time Available Next Month**	**Units**		**Total Benefit**	
7								**Nonnegativity**		**B1, B2, B3, B4 ≥ 0**
8	Batching Mixing	1	2	10	16	**1001**	hours	**Batching Mixing**	**1001**	
9	Mold Vibrating	1.5	2	4	5	1000	hours	**Mold Vibrating**	**1000**	
10	Inspection	0.5	0.6	1	2	340	hours	**Inspection**	**300.0454545**	
11										
12	Profit	8	14	30	50		$/pallet			
13										
14	**MODEL:**									
15	**Decision Variables**	**B1**	**B2**	**B3**	**B4**					
16	Pallets Produced	0	**499.773**	**0**	**0.0909091**		**Pallets**			
17										
18	**Objective Function:**		max { (B1*8) + (B2*14) + (B3*30) + (B4*50) **- 4*201** }							
19	Maximize Profit				***6197.36***	[$]				