Case 3.2 Report: Cutting Cafeteria Costs

Prepared for

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Executive Summary

The cafeteria at All-State University serves a casserole dish every Thursday at noon. Our services were requested to help reduce the overall cafeteria's operating costs this coming year by selecting cheaper casserole ingredients. In particular, we aimed to reduce the weekly cost of purchasing kilograms of potatoes and green beans while ensuring the resulting dish meets the following nutritional, taste, and demand requirements:

- **Nutritional requirements.** All-State University requires that cafeteria dished contain at least 180 grams (g) of protein, 80 milligrams (mg) (or 0.08 g) of iron, and 1,050 mg (or 1.050 g) of vitamin C.
- **Taste requirements.** The cafeteria cook indicates that a minimum 6 to 5 ratio in the weight of potatoes to green beans is required for the casserole to be edible.
- **Demand requirements.** The cafeteria manager requires that a minimum of 10 kilograms (kg) of casserole be prepared each week to meet the cafeteria demand.

We found that my purchasing 6.15 kg of potatoes and 5.13 kg of green beans per week, the dish can meet all requirements for a minimal price of $16.75.

In this report, we also considered alternatives to further reduce the weekly cost of preparing the dish. First, by reducing the ratio of potatoes to green beans, the cost of the dish can further be reduced to $16.26 per week. On the other hand, by simply considering that other ingredients in the dish may also contain iron and accordingly reducing the requirements for iron from potatoes and green beans, the weekly cost of the dish can be further lowered to $14.32 per week. Furthermore, while purchasing green beans at a discounted price allows the cafeteria to prepare the dish for a sum of $10.21 per week, replacing green beans with more nutrient-rich lima beans can bring the price of the dish down to $9.38 per week. However, the resulting casserole is mostly composed of potatoes, which may have an adverse effect on taste. Finally, if the nutritional requirements were to increase according the student task force's proposal, the weekly cost of preparing the casserole using lima beans would increase to $10.69.

The above results are based on the following assumptions:

(1) The cafeteria only considers potatoes and green, or lima, beans to fulfill the nutrient requirements of the casserole meal.

(2) There is no limit to the amount of casserole to prepare since leftovers can be served in following days, or used to create new dishes.

(3) Green beans and lima beans taste the same and can be used interchangeably within the casserole.

(4) The wholesaler can provide fractions of kilograms of potatoes, green beans, and lima beans.

Contents

Introduction

Every week, the cafeteria at All-State University serves a casserole prepared largely from a mix of potatoes and green beans, and also includes onions and cream of mushroom soup. Far from being a random mix of ingredients, the dish must meet all requirements defined by various authorities:

- **Nutritional requirements.** All-State University requires that cafeteria dished contain at least 180 grams (g) of protein, 80 milligrams (mg) (or 0.08 g) of iron, and 1,050 mg (or 1.050 g) of vitamin C.
- **Taste requirements.** The cafeteria cook indicates that a minimum 6 to 5 ratio in the weight of potatoes to green beans is required for the casserole to be edible.
- **Demand requirements.** The cafeteria manager requires that a minimum of 10 kilograms (kg) of casserole be prepared each week to meet the cafeteria demand.

The key objective for the consultation was to minimize the weekly cost of preparing the dish while meeting the nutritional, taste, and demand requirements.

In order to conduct this analysis, the following assumptions were made:

(1) The cafeteria only considers potatoes and green, or lima, beans to fulfill the nutrient, taste, and demand requirements and to calculate the cost of the casserole meal.

(2) There is no maximum amount of casserole that may be prepared since leftovers can be served in following days, or used to create new dishes.

(3) Green beans and lima beans can be used interchangeably within the casserole.

(4) The cafeteria's wholesaler can provide fractions of kilograms of potatoes, green beans, and lima beans.

The present report contains the consultants' analysis and recommendations for cutting cafeteria costs.

Analysis

Linear Programming Model

Decision Variables: let

PT - denote the quantity of potatoes (in kilograms) to purchase per week; and

GB - denote the quantity of green beans (in kilograms) to purchase per week.

Objective Function:

We seek to minimize the cost of purchasing potatoes and green beans while ensuring the casserole will meet nutritional, taste and demand requirements. The All-State University cafeteria purchases potatoes at a cost of $0.40 per pound, or $0.88 per kilogram, and green beans at a cost of $1.00 per pound, or $2.21 per kilogram, as calculated in Appendix A.

Therefore, we seek:

$$\mathbf{min}\ \{\ \mathbf{0.88}\ \boldsymbol{PT} + \mathbf{2.21}\ \boldsymbol{GB}\ \}\quad [\$]$$

Constraints:

The All-State University cafeteria provided the nutritional content of per 100 g of potatoes and per 10 ounces (oz.) of green beans, as shown in Table 1. To facilitate the analysis, nutritional content is also indicated per kg of each ingredient; calculations for each conversion are presented in Appendix B.

Table 1. Nutritional content of potatoes and green beans. All-State University provided the data, originally sourced from the web. Conversion calculations are presented in Appendix B.

	Nutritional Content of Potatoes		Nutritional Content of Green Beans	
Nutrient	**Per 100 grams of potatoes (All-State Data)**	**Per kilogram of potatoes (Converted)**	**Per 10 ounces of green beans (All-State Data)**	**Per kilogram of green beans (Converted)**
Protein	1.5 g per 100 g	15 g per kg	5.67 g per 10 oz.	20 g per kg
Iron	0.3 mg per 100 g	0.003 g per 100 kg	3.402 mg per 10 oz.	0.012 g per kg
Vitamin C	12 mg per 100 g	0.12 mg per kg	28.35 mg per 10 oz.	0.10 g per kg

The following constraints define the nutritional, taste, and demand requirements the dish must meet each week:

1. **Non-negativity:** PT, GB ≥ 0 [kilograms]
   -The number of kilograms of potatoes and green beans purchased per week cannot be negative.
2. **Ratio of Potatoes to Green Beans:** $\frac{PT}{GB} \geq \frac{6}{5} \equiv$ (PT*5) – (GB*6) ≥ 0 [kilograms]
   -The ratio of kilograms of potatoes to green beans must be at least 6/5.
3. **Amount of Protein:** (PT*15) + (GB*20) ≥ 180 [grams]
   -The total grams of protein contained in each meal is provided by potatoes and green beans and must be at least 180 grams.
4. **Amount of Iron:** (PT*0.03) + (GB*0.012) ≥ 0.080 [grams]
   -The total grams of iron contained in each meal is provided by potatoes and green beans and must be at least 0.080 grams.
5. **Amount of Vitamin C:** (PT*0.12) + (GB*0.10) ≥ 1.050 [grams]
   -The total grams of vitamin C contained in each meal is provided by potatoes and green beans must be at least 1.050 grams.
6. **Amount of total Mass:** PT + GB ≥ 10 [kilograms]
   -The total mass of potatoes and green beans purchased each week must be at least 10 kilograms.

The above data was entered in an Excel spreadsheet to facilitate manipulation of the model. A screenshot of the representation of the model in Excel model is provided in Appendix C.

Optimal Solution

Using the simplex method built-in to Excel's Solver add-on, we find that, in order to meet all nutritional, taste, and demand requirements at minimal cost, 6.15 kg of potatoes and 5.13 kg of green beans should be purchased weekly to prepare the dish, for a total weekly cost of $16.75.

Considering Variants Which May Further Reduce Costs

Per the request of the client, we also considered variants on the ratio of potatoes to green beans, minimal requirements for content in iron, and type of bean used in the dish. In the following section, we present the potential cost savings for various scenarios. Annotated screenshots of the model variants are provided in Appendix D.

Changing Taste Requirements

The optimal solution satisfying all original constraints yields a total of 11.28 kg of casserole per week. Given that green beans have higher protein and iron content than potatoes, changing the recipe to allow for a smaller ratio of potatoes to green beans could reduce the quantity of produce needed to meet nutritional requirements while still meeting minimal demand requirements. Indeed, by allowing the ratio of the weight of potatoes to the weight of green beans to be as low as 1 to 2, only 4.67 kg of potatoes and 5.50 kg of green beans, for a total yield of 10.17 kg of casserole, are needed to meet nutritional requirements. This change would result in a total savings of $0.49 per week.

Considering Other Sources of Iron

Alternatively, other elements included in the casserole, such as the onions and cream of mushroom soup, could be considered for their nutritional content. Assuming that other elements of the casserole contribute a minimum of 0.015 g of iron per 10 kg of casserole, we find that purchasing 7.17 kg of potatoes and 3.63 kg of green beans per week would suffice to meet all requirements, for a total cost of $14.32.

Accounting for a Discount on Green Beans

The cafeteria's wholesaler is currently offering green beans at half price. Considering the previous solution, the cost of purchasing produce would drop to $10.30, calculated as follows:

$$7.17\ kg\ potatoes\ \times 0.88\ {}^{\$}/_{kg\ potatoes} + 3.63\ kg\ green\ beans \times \frac{1}{2}\left(2.21\ {}^{\$}/_{kg\ green\ beans}\right) = \$10.30$$

However, we find further savings can be achieved while meeting all requirements by purchasing 5.68 kg of potatoes and 4.74 kg of green beans, for a total of $10.21.

Using Lima Beans Instead of Green Beans

All-State University cafeteria should also consider using lima beans instead of green beans in its casserole dish, assuming that they can be used interchangeably without affecting taste. While slightly more expensive than discounted green beans, lima beans contain 4 times the amount of protein and twice as much iron as the equivalent weight of green beans. Consequently, all nutritional and demand requirements could be met by using 8.75 kg of potatoes and 1.61 kg of lima beans per week, for a total minimal weekly cost of $9.83. This translates to a total weekly savings of $0.38 compared to using discounted green beans, or $4.49 compared to using full-priced green beans.

However, using lima beans rather than green beans at the suggested quantity of 1.61 kg in each casserole results in a potato-to-bean ratio in excess of 5 to 1. While this respects the cafeteria cook's recommended minimum ratio of 6 to 5 in the weight of potatoes to green beans, given the important difference between the cook's recommendation and our weekly purchase recommendations, the taste of the dish may be adversely affected by the change in the recipe. Therefore, we recommend that the cook be consulted before choosing to purchase 8.75 kg of potatoes and 1.61 kg of lima beans per week to prepare the casserole.

Meeting New Nutritional Requirements Proposed by the Student Task Force

A student task force is urging All-State University to adopt a new policy increasing the minimum required amount of iron to 120 mg per dish and decreasing the required amount of vitamin C to 500 mg per dish. To meet these new nutritional requirements, the minimal weekly cost of preparing the casserole will increase to $10.69, if the cafeteria chooses to use lima beans instead of green beans. Specifically, 5.71 kg of potatoes and 4.29 kg of lima beans will be required each week to prepare the dish.

Appendices

Appendix A. Cost Conversions

Cost of Potatoes and Green Beans from $/lb to $/kg:

$$Cost\ of\ potatoes\ in\ \frac{\$}{kg} = 0.40\ \frac{\$}{lb} * \frac{2.205\ lb}{1\ kg} = 0.88\ \frac{\$}{kg}$$

$$Cost\ of\ green\ beans\ in\ \frac{\$}{kg} = 1.00\ \frac{\$}{lb} * \frac{2.205\ lb}{1\ kg} = 2.21\ \frac{\$}{kg}$$

Appendix B. Nutritional Requirement Conversions

Nutritional Content Conversions for Potatoes:

$$grams\ of\ protein\ per\ kilogram\ of\ potatoes = \frac{1.5}{100}\ \frac{g\ protein}{g\ potatoes} * \frac{1000\ g}{1\ kg} = 15\ \frac{g\ protein}{kg\ potatoes}$$

$$grams\ of\ iron\ per\ kilogram\ of\ potatoes = \frac{0.3}{100}\ \frac{mg\ iron}{g\ potatoes} * \frac{1000\ g}{1\ kg} * \frac{1\ g}{1000\ mg} = 0.003\ \frac{g\ iron}{kg\ potatoes}$$

$$grams\ of\ vitamin\ C\ per\ kilogram\ of\ potatoes = \frac{12}{100}\ \frac{mg\ vitamin\ C}{g\ potatoes} * \frac{1000\ g}{1\ kg} * \frac{1\ g}{1000\ mg}$$

$$= 0.12\ \frac{g\ protein}{kg\ potatoes}$$

Nutritional Content Conversions for Green Beans:

$$grams\ of\ protein\ per\ kilogram\ of\ green\ beans = \frac{5.67}{10}\ \frac{g\ protein}{oz\ green\ beans} * \frac{35.273\ oz}{1\ kg}$$

$$= 20\ \frac{g\ protein}{kg\ green\ beans}$$

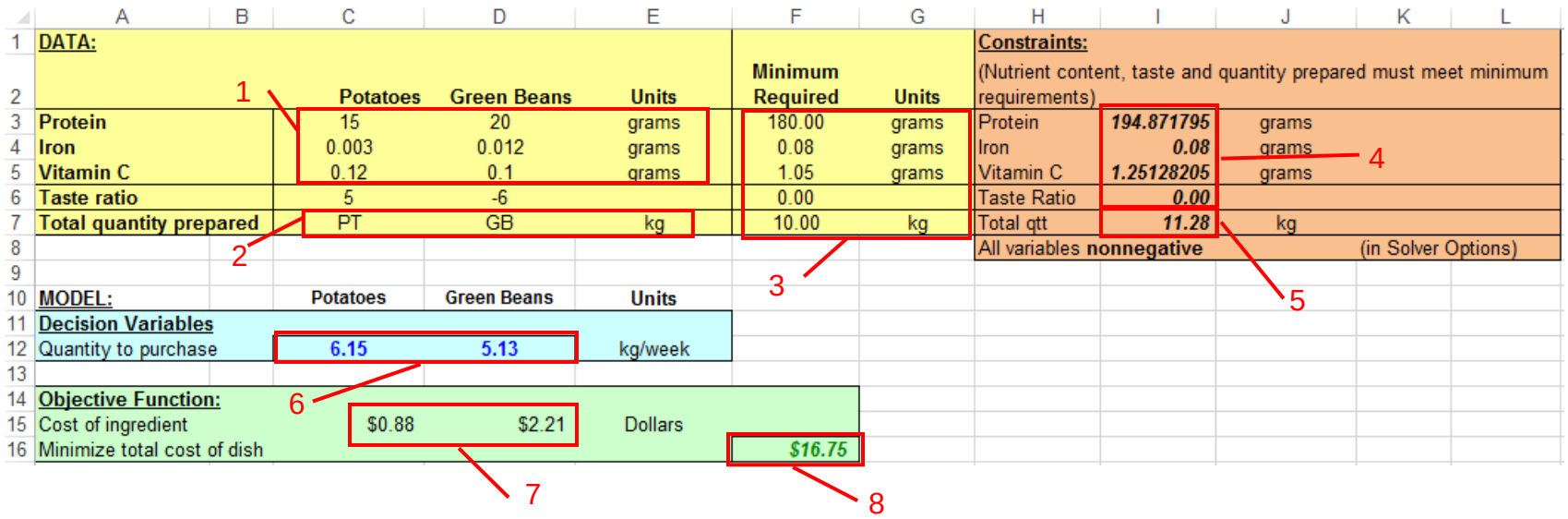
$$grams\ of\ iron\ per\ kilogram\ of\ green\ beans = \frac{3.402}{10}\ \frac{mg\ iron}{oz\ green\ beans} * \frac{35.273\ oz}{1\ kg} * \frac{1\ g}{1000\ mg}$$

$$= 0.012\ \frac{g\ iron}{kg\ green\ beans}$$

$$grams\ of\ vitamin\ C\ per\ kilogram\ of\ green\ beans = \frac{28.35}{10}\ \frac{mg\ vitmin\ C}{oz\ green\ beans} * \frac{35.273\ oz}{1\ kg} * \frac{1\ g}{1000\ mg}$$

$$= 0.100\ \frac{g\ vitamin\ C}{kg\ green\ beans}$$

Appendix C. Representation of Model in Excel (with optimal solution)



	A	B	C	D	E	F	G	H	I	J	K	L
1	**DATA:**							**Constraints:**				
2			**Potatoes**	**Green Beans**	**Units**	**Minimum Required**	**Units**	(Nutrient content, taste and quantity prepared must meet minimum requirements)				
3	**Protein**		15	20	grams	180.00	grams	Protein	***194.871795***	grams		
4	**Iron**		0.003	0.012	grams	0.08	grams	Iron	***0.08***	grams		
5	**Vitamin C**		0.12	0.1	grams	1.05	grams	Vitamin C	***1.25128205***	grams		
6	**Taste ratio**		5	-6		0.00		Taste Ratio	***0.00***			
7	**Total quantity prepared**		PT	GB	kg	10.00	kg	Total qtt	***11.28***	kg		
8								All variables **nonnegative**				(in Solver Options)
9												
10	**MODEL:**		**Potatoes**	**Green Beans**	**Units**							
11	**Decision Variables**											
12	Quantity to purchase		6.15	5.13	kg/week							
13												
14	**Objective Function:**											
15	Cost of ingredient		$0.88	$2.21	Dollars							
16	Minimize total cost of dish					***$16.75***						

1. Nutritional content of potatoes and green beans
2. Refers to the quantities purchased (see 6 below)
3. Nutritional, taste and demand requirements
4. SUMPRODUCT function, sum of the products of the nutritional content (or taste coefficient) by quantity of each ingredient for total nutrient content (or taste ratio)
5. SUM of quantities of potatoes and green beans purchased (see 6 below)
6. Quantity of potatoes and green beans to be purchased weekly (values generated by the Solver)
7. Cost of each ingredient
8. SUMPRODUCT function: sum of the products of the ingredient quantities by the price of ingredients for total cost of preparing the dish

Appendix D. Variants on the original model

Changing Taste Requirements

	A	B	C	D	E	F	G	H	I	J	K	L
1	**DATA:**							**Constraints:**				
2			**Potatoes**	**Green Beans**	**Units**	**Minimum Required**	**Units**	(Nutrient content, taste and quantity prepared must meet minimum requirements)				
3	**Protein**		15	20	grams	180.00	grams	Protein	***180***	grams		
4	**Iron**		0.003	0.012	grams	0.08	grams	Iron	***0.08***	grams		
5	**Vitamin C**		0.12	0.1	grams	1.05	grams	Vitamin C	***1.11***	grams		
6	**Taste ratio**		2	-1		0.00		Taste Ratio	***3.83***			
7	**Total quantity prepared**		PT	GB	kg	10.00	kg	Total qtt	***10.17***	kg		
8								All variables **nonnegative**				(in Solver Options)
9												
10	**MODEL:**		**Potatoes**	**Green Beans**	**Units**							
11	**Decision Variables**											
12	Quantity to purchase		4.67	5.50	kg/week							
13												
14	**Objective Function:**											
15	Cost of ingredient		$0.88	$2.21	Dollars							
16	Minimize total cost of dish					***$16.26***						

New ratio of potatoes to green beans:

$$\frac{PT}{GB} \geq \frac{1}{2} \equiv (PT*2) - (GB*1) \geq 0$$

Considering Other Sources of Iron

	A	B	C	D	E	F	G	H	I	J	K	L
1	**DATA:**							**Constraints:**				
2			**Potatoes**	**Green Beans**	**Units**	**Minimum Required**	**Units**	(Nutrient content, taste and quantity prepared must meet minimum requirements)				
3	**Protein**		15	20	grams	180.00	grams	Protein	***180***	grams		
4	**Iron**		0.003	0.012	grams	0.065	grams	Iron	***0.065***	grams		
5	**Vitamin C**		0.12	0.1	grams	1.05	grams	Vitamin C	***1.2225***	grams		
6	**Taste ratio**		5	-6		0.00		Taste Ratio	***14.08***			
7	**Total quantity prepared**		PT	GB	kg	10.00	kg	Total qtt	***10.79***	kg		
8								All variables **nonnegative**				(in Solver Options)
9												
10	**MODEL:**		**Potatoes**	**Green Beans**	**Units**							
11	**Decision Variables**											
12	Quantity to purchase		7.17	3.63	kg/week							
13												
14	**Objective Function:**											
15	Cost of ingredient		$0.88	$2.21	Dollars							
16	Minimize total cost of dish					***$14.32***						

New minimum requirement for iron from potatoes and green beans:

$$65\ mg\ iron \times \frac{1000\ \text{mg}}{1\ g} = 0.065\ g\ iron$$

Accounting for a Discount on Green Beans

Maintain lowered iron requirements

	A	B	C	D	E	F	G	H	I	J	K	L
1	**DATA:**							**Constraints:**				
2			**Potatoes**	**Green Beans**	**Units**	**Minimum Required**	**Units**	(Nutrient content, taste and quantity prepared must meet minimum requirements)				
3	**Protein**		15	20	grams	180.00	grams	Protein	***180***	grams		
4	**Iron**		0.003	0.012	grams	0.065	grams	Iron	***0.07389474***	grams		
5	**Vitamin C**		0.12	0.1	grams	1.05	grams	Vitamin C	***1.15578947***	grams		
6	**Taste ratio**		5	-6		0.00		Taste Ratio	***0.00***			
7	**Total quantity prepared**		PT	GB	kg	10.00	kg	Total qtt	***10.42***	kg		
8								All variables **nonnegative**				(in Solver Options)
9												
10	**MODEL:**		**Potatoes**	**Green Beans**	**Units**							
11	**Decision Variables**											
12	Quantity to purchase		5.68	4.74	kg/week							
13												
14	**Objective Function:**											
15	Cost of ingredient		$0.88	$1.10	Dollars							
16	Minimize total cost of dish					***$10.21***						

New discounted price of green beans:

$$0.50\ \frac{\$}{lb} \times \frac{2.205\ lb}{1\ kg} = 1.10\ \frac{\$}{kg}$$

Using Lima Beans Instead of Green Beans

Nutritional content of lima beans:

Protein: $\frac{22.68}{10}\frac{g\ protein}{oz\ lima\ beans} \times \frac{35.273\ oz}{1\ kg} = 80\ \frac{g\ protein}{kg\ lima\ beans}$

Iron: $\frac{6.804}{10}\frac{mg\ iron}{oz\ lima\ beans} \times \frac{35.273\ oz}{1\ kg} \times \frac{1\ g}{1000\ mg} = 0.024\ \frac{g\ iron}{kg\ lima\ beans}$

Vitamin C: $\frac{0}{10}\frac{mg\ vitmin\ C}{oz\ lima\ beans} \times \frac{35.273\ oz}{1\ kg} \times \frac{1\ g}{1000\ mg} = 0\ \frac{g\ vitamin\ c}{kg\ lima\ beans}$

Maintain lowered iron requirements

	A	B	C	D	E	F	G	H	I	J	K	L
1	DATA:							Constraints:				
2			Potatoes	Lima Beans	Units	Minimum Required	Units	(Nutrient content, taste and quantity prepared must meet minimum requirements)				
3	Protein		15	80	grams	180.00	grams	Protein	260.416667	grams		
4	Iron		0.003	0.024	grams	0.065	grams	Iron	0.065	grams		
5	Vitamin C		0.12	0	grams	1.05	grams	Vitamin C	1.05	grams		
6	Taste ratio		5	-6		0.00		Taste Ratio	34.06			
7	Total quantity prepared		PT	GB	kg	10.00	kg	Total qtt	10.36	kg		
8								All variables nonnegative			(in Solver Options)	
9												
10	MODEL:		Potatoes	Lima Beans	Units							
11	Decision Variables											
12	Quantity to purchase		8.75	1.61	kg/week							
13												
14	Objective Function:											
15	Cost of ingredient		$0.88	$1.32	Dollars							
16	Minimize total cost of dish					$9.83						

Price of lima beans:

$0.60\ \frac{\$}{lb} \times \frac{2.205\ lb}{1\ kg} = 1.32\ \frac{\$}{kg}$

Meeting New Nutritional Requirements Proposed by the Student Task Force

	A	B	C	D	E	F	G	H	I	J	K	L
1	DATA:							Constraints:				
2			Potatoes	Lima Beans	Units	Minimum Required	Units	(Nutrient content, taste and quantity prepared must meet minimum requirements)				
3	Protein		15	80	grams	180.00	grams	Protein	428.571429	grams		
4	Iron		0.003	0.024	grams	0.120	grams	Iron	0.12	grams		
5	Vitamin C		0.12	0	grams	0.500	grams	Vitamin C	0.68571429	grams		
6	Taste ratio		5	-6		0.00		Taste Ratio	2.86			
7	Total quantity prepared		PT	GB	kg	10.00	kg	Total qtt	10.00	kg		
8								All variables nonnegative			(in Solver Options)	
9												
10	MODEL:		Potatoes	Lima Beans	Units							
11	Decision Variables											
12	Quantity to purchase		5.71	4.29	kg/week							
13												
14	Objective Function:											
15	Cost of ingredient		$0.88	$1.32	Dollars							
16	Minimize total cost of dish					$10.69						

Maintain use of lima beans

New nutritional requirements:

$120\ mg\ iron\ \times \frac{1000\ \text{mg}}{1\ g} = 0.120\ g\ iron$

$500\ mg\ vitamin\ C\ \times \frac{1000\ \text{mg}}{1\ g} = 0.500\ g\ vitmin\ C$