Homework Assignment 2

Due Wednesday September 21, 9:00 am

1. Do the following **Exercise**. (Please refer to the instructions on Homework Assignments in the course outline.) The Concrete Blocks Corporation (CBC) can produce 4 types of concrete blocks, B1 to B4. Each block is subjected to several processes, of which the following 3 have limited capacity: batch mixing, mold vibrating, and inspection. The table below indicates the number of hours of each process required per pallet of blocks of each type; the number of hours available next month for each process; and the unit profit per pallet for of each type of blocks. The plant manager desires to maximize profits during the next month.

	Process time (hours per pallet)				Hours available
Block type:	B1	B2	B3	B4	next month
Batch mixing	1	2	10	16	800
Mold vibrating	1.5	2	4	5	1,000
Inspection	0.5	0.6	1	2	340
Profit ($/batch)	8	14	30	50	

(a) Formulate as a linear programming problem.

(b) Solve using the computer.

Answer the following questions using, as much as possible, sensitivity analysis. When sensitivity analysis only gives you lower and/or upper bounds for a numerical answer, obtain an exact answer by solving an appropriately modified LP model. Note that the changes considered in each question below are non-cumulative, i.e., independent: each change only applies to the question where it is considered.

(c) By how much must the profit per pallet on Type 3 blocks be increased before it would be profitable to produce them?

(d) What minimum profit per pallet on Type 2 blocks must be realized so that they remain in an optimum production schedule?

(e) If the 800 machine-hours capacity on the batch mixer is uncertain, for what range of machine-hours will it remain optimal to produce Type 1 and Type 2 blocks?

(f) A competitor located next door has offered CBC additional batch mixing time at a rate of $4.00 per hour. Should CBC accept this offer? If yes, for how many hours, and what will be the net impact on total profit? If no, what is the maximum price CBC should be willing to accept for this additional time?

(g) CBC is considering the production of a new type of block that would require 4 hours of batch mixing, 4 hours of mold vibrating, and 1 hour of inspection per pallet. What profit per pallet is required so it is optimal to produce this new block type next month?

2. Write a **Case Report** for the *Farm Management* Case (Case 6.2 in the Hillier & Lieberman text; copy of this case below.) Include brief answers to all Questions (a) to (k) in the Executive Summary, which for this Case may be up to 2 pages.

Please note:
1) Livestock can be purchased at beginning of the year, but cannot be sold.
2) The given labor requirements are person-hours per month for cows and hens (top paragraph of next page); and per half-year for each crop (first Table).
3) For simplicity, assume that crops used for livestock (corn for cows, and wheat for hens) contribute to crop value.

CASE 6.2 FARM MANAGEMENT

The Ploughman family owns and operates a 640-acre farm that has been in the family for several generations. The Ploughmans always have had to work hard to make a decent living from the farm and have had to endure some occasional difficult years. Stories about earlier generations overcoming hardships due to droughts, floods, etc., are an important part of the family history. However, the Ploughmans enjoy their self-reliant lifestyle and gain considerable satisfaction from continuing the family tradition of successfully living off the land during an era when many family farms are being abandoned or taken over by large agricultural corporations.

John Ploughman is the current manager of the farm while his wife Eunice runs the house and manages the farm's finances. John's father, Grandpa Ploughman, lives with them and still puts in many hours working on the farm. John and Eunice's older children. Frank, Phyllis, and Carl, also are given heavy chores before and after school.

The entire family can produce a total of 4,000 person-hours worth of labor during the winter and spring months and 4,500 person-hours during the summer and fall. If any of these person-hours are not needed, Frank, Phyllis, and Carl will use them to work on a neighboring farm for $5 per hour during the winter and spring months and $5.50 per hour during the summer and fall.

The farm supports two types of livestock: dairy cows and laying hens, as well as three crops: soybeans, corn, and wheat. (All three are cash crops, but the corn also is a feed crop for the cows and the wheat also is used for chicken feed.) The crops are harvested during the late summer and fall. During the winter months, John, Eunice, and Grandpa make a decision about the mix of livestock and crops for the coming year.

Currently, the family has just completed a particularly successful harvest which has provided an investment fund of $20,000 that can be used to purchase more live- stock. (Other money is available for ongoing expenses, including the next planting of crops.) The family currently has 30 cows valued at $35,000 and 2,000 hens valued at $5,000. They wish to keep all this livestock and perhaps purchase more. Each new cow would cost $1,500, and each new hen would cost $3.

Over a year's time, the value of a herd of cows will decrease by about 10 percent and the value of a flock of hens will decrease by about 25 percent due to aging.

Each cow will require 2 acres of land for grazing and 10 person-hours of work per month, while producing a net annual cash income of $850 for the family. The corre- sponding figures for each hen are: no significant acreage, 0.05 person-hour per month, and an annual net cash income of $4.25. The chicken house can accommodate a max-imum of 5,000 hens, and the size of the barn limits the herd to a maximum of 42 cows.

For each acre planted in each of the three crops, the following table gives the number of person-hours of work that will be required during the first and second halves of the year, as well as rough estimate of the crop's net value (in either income or sav-ings in purchasing feed for the livestock).

Data per acre planted

	Soybeans	Corn	Wheat
Winter and spring, person-hours	1.0	0.9	0.6
Summer and fall, person-hours	1.4	1.2	0.7
Net value	$70	$60	$40

To provide much of the feed for the livestock, John wants to plant at least 1 acre of corn for each cow in the coming year's herd and at least 0.05 acre of wheat for each hen in the coming year's flock.

John, Eunice and Grandpa now are discussing how much acreage should be planted in each of the crops and how many cows and hens to have for the coming year. Their objective is to maximize the family's monetary worth at the end of the coming year (the *sum* of the net income from the livestock for the coming year, *plus* the net value of the crops for the coming year, *plus* what remains from the investment fund, *plus* the value of the livestock at the end of the coming year, *plus* any income from working on a neighboring farm, *minus* living expenses of $40,000 for the year).

(a) Identify verbally the components of a linear programming model for this problem.

(b) Formulate this model. (Either an algebraic or a spreadsheet formulation is acceptable.)

(c) Obtain an optimal solution and generate the additional output provided for performing postoptimality analysis (e.g., the Sensitivity Report when using Excel). What does the model predict regarding the family's monetary worth at the end of the coming year?

(d) Find the allowable range to stay optimal for the net value, per acre planted for each of the three crops.

The above estimates of the net value per acre planted in each of the three crops assumes good weather conditions. Adverse weather conditions would harm the crops and greatly reduce the resulting value. The scenarios particularly feared by the family are a

drought, a flood, an early frost, *both* a drought and an early frost, and *both* a flood and an early frost. The estimated net values for the year under these scenarios are shown on the next page.

(e) Find an optimal solution under each scenario after making the necessary adjustments to the linear programming model formulated in part (*b*). In each case, what is the prediction regarding the family's monetary worth at the end of the year?

(f) For the optimal solution obtained under each of the six scenarios [including the good weather scenario considered in parts (a) to (d)], calculate what the family's monetary worth would be at the end of the year if each of the other five scenarios occur instead. In your judgment, which solution provides the best balance between yielding a large monetary worth under good weather conditions and avoiding an overly small monetary worth under adverse weather conditions?

	Net Value per Acre Planted		
Scenario	**Soybeans**	**Corn**	**Wheat**
Drought	– $10	– $15	0
Flood	$15	$20	$10
Early frost	$50	$40	$30
Drought and early frost	– $15	– $20	– $10
Flood and early frost	$10	$10	$ 5

Grandpa has researched what the weather conditions were in past years as far back as weather records have been kept, and obtained the following data.

Scenario	**Frequency**
Good weather	40%
Drought	20%
Flood	10%
Early frost	15%
Drought and early frost	10%
Flood and early frost	5%

With these data, the family has decided to use the following approach to making its planting and livestock decisions. Rather than the optimistic approach of assuming that good weather conditions will prevail [as done in parts (*a*) to (*d*)], the *average* net value under all weather conditions will be used for each crop (weighting the net values under the various scenarios by the frequencies in the above table).

(g) Modify the linear programming model formulated in part (*b*) to fit this new approach.

(h) Repeat part (c) for this modified model.

(i) Use a shadow price obtained in part (*h*) to analyze whether it would be worthwhile for the family to obtain a bank loan with a 10 percent interest rate to purchase more livestock now beyond what can be obtained with the $20,000 from the investment fund.

(j) For each of the three crops, use the postoptimality analysis information obtained in part (*h*) to identify how much latitude for error is available in estimating the net value per acre planted for that crop without changing the optimal solution. Which two net values need to be estimated most carefully? If both estimates are incorrect simultaneously, how close do the estimates need to be to guarantee that the optimal solution will not change?

This problem illustrates a kind of situation that is frequently faced by various kinds of organizations. To describe the situation in general terms, an organization faces an uncertain future, where any one of a number of scenarios may unfold. Which one will occur depends on conditions that are outside the control of the organization. The organization needs to choose the level of various activities, but the unit contribution of each activity to the overall measure of performance is greatly affected by which scenario unfolds. Under these circumstances, what is the best mix of activities?

(k) Think about specific situations outside of farm management that fit this description. Describe one.