Homework Assignment 3

Due Wednesday November 16, 9:00 am

Do all problems below. *Please refer to the instructions for Homework in the Course Outline. A spreadsheet containing the data for this homework set is posted on the course web site. All integer programming models in this homework must have linear constraints and objective. Make sure to produce and explain all relevant computer printouts. Interrupt a computer solution run if it exceeds 300 seconds.*

1. Solving a Cutting Stock instance. Consider the instance of the Cutting Stock problem, to satisfy the following demands for small rolls using the smallest possible number of big rolls, where each big roll has width $W = 240$ cm.

Small roll type:	1	2	3	4	5	6
Width (cm)	29	43	47	50	26	23
Number required (rolls)	81	56	96	94	118	121

(a) Define the initial restriction for the Gilmore & Gomory approach, using only patterns each consisting of as many small rolls as possible of a single type. Solve its LP relaxation. Does the optimum objective value of this LP relaxation give, in general, a lower bound or an upper bound on the optimum value of the integer program? Explain.

(b) Solve using the Restriction & Pricing (aka, Column Generation) method, starting from this initial restriction. Detail and explain your calculations.

2. Product mix with learning. The Acme Company is planning the production of its two products, the popular Widget and the new SuperWidget soon to be introduced to the market. Each Widget sells for \$40 and requires \$7 for materials. Each SuperWidget will require \$8 of materials and, during the planning period, will be sold at the promotional price of \$30. Widgets and SuperWidgets each go through two machining operations. For the planning period, machine 1 is limited to a total of 320 hours and machine 2 to a total of 240 hours. Labour costs \$5 per hour on each machine. Each Widget takes four hours on machine 1, and two hours on machine 2. Producing a total of x SuperWidgets during the planning period will take a total of $t(x)$ hours on machine 1, and the same amount $t(x)$ on machine 2. The function $t(.)$ is a continuous, increasing function of the cumulative production x and, because of learning effects, it grows less than linearly. Namely, the larger the accumulated production x of SuperWidgets, and the more experienced the workers will be in machining the new product. Thus $t(.)$ is concave and increasing. Based on past experience with similar products, the following time estimates are available:

x (SuperWidgets)	0	30	60	120	240
$t(x)$ (hours)	0	60	96	153.6	245.76

Assume that any quantity of both products that Acme can produce will be sold during the planning period at the specified prices. The (short-sighted) goal of the Acme Company is to produce the combination of Widgets and SuperWidgets that will maximize its total contribution margin (revenues from sales, minus materials and labour costs) for the planning period. Using an appropriate (concave and increasing) approximation for $t(.)$, formulate as a mixed integer linear programming problem. (Note: there is no need to constrain the quantities of Widgets and SuperWidgets to be integer, a solution with fractional values is acceptable; i.e., you may use continuous variables for these.) Solve using the computer.

3. Selecting the Dream Team. Write a Case Report for the *Selecting the Dream Team* case below. Please follow the instructions below in presenting your Case Report. Include brief answers to Questions (c), (d) and (e) in your one-page Executive Summary.

SELECTING THE DREAM TEAM

The coach of the national basketball team is faced with the decision of selecting a "Dream Team" of 12 players for the upcoming international tournament. He has limited his final selection to a "short list" of 20 players, $P_1,\ldots, P_{20}$. For each player P_i the coach has collected several statistics that can be summarized as follows: his rebounding average r_i, his assist average a_i, his height h_i, his scoring average s_i, and his overall defense ability d_i. These 20 players have been divided into four categories: play makers (PM) $P_1,\ldots, P_5$; shooting guards (SG) $P_4,\ldots, P_{11}$; forwards (F) $P_9,\ldots, P_{16}$; and centers (C) $P_{16},\ldots, P_{20}$. Notice that some players can be used in multiple roles (for example, player P_4 can be used both as a play maker and as a shooting guard). Players P_4, P_8, P_{15} and P_{20} play in the NCAA (college level), while the other 16 players play in the NBA (professional level). For balance purposes, the Dream Team should consist of at least 3 play makers, 4 shooting guards, 4 forwards, and 3 centers; this implies that some players with dual roles should be selected. In addition, at least two players from the NCAA should be selected, while the mean rebound, assist, height, and defense abilities of the Dream Team should be at least $\underline{r}$, $\underline{a}$, $\underline{h}$, and $\underline{d}$, respectively. The problem is further complicated by the fact that there are incompatibility problems among some players. Player P_5 has declared that if player P_9 is selected, then he does not want to be in the Dream Team. Also, players P_2 and P_9 can only be selected together, as they have been playing in the same team for years and feel that they are much more effective together. Finally, at most three players should be selected from any professional team, so that the coach is not accused of favoritism; players P_1, P_7, P_{12} and P_{16} play for one team, while players P_2, P_3, P_9 and P_{19} play for another team. Faced with these difficulties, the coach has decided he would like to maximize the mean scoring of the Dream Team, while satisfying all above constraints (if the latter is possible).

(a) Formulate an integer programming model for the coach's problem.

After some careful thought, the coach would also like to decide how much time to give to each selected player, as some of the players in the initial list of 20, although extremely talented, were returning from long injuries or were aging. For such reasons, the coach has also set an upper bound u_i on the average number of minutes per game each player P_i can play in the Dream Team. In international tournaments, the duration of a game is 40 minutes. The coach has decided to use two team compositions in the tournament, depending on the type of opponent and circumstances of a game: (PM, SG, SG, F, C) and (PM, SG, F, F, C). Looking at the schedule, he expects to use each of these two schemes about equally in the tournament. To keep things simple, the coach decides to ignore game-to-game variability, and just to plan for an "average game" in which the total play time will therefore be 40 minutes for the play makers; 60 minutes for the shooting guards; 60 minutes for the forwards; and 40 minutes for the centers. In addition, the mean scoring, rebound, assist, height, and defense abilities of the Dream Team should now be computed as *playtime-weighted averages*. The other constraints from question (a) (i.e., those that do not concern these team mean abilities) still refer to the *composition* of the team (i.e., who is selected), irrespective of the planned playtime allocations.

(b) Formulate an integer programming model for the coach's problem of selecting the players and their planned playtime allocations, in order to maximize the playtime-weighted scoring average of the Dream Team. Please make sure to clearly explain and justify any additional assumption you feel is needed in order to formulate such a model.

The coach has finally obtained the following player statistics:

Player *i*	Rebound r_i	Assist a_i	Height h_i	Scoring s_i	Defense d_i	Maximum playtime u_i
1	8	8	1.92	3	7	35
2	6	7	1.87	6	6	40
3	4	7	1.84	8	4	40
4	8	9	1.91	7	9	40
5	9	6	2.02	8	10	25
6	8	6	1.93	6	9	35
7	7	8	1.86	9	8	40
8	7	5	1.83	9	8	40
9	6	7	1.98	6	9	40
10	10	7	2.05	2	9	20
11	9	10	2.00	5	10	35
12	9	5	2.01	6	5	20
13	7	7	1.94	8	6	35
14	4	9	1.89	10	4	40
15	3	7	1.87	9	2	40
16	7	8	2.03	4	6	30
17	6	7	1.88	7	5	40
18	4	7	1.93	5	4	35
19	3	8	1.95	6	3	40
20	6	9	1.99	8	4	30

The coach sets the minimum value for the average team height at 1.92 m and all other minimum values $\underline{r}$, $\underline{a}$, and $\underline{d}$, at 7.

(c) Solve your model in (a) using these data. Compare with the LP solution: what do you observe?

(d) Now repeat question (c) for your model in (b).

(e) Should the coach be satisfied with the solution obtained in (d)? Why or why not? What would you suggest in order to obtain a more sensible solution? (Do *not* implement your suggestions.)