Case Report: Selecting the Dream Team

Prepared for

Coach
National Basketball Team

Date: November 16, 2016

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1 Executive Summary

The coach of the national basketball team is selecting a "Dream Team" consisting of twelve players who will compete in the upcoming international tournament. After careful consideration, he has narrowed down his choices to twenty people based on their summary statistics. In particular, some players can play in more than one of the four traditional basketball player roles, there are incompatibilities amongst players, and players from the NBA and NCAA can be selected on the same team. Our services were requested to formulate an integer programming model for the coach's problem to select the "Dream Team" which fulfills all of his criteria while maximizing the team's mean score.

This report discusses the binary integer programming models we formulated under various scenarios and our recommendations for the coach to optimize their "Dream Team" selection strategy. We first considered how to meet all of the coach's identified constraints to select 12 players from his top 20 picks. Next, we reformulated our integer-programming model to assist the coach in determining how much time to give each selected player if two team schemes would be applied in the international tournament. Moreover, we then solved our integer programming models.

The key objective here for the consultation was to maximize the "Dream Team's" average score under various conditions. Given the aforementioned goals, we determined players {3,4,5,6,7,8,11,12,13,16,17,20} should be chosen without considering time-weighted averages (See Table 2 Section 3.2 of report). In particular, the average score of the "Dream Team" is 7.083 per game.

Next, we maximized the "Dream Team's" average score using time-weighted averages. Given the aforementioned goals, we determined players {1,3,5,7,8,11,13,14,15,17,18,20} should be used considering time-weighted averages (See Table 3, Section 3.4 of report). In particular, the average score of the "Dream Team" is 43.75.

The LP Relaxations of both formulations yield an upper bound on the maximum average score. However, it is not particularly useful to the coach when it comes to making team selections as LP selections differ substantially from the IP optimal selections, with no generalizable rounding convention relating one solution to the other.

Our analysis was conducted while meeting the following requirements:

1. **Play Makers:** The total number of play makers must be at least 3.
2. **Shooting Guards:** The total number of shooting guards must be at least 4.
3. **Forwards:** The total number of forwards must be at least 4.
4. **Centers:** The total number of centers must be at least 3.
5. **NCAA:** At least 2 NCAA players must be selected for the "Dream Team."
6. **P5 and P9:** At most one of player P5 and P9 are selected as they cannot play together.
7. **P2 and P9:** Both players P2 and P9 are selected together or not at all.
8. **NBA:** At most, 3 players from the same NBA team can be selected together.
9. **Mean Rebound Value:** The mean rebound average of the "Dream Team" should be at least 7.
10. **Mean Assist Value:** The mean assist average of the "Dream Team" should be at least 7.
11. **Mean Height Value:** The mean height average of the "Dream Team" should be at least 1.92m.
12. **Mean Defence Value:** The mean defence average of the "Dream Team" should be at least 7.
13. **12 Players Total:** The total number of players on the "Dream Team" is 12.
14. **Binary Constraint:** The values of P_i are binary.

Furthermore, we made the following assumptions to conduct our analysis:

1. Some players are capable of playing multiple positions.
2. The coach has selected to ignore game-to-game variability and just plan for the "average game."
3. Players that are selected for the Dream Team will not necessarily be allocated playtime
4. Players can only occupy one position type at a time (i.e. cannot play forward and center simultaneously)

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2 Introduction

The coach of the national basketball team is selecting a "Dream Team" consisting of twelve players who will compete in the upcoming international tournament. After careful consideration, he has narrowed down his choices to twenty people based on their summary statistics. In particular, some players can play in more than one of the four traditional basketball player roles, there are incompatibilities amongst players, and players from the NBA and NCAA can be selected on the same team. Our services were requested to formulate an integer programming model for the coach's problem to select the "Dream Team" which fulfills all of his criteria while maximizing the team's mean score.

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1. Some players are capable of playing multiple positions.
2. The coach has selected to ignore game-to-game variability and just plan for the "average game."

2.1 Information Provided

Player Statistics

The coach as narrowed down his selection to 20 players, which we have denoted as P1, …, P20 . For each player Pi the coach has collected several statistics that can be summarized in Table 1. In particular, each player's rebounding average r_i, assist average a_i, height h_i, scoring average s_i, and overall defense ability d_i can be found in Table 1.

TABLE 1: Player statistics.

Player i	Rebound (r_i)	Assist (a_i)	Height (h_i)	Scoring (s_i)	Defence (s_i)	Maximum Playtime (u_i)
1	8	8	1.92	3	7	35
2	6	7	1.87	6	6	40
3	4	7	1.84	8	4	40
4	8	9	1.91	7	9	40
5	9	6	2.02	8	10	25
6	8	6	1.93	6	9	35
7	7	8	1.86	9	8	40
8	7	5	1.83	9	8	40
9	6	7	1.98	6	9	40
10	10	7	2.05	2	9	20
11	9	10	2.00	5	10	35
12	9	5	2.01	6	5	20
13	7	7	1.94	8	6	35
14	4	9	1.89	10	4	40
15	3	7	1.87	9	2	40
16	7	8	2.03	4	6	30
17	6	7	1.88	7	5	40
18	4	7	1.93	5	4	35
19	3	8	1.95	6	3	40
20	6	9	1.99	8	4	30

4 Player Categories

The twenty players can take on four roles: (1) play makers (PM) P1 ,…, P5; (2) shooting guards (SG) P4 ,…, P11; (3) forwards (F) P9 ,…, P16; (4) and centers (C) P16 ,…, P20. In particular, some players can play multiple roles.

College Vs. Professional Level

Four players play in the NCAA (i.e. college level) where as the remaining 16 players play in the NBA (i.e. professional level). For instance, players P4, P8 , P15 and P20 play in the NCAA. For balance purposes, the “Dream Team” should consist of at least 3 PMs, 4 SGs, 4 Fs, and 3 Cs; this implies that some players with dual roles should be selected. In addition, at least two players from the NCAA should be selected such that the mean rebound ($\bar{r}$), assist ($\bar{a}$), height ($\bar{h}$), and defense ($\bar{d}$) abilities of the “Dream Team” should be at least 7, 7, 1.92 m, and 7, respectively.

Player Incompatibilities

Unfortunately, some players have team-mate preferences. First, player P5 will not join the “Dream Team” if player P9 is chosen. Also, players P2 and P9 can only be selected together, as they have been playing in the same team for years and feel that they are much more effective together. Finally, the coach wants to avoid being accused of favoritism; at most three players should be selected from any professional team.

3 Analysis

3.1 Binary Integer Programming Model [Question (a)]

Considering the aforementioned considerations in the Introduction of this report, we formulated the following binary integer programming model to maximize the mean scoring of the "Dream Team."

Constants: Based on Table 1, the summary statistics will be referred to as follows:

Rebound (r_i) where i ϵ 1, 2, … 20

Assist (a_i) where i ϵ 1, 2, … 20

Height (h_i) where i ϵ 1, 2, … 20

Scoring (s_i) where i ϵ 1, 2, … 20

Defence (s_i) where i ϵ 1, 2, … 20

Maximum Playtime (u_i) where i ϵ 1, 2, … 20

Binary Decision Variables: Whether or not each of the 20 players (i.e. whose statistics are detailed in Table 1), P_i where i = 1, 2… 20, are selected for the "Dream Team" by letting P_i equal to:

1	denote the player is selected.	[none]
0	denote the player is not selected.	[none]

Constraints: The following constraints define the binary integer programming model to find the optimal solution:

1. **Play Makers:** The total number of play makers must be at least 3.
   $\sum_{i=1}^{5} P_i \geq 3$ [players]

2. **Shooting Guards:** The total number of shooting guards must be at least 4.
   $\sum_{i=4}^{11} P_i \geq 4$ [players]

3. **Forwards:** The total number of forwards must be at least 4.
   $\sum_{i=9}^{16} P_i \geq 4$ [players]

4. **Centers:** The total number of centers must be at least 3.
   $\sum_{i=16}^{20} P_i \geq 3$ [players]

5. **NCAA:** At least 2 NCAA players must be selected for the "Dream Team."
   $P_5 + P_9 = 1$ [players]

6. **P5 and P9:** At most one of player P5 and P9 are selected as they cannot play together.
   $P_5 + P_9 = 1$ [players]

7. **P2 and P9:** Both players P2 and P9 are selected together or not at all.
   $P_2 - P_9 = 0$ [players]

8. **NBA:** At most, 3 players from the same NBA team can be selected together.
   $P_1 + P_7 + P_{12} + P_{16} \geq 3$ [players]
   $P_2 + P_3 + P_9 + P_{19} \geq 3$ [players]

9. **Mean Rebound Value:** The mean rebound average of the "Dream Team" should be at least 7.
   $\frac{1}{12}\sum_{i=1}^{12} P_i * r_i \geq 7$ [rebound score]

10. **Mean Assist Value:** The mean assist average of the "Dream Team" should be at least 7.
    $\frac{1}{12}\sum_{i=1}^{12} P_i * a_i \geq 7$ [assist score]

11. **Mean Height Value:** The mean height average of the "Dream Team" should be at least 1.92m.
    $\frac{1}{12}\sum_{i=1}^{12} P_i * h_i \geq 1.92$ [m]

12. **Mean Defence Value:** The mean defence average of the "Dream Team" should be at least 7.
    $\frac{1}{12}\sum_{i=1}^{12} P_i * d_i \geq 7$ [defence score]

13. **12 Players Total:** The total number of players on the "Dream Team" is 12.
    $\sum_{i=1}^{12} P_i = 12$ [players]

14. **Binary Constraint:** The values of P_i are binary.

<u>Objective Function</u>:

The objective is to maximize the "Dream Teams" maximum mean score while fulfilling all of the aforementioned constraints. Therefore, we seek to:

max Z = $\{ \frac{1}{12}\sum_{i=1}^{12} P_i * s_i \}$ [score]

3.2 Optimal Solution of Original Formulation

Question (c): Optimal Solution to Maximize the Mean "Dream Team's" Score from Part (a).

Using the Gurobi Solver built-in to AMPL, we find that all aforementioned requirements can be met to select players P3, P4, P5, P6, P7, P8, P11, P12, P13, P16, P17 & P20 for an average "Dream Team" score of 7.083333. This can be achieved by The constructed binary integer programming model can be found as an AMPL output is found in Figure 1 in Appendix A.

TABLE 2: List of Players.

Player i	Rebound (r_i)	Assist (a_i)	Height (h_i)	Scoring (s_i)	Defence (s_i)	Maximum Playtime (u_i)
3	4	7	1.84	8	4	40
4	8	9	1.91	7	9	40
5	9	6	2.02	8	10	25
6	8	6	1.93	6	9	35
7	7	8	1.86	9	8	40
8	7	5	1.83	9	8	40
11	9	10	2.00	5	10	35
12	9	5	2.01	6	5	20
13	7	7	1.94	8	6	35
16	7	8	2.03	4	6	30
17	6	7	1.88	7	5	40
20	6	9	1.99	8	4	30

When we compare the IP solution to its LP relaxation (switch from Gurobi Solver to MINOS in AMPL) we obtain the following output:

```
ampl: solve;
MINOS 5.51: ignoring integrality of 20 variables
MINOS 5.51: optimal solution found.
28 iterations, objective 7.242424242
ampl: display p;
p [*] :=
 1   0.272727
 2  -1.12751e-16
 3   0.727273
 4   1
 5   1
 6   1
 7   1
 8   1
 9   0
10   0
11   1
12   0.181818
13   1
14   0.818182
15   0
16   1
17   1
18  -7.17461e-17
19   0
20   1
;
```

FIGURE 1: LP RELAXATION OF PART A) FORMULATION

As expected, the LP optimum is higher than the IP optimum (but only fractionally). The choices as to which players should be selected for the Dream Team are different in the LP relaxation and there is no generalizable rounding convention that we could use to approximate the true IP solution. For instance, P1 is not selected in the IP optimal solution, however P12 is. But in the LP optimal solution both these players are assigned values that would be rounded to zero (0.27 and 0.18 respectively). P14 is not selected in the IP solution, but the LP relaxation assigns him a value that would be rounded up (0.82).

3.3 Integer Programming Model for Time-weighted Conditions [Question (b)]

Constraint Information for Time-weighted Formulation

Per request of the client, we found an additional integer programming model to advise the coach on how much time he should give to each selected player. Since some players were aging or returning from long injuries, the coach asked us to set an upper bound u_i on the average number of minutes per game each player p_i can play in the "Dream Team." In particular, the coach wants to apply two team formulations for the 40 minute games which meet the following specification: (PM, SG, SG, F, C) and (PM, SG, F, F, C). We have been asked by the coach to ignore game-to-game variability, and just plan for an "average game" in which the total play time will therefore be 40 minutes for the PMs; 60 minutes for the SGs; 60 minutes for the Fs; and 40 minutes for the Cs. Finally, the scoring abilities of the "Dream Team" are now computed as playtime-weighted averages.

The following additions and modifications are made to the previous formulation:

NEW Decision Variables

$T_{i,q}$= 26 Continuous variables represents the minutes of game-time the ith selected player in the qth position is assigned, $i \in [1,20], q \in [PM, SG, F, C]$ (accounting for the possibility of 6 players taking on two position in a game).

NEW constraints

1) **Valid Game-time Assignments:** Only players that are selected for the Dream Team can be allocated game-time minutes (but selected players can be assigned zero minutes of play). Selected players cannot be given more minutes than their maximum playtime time U_i allows.

$$\sum_{\forall q} T_{i,q} \leq \mathrm{P}_i * \mathrm{U}_i \quad \forall i \in [1,20]$$

2) **Upper limit on play time for PM**: the total number of minutes played by Play Makers must be 40 minutes

$$\sum_{i=1}^{5} T_{i,PM} = 40$$

3) **Upper limit on play time for SG**: the total number of minutes played by Shooting Guards must be 60 minutes

$$\sum_{i=4}^{11} T_{i,SG} = 60$$

4) **Upper limit on play time for F**: the total number of minutes played by Forwards must be 60 minutes

$$\sum_{i=9}^{16} T_{i,F} = 60$$

5) **Upper limit on play time for C**: the total number of minutes played by Centers must be 40 minutes

$$\sum_{i=16}^{20} T_{i,C} = 40$$

<u>Modified Constraints</u>

6) **Time-weighted rebound average**: The time weighted rebound average per game (40 minutes in length) of the selected team should be at least $\bar{r}$ (solved in AMPL for r=7).

$$\frac{1}{40}\sum_{\forall q} \sum_{i=1}^{20} T_{i,q} * r_i \geq \bar{r}$$

7) **Time-weighted defense average**: The time weighted defense average per game (40 minutes in length) of the selected team should be at least $\bar{d}$ (solved in AMPL for d=7).

$$\frac{1}{40}\sum_{\forall q} \sum_{i=1}^{20} T_{i,q} * d_i \geq \bar{d}$$

8) **Time-weighted assist average**: The time weighted assist average per game (40 minutes in length) of the selected team should be at least $\bar{a}$ (solved in AMPL for a=7).

$$\frac{1}{40}\sum_{\forall q} \sum_{i=1}^{20} T_{i,q} * a_i \geq \bar{a}$$

9) **Time-weighted height average**: The time weighted rebound average per game (40 minutes in length) of the selected team should be at least $\bar{h}$ (solved in AMPL for h=1.92 meters).

$$\frac{1}{40}\sum_{\forall q} \sum_{i=1}^{20} T_{i,q} * h_i \geq \bar{h}$$

<u>Modified Objective Function</u>

We want to select the number of minutes $T_{i,q}$ every i[th] player plays in the q[th] position so as to maximize the time-weighted scoring average per game:

$$\textbf{max} \ \frac{1}{40}\sum_{\forall q} \sum_{i=1}^{20} T_{i,q} * s_i$$

For the sake of brevity we will not rewrite out the full formulation here. Aside from the above additions/modifications, the problem formulation remains the same.

3.4 Optimal Solution for Updated Formulation

Question (d): Optimal Solution to Maximize the Mean "Dream Team's" Score from Part (b).

Using the Gurobi Solver built-in to AMPL, we find that all aforementioned requirements can be met to select players P1, P3, P5, P7, P8, P11, P13, P14, P15, P17, P18 & P20 for time-weighted average "Dream Team" score of **43.75**. This can be achieved by The constructed binary integer programming model can be found as an AMPL output is found in Figure 2 in Appendix B.

TABLE 3: List of Players Selected for Dream Team and their Allocated Playtimes

Player i	**Rebound (r_i)**	**Assist (a_i)**	**Height (h_i)**	**Scoring (s_i)**	**Defence (s_i)**	**Maximum Playtime (u_i)**	**Time Allocation (minutes)**
1	8	8	1.92	3	7	35	0
3	4	7	1.84	8	4	40	40
5	9	6	2.02	8	10	25	0
7	7	8	1.86	9	8	40	20
8	7	5	1.83	9	8	40	40
11	9	10	2.00	5	10	35	0
13	7	7	1.94	8	6	35	0
14	4	9	1.89	10	4	40	40
15	3	7	1.87	9	2	40	20
17	6	7	1.88	7	5	40	10
18	4	7	1.93	5	4	35	0
20	6	9	1.99	8	4	30	30

All other players were not selected for the Dream Team and as such were assigned times of zero. Players P_1,P_5,P_{11},P_{13} and P_{18} Were selected to be on the team, however the coach would not prefer to put them into play during the optimal average game. So they are all assigned times of zero. As it would happen, of the players selected, P_5 and P_{11} could play two positions (PM & SG, F & SG respectively) but it would have been suboptimal to allocate them any game-time.

We compare this MIP solution to the LP Relaxation (change AMPL solver from Gurobi to MINOS). We obtain the following objective value and optimal solution:

```
ampl: solve;
MINOS 5.51: ignoring integrality of 20 variables
MINOS 5.51: optimal solution found.
44 iterations, objective 43.75
ampl: display p,t;
:        p            t      :=
1     1               0
2     1.44416e-16     0
3     0.375          15
4     1               .
5     1               .
6    -1.35353e-16     0
7     0.625          20
8     1              40
9     3.27272e-16     .
10    1               .
11    0               .
12    1               0
13    0               0
14    1              40
15    1              20
16    0               .
17    1              10
18    1               0
19    0               0
20    1              30
;
```

```
ampl: display tpmsg,tsgf,tcf;
:      tpmsg tsgf tcf    :=
4  pm    0     .    .
4  sg    0     .    .
5  pm   25     .    .
5  sg    0     .    .
9  f     .     0    .
9  sg    .     0    .
10 f     .     0    .
10 sg    .     0    .
11 f     .     0    .
11 sg    .     0    .
16 c     .     .    0
16 f     .     .    0
;
```

FIGURE 2: LP RELAXATION OF PART B) FORMULATION

We note that the LP maximized objective value is the same as the MIP's. In general, the LP objective will always form an upper bound for the IP objective in a maximization problem (IP can as good as or worse than the LP relaxation). Again, the LP is not particularly helpful in finding the optimal team assignment; the optimal solution deviates from the IP solution. There is no generalizable rounding convention that can lead us to making the optimal IP selections.

Question (e): Our Recommended Formulation.

In the second formulation of the coach's problem, we address the issue of selecting the optimal team of twelve as well as the problem of deciding playtime allocations. One issue is we only have 5 players selected to play in the average match (the best choices the coach would want to put into play for the average match of the tournament- given his criteria). It is not realistic to only assign game time to 5 players in a team of 12. We need to plan for contingencies like injuries, player exhaustion, change in opponent strategy, etc. We would suggest formulating a stochastic program that takes into account these factors and also the likelihood of facing opponents of differing team structures. In essence we would create a dynamic schedule that takes uncertainty into account. Only then can the coach make use of the Dream Teams diverse abilities.

4 Appendices

Appendix A. Representation of the Binary Integer Programming Model in AMPL with the Optimal Solution

FIGURE 3: IP Optimal solution for BIP formulation

Optimum solution yields the 12 players which should be selected for the “Dream Team” for the original formulation without considering time-weighted averages.

```
ampl: options solver gurobi;
ampl: reset;
ampl: model '\\Mac\Home\Desktop\Period 2 - Courses\BAMS 508 - Discrete Optimization\2016-11-03_HW3\DreamTeam1.mod';
ampl: data '\\Mac\Home\Desktop\Period 2 - Courses\BAMS 508 - Discrete Optimization\2016-11-03_HW3\DreamTeam.dat';
ampl: solve;
Gurobi 6.5.1: optimal solution; objective 7.083333333
10 simplex iterations
ampl: display p;
p [*] :=
 1  0
 2  0
 3  1
 4  1
 5  1
 6  1
 7  1
 8  1
 9  0
10  0
11  1
12  1
13  1
14  0
15  0
16  1
17  1
18  0
19  0
20  1
;
```

DreamTeam1.mod | DreamTeam.dat

```
set players := {1..20} ;
set attributes := {'rebound','assist','height','scoring','defense','maxplay'};

var p {players} binary;

param stats {players,attributes};


### OBJECTIVE
maximize mean_scoring: (1/12)*( sum{i in players} p[i]*stats[i,'scoring']);

### CONSTRAINTS

# 1) At least 3 playmakers (players 1-5)
subject to PM: sum{i in 1..5} p[i] >= 3 ;

# 2) at least 4 shooting guards (players 4 -11)
subject to SG: sum{i in 4..11} p[i] >= 4;

# 3) at least 4 Forwards (players 9 - 16)
subject to F: sum{i in 9..16} p[i] >= 4;

# 4) at least 3 centers (players 16-20)
subject to C: sum{i in 16..20} p[i] >= 3;

# 5) 2 NCAA (amateur) players selected
subject to NCAA: p[4] + p[8] + p[15] + p[20] >= 2;

# 6) at most 1 of players 5 and 9 are selected
subject to p5_or_p9: p[5] + p[9] <= 1 ;

# 7) players 2 and 9 are selected together, if at all
subject to p2_and_p9: p[2] = p[9] ;

# 8) at most 3 players from the same professional team
subject to nba1: p[1] + p[7] + p[12] + p[16] <= 3 ;
subject to nba2: p[2] + p[3] + p[9] + p[19] <= 3 ;

# 9) mean rebound ability of team is atleast 7
subject to r: (1/12)*sum{i in players} p[i]*stats[i,'rebound'] >= 7 ;

# 10) mean assist ability of team is atleast 7
subject to a: (1/12)*sum{i in players} p[i]*stats[i,'assist'] >= 7 ;

# 11) mean defense ability of team is atleast 7
subject to d: (1/12)*sum{i in players} p[i]*stats[i,'defense'] >= 7 ;

# 12) mean height of team is atleast 1.92
subject to h: (1/12)*sum{i in players} p[i]*stats[i,'height'] >= 1.92 ;

# 13) exactly 12 people on the basketball team
subject to team_size: sum{i in players} p[i] = 12;
```

DreamTeam1.mod | DreamTeam.dat

```
param stats:
    rebound  assist   height    scoring   defense  maxplay :=
1       8       8       1.92        3        7       35
2       6       7       1.87        6        6       40
3       4       7       1.84        8        4       40
4       8       9       1.91        7        9       40
5       9       6       2.02        8        10      25
6       8       6       1.93        6        9       35
7       7       8       1.86        9        8       40
8       7       5       1.83        9        8       40
9       6       7       1.98        6        9       40
10      10      7       2.05        2        9       20
11      9       10      2.00        5        10      35
12      9       5       2.01        6        5       20
13      7       7       1.94        8        6       35
14      4       9       1.89        10       4       40
15      3       7       1.87        9        2       40
16      7       8       2.03        4        6       30
17      6       7       1.88        7        5       40
18      4       7       1.93        5        4       35
19      3       8       1.95        6        3       40
20      6       9       1.99        8        4       30 ;
```

Appendix B. Variants of the Binary Integer Programming Model in AMPL with the Optimal Solutions

FIGURE 4: Optimal solution to MIP formulation

Optimum solution yields the 12 players which should be selected for the "Dream Team" for the original formulation with considering time-weighted averages.

```
ampl: solve;
Gurobi 7.0.0: optimal solution; objective 43.75
33 simplex iterations
plus 3 simplex iterations for intbasis
ampl: display p,t;
:    p    t     :=
1    1    0
2    0    0
3    1   40
4    0    .
5    1    .
6    0    0
7    1   20
8    1   40
9    0    .
10   0    .
11   1    .
12   0    0
13   1    0
14   1   40
15   1   20
16   0    .
17   1   10
18   1    0
19   0    0
20   1   30
;
```

```
ampl: display tpmsg,tsgf,tcf;
:      tpmsg tsgf tcf    :=
4  pm    0     .    .
4  sg    0     .    .
5  pm    0     .    .
5  sg    0     .    .
9  f     .     0    .
9  sg    .     0    .
10 f     .     0    .
10 sg    .     0    .
11 f     .     0    .
11 sg    .     0    .
16 c     .     .    0
16 f     .     .    0
;
```

```
set players := {1..20} ;
set singles := players diff {4,5,9,10,11,16} ;
set pmsg := {4,5};
set sgf := {9,10,11};
set cf := {16} ;

set attributes :=
{'rebound','assist','height','scoring','defense','maxplay'};

# single role players time
var t {singles} >=0;

# pm and sg dual role players time
var tpmsg {{4,5},{'pm','sg'}} >=0;
# st and f dual role players time
var tsgf {{9,10,11},{'sg','f'}} >=0;
# c and f dual role players
var tcf {{16},{'f','c'}} >=0;

param stats {players,attributes};


### OBJECTIVE
maximize pwas:  (1/40)*(sum{i in singles} t[i]*stats[i,'scoring']) +
```

```
                 (1/40)*(sum{i in pmsg, j in {'pm','sg'}}
                 tpmsg[i,j]*stats[i,'scoring']) +

                 (1/40)*(sum {i in sgf,j in {'sg','f'}}
                 tsgf[i,j]*stats[i,'scoring']) +

                 (1/40)*(tcf[16,'f']*stats[16,'scoring']) +
                 (1/40)*(tcf[16,'c']*stats[16,'scoring']) ;

### CONSTRAINTS

# 1) At least 3 playmakers (players 1-5)
subject to PM: sum{i in 1..5} p[i] >= 3 ;

# 2) at least 4 shooting guards (players 4 -11)
subject to SG: sum{i in 4..11} p[i] >= 4;

# 3) at least 4 Forwards (players 9 - 16)
subject to F: sum{i in 9..16} p[i] >= 4;

# 4) at least 3 centers (players 16-20)
subject to C: sum{i in 16..20} p[i] >= 3;

# 5) 2 NCAA (amateur) players selected
subject to NCAA: p[4] + p[8] + p[15] + p[20] >= 2;

# 6) at most 1 of players 5 and 9 are selected
subject to p5_or_p9: p[5] + p[9] <= 1 ;

# 7) players 2 and 9 are selected together, if at all
subject to p2_and_p9: p[2] = p[9] ;

# 8) at most 3 players from the same professional team
subject to nba1: p[1] + p[7] + p[12] + p[16] <= 3 ;
subject to nba2: p[2] + p[3] + p[9] + p[19] <= 3 ;

# 13) exactly 12 people on the basketball team
subject to team_size: sum{i in players} p[i] = 12;




### TEAM STATS ADJUSTED IN PART 2

# 9) mean rebound ability of team is atleast 7
subject to r: (sum{i in singles} (1/40)*t[i]*stats[i,'rebound']) +
                  (sum{i in pmsg,j in {'pm','sg'}}
(1/40)*tpmsg[i,j]*stats[i,'rebound']) +
                  (sum{i in sgf,j in {'sg','f'}}
(1/40)*tsgf[i,j]*stats[i,'rebound']) +
                  (sum{i in cf, j in {'c','f'}}
(1/40)*tcf[i,j]*stats[i,'rebound'])  >= 7 ;
```

```
# 10) mean assist ability of team is atleast 7
subject to a: (sum{i in singles} (1/40)*t[i]*stats[i,'assist']) +
                   (sum{i in pmsg,j in {'pm','sg'}}
(1/40)*tpmsg[i,j]*stats[i,'assist']) +
                   (sum{i in sgf,j in{'sg','f'}}
(1/40)*tsgf[i,j]*stats[i,'assist']) +
                   (sum{i in cf, j in {'c','f'}}
(1/40)*tcf[i,j]*stats[i,'assist'])  >= 7 ;

# 11) mean defense ability of team is atleast 7
subject to d: (sum{i in singles} (1/40)*t[i]*stats[i,'defense']) +
                   (sum{i in pmsg,j in {'pm','sg'}}
(1/40)*tpmsg[i,j]*stats[i,'defense']) +
                   (sum{i in sgf, j in {'sg','f'}}
(1/40)*tsgf[i,j]*stats[i,'defense']) +
                   (sum{i in cf, j in {'c','f'}}
(1/40)*tcf[i,j]*stats[i,'defense'])  >= 7 ;

# 12) mean height of team is atleast 1.92
subject to h: (sum{i in singles} (1/40)*t[i]*stats[i,'height']) +
                   (sum{i in pmsg,j in {'pm','sg'}}
(1/40)*tpmsg[i,j]*stats[i,'height']) +
                   (sum{i in sgf,j in {'sg','f'}}
(1/40)*tsgf[i,j]*stats[i,'height']) +
                   (sum{i in cf, j in {'c','f'}}
(1/40)*tcf[i,j]*stats[i,'height'])  >= 1.92 ;



#### PART 2 TIME CONSTRAINTS

# 14) total play duration for PMs
subject to PMtime: (sum{i in 1..3} t[i]) + tpmsg[4,'pm']+tpmsg[5,'pm']
= 40 ;

# 15) total play duration for SGs
subject to SGtime: (sum{i in 6..8} t[i]) +
                   tpmsg[4,'sg']+tpmsg[5,'sg'] +
                   tsgf[9,'sg']+ tsgf[10,'sg']+ tsgf[11,'sg'] = 60;

# 16) total play duration for F
subject to Ftime: (sum{i in 12..15} t[i]) +
                 tsgf[9,'f']+ tsgf[10,'f']+ tsgf[11,'f'] +
                 tcf[16,'f']  = 60;

# 17) total play duration for Cs
subject to Ctime: (sum{i in 17..20} t[i]) + tcf[16,'c']= 40;



# 18) maximum minutes of play time per game
```

```
subject to playtime1 {i in singles}:
                      t[i] <= p[i]*stats[i,'maxplay'];

subject to playtime2 {i in pmsg}:
      sum{j in {'pm','sg'}}  tpmsg[i,j] <= p[i]*stats[i,'maxplay'];

subject to playtime3 {i in sgf}:
      sum{j in {'f','sg'}}  tsgf[i,j] <= p[i]*stats[i,'maxplay'];

subject to playtime4 {i in cf}:
     sum{j in {'f','c'}} tcf[i,j] <= p[i]*stats[i,'maxplay'];
```