BAMS 506 - Homework Assignment 3
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Note: All models in this homework set can be formulated as linear or smooth nonlinear programs. Thus you should not use integer or binary variables; nor discontinuous or non-smooth functions (such as MAX, MIN, ABS, etc.,) or logical operators (such as IF, OR, etc.), when their arguments depend on variables.

1. Estimating Learning Effects (a) – (d)

The production of a new product usually involves some learning: the larger the accumulated production x of the product, and the more experienced the workers will be in manufacturing the product. Let t(x) denote the total time (hours) it takes to produce the first x units of the product. A frequently used model for such learning effects is a power function $t(x) = ax^p$. The following five observations of cumulative production times for Widgets are available:

Observation i	1	2	3	4	5
Cumulative production x_i	1	30	60	120	240
Cumulative time t_i	6	60	100	150	250

(a) Define an unconstrained optimization model to find the parameters a and p to minimize the sum of the squared deviations between the observed values t_i and the predicted values ax_i^p. Is the objective function a convex function of a and p?

<u>Decision Variables</u>:
Let a denotes the value of alpha in the power equation *[hours per unit of product]*
Let p denotes the value of p in the power equation *[unitless]*

<u>Objective</u>: To minimize the sum of the square deviations between the observed values of t_i, the total time to produce the first x units of Widgets, and the predicted values of ax^p, by determining the parameters a [hours per unit of product] and p.

minimize $f(y) = \sum_{i=1}^{5}\left(t_i - ax_i^p \right)^2$ *[hours]*
$= (6 - a(1)^p)^2 + (60 - a(30)^p)^2 + (100 - a(60)^p)^2 + (150 - a(120)^p)^2 + (250 - a(240)^p)^2$
where y = (a, p, t, x)

This objective function is non-linear.

<u>Constraints</u>:

1. **Non-negativity:** $x, t, a \geq 0$ *[units of product, hours, hours per unit of product]*
   The total time to produce x units, or the number of units themselves, cannot be a negative number.

Question 1(b) Solve using the computer.

Figure 1 (below) shows an optimal solution that was found using the **GRG Nonlinear Solver** and model described in Question 1(a). This optimal solution indicates that we expect to minimize the sum of the square deviations between the observed values of t_i and the predicted values of ax^p, by setting **a = 5.8570338**, and **p = 0.68384**. The minimum sum of squares time was calculated to be **38.01 hours squared**. The results corresponding sensitivity report is found below in Figure 2.

Figure 1

5	DATA:	Benefits:						Constraints:	
6	Observation i	1	2	3	4	5	Units		
7								Nonnegativity	a, t, x ≥ 0
8	Cumulative Production xi	1	30	60	120	240	units		
9	Cumulative Time ti	6	60	100	150	250	hours		
10									
11									
12	MODEL:								
13	Decision Variables	a	p						
14	values	5.8570338	0.68384						
15		hours/ unit	none		Units				
16									
17	Objective Function:	min { (6 - a(1^p))^2 + (60 - a(30^p))^2 + (100 - a(60^p))^2 + (150 - a(120^p))^2 + (250 - a(240^p))^2 }							
18	Predicted ax^p	5.8570338	59.9498	96.3	154.7	248.516			
19	Minimize sum of squares time						38.01	Hours	

Figure 2

	A	B	C	D	E	F	G
1	Microsoft Excel 15.0 Sensitivity Report						
2	Worksheet: [Assignment 3 - Question 1.xlsx]Question 1-b						
3	Report Created: 9/26/2016 6:36:54 PM						
4							
5							
6	Variable Cells						
7					Final	Reduced	
8		Cell	Name	Value	Gradient		
9		$B$14	values a	5.857039961	0		
10		$C$14	values p	0.683838993	0		
11							
12	Constraints						
13		NONE					

In order to verify the coefficients of my optimized solution, a power regression model was fit into the original data as found in Figure 3.

Figure 3

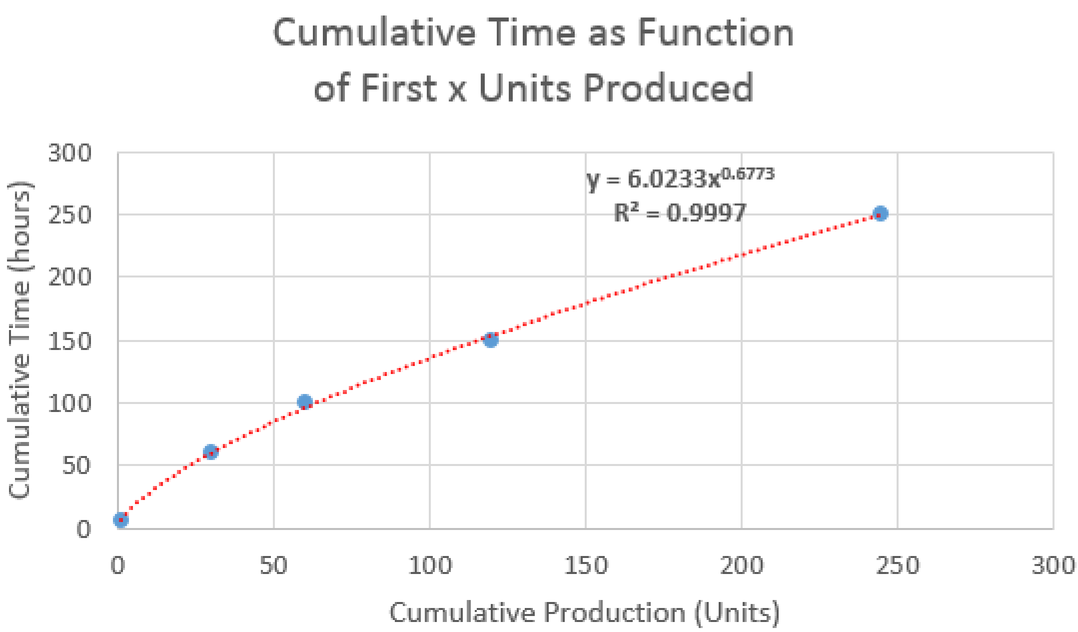

Table 1 below was constructed by selecting values of a, ranging from 0.1 to 1000, and p ranging from 0.1 to 10 while denoting the sum of the square deviations. Increasing the value of p had a much greater influence on the final value of the sum of square deviations than an increase in the value of a.

Table 1: Ploughman family's predicted monetary worth for each weather condition.

Minimum Sum of Square Deviations	a	p
98452	0.1	0.1
97307	0.1	0.5
2.19E12	0.1	3
4.68E26	0.1	6
6.07E45	0.1	10
96814	1	0.1
85760	1	0.5
2.19E+14	1	3
4.68E+28	1	6
6.07E+47	1	10
8.14E+04	10	0.1
1.09E+04	10	0.5
2.19E+16	10	3
4.68E+30	10	6
6.07E+49	10	10
2.39E+04	100	0.1
3.33E+06	100	0.5
2.19E+18	100	3
4.68E+32	100	6
6.07E+51	100	10
9.12E+06	1000	0.1

4.43E+08	1000	0.5
2.19E+20	1000	3
4.68E+34	1000	6
6.07E+53	1000	10

(c) For estimating such power functions, statisticians recommend the use of a logarithmic transformation, that is, ln(*t*(*x*)) = ln(*a*) + *p*ln(*x*). Define an unconstrained optimization model to find the parameters *b* (which is meant to represent ln(*a*)) and *p* to minimize the sum of the squared deviations between the logarithms ln(*ti*) of the observed times and the predicted values *b* + *p*ln(*xi*). Is the objective function a convex function of *b* and *p*?

Now, we will perform a logarithmic transformation.

t(x) = ax^p [original units of hours]

ln(t(x)) = ln(a) + p*ln(x) [log-transformed units of hours]

= b + p*ln(x)

Decision Variables:

Let b denotes the value of ln (a) in the log-transformed power equation *[log-transformed hours]*

Let p denotes the value of p in the log-transformed power equation *[log-transformed hours/log-units]*

Objective: To minimize the sum of the square deviations between the observed values of ln (t_i), the total time to produce the first x units of Widgets in log-transformed units, and the predicted values of b + p*ln(x), by determining the parameters b (log-transformed hours) and p (log-transformed hours/log-units).

minimize f (y) = $\sum_{i=1}^{5}(\ln(t_i) - \mathrm{b} - \mathrm{p} * \ln(x_i))^2$ *[log-transformed hours]*

= [ln(6) – b – p*ln(1)]^2 + [ln(60) – b – p*ln(30)]^2 + [ln(100) – b – p*ln(60)]^2

+ [ln(150) – b – p*ln(120)]^2 + [ln(250) – b – p*ln(240)]^2

where y = (b, p, t, x)

This objective function is non-linear.

Constraints: While this is an unconstrained optimization model of parameters b and p, the ln() function has constraints as follows:

1. **Natural Logarithm:** b ≥ 0 [log-transformed units of hours]

   The natural log of negative number does not exist. Consequently, a must not be negative.

(d) Use the computer to solve your model in question (c). Compare the results with those obtained in question (b). What do you think of the statisticians' recommendation?

Figure 4 (below) shows an optimal solution that was found using the **GRG Nonlinear Solver** and model described in Question 1(c). This optimal solution indicates that we expect to minimize the sum of the square deviations between the observed values ln (t_i), in log-transformed units, and the predicted values of b + p*ln(x), by setting **b = 1.793171,** and **p = 0.67877379**. These values are, in fact, different than my calculated values found in question 1(b). The results corresponding sensitivity report is found below in Figure 5.

Figure 4

5	**DATA:**	**Benefits:**						**Constraints:**
6	Observation i	**1**	**2**	**3**	**4**	**5**	**Units**	
7								**Nonnegativity** b, t, x ≥ 0
8	Cumulative Production xi	1	30	60	120	240	units	
9	Cumulative Time ti	6	60	100	150	250	hours	
10	ln (ti)	1.7917595	4.0943446	4.6052	5.011	5.52146	log-hours	
11	ln (xi)	0	3.4011974	4.0943	4.787	5.48064	log-units	
12								
13								
14	**MODEL:**							
15	**Decision Variables (b, p)**	**b**	**p**	a				
16	values	**1.793171**	**0.6787738**	6.0085				
17		**Log-hrs**	**Log-hrs/unit**	Units	**Units**			
18								
19	**Objective Function:**	min { (ln(6) - b - p*ln(1))^2 + (ln(60) - b - p*ln(30))^2 + (ln(100) - b - p*ln(60))^2 + (ln(150) - b - p*ln(120))^2 + (ln(250) - b - p*ln(245))^2 }						
20	**Predicted -b - p*ln(x)**	-1.793171	-4.1018146	-4.5723	-5.043	-5.5133		
21	Minimize sum of squares time						*0.00223902*	Log-hours

Figure 5

	A	B	C	D	E	F
1	**Microsoft Excel 15.0 Sensitivity Report**					
2	**Worksheet: [Assignment 3 - Question 1.xlsx]Question 1-c**					
3	**Report Created: 9/26/2016 6:44:24 PM**					
4						
5						
6	Variable Cells					
7				**Final**	**Reduced**	
8		**Cell**	**Name**	**Value**	**Gradient**	
9		$B$16	values b	1.793171012	0	
10		$C$16	values p	0.678773789	0	
11						
12	Constraints					
13		NONE				

In order to determine the value of a, and the minimum sum of squares time in original units, I back calculated my values using the exponential function. My calculations are as follows:

a = $e^b = e^{1.793171} = 6.0085$ [hours per unit]
minimum sum of squared deviations = $e^{0.00223902}$ = 1.00224 [hours]

Hence, I calculated **a = 6.0085 [hours per unit]** and the **minimum square of squared deviations is much smaller than it was in part (b)**. The minimum value of the squared deviations decreased significantly and now matches Excel's solution for fitting a power function. Thus, **I feel the statistician's recommendation is a very good one**. **One should use a logarithmic transformation when trying to fit a power function.**

BAMS 506 Assignment #3 Problem #2

(a).

Decision Variables

C denotes the production amount of chairs for the sales of next week.
T denotes the production amount of tables for the sales of next week.
S denotes the production amount of stools for the sales of next week.
Pc denotes the prices of chairs for the sales of next week
Pt denotes the prices of tables for the sales of next week
Ps denotes the prices of stools for the sales of next week

Constraints

a. Assembly hours (AH)
The total number of assembly hours that are spent on producing chairs, tables, and stools should not exceed 130 hours for the sales of next week.

$$0.1C + 0.2T + 0.15S \leq 130 \text{ (hours)}$$

b. Finishing hours (FH)
The total number of finishing hours that are spent on producing chairs, tables, and stools should not exceed 370 hours for the sales of next week.

$$0.4C + 0.3T + 0.2S \leq 370 \text{ (hours)}$$

c. Number of tabletops (TT)
The total number of tables produced could not exceed the 500 of tabletops that are necessary for the production of tables with rate of 1 tabletop per table.

$$T \leq 500 \text{ (tabletops)}$$

d. Demand of chairs (DC)
The total number of chairs produced could not exceed the total demand of chairs (which is determined by the unit price of chairs) in the coming week.

$$C \leq 1000 - \frac{1000}{13.5 \times 0.64}(Pc - 13.5) \text{ (Chairs)}$$

e. Demand of tables (DT)
The total number of tables produced could not exceed the total demand of tables (which is determined by the unit price of tables) in the coming week.

$$T \leq 500 - \frac{500}{16.5 \times 0.64}(Pt - 16.5) \text{ (Tables)}$$

f. Demand of stools (DS)
The total number of tables produced could not exceed the total demand of tables (which is determined by the unit price of tables) in the coming week.

$$S \leq 350 - \frac{350}{7.50 \times 0.64}(Ps - 7.50) \text{ (Stools)}$$

g. Price bounds (PB)
The number of salvaged tables should be less than or equal to the difference between weekly demand of tables and production amount of tables.

13.5$ ≤ Pc ≤ 22.14$
16.5$ ≤ Pt ≤ 27.06$
7.50$ ≤ Ps ≤ 12.3 $

h. Non-negativity (NN)
The number of production for chairs, tables, and stools should not be smaller than 0.
C, T, S ≥ 0

Objective function
The objective of this optimization model is to maximize the profit that Maple Products makes through the sales of chairs, table, and stools in the coming week.

Max
$(Pc - 13.5)C + (Pt - 16.5)T + (Ps - 7.5)S - 3800$ ($)

Summary
Find the total number of chairs, tables, and stools to produce and their price for sales in the coming week, so as to
Max: $(Pc - 13.5)C + (Pt - 16.5)T + (Ps - 7.5)S - 3800$ ($)

Subject to:

a. (AH) 0.1C + 0.2T + 0.15S ≤ 130 (hours)
b. (FH) 0.4C + 0.3T + 0.2S ≤ 370 (hours)
c. (TT) T ≤ 500 (tabletops)
d. (DC) $C \leq 1000 - \frac{1000}{13.5 \times 0.64}(Pc - 13.5)$ (Chairs)
e. (DT) $T \leq 500 - \frac{500}{16.5 \times 0.64}(Pt - 16.5)$ (Tables)

f. (DS) $S \leq 350 - \frac{350}{7.50 \times 0.64}(Ps - 7.50)$ (Stools)
g. (PB) 13.5$ ≤ Pc ≤ 22.14$
16.5$ ≤ Pt ≤ 27.06$
7.50$ ≤ Ps ≤ 12.3 $
h. (NN) C, T, S ≥ 0

Model results

We established the model in MS Excel, used solver and obtained results below:

Data						Decision variables	
						Chairs(C)	500.00028
	Chair	Table	Stool	Resource Limit		Talbes(T)	250.0001
Assmebly Hours	0.1	0.2	0.15	130		Stools(S)	175.00011
Finishing Hours	0.4	0.3	0.2	370		Price of chairs (Pc)	17.819998
Unit production cost($)	13.5	16.5	7.5			Price of tables (Pt)	21.779998
Current market size	1000	500	350			Price of stools (Ps)	9.8999985
Limt of table tops	500		Max profit	0.64			
Overhead cost	3800						
Constraints	Values	RHS				Objective Function Value	
a). Assembly hours(AH)	126.25006	130				100	
b). Finishing hours(FH)	310.00017	370					
c). Number of tabletops (TT)	250.0001	500					
d). Demand of chairs (DC)	500.00028	500.00028					
e). Demand of tables (DT)	250.0001	250.0001					
f). Demand of stools (DS)	175.00011	175.00011					
g). Price bounds (PB)							
Pc lower	17.819998	13.5					
Pc upper	17.819998	22.14					
Pt lower	21.779998	16.5					
Pt upper	21.779998	27.06					
Ps lower	9.8999985	7.5					
Ps upper	9.8999985	12.3					
h). Non-negativity (NN)							

The optimization model results suggest that for the coming week, we should produce 500 chairs, 250 tables, and 175 stools. The price for chairs is set to be 17.82 dollars; price of tables is set to be 21.78 dollars; and the price of stools is set to be 9.9 dollars. Such decision would yield an optimized objective value of 100 dollars.

The binding constraints are: d) demand of chairs; e) demand of tables; f) demand of stools.

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	a). Assembly hours(AH) Values	126.2500648	$B$15<=$C$15	Not Binding	3.749935171
$B$16	b). Finishing hours(FH) Values	310.0001653	$B$16<=$C$16	Not Binding	59.99983469
$B$17	c). Number of tabletops (TT) Values	250.0001018	$B$17<=$C$17	Not Binding	249.9998982
$B$18	d). Demand of chairs (DC) Values	500.000283	$B$18<=$C$18	Binding	0
$B$19	e). Demand of tables (DT) Values	250.0001018	$B$19<=$C$19	Binding	0
$B$20	f). Demand of stools (DS) Values	175.0001078	$B$20<=$C$20	Binding	0
$B$22	Pc lower Values	17.81999755	$B$22>=$C$22	Not Binding	4.319997555
$B$23	Pc upper Values	17.81999755	$B$23<=$C$23	Not Binding	4.320002445
$B$24	Pt lower Values	21.77999785	$B$24>=$C$24	Not Binding	5.27999785
$B$25	Pt upper Values	21.77999785	$B$25<=$C$25	Not Binding	5.28000215
$B$26	Ps lower Values	9.899998522	$B$26>=$C$26	Not Binding	2.399998522
$B$27	Ps upper Values	9.899998522	$B$27<=$C$27	Not Binding	2.400001478
$H$2	Chairs(C)	500.000283	$H$2>=0	Not Binding	500.000283
$H$3	Talbes(T)	250.0001018	$H$3>=0	Not Binding	250.0001018
$H$4	Stools(S)	175.0001078	$H$4>=0	Not Binding	175.0001078

(b).

Now the Maple Product Company is facing an uncertainty in the demand of tables. There is a 0.6 probability that the demand for tables will increase by 20% in the coming week; and there is a 0.4 probability that the demand for tables will decrease by 20% in the coming week.
Additionally the company is now able to salvage the unsold productions at 10% below their unit production costs.

<u>**Decision Variables**</u>

C denotes the production amount of chairs for the sales of next week.
T denotes the production amount of tables for the sales of next week.
S denotes the production amount of stools for the sales of next week.
Pc denotes the prices of chairs for the sales of next week
Pt denotes the prices of tables for the sales of next week
Ps denotes the prices of stools for the sales of next week
St denotes the amount of tables salvaged at the end of next week

<u>**Constraints**</u>

a. Assembly hours (AH)
The total number of assembly hours that are spent on producing chairs, tables, and stools should not exceed 130 hours for the sales of next week.

$$0.1C + 0.2T + 0.15S \leq 130 \text{ (hours)}$$

b. Finishing hours (FH)
The total number of finishing hours that are spent on producing chairs, tables, and stools should not exceed 370 hours for the sales of next week.

$$0.4C + 0.3T + 0.2S \leq 370 \text{ (hours)}$$

c. Number of tabletops (TT)
The total number of tables produced could not exceed the 500 of tabletops that are necessary for the production of tables with rate of 1 tabletop per table.

$$T \leq 500 \text{ (tabletops)}$$

d. Demand of chairs (DC)
The total number of chairs produced could not exceed the total demand of chairs (which is determined by the unit price of chairs) in the coming week.

$$C \leq 1000 - \frac{1000}{13.5 \times 0.64}(Pc - 13.5) \text{ (Chairs)}$$

e. Demand of stools (DS)
The total number of tables produced could not exceed the total demand of tables (which is determined by the unit price of tables) in the coming week.

$$S \leq 350 - \frac{350}{7.50 \times 0.64}(Ps - 7.50) \text{ (Stools)}$$

f. Price bounds (PB)
The number of salvaged tables should be less than or equal to the difference between weekly demand of tables and production amount of tables.

13.5$ ≤ Pc ≤ 22.14$
16.5$ ≤ Pt ≤ 27.06$
7.50$ ≤ Ps ≤ 12.3 $

g. Non-negativity (NN)
The number of production for chairs, tables, and stools, as well as number of salvaged tables should not be smaller than 0.
C, T, S, St ≥ 0

<u>Objective functions</u>

The objective of this optimization model is to maximize the profit that Maple Products makes through the sales of chairs, table, and stools in the coming week.

Max

$$(Pc - 13.5)C + (Pt - 16.5)(T - St) + (Ps - 7.5)S - 0.1(16.5) * St - 3800$$

<u>Summary</u>
Find the total number of chairs, tables, and stools to produce and their price for sales in the coming week, so as to
Max: $(Pc - 13.5)C + (Pt - 16.5)(T - St) + (Ps - 7.5)S - 0.1\ (16.5) \times\ St - 3800$ ($)

Subject to:
a. (AH) 0.1C + 0.2T + 0.15S ≤ 130 (hours)
b. (FH) 0.4C + 0.3T + 0.2S ≤ 370 (hours)
c. (TT) T ≤ 500 (tabletops)
d. (DC) $C \leq 1000 - \frac{1000}{13.5 \times 0.64}(Pc - 13.5)$ (Chairs)
e. (DS) $S \leq 350 - \frac{350}{7.50 \times 0.64}(Ps - 7.50)$ (Stools)
f. (PB) 13.5$ ≤ Pc ≤ 22.14$
16.5$ ≤ Pt ≤ 27.06$
7.50$ ≤ Ps ≤ 12.3 $
g. (NN) C, T, S, St ≥ 0

Model Results

Due to the fact that the market size of table for next week could only increase by 20% with probability of 60% or decrease by 20% with probability of 40%, we used Excel solver and calculated the optimal solution for each scenario. The solution of +20% in the market size of table case carries out 356.297 dollars total profit. The same solution applied on the -20% case carries out -327.858 dollars total profit. The **expected profit** taking account of the probabilities equals

356.297 * 0.6 + (-327.858)*0.4 **= 82.635 dollars**

	Market size of tables	Total profit ($)	Probability
Market increases by 20%	600	356.297	60%
Market decreases by 20%	400	-327.858	40%

The figure below shows the optimal decision variable values and their results in the +20% market.

Data	Chair	Table	Stool	Resource Limit
Assmebly Hours	0.1	0.2	0.15	130
Finishing Hours	0.4	0.3	0.2	370
Unit production cost($)	13.5	16.5	7.5	
Current market size	1000	600	350	

Limt of table tops	500	Max profit	0.64
Overhead cost	3800		

Constraints	Values	RHS
a). Assembly hours(AH)	130	130
b). Finishing hours(FH)	312.39655	370
c). Number of tabletops (TT)	286.00574	500
d). Demand of chairs (DC)	485.73633	485.73633
e). Demand of tables (DT)	286.00574	286.00574
f). Demand of stools (DS)	161.50147	161.50147
g). Price bounds (PB)		
Pc lower	17.943238	13.5
Pc upper	17.943238	22.14
Pt lower	22.026299	16.5
Pt upper	22.026299	27.06
Ps lower	10.085123	7.5
Ps upper	10.085123	12.3
h). Non-negativity (NN)		

Decision variables	
Chairs(C)	485.73633
Tables(T)	286.00574
Stools(S)	161.50147
Price of chairs (Pc)	17.943238
Price of tables (Pt)	22.026299
Price of stools (Ps)	10.085123
Tables Salvaged(St)	0

Objective Function Value
356.2965117

Optimal Decision		Rounded integer
Chairs(C)	485.7363274	485
Tables (T)	286.0057359	286
Stools (S)	161.5014672	161
Price of chairs (Pc)	17.94323813	
Price of tables (Pt)	22.02629905	
Price of stools (Ps)	10.08512274	

The figure below shows the profit of maintaining the optimal decision while the market size decreases by 20%.

Data						Decision variables	
						Chairs(C)	485.73633
	Chair	Table	Stool	Resource Limit		Tables(T)	286.00574
Assmebly Hours	0.1	0.2	0.15	130		Stools(S)	161.50147
Finishing Hours	0.4	0.3	0.2	370		Price of chairs (Pc)	17.943238
Unit production cost($)	13.5	16.5	7.5			Price of tables (Pt)	22.026299
Current market size	1000	400	350			Price of stools (Ps)	10.085123
						Tables Salvaged(St)	95.335245
Limt of table tops	500		Max profit	0.64			
Overhead cost	3800						
						Objective Function Value	
Constraints	Values	RHS				-327.8577175	
a). Assembly hours(AH)	130	130					
b). Finishing hours(FH)	312.3965452	370					
c). Number of tabletops (TT)	286.0057359	500					
d). Demand of chairs (DC)	485.7363274	485.73633					
e). Demand of tables (DT)	286.0057359	190.67049					
f). Demand of stools (DS)	161.5014672	161.50147					
g). Price bounds (PB)							
Pc lower	17.94323813	13.5					
Pc upper	17.94323813	22.14					
Pt lower	22.02629905	16.5					
Pt upper	22.02629905	27.06					
Ps lower	10.08512274	7.5					
Ps upper	10.08512274	12.3					
h). Non-negativity (NN)							

We observe a decrease in demand of tables as a result of decrease in market size. It causes 95 tables being salvaged and the total profit decreases to -327.858 dollars. Therefore **the probability distribution is:**

P (356.297) = 60%
P (-327.858) = 40%

The table below summarizes the two scenarios and gives the realized sales and salvaged numbers:

	Realized sales		
	Chairs	Tables	Stools
Market increases by 20%	485	286	161
Market decreases by 20%	485	190	161

	Salvaged numbers		
	Chairs	Tables	Stools
Market increases by 20%	0	0	0
Market decreases by 20%	0	95	0

The binding constraints are: a) Assembly hours; d) the demand of chairs (500); f) demand of stools (175); and the number of tables salvaged (one of the non-negativity constraints, equals 0)

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	a). Assembly hours(AH) Values	130	$B$15<=$C$15	Binding	0
$B$16	b). Finishing hours(FH) Values	312.3965452	$B$16<=$C$16	Not Binding	57.60345483
$B$17	c). Number of tabletops (TT) Values	286.0057359	$B$17<=$C$17	Not Binding	213.9942641
$B$18	d). Demand of chairs (DC) Values	485.7363274	$B$18<=$C$18	Binding	0
$B$20	f). Demand of stools (DS) Values	161.5014672	$B$20<=$C$20	Binding	0
$B$22	Pc lower Values	17.94323813	$B$22>=$C$22	Not Binding	4.443238131
$B$23	Pc upper Values	17.94323813	$B$23<=$C$23	Not Binding	4.196761869
$B$24	Pt lower Values	22.02629905	$B$24>=$C$24	Not Binding	5.526299045
$B$25	Pt upper Values	22.02629905	$B$25<=$C$25	Not Binding	5.033700955
$B$26	Ps lower Values	10.08512274	$B$26>=$C$26	Not Binding	2.585122736
$B$27	Ps upper Values	10.08512274	$B$27<=$C$27	Not Binding	2.214877264
$H$9	Tables Salvaged(St)	0	$H$9>=0	Binding	0
$H$2	Chairs(C)	485.7363274	$H$2>=0	Not Binding	485.7363274
$H$3	Tables(T)	286.0057359	$H$3>=0	Not Binding	286.0057359
$H$4	Stools(S)	161.5014672	$H$4>=0	Not Binding	161.5014672

(C).

Decision Variables

C denotes the production amount of chairs for the sales of next week.
T denotes the production amount of tables for the sales of next week.
S denotes the production amount of stools for the sales of next week.
Pc denotes the prices of chairs for the sales of next week
Pt denotes the prices of tables for the sales of next week
Ps denotes the prices of stools for the sales of next week
Sc denotes the number of salvaged chairs after next week
St denotes the number of salvaged tables after next week
Ss denotes the number of salvaged stools after next week

Constraints

a. Assembly hours (AH)
The total number of assembly hours that are spent on producing chairs, tables, and stools should not exceed 130 hours for the sales of next week.

$$0.1C + 0.2T + 0.15S \leq 130 \text{ (hours)}$$

b. Finishing hours (FH)
The total number of finishing hours that are spent on producing chairs, tables, and stools should not exceed 370 hours for the sales of next week.

$$0.4C + 0.3T + 0.2S \leq 370 \text{ (hours)}$$

c. Number of tabletops (TT)
The total number of tables produced could not exceed the 500 of tabletops that are necessary for the production of tables with rate of 1 tabletop per table.

$$T \leq 500 \text{ (tabletops)}$$

d. Price bounds (PB)
The number of salvaged tables should be less than or equal to the difference between weekly demand of tables and production amount of tables.

$$13.5\$ \leq Pc \leq 22.14\$$$
$$16.5\$ \leq Pt \leq 27.06\$$$
$$7.50\$ \leq Ps \leq 12.3\ \$$$

e. Non-negativity (NN)
The number of production and salvage for chairs, tables, and stools should not be smaller than 0.

$$C, T, S, Sc, St, Ss \geq 0$$

Objective functions

The objective of this optimization model is to maximize the profit that Maple Products makes through the sales and salvages of chairs, table, and stools in the coming week.

Max

$(Pc - 13.5)(C - Sc) + (Pt - 16.5)(T - St) + (Ps - 7.5)(S - Ss) - 0.1[(13.5) \times 0.1 + (16.5) \times St + (7.5) \times Ss)] - 3800$ (\$)

Summary

Find the total number of chairs, tables, and stools to produce and their price for sales in the coming week, so as to

Max: $(Pc - 13.5)(C - Sc) + (Pt - 16.5)(T - St) + (Ps - 7.5)(S - Ss) - 0.1[(13.5) \times 0.1 + (16.5) \times St + (7.5) \times Ss)] - 3800$ (\$)

Subject to:

a. (AH) $0.1C + 0.2T + 0.15S \leq 130$ (hours)
b. (FH) $0.4C + 0.3T + 0.2S \leq 370$ (hours)
c. (TT) $T \leq 500$ (tabletops)
d. (PB) $13.5\$ \leq Pc \leq 22.14\$$
$16.5\$ \leq Pt \leq 27.06\$$
$7.50\$ \leq Ps \leq 12.3\ \$$
e. (NN) $C, T, S, St \geq 0$

Model Results

Due to the fact that the market size of all three products for next week could only increase by 20% simultaneously with probability of 60% or decrease simultaneously by 20% with probability of 40%, we used Excel solver and calculated the optimal solution for each scenario. The solution of +20% in the market size case carries out 797.868 dollars total profit. The same solution applied on the -20% case carries out -1175.800 dollars total profit. The **expected profit** taking account of the probabilities equals

797.868 * 0.6 + (-1175.800)*0.4 **= 8.402 dollars**

The figure below shows the optimization model and its results by Excel Solver

Data						
	Chair	Table	Stool	Resource Limit		
Assmebly Hours	0.1	0.2	0.15	130		
Finishing Hours	0.4	0.3	0.2	370		
Unit production cost($)	13.5	16.5	7.5			
Current market size	1200	600	420			
Limt of table tops	500		Max profit	0.64		
Overhead cost	3800					

Decision variables	
Chairs(C)	545.19223
Talbes(T)	257.16232
Stools(S)	160.32208
Price of chairs (Pc)	18.214615
Price of tables (Pt)	22.533939
Price of stools (Ps)	10.467742
Salvaged chairs (SC)	0
Salvaged tables (ST)	0
Salvaged stools (SS)	0
Objective Function Value	
797.8679899	

Constraints	Values	RHS
a). Assembly hours(AH)	130	130
b). Finishing hours(FH)	327.29001	370
c). Number of tabletops (TT)	257.16232	500
d). Demand of chairs (DC)	545.19223	545.19232
e). Demand of tables (DT)	257.16232	257.16254
f). Demand of stools (DS)	160.32208	160.3226
g). Price bounds (PB)		
Pc lower	18.214615	13.5
Pc upper	18.214615	22.14
Pt lower	22.533939	16.5
Pt upper	22.533939	27.06
Ps lower	10.467742	7.5
Ps upper	10.467742	12.3
h). Non-negativity (NN)		

From the solver results we obtained the optimal solution:

Decision variables		Rounded Integer
Chairs(C)	545.192231	545
Tables (T)	257.1623219	257
Stools (S)	160.3220835	160
Price of chairs (Pc) ($)	18.21461526	
Price of tables (Pt) ($)	22.53393931	
Price of stools (Ps) ($)	10.46774171	

Applying this solution under the scenario of 20% simultaneous decrease in the market size of all 3 products:

Data						Decision variables	
						Chairs(C)	545.19223
	Chair	Table	Stool	Resource Limit		Talbes(T)	257.16232
Assmebly Hours	0.1	0.2	0.15	130		Stools(S)	160.32208
Finishing Hours	0.4	0.3	0.2	370		Price of chairs (Pc)	18.214615
Unit production cost($)	13.5	16.5	7.5			Price of tables (Pt)	22.533939
Current market size	800	400	270			Price of stools (Ps)	10.467742
						Salvaged chairs (SC)	181.73068
						Salvaged tables (ST)	85.720629
Limt of table tops	500		Max profit	0.64		Salvaged stools (SS)	57.257555
Overhead cost	3800						
Constraints	Values	RHS				Objective Function Value	
a). Assembly hours(AH)	130	130				-1175.799583	
b). Finishing hours(FH)	327.29001	370					
c). Number of tabletops (TT)	257.16232	500					
d). Demand of chairs (DC)	545.19223	363.46155					
e). Demand of tables (DT)	257.16232	171.44169					
f). Demand of stools (DS)	160.32208	103.06453					
g). Price bounds (PB)							
Pc lower	18.214615	13.5					
Pc upper	18.214615	22.14					
Pt lower	22.533939	16.5					
Pt upper	22.533939	27.06					
Ps lower	10.467742	7.5					
Ps upper	10.467742	12.3					
h). Non-negativity (NN)							

The optimal decision with 20% simultaneous decrease in market size results in reduced demand for all three products, which in turn causes the salvaged number for chairs, tables, and stools be 181, 85, and 57.

Probability distribution of total profits:
P (797.868) = 60%
P (-1175.8) = 40%

The following tables summarize the realized sales and salvaged numbers of different scenarios:

	Market size of tables			Total profit ($)	Probability
	Chairs	Tables	Stools		
Market increases by 20%	1200	600	420	**797.87**	60%
Market decreases by 20%	800	400	280	**-1175.8**	40%

	Realized sales		
	Chairs	Tables	Stools
Market increases by 20%	545	257	160
Market decreases by 20%	400	200	140

	Salvaged numbers		
	Chairs	Tables	Stools
Market increases by 20%	0	0	0
Market decreases by 20%	363	171	103

Expected profit 8.402

The binding constraints are: the assembly hours and the number salvaged chairs, tables, and stools.

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	a). Assembly hours(AH) Values	130	$B$15<=$C$15	Binding	0
$B$16	b). Finishing hours(FH) Values	327.7700664	$B$16<=$C$16	Not Binding	42.22993357
$B$17	c). Number of tabletops (TT) Values	256.4922557	$B$17<=$C$17	Not Binding	243.5077443
$B$22	Pc lower Values	18.20105124	$B$22>=$C$22	Not Binding	4.701051237
$B$23	Pc upper Values	18.20105124	$B$23<=$C$23	Not Binding	3.938948763
$B$24	Pt lower Values	22.54573629	$B$24>=$C$24	Not Binding	6.04573629
$B$25	Pt upper Values	22.54573629	$B$25<=$C$25	Not Binding	4.51426371
$B$26	Ps lower Values	10.47189125	$B$26>=$C$26	Not Binding	2.971891251
$B$27	Ps upper Values	10.47189125	$B$27<=$C$27	Not Binding	1.828108749
$H$9	Salvaged chairs (SC)	0	$H$9>=0	Binding	0
$H$10	Salvaged tables (ST)	0	$H$10>=0	Binding	0
$H$11	Salvaged stools (SS)	0	$H$11>=0	Binding	0