

Homework Assignment 1

Due Wednesday November 2, 9:00 am

Do all problems below. *Please refer to the instructions for Homework Assignments and Case Reports in the Course Outline. The data for problems 1 and 3 are posted on the course web site. All integer programming models in this homework must have linear constraints and objective. Make sure to produce and explain all relevant computer printouts. Interrupt a computer solution run if it exceeds 300 seconds.*

1. Bus drivers shift scheduling. The Urban Bus Co. (UBC) is considering the acquisition of the Suburban Bus Co. (SBC). As an optimization specialist for UBC you are being asked to evaluate possible labour cost savings that may result from this acquisition. UBC and SBC currently have very similar agreements with the bus drivers union, which will continue to be in effect for some time after the acquisition. These agreements specify the two types of bus drivers work shifts in effect at each company:

- A *normal shift*, consisting of 4 consecutive hours of work, followed by a one-hour paid break, immediately followed by 4 other consecutive hours of work;
- A *short shift*, consisting of only 4 consecutive hours of work.

The union has indicated that after the acquisition they will agree to a third shift type:

- A *dual shift*, consisting of 4 consecutive hours of work in one of the companies (UBC or SBC), followed by a one-hour paid break, immediately followed by 4 consecutive hours of work at the other company.

You have estimated the following daily minimum staffing requirements at each company:

From	To	Min. drivers req'd	
		UBC	SBC
5:00 AM	6:00 AM	3	5
6:00 AM	7:00 AM	10	12
7:00 AM	8:00 AM	12	20
8:00 AM	9:00 AM	12	20
9:00 AM	10:00 AM	10	15
10:00 AM	11:00 AM	10	8
11:00 AM	12:00 PM	15	6
12:00 PM	1:00 PM	19	6
1:00 PM	2:00 PM	18	6
2:00 PM	3:00 PM	15	8

From	To	Min. drivers req'd	
		UBC	SBC
3:00 PM	4:00 PM	12	12
4:00 PM	5:00 PM	10	15
5:00 PM	6:00 PM	8	21
6:00 PM	7:00 PM	6	20
7:00 PM	8:00 PM	5	12
8:00 PM	9:00 PM	5	6
9:00 PM	10:00 PM	5	4
10:00 PM	11:00 PM	4	3
11:00 PM	12:00 AM	3	2
12:00 AM	1:00 AM	2	2

- (a) Define an integer programming (IP) model to determine a minimum cost daily staff schedule after the acquisition. Set up your model in an Excel spreadsheet and solve it with the Excel

Solver. What do you observe about the mix of shift types in each company? Briefly explain why this is so.

- (b) In fact, the current work agreements require that in each company at least 65% of the shifts must be normal shifts. The union has also indicated that after the acquisition they will agree to change these clauses to: “in each company at least 65% of the shifts must be normal or dual shifts”. Modify your IP model of question (a) above, to take these two constraints into account. Try and solve with the *Excel Solver*, using the standard (default) version of the *Solver*. What happens?
- (c) Now try and solve your model of question (b) using the *OpenSolver*. What happens?
- (d) Now set up and solve your models in questions (a) and (b) using a mathematical programming system with an algebraic modeling language. Compare the results with those in (a) and (c). Comment on the relative advantages and disadvantages of these two optimization systems (spreadsheet-based, vs. mathematical programming system with an algebraic modeling language) for setting up and solving these IP models.
- (e) Use your results in (c) or (d) above to estimate the labour cost savings, if any, that may result from the use of dual shifts in case of the SBC acquisition.

2. LP-based Branch-and-Bound, graphical example. Solve the following two-variable integer program by the LP-based Branch-and-Bound method explained in class. (Note the direction of the third inequality). Solve each LP relaxation using the graphical method in the (x_1, x_2) -space. Detail all calculations and display the branch-and-bound tree as it evolves.

$$\begin{aligned}
 &\text{maximize } 9x_1 + 5x_2 \\
 &\text{subject to } 4x_1 + 9x_2 \leq 35 \\
 &\quad \quad \quad x_1 \leq 6 \\
 &\quad \quad \quad x_1 - 3x_2 \geq 1 \\
 &\quad \quad \quad 3x_1 + 2x_2 \leq 19 \\
 &\quad \quad \quad x_1 \text{ and } x_2 \geq 0, \text{ integer.}
 \end{aligned}$$

3. Assigning Art. Write a **Case Report** for the *Assigning Art* case (Case 11.2 in the Hillier & Lieberman text; copy of this case attached; the female artists are Rita Losky, Candy Tate, Angie Oldman, and Helen Row). Please follow the instructions in the Course Outline for presenting your Case Report. Include brief answers to all Questions (a) to (c) in your one-page Executive Summary.

CASE 11.2 ASSIGNING ART

It had been a dream come true for Ash Briggs, a struggling artist living in the San Francisco Bay Area. He had made a trip to the corner grocery store late one Friday afternoon to buy some milk, and on impulse, he had also purchased a California lottery ticket. One week later, he was a millionaire.

Ash did not want to squander his winnings on materialistic, trivial items. Instead he wanted to use his money to support his true passion: art. Ash knew all too well the difficulties of gaining recognition as an artist in this postindustrial, technological society where artistic appreciation is rare and financial support even rarer. He therefore decided to use the money to fund an exhibit of up-and-coming modern artists at the San Francisco Museum of Modern Art.

Ash approached the museum directors with his idea, and the directors became excited immediately after he informed them that he would fund the entire exhibit in addition to donating \$1 million to the museum. Celeste McKenzie, a museum director, was assigned to work with Ash in planning the exhibit. The exhibit was slated to open one year from the time Ash met with the directors, and the exhibit pieces would remain on display for two months.

Ash began the project by combing the modern art community for potential artists and pieces. He presented the following list of artists, their pieces, and the price of displaying each piece¹ to Celeste.

Artist	Piece	Description of Piece	Price
Colin Zweibell	"Perfection"	A wire mesh sculpture of the human body	\$300,000
	"Burden"	A wire mesh sculpture of a mule	\$250,000
	"The Great Equalizer"	A wire mesh sculpture of a gun	\$125,000
Rita Losky	"Chaos Reigns"	A series of computer-generated drawings	\$400,000
	"Who Has Control?"	A computer-generated drawing intermeshed with lines of computer code	\$500,000
	"Domestication"	A pen-and-ink drawing of a house	\$400,000
	"Innocence"	A pen-and-ink drawing of a child	\$550,000
Norm Marson	"Aging Earth"	A sculpture of trash covering a larger globe	\$700,000
	"Wasted Resources"	A collage of various packaging materials	\$575,000
Candy Tate	"Serenity"	An all blue watercolor painting	\$200,000
	"Calm Before the Storm"	A painting with an all blue watercolor background and a black watercolor center	\$225,000
Robert Bayer	"Void"	An all black oil painting	\$150,000
	"Sun"	An all yellow oil painting	\$150,000
David Lyman	"Storefront Window"	A photo-realistic painting of a jewelry store display window	\$850,000
	"Harley"	A photo-realistic painting of a Harley-Davidson motorcycle	\$750,000
Angie Oldman	"Consumerism"	A collage of magazine advertisements	\$400,000
	"Reflection"	A mirror (considered a sculpture)	\$175,000
	"Trojan Victory"	A wooden sculpture of a condom	\$450,000
Rick Rawls	"Rick"	A photo-realistic self-portrait (painting)	\$500,000
	"Rick II"	A cubist self-portrait (painting)	\$500,000
	"Rick III"	An expressionist self-portrait (painting)	\$500,000

¹The display price includes the cost of paying the artist for loaning the piece to the museum, transporting the piece to San Francisco, constructing the display for the piece, insuring the piece while it is on display, and transporting the piece back to its origin.

Artist	Piece	Description of Piece	Price
Bill Reynolds	"Beyond"	A science fiction oil painting depicting Mars colonization	\$650,000
	"Pioneers"	An oil painting of three astronauts aboard the space shuttle	\$650,000
Bear Canton	"Wisdom"	A pen-and-ink drawing of an Apache chieftain	\$250,000
	"Superior Powers"	A pen-and-ink drawing of a traditional Native American rain dance	\$350,000
	"Living Land"	An oil painting of the Grand Canyon	\$450,000
Helen Row	"Study of a Violin"	A cubist painting of a violin	\$400,000
	"Study of a Fruit Bowl"	A cubist painting of a bowl of fruit	\$400,000
Ziggy Lite	"My Namesake"	A collage of Ziggy cartoons	\$300,000
	"Narcissism"	A collage of photographs of Ziggy Lite	\$300,000
Ash Briggs	"All That Glitters"	A watercolor painting of the Golden Gate Bridge	\$50,000*
	"The Rock"	A watercolor painting of Alcatraz	\$50,000
	"Winding Road"	A watercolor painting of Lombard Street	\$50,000
	"Dreams Come True"	A watercolor painting of the San Francisco Museum of Modern Art	\$50,000

*Ash does not require personal compensation, and the cost for moving his pieces to the museum from his home in San Francisco is minimal. The cost of displaying his pieces therefore only includes the cost of constructing the display and insuring the pieces.

Ash possesses certain requirements for the exhibit. He believes the majority of Americans lack adequate knowledge of art and artistic styles, and he wants the exhibit to educate Americans. Ash wants visitors to become aware of the collage as an art form, but he believes collages require little talent. He therefore decides to include only one collage. Additionally, Ash wants viewers to compare the delicate lines in a three-dimensional mesh sculpture to the delicate lines in a two-dimensional computer-generated drawing. He therefore wants at least one wire mesh sculpture displayed if a computer-generated drawing is displayed. Alternatively, he wants at least one computer-generated drawing displayed if a wire mesh sculpture is displayed. Furthermore, Ash wants to expose viewers to all painting styles, but he wants to limit the number of paintings displayed to achieve a balance in the exhibit between paintings and other art forms. He therefore decides to include at least one photo-realistic painting, at least one cubist painting, at least one expressionist painting, at least one watercolor painting, and at least one oil painting. At the same time, he wants the number of paintings to be no greater than twice the number of other art forms.

Ash wants all his own paintings included in the exhibit since he is sponsoring the exhibit and since his paintings celebrate the San Francisco Bay Area, the home of the exhibit.

Ash possesses personal biases for and against some artists. Ash is currently having a steamy affair with Candy Tate, and he wants both of her paintings displayed. Ash counts both David Lyman and Rick Rawls as his best friends, and he does not want to play favorites among these two artists. He therefore decides to display as many pieces from David Lyman as from Rick Rawls and to display at least one piece from each of them. Although Ziggy Lite is very popular within art circles, Ash believes Ziggy makes a mockery of art. Ash will therefore only accept one display piece from Ziggy, if any at all.

Celeste also possesses her own agenda for the exhibit. As a museum director, she is interested in representing a diverse population of artists, appealing to a wide audience, and creating a politically correct exhibit. To advance feminism, she decides to include at least one piece from a female artist for every two pieces included from a male artist. To advance environmentalism, she decides to include either one or both of the pieces "Aging Earth" and "Wasted Resources." To advance Native American rights, she decides to include at least one

piece by Bear Canton. To advance science, she decides to include at least one of the following pieces: "Chaos Reigns," "Who Has Control," "Beyond," and "Pioneers."

Celeste also understands that space is limited at the museum. The museum only has enough floor space for four sculptures and enough wall space for 20 paintings, collages, and drawings.

Finally, Celeste decides that if "Narcissism" is displayed, "Reflection" should also be displayed since "Reflection" also suggests narcissism.

Please explore the following questions independently except where otherwise indicated.

- (a) Ash decides to allocate \$4 million to fund the exhibit. Given the pieces available and the specific requirements from Ash and Celeste, formulate and solve a BIP model to maximize the number of pieces displayed in the exhibit without exceeding the budget. How many pieces are displayed? Which pieces are displayed?
- (b) To ensure that the exhibit draws the attention of the public, Celeste decides that it must include at least 20 pieces. Formulate and solve a BIP model to minimize the cost of the exhibit while displaying at least 20 pieces and meeting the requirements set by Ash and Celeste. How much does the exhibit cost? Which pieces are displayed?
- (c) An influential patron of Rita Losky's work who chairs the Museum Board of Directors learns that Celeste requires at least 20 pieces in the exhibit. He offers to pay the minimum amount required on top of Ash's \$4 million to ensure that exactly 20 pieces are displayed in the exhibit and that all of Rita's pieces are displayed. How much does the patron have to pay? Which pieces are displayed?