Case 12.2 Report: International Investments

Prepared for

Charles Rosen
Business Administrator

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by Gurpal Bisra and Leon Zhu
Center for Operations Excellence

1 Executive Summary

Four-years-ago Charles Rosen, the "Client," invested 30,000 German Marks (GM) in B-Bonds that would mature over seven years. Initially, the Client invested since he was offered high interest rates and told he could sell the B-Bonds any time after the first year. However, last year, the German federal government introduced a capital gains tax of 30% on interest income. Our services were requested to help maximize Charles Rosen's **investment interest return**, three years from now, by determining his optimal investment strategy.

This report discusses the linear programming models we formulated under various scenarios and our recommendations for the Client's optimal investment strategy. First, we determined the Client's maximum interest return is DM 19,098.62 without investing in certificates of deposit (CD) or US municipal funds. This interest return can be achieved if the Client sells DM 9604.79 at the end of year 6 and DM 20395.21 at the end of year 7. Hence, we noted the Client's investment analyst was incorrect. The Client's investment analyst did not consider the fact Mr. Rosen pays a capital gains tax on all of the previous year's interest income he accumulates. Had Mr. Rosen followed his investment analyst's advice, we determined he would have made an interest income of DM 18,258.30 - a loss of $840.32 when compared to our derived optimal solution. To investigate the matters further, formulated a new model to take CDs into account. Our analysis indicated a higher maximum interest income of **DM 19,997.86** is gained when investing in CDs. This can be achieved by selling: (1) DM 12197.60 at the end of year 5 which is not taxed; (2) DM 9604.79 at the end of year 6 which is not taxed; (3) DM 18,297.56 of CDs at the end of year 6 which is taxed; (4) DM 34,514.68 of CDs at the end of year 7 which is taxed; (5) DM 7,797.52 at the end of year 7 which is not taxed; and (6) DM 400.13 at the end of year 7 which is taxed.

As per request of the Client, we explored deviations from this optimal solution by considering whether the Client initially invests 50,000, gets married, and purchases US municipal bonds. Had the Client initially invested 50,000, a maximum interest income of DM 30,950.06 would be gained when considering CDs but not US municipal funds. Our models also enabled us to determine that if the Client marries this year, he can produce a maximum interest income of **DM 22,008.09** when considering CDs but not US municipal funds. This means he can **saving an additional DM 2010.23**. Finally, we explored the prospect of the Client investing in US municipal bonds. We determined the Client could produce a maximum interest income of USD 30,478.23 when considering both CDs and US municipal funds.

Given the aforementioned findings, the Client can maximize his profits if he marries this year, invests in CDs, and invests in US municipal funds this year while the exchange rate is still strong. Our analysis was conducted while meeting the following requirements:

1. **Non-negativity:** The sold price of B-Bonds, whether or not they are taxed, cannot be negative amounts.
2. **Total B-Bonds:** The sum of B-Bonds sold must equal initial investment of DM 30,000.
3. **Year-5 Tax:** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 50.01% return, must be less than 6,100.
4. **Year-6 Tax:** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 63.51% return, must be less than 6,100.
5. **Year-7 Tax:** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 78.23% return, must be less than 6,100.
6. **Year-6 CD:** Mr. Rosen can invest an amount which equals the interest income of B-Bonds sold at the end of year 5.
7. **Year-7 CD:** Mr. Rosen can invest an amount which equals the interest income of B-Bonds, and CDs, sold at the end of year 6.

Additional requirements are listed in Section 3.2 of the report when we analyzed the effect investing in US municipal bonds would have on Mr. Rosen's interest return. The key objective here for the consultation was to maximize the interest return of selling the B-Bonds while meeting the aforementioned requirements. To conduct our analysis, the following assumptions were made:

1. When Charles sells a portion of his bonds, he cannot earn interest for questions (a) to (c).
2. Charles can invest proceeds of B-Bonds in a Certificate of Deposit (CD) for questions (d) onward.
3. All investments (i.e. B-Bonds, CD) are perfectly divisible: Charles can invest any nonnegative amount in them which are not necessarily integer multiples of 100 DM.
4. The 3.6% interest on US Municipal Bonds is annual, and is compounded annually.
5. A CD (certificate of deposit) is a 1-year security. If it is kept more than one year then its interest is compounded annually.
6. The interest from a CD investment is taxable. The 6,100 DM allowance and the 30% tax rate apply to the total revenue from all investments.
7. Interest tax is paid the same year the investment is redeemed (i.e. it can be withheld at the source).

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2 Introduction

Four-years-ago Charles Rosen, the "Client," invested 30,000 German Marks (GM) in B-Bonds that would mature over seven years. Initially, the Client invested since he was offered high interest rates and told he could sell the B-Bonds any time after the first year. However, last year, the German federal government introduced a capital gains tax of 30% on interest income. Our services were requested to help maximize Charles Rosen's **investment interest return**, three years from now, by determining his optimal investment strategy.

Our analysis was conducted while meeting the following requirements:

1. **Non-negativity:** The sold price of B-Bonds, whether or not they are taxed, cannot be negative amounts.
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4. **Year-6 Tax:** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 63.51% return, must be less than 6,100.
5. **Year-7 Tax:** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 78.23% return, must be less than 6,100.
6. **Year-6 CD:** Mr. Rosen can invest an amount which equals the interest income of B-Bonds sold at the end of year 5.
7. **Year-7 CD:** Mr. Rosen can invest an amount which equals the interest income of B-Bonds, and CDs, sold at the end of year 6.

Additional requirements are listed in Section 3.2 when we analyzed the effect investing in US municipal bonds would have on Mr. Rosen's interest return. The key objective here for the consultation was to maximize the Client's interest return of selling the B-Bonds while meeting the aforementioned requirements.

To conduct the analysis, the following assumptions were made:

1. When Charles sells a portion of his bonds, he cannot earn interest for questions (a) to (c).
2. Assume Charles can invest proceeds of B-Bonds in a Certificate of Deposit (CD) for questions (d) onward.
3. All investments (i.e. B-Bonds, CD) are perfectly divisible: Charles can invest any nonnegative amount in them which are not necessarily integer multiples of 100 DM.
4. The 3.6% interest on US Municipal Bonds is annual, and is compounded annually.
5. A CD (certificate of deposit) is a 1-year security. If it is kept more than one year then its interest is compounded annually.
6. The interest from a CD investment is taxable. The 6,100 DM allowance and the 30% tax rate apply to the total revenue from all investments.
7. Interest tax is paid the same year the investment is redeemed (i.e. it can be withheld at the source).

2.1 Information Provided

Client's Investment

Mr. Rosen invested 30,000 German Marks (GM) in B-bonds that would mature in seven years. Initially, he was attracted to the investment opportunity by its prospect of high interest rates and the ability for Mr. Rosen to sell the B-Bonds any time after one year. The interest rates for the B-Bonds over the seven years is depicted in Table 1. In particular, the amount paid includes the principle plus interest. For example, if Mr. Rosen sold DM 100 on December 31 of the sixth year, after investing in the B-Bonds, he would be paid DM 163.51. Here, the principle is DM 100 while the interest DM is 63.51. At present, Mr. Rosen has not sold any of his B-Bonds. More specifically, Table 2 illustrates the total return on 100 DM over the 7-year period.

German Federal Government's Introduced 30% Income Tax

Last year, much to the Client's dismay, the German federal government introduced a capital gains tax of 30% on interest income (i.e. decreasing marginal returns). More specifically, the first DM 6,100 made by a single individual earns an interest per year would be tax-free and taxes are only paid once one sells their B-Bonds. For example, Mr. Rosen would be required to pay DM 1,200 for earning DM 10,100 and be left with DM 8,900.

TABLE 1: Interest rates over 7 years.

Year	Interest Rate (%)	Annual Percentage Yield (%)
1	7.50	7.50
2	8.50	8.00
3	8.50	8.17
4	8.75	8.31
5	9.00	8.45
6	9.00	8.54
7	9.00	8.61

TABLE 2: Total return on 100 DM.

Year	Interest Rate (%)
1	107.50
2	116.64
3	126.55
4	137.62
5	150.01
6	163.51
7	178.23

Client's Decisions and Alternatives

Our services were requested to assist Mr. Rosen re-evaluate his investment since the capital gains tax potentially affects his maximum return from selling B-Bonds. We considered the effects of selling portions of his B-Bonds across the next three years, not selling any of his B-Bonds until the end of year 7, and whether he should purchase certificates of deposit or US municipal funds. The Client's investment analyst suggested he keep all of his B-Bonds until the end of the seventh year because he would earn 0.7823 times DM 30,000 in interest income, but pay heavy taxes that year. In particular, this advisor argued their strategy would pay out 6.30% interest which his higher than that for CDs (i.e. 4.00%).

Client's Timeline and Future

The Client informed us he wants to conduct all of his transactions on December 31 of any given year. He will attend business school in America in fall of seventh year, and plans to pay for his tuition with his investment. He will not keep money in Germany after he attends school.

3 Analysis

3.1 Linear Programming Model

Decision Variables: For the coming year, let:

Y5NO	denote the B-Bonds sold at the end of year five which is not taxed.	[DM]
Y5TAX	denote the B-Bonds sold at the end of year five which is taxed.	[DM]
Y6NO	denote the B-Bonds sold at the end of year six which is not taxed.	[DM]
Y6TAX	denote the B-Bonds sold at the end of year six which is taxed.	[DM]
Y7NO	denote the B-Bonds sold at the end of year seven which isn't taxed.	[DM]
Y7TAX	denote the B-Bonds sold at the end of year seven which is taxed.	[DM]

Constraints: The following constraints define the linear model to find the optimal solution:

1. **Non-negativity (NN):** The sold price of B-Bonds, whether or not they are taxed, cannot be negative amounts.
   Y5NO, Y5TAX, Y6NO, Y6TAX, Y7NO, Y7TAX ≥ 0 [DM]

2. **Total B-Bonds (TBB):** The sum of B-Bonds sold must equal initial investment of DM 30,000.
   Y5NO + Y5TAX + Y6NO + Y6TAX + Y7NO + Y7TAX = 30,000 [DM]

3. **Year-5 Tax (Y5T):** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 50.01% return, must be less than 6,100. The calculation for this constraint can be found in Appendix A.
   Y5NO*(0.5001) ≤ 6100 [DM]

4. **Year-6 Tax (Y6T):** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 63.51% return, must be less than 6,100. The calculation for this constraint can be found in Appendix A.
   Y6NO*(0.6351) ≤ 6100 [DM]

5. **Year-7 Tax (Y7T):** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 78.23% return, must be less than 6,100. The calculation for this constraint can be found in Appendix A.
   Y7NO*(0.7823) ≤ 6100 [DM]

Objective Function:

The objective is to maximize the return of Mr. Rosen's investment as interest income at the end of the seven-year-period. Therefore, we seek to:

Max { Interest income in years 5, 6, and 7 } = Z
= { (Y5NO*IY5NO) + (Y6NO*IY6NO) + (Y7NO*IY7NO)
+ (Y5TAX*IY5TAX) + (Y6TAX*IY6YES) + (Y7TAX*IY7TAX) }

where:

IY5NO = interest rate of return assuming no capital taxes incurred. [%]

IY5TAX = interest rate of return assuming capital taxes incurred. [%]

IY6NO = interest rate of return assuming no capital taxes incurred. [%]

IY6TAX = interest rate of return assuming capital taxes incurred. [%]

IY7NO = interest rate of return assuming no capital taxes incurred. [%]

IY7TAX = interest rate of return assuming capital taxes incurred. [%]

Detailed calculations for interest rates are listed in Appendix A. The above data was entered in an Excel spreadsheet to facilitate manipulation of the model. A screenshot of the representation of the model in Excel model is provided in Appendix B.

3.2 Optimal Solution

Using the simplex method built-in to Excel's Solver add-on, we find that all aforementioned requirements can be met to produce as maximum interest income of **DM 19,098.62**. This can be achieved by selling DM 9604.79 at the end of year 6 which is not taxed, DM 7797.52 at the end of year 7 which is not taxed, and DM 12,597.69 at the end of year 7 which is subject to the 30% capital tax. The constructed linear model can be found as an Excel spreadsheet is found in Figure 1 in Appendix B. Additionally, as per request by the Client, we generated an additional sensitivity output, of post-optimality analysis, which can be found in Appendix B as Figure 2.

3.3 Optimal Solution for Various Conditions

Question (c): Mr. Rosen's Investment Advisor's Error

We have determined an optimal solution which contradicts the solution provided by Mr. Rosen's investment analyst. For example, Mr. Rosen's investment analyst had suggested he keep all of his bonds until the end of the seventh year because he would earn 0.7823 times DM 30,000 in interest income, despite the fact he would incur heavy taxes. However, his investment analyst did not consider the fact Mr. Rosen pays a capital gains tax on all of the previous year's interest income. Had Mr. Rosen followed his investment analyst's advice, he would have made an interest income of DM 18,258.30, which would have been a loss of $840.32 when compared to our derived optimal solution. In particular, Mr. Rosen would only make 54.761% on taxable income in year 7 which is, in fact, lower than the 63.51% he would make on the taxable income in year 6. Therefore, the optimal solution can only be met if Mr. Rosen sells portions of his share in year 6. Annotated screenshots of the model variants, depicting Mr. Rosen's investment analyst's advice is provided in Appendix C as Figure 3.

Question (d): Optimal Solution – Investment in Certificate of Deposit

As per request by the Client, we were tasked to determine Mr. Rosen's optimal investment strategy when considering investment in certificates of deposit (CD). First, Mr. Rosen should only consider investing in CDs after he receives his maximum non-taxable interest incomes from selling B-Bonds. For instance, CDs only pay Mr. Rosen 4.00% interest. However, this rate is lower than any rate received by Mr. Rosen for selling his B-Bonds in years 5 through 7 irrespective of whether taxation occurs. Therefore, Mr. Rosen's earnings from CDs should always be taxed. Consequently, Mr. Rosen's return on CDs is actually 2.80%, after the capital gains tax is applied, as calculated in Appendix A. Furthermore, the Client needs to buy the CDs the same day he sells his B-Bonds in a given year; otherwise, he would lose money by missing the opportunity to obtain tax-free interest income. Since our optimal solution indicated Mr. Rosen should not sell any B-Bonds in year 5, it is logical to assume CDs may only be purchased in year 6 or 7.

The aforementioned considerations led us to formulate an updated model, which takes into account the CDs, whose additional decision variables and constraints are found below.

Decision Variables: In addition to the model formulated in Section 3.1, let:

Y6CD	denote the certificate of deposit invested at the start of year 6.	[DM]
Y7CD	denote the certificate of deposit invested at the start of year 7.	[DM]

Constraints: The following constraints, in addition to those formulated in Section 3.1, define the linear model to find the optimal solution:

6. **Year-6 CD (6CD):** Mr. Rosen can invest an amount which equals the interest income of B-Bonds sold at the end of year 5.
   Y6CD = (Y5NO*(1+IY5NO)) + (Y5TAX*(1+IY5TAX)) [DM]

7. **Year-7 CD (7CD):** Mr. Rosen can invest an amount which equals the interest income of B-Bonds, and CDs, sold at the end of year 6.
   Y7CD = (Y6NO*(1+IY6NO)) + (Y6TAX*(1+IY6TAX)) + (CDTAX*Y6CD) [DM]

Objective Function:

The objective is to maximize the return of Mr. Rosen's investment as interest income at the end of the seven-year-period. Therefore, we seek to:

Max { Interest income in years 5, 6, and 7 } = Z
= { (Y5NO*IY5NO) + (Y6NO*IY6NO) + (Y7NO*IY7NO)
+ (Y5TAX*IY5TAX) + (Y6TAX*IY6YES) + (Y7TAX*IY7TAX)
+ (CDTAX*Y6CD) + (CDTAX*Y7CD) }

Using the simplex method built-in to Excel's Solver add-on, we find that all aforementioned requirements can be met to produce as maximum interest income of **DM 19,997.86**. This can be achieved by selling DM 12197.60 at the end of year 5 which is not taxed, DM 9604.79 at the end of year 6 which is not taxed, DM 18,297.56 of CDs at the end of year 6 which is taxed, DM 34,514.68 of CDs at the end of year 7 which is taxed, DM 7,797.52 at the end of year 7 which is not taxed, and DM 400.13 at the end of year 7 which is taxed. The constructed linear model can be found as an Excel spreadsheet is found in Figure 4 in Appendix C. Additionally, we generated an additional sensitivity output, of post-optimality analysis, which can be found in Appendix C as Figure 5.

Question (e): Optimal Solution – Initial Investment of DM 50,000

Per request of the Client, we found an optimal solution if Mr. Rosen had instead originally invested DM 50,000. Constraint 2 in Section 3.1 was adapted to the following:

2. **Total B-Bonds (TBB):** The sum of B-Bonds sold must equal initial investment of DM 50,000.
   Y5NO + Y5TAX + Y6NO + Y6TAX + Y7NO + Y7TAX = 50,000 [DM]

Using the simplex method built-in to Excel's Solver add-on, we find that all aforementioned requirements can be met to produce as maximum interest income of **DM 30,950.06**. This can be achieved by selling DM 12197.60 at the end of year 5 which is not taxed, DM 9604.79 at the end of year 6 which is not taxed, DM 18,297.56 of CDs at the end of year 6 which is taxed, DM 34,514.68 of CDs at the end of year 7 which is taxed, DM 7,797.52 at the end of year 7 which is not taxed, and DM 20400.13 at the end of year 7 which is taxed. The constructed linear model can be found as an Excel spreadsheet is found in Figure 6 in Appendix C. Furthermore, we generated an additional sensitivity output, of post-optimality analysis, which can be found in Appendix C as Figure 7.

Question (f): Optimal Solution – Savings Earned if Married

We considered what effect Mr. Rosen's wedding would have if he gets married at the end of this year. We formulated our model by setting the tax-free amount of interest earnings each year to DM 12,000. To illustrate, the following constraints in Section 3.1 were adapted to the following:

3. **Year-5 Tax (Y5T):** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 50.01% return, must be less than 12,200. The calculation for this constraint can be found in Appendix A.
   $Y5NO*(0.5001) \leq 12,200$ [DM]

4. **Year-6 Tax (Y6T):** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 63.51% return, must be less than 12,200. The calculation for this constraint can be found in Appendix A.
   $Y6NO*(0.6351) \leq 12,200$ [DM]

5. **Year-7 Tax (Y7T):** The income interest accumulated from non-taxed portions of B-Bonds sold, at a rate of 78.23% return, must be less than 12,200. The calculation for this constraint can be found in Appendix A.
   $Y7NO*(0.7823) \leq 12,200$ [DM]

Using the simplex method built-in to Excel's Solver add-on, we find that all aforementioned requirements can be met to produce as maximum interest income of DM 22,008.09. Therefore, Mr. Rosen would make a profit of **DM 2010.23**, as calculated in Appendix A. This can be achieved by selling DM 14,404.96 at the end of year 6 which is not taxed, DM 23,553.55 of CDs at the end of year 7 which is taxed, and DM 15,595.04 at the end of year 7 which is not taxed. The constructed linear model can be found as an Excel spreadsheet is found in Figure 8 in Appendix C. Moreover, we generated an additional sensitivity output, of post-optimality analysis, which can be found in Appendix C as Figure 9.

Question (g): Optimal Solution – Investments in American Bonds in Year 6

The Client tasked us with determining whether he should invest in American municipal funds at the beginning of this year which pay back with a 3.60% interest rate. The Client reported from his investment bank that the US dollars to DM exchange rate will increase from 1:1.5 in the fifth year of analysis to 1:1.8 in the seventh year. As per requested by the Client, analyzed the option of investing in the American municipal funds, and recommend how much of the Client's B-Bond should be sold in the fifth year for the investment in American municipal funds, so that Client's profit is maximized.

The aforementioned consideration led us to a modified optimization model based on the American municipal fund's interest rate and USD to DM exchange rate. The additional decision variables and constraints are presented below:

Decision Variables: In addition to the model formulated in Section 3.1, let:

USMB	denote the US municipal bonds invested (in USD) at the beginning of year 6.	[USD]
US	denote the US dollars that the Client can convert from the income of selling B-Bonds in years 6 and 7.	[USD]

Constraints: The following constraints, in addition to those formulated in Section 3.1, define the linear model to find the optimal solution:

8. **US Municipal Bonds**: Mr. Rosen can invest in US municipal bonds with the income from selling B-Bonds at end of year 5.

   USMB =[(Y5NO*(1+IY5NO)) + (Y5TAX*(1+IY5TAX))] / 1.5 [USD]

9. **Total US dollars converted**: convert the total income from selling B-Bonds at year 6 and 7 to US Dollars.

   US =[(Y6NO*(1+IY6NO)+(Y7NO*(1+IY7NO)+Y6TAX*(1+IY6YES)+Y7TAX*(1+IY7TAX)]/1.8 [USD]

Objective Function:

The objective is to maximize the return of the Client's investment as interest income at the end of the seven-year-period. Therefore, we seek to:

$$\textbf{Max } \{ \textbf{USMB}*I_{USMB} + \textbf{US} \} = \textbf{Z1} \quad \text{[USD]}$$

where:

I_{USMB} denotes the interest rate for investing in American municipal bonds for two years.

$I_{USMB} = 1.036^2 = 1.073296$

Using the simplex method built-in to Excel's Solver add-on, we find that all aforementioned requirements can be met to produce as maximum interest income of **USD 30478.23**. This can be achieved by selling DM 12197.60 at the end of year 5 which is not taxed, DM 10004.9 at the end of year 5 which is taxed, DM 7707.5201 at the end of year 7 which is not taxed, invest 21203.268 USD at the beginning of year six, convert total of USD 7720.84 at the end of year 7. The constructed linear model can be found as an Excel spreadsheet is found in Figure 10 in Appendix C. Conjointly, we generated an additional sensitivity output which can be found in Appendix C as Figure 11.

4 Appendices

Appendix A. Cost Conversions

Interest values, based off of Table 2, were calculated as follows:

IY5NO = (150.01/100)*100% = 50.01% [%]

IY5TAX = I5YNO*(0.7) = 35.007% [%]

IY6NO = (163.51/100)*100% = 63.51% [%]

IY6TAX = I6YNO*(0.7) = 44.457% [%]

IY7NO = (178.23/100)*100% = 78.23% [%]

IY7TAX = I7YNO*(0.7) = 54.761% [%]

Interest rate of certificate of deposit after the 30% capital gains tax is applied:

CDTAX = (0.04)*(0.7) = 2.80% [DM]

Mr. Rosen's potential increase in interest income if he gets married in year 5:

22,008.09 – 19,997.86 = 2010.23 [DM]

Appendix B. Representation of Model in Excel with the Optimal Solution

FIGURE 1: Maximum interest income of DM 19,098.62 without considering certificates of deposit or US municipal funds.

DATA:				Constraints:				Slack:
Year	Interest Rate	Annual % Yield	DM					
1	0.075	0.075	107.5	1. NN	All 6 Decision Variables	≥	0	
2	0.085	0.08	116.64	2. TBB	30000	=	30000	
3	0.085	0.0817	126.55	3. Y5T	0	≤	6100	
4	0.0875	0.0831	137.62	4. Y6T	6100	≤	6100	
5	0.09	0.0845	150.01	5. Y7T	6100	≤	6100	
6	0.09	0.0854	163.51					
7	0.09	0.0861	178.23					

Benefits:	IY5NO	IY5TAX	IY6NO	IY6TAX	IY7NO	IY7TAX
Interest Rate	0.5001	0.35007	0.6351	0.44457	0.7823	0.54761
Interest Limit	6100	0	6100	0	6100	0

MODEL:							
Decision Variables	Y5NO	Y5TAX	Y6NO	Y6TAX	Y7NO	Y7TAX	
values	0	0	9604.7866	0	7797.52	12598	
units	DM	DM	DM	DM	DM	DM	
Objective Function:	max { (Y5NO*IY5NO) + (IY5TAX*Y5TAX)+ (IY6NO*Y6NO) + (IY6TAX*Y6TAX) + (Y7NO*IY7NO) + (IY7TAX*Y7TAX) }						
Decision Variables	0	0	6100	0	6100	6898.6	
Overhead							
Maximize Profit					19098.62	$	

FIGURE 2: Sensitivity report corresponding to producing a maximum interest income of DM 19,098.62 without considering certificates of deposit or US municipal funds.

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [Assignment 3 - Case 12.2 - International Investment.xlsx]Question 1-b
Report Created: 9/25/2016 11:08:59 PM

Variable Cells

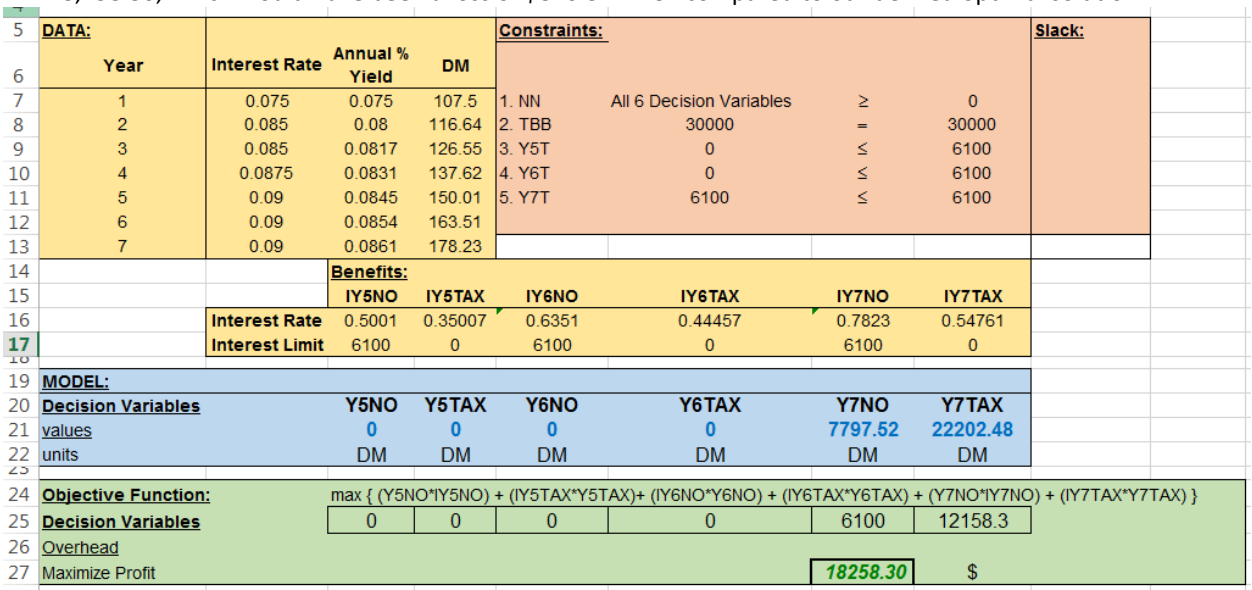
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C21	values Y5NO	0	-0.04751	0.5001	0.04751	1E+30
D21	values Y5TAX	0	-0.19754	0.35007	0.19754	1E+30
E21	values Y6NO	9604.786648	0	0.6351	1E+30	0.08749
F21	values Y6TAX	0	-0.10304	0.44457	0.10304	1E+30
G21	values Y7NO	7797.520133	0	0.7823	1E+30	0.23469
H21	values Y7TAX	12597.69322	0	0.54761	0.08749	0.04751

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F8	2. TBB All 6 Decision Variables	30000	0.54761	30000	1E+30	12597.69322
F9	3. Y5T All 6 Decision Variables	0	0	6100	1E+30	6100
F10	4. Y6T All 6 Decision Variables	6100	0.137757833	6100	8000.794964	6100
F11	5. Y7T All 6 Decision Variables	6100	0.3	6100	9855.175405	6100

Appendix C. Variants of the Original Model

FIGURE 3: Had Mr. Rosen followed his investment analyst's advice, he would have made an interest income of DM 18,258.30, which would have been a loss of $840.32 when compared to our derived optimal solution.



DATA: Year	Interest Rate	Annual % Yield	DM
1	0.075	0.075	107.5
2	0.085	0.08	116.64
3	0.085	0.0817	126.55
4	0.0875	0.0831	137.62
5	0.09	0.0845	150.01
6	0.09	0.0854	163.51
7	0.09	0.0861	178.23

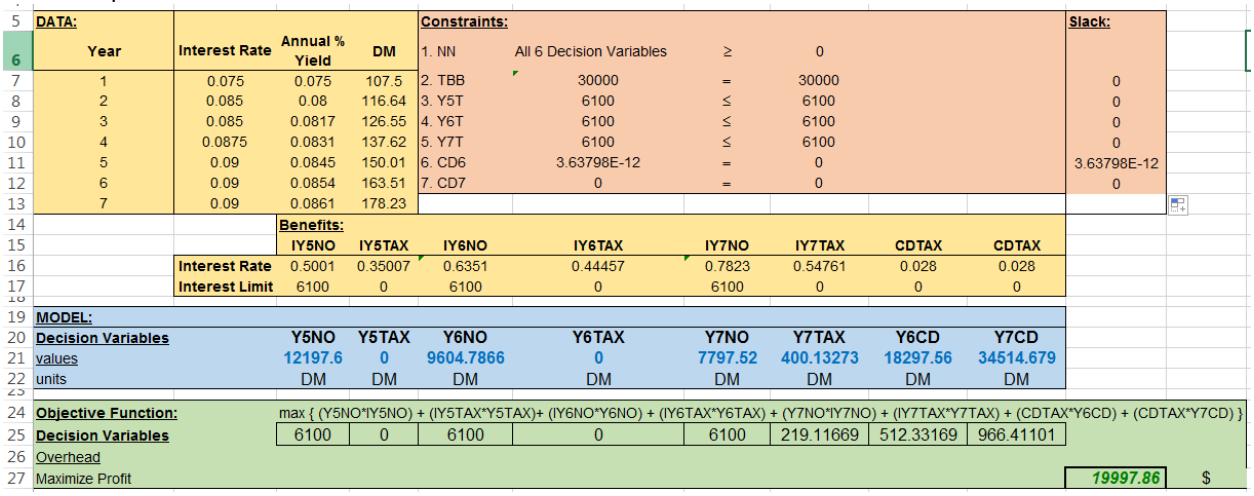
Constraints:				Slack:
1. NN	All 6 Decision Variables	≥	0	
2. TBB	30000	=	30000	
3. Y5T	0	≤	6100	
4. Y6T	0	≤	6100	
5. Y7T	6100	≤	6100	

Benefits:	IY5NO	IY5TAX	IY6NO	IY6TAX	IY7NO	IY7TAX
Interest Rate	0.5001	0.35007	0.6351	0.44457	0.7823	0.54761
Interest Limit	6100	0	6100	0	6100	0

MODEL: Decision Variables	Y5NO	Y5TAX	Y6NO	Y6TAX	Y7NO	Y7TAX
values	0	0	0	0	7797.52	22202.48
units	DM	DM	DM	DM	DM	DM

Objective Function:	max { (Y5NO*IY5NO) + (IY5TAX*Y5TAX)+ (IY6NO*Y6NO) + (IY6TAX*Y6TAX) + (Y7NO*IY7NO) + (IY7TAX*Y7TAX) }					
Decision Variables	0	0	0	0	6100	12158.3
Overhead						
Maximize Profit					18258.30	$

FIGURE 4: Maximum interest income of DM 19,997.86 is gained when considering certificates of deposit but not US municipal funds.



DATA: Year	Interest Rate	Annual % Yield	DM
1	0.075	0.075	107.5
2	0.085	0.08	116.64
3	0.085	0.0817	126.55
4	0.0875	0.0831	137.62
5	0.09	0.0845	150.01
6	0.09	0.0854	163.51
7	0.09	0.0861	178.23

Constraints:				Slack:
1. NN	All 6 Decision Variables	≥	0	
2. TBB	30000	=	30000	0
3. Y5T	6100	≤	6100	0
4. Y6T	6100	≤	6100	0
5. Y7T	6100	≤	6100	0
6. CD6	3.63798E-12	=	0	3.63798E-12
7. CD7	0	=	0	0

Benefits:	IY5NO	IY5TAX	IY6NO	IY6TAX	IY7NO	IY7TAX	CDTAX	CDTAX
Interest Rate	0.5001	0.35007	0.6351	0.44457	0.7823	0.54761	0.028	0.028
Interest Limit	6100	0	6100	0	6100	0	0	0

MODEL: Decision Variables	Y5NO	Y5TAX	Y6NO	Y6TAX	Y7NO	Y7TAX	Y6CD	Y7CD
values	12197.6	0	9604.7866	0	7797.52	400.13273	18297.56	34514.679
units	DM	DM	DM	DM	DM	DM	DM	DM

Objective Function:	max { (Y5NO*IY5NO) + (IY5TAX*Y5TAX)+ (IY6NO*Y6NO) + (IY6TAX*Y6TAX) + (Y7NO*IY7NO) + (IY7TAX*Y7TAX) + (CDTAX*Y6CD) + (CDTAX*Y7CD) }							
Decision Variables	6100	0	6100	0	6100	219.11669	512.33169	966.41101
Overhead								
Maximize Profit								19997.86 $

FIGURE 5: Sensitivity report corresponding to producing a maximum interest income of DM 19,997.86 when considering certificates of deposit but not US municipal funds.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C21	values Y5NO	12197.56049	0	0.5001	1E+30	0.037671678
D21	values Y5TAX	0	-0.120877625	0.35007	0.120877625	1E+30
E21	values Y6NO	9604.786648	0	0.6351	1E+30	0.1332728
F21	values Y6TAX	0	-0.06259204	0.44457	0.06259204	1E+30
G21	values Y7NO	7797.520133	0	0.7823	1E+30	0.23469
H21	values Y7TAX	400.1327314	0	0.54761	0.037671678	0.06259204
I21	values Y6CD	18297.56049	0	0.028	0.089534339	0.025112778
J21	values Y7CD	34514.67883	0	0.028	0.043329184	0.024428772

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F11	6. CD6 All 6 Decision Variables	3.63798E-12	-0.056784	0	18297.56049	1E+30
F12	7. CD7 All 6 Decision Variables	0	-0.028	0	34514.67883	1E+30
F7	2. TBB All 6 Decision Variables	30000	0.54761	30000	1E+30	400.1327314
F8	3. Y5T All 6 Decision Variables	6100	0.075328291	6100	200.106379	6100
F9	4. Y6T All 6 Decision Variables	6100	0.209845379	6100	254.1242977	6100
F10	5. Y7T All 6 Decision Variables	6100	0.3	6100	313.0238358	6100

FIGURE 6: Had the Client initially invested DM 50,000, a maximum interest income of DM 30,950.06 would be gained when considering certificates of deposit but not US municipal funds.

DATA: Year	Interest Rate	Annual % Yield	DM	Constraints:				Slack:
				1. NN	All 6 Decision Variables	≥	0	
1	0.075	0.075	107.5	2. TBB	50000	=	50000	0
2	0.085	0.08	116.64	3. Y5T	6100	≤	6100	0
3	0.085	0.0817	126.55	4. Y6T	6100	≤	6100	0
4	0.0875	0.0831	137.62	5. Y7T	6100	≤	6100	0
5	0.09	0.0845	150.01	6. CD6	3.63798E-12	=	0	3.63798E-12
6	0.09	0.0854	163.51	7. CD7	0	=	0	0
7	0.09	0.0861	178.23					

Benefits:	IY5NO	IY5TAX	IY6NO	IY6TAX	IY7NO	IY7TAX	CDTAX	CDTAX
Interest Rate	0.5001	0.35007	0.6351	0.44457	0.7823	0.54761	0.028	0.028
Interest Limit	6100	0	6100	0	6100	0	0	0

MODEL:								
Decision Variables	Y5NO	Y5TAX	Y6NO	Y6TAX	Y7NO	Y7TAX	Y6CD	Y7CD
values	12197.6	0	9604.7866	0	7797.52	20400.133	18297.56	34514.679
units	DM	DM	DM	DM	DM	DM	DM	DM

Objective Function:	max { (Y5NO*IY5NO) + (IY5TAX*Y5TAX)+ (IY6NO*Y6NO) + (IY6TAX*Y6TAX) + (Y7NO*IY7NO) + (IY7TAX*Y7TAX) + (CDTAX*Y6CD) + (CDTAX*Y7CD) }							
Decision Variables	6100	0	6100	0	6100	11171.317	512.33169	966.41101
Overhead								
Maximize Profit							30950.06	$

FIGURE 7: Sensitivity report corresponding to if the Client initially invested DM 50,000 to produce a maximum interest income of DM 30,950.06 when considering certificates of deposit but not US municipal funds.

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C21	values Y5NO	12197.56049	0	0.5001	1E+30	0.037671678
D21	values Y5TAX	0	-0.120877625	0.35007	0.120877625	1E+30
E21	values Y6NO	9604.786648	0	0.6351	1E+30	0.1332728
F21	values Y6TAX	0	-0.06259204	0.44457	0.06259204	1E+30
G21	values Y7NO	7797.520133	0	0.7823	1E+30	0.23469
H21	values Y7TAX	20400.13273	0	0.54761	0.037671678	0.06259204
I21	values Y6CD	18297.56049	0	0.028	0.089534339	0.025112778
J21	values Y7CD	34514.67883	0	0.028	0.043329184	0.024428772

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F11	6. CD6 All 6 Decision Variables	3.63798E-12	-0.056784	0	18297.56049	1E+30
F12	7. CD7 All 6 Decision Variables	0	-0.028	0	34514.67883	1E+30
F7	2. TBB All 6 Decision Variables	50000	0.54761	50000	1E+30	20400.13273
F8	3. Y5T All 6 Decision Variables	6100	0.075328291	6100	10202.10638	6100
F9	4. Y6T All 6 Decision Variables	6100	0.209845379	6100	12956.1243	6100
F10	5. Y7T All 6 Decision Variables	6100	0.3	6100	15959.02384	6100

FIGURE 8: If the Client marries this year, and he initially invested DM 30,000, a maximum interest income of DM 22,008.09 would be gained when considering certificates of deposit but not US municipal funds.

DATA: Year	Interest Rate	Annual % Yield	DM
1	0.075	0.075	107.5
2	0.085	0.08	116.64
3	0.085	0.0817	126.55
4	0.0875	0.0831	137.62
5	0.09	0.0845	150.01
6	0.09	0.0854	163.51
7	0.09	0.0861	178.23

Constraints:				Slack:
1. NN	All 6 Decision Variables	≥	0	
2. TBB	30000	=	30000	0
3. Y5T	0	≤	12200	-12200
4. Y6T	9148.589927	≤	12200	-3051.410073
5. Y7T	12200	≤	12200	0
6. CD6	0	=	0	0
7. CD7	7.27596E-12	=	0	7.27596E-12

Benefits:	IY5NO	IY5TAX	IY6NO	IY6TAX	IY7NO	IY7TAX	CDTAX	CDTAX
Interest Rate	0.5001	0.35007	0.6351	0.44457	0.7823	0.54761	0.028	0.028
Interest Limit	6100	0	6100	0	6100	0	0	0

MODEL: Decision Variables	Y5NO	Y5TAX	Y6NO	Y6TAX	Y7NO	Y7TAX	Y6CD	Y7CD
values	0	0	14404.96	0	15595.04	0	0	23553.55
units	DM	DM	DM	DM	DM	DM	DM	DM

Objective Function:	max { (Y5NO*IY5NO) + (IY5TAX*Y5TAX)+ (IY6NO*Y6NO) + (IY6TAX*Y6TAX) + (Y7NO*IY7NO) + (IY7TAX*Y7TAX) + (CDTAX*Y6CD) + (CDTAX*Y7CD) }							
Decision Variables	0	0	9148.5899	0	12200	0	0	659.49939

Overhead

Maximize Profit: 22008.09 $

FIGURE 9: Sensitivity report corresponding to if the Client marries this year, and he initially invested DM 30,000, a maximum interest income of DM 22,008.09 would be gained when considering certificates of deposit but not US municipal funds

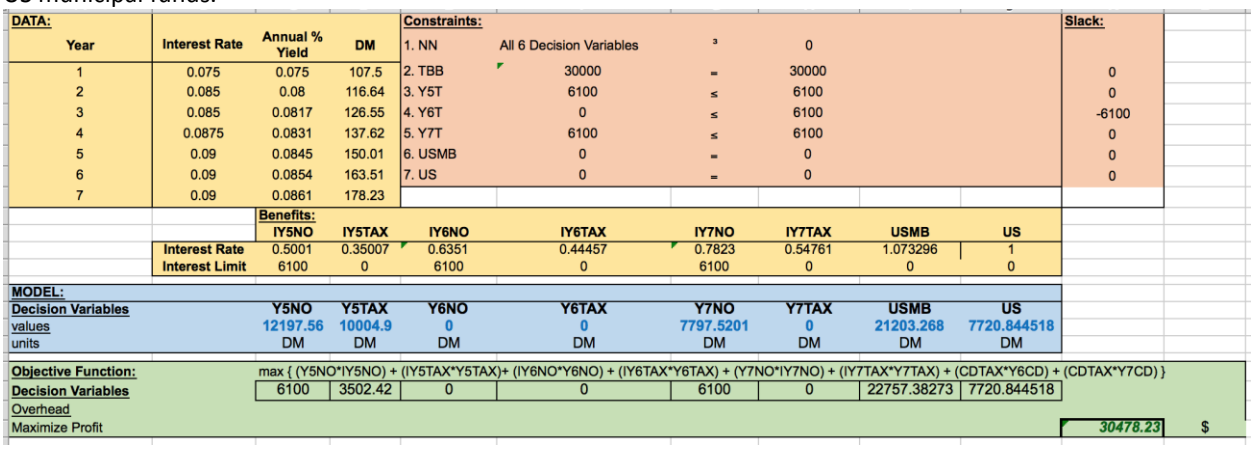
Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C21	values Y5NO	0	0	0.5001	0.095601122	0.186792423
D21	values Y5TAX	0	-0.16811069	0.35007	0.16811069	1E+30
E21	values Y6NO	14404.95973	0	0.6351	0.1014172	0.095601122
F21	values Y6TAX	0	-0.19586484	0.44457	0.19586484	1E+30
G21	values Y7NO	15595.04027	0	0.7823	1E+30	0.1014172
H21	values Y7TAX	0	-0.1332728	0.54761	0.1332728	1E+30
I21	values Y6CD	0	-0.063729832	0.028	0.063729832	1E+30
J21	values Y7CD	23553.54966	0	0.028	0.062025075	0.081507431

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F11	6. CD6 All 6 Decision Variables	0	-0.120513832	0	21608.8801	0
F12	7. CD7 All 6 Decision Variables	7.27596E-12	-0.028	0	23553.54966	1E+30
F7	2. TBB All 6 Decision Variables	30000	0.6808828	30000	4804.613561	14404.95973
F8	3. Y5T All 6 Decision Variables	0	0	12200	1E+30	12200
F9	4. Y6T All 6 Decision Variables	9148.589927	0	12200	1E+30	3051.410073
F10	5. Y7T All 6 Decision Variables	12200	0.12963978	12200	11269	3758.649189

FIGURE 10: Maximum interest income of USD 30,478.23 is gained when considering certificates of deposit and US municipal funds.



DATA: Year	Interest Rate	Annual % Yield	DM
1	0.075	0.075	107.5
2	0.085	0.08	116.64
3	0.085	0.0817	126.55
4	0.0875	0.0831	137.62
5	0.09	0.0845	150.01
6	0.09	0.0854	163.51
7	0.09	0.0861	178.23

Constraints:				Slack:
1. NN	All 6 Decision Variables	³	0	
2. TBB	30000	=	30000	0
3. Y5T	6100	≤	6100	0
4. Y6T	0	≤	6100	-6100
5. Y7T	6100	≤	6100	0
6. USMB	0	=	0	0
7. US	0	=	0	0

Benefits:	IY5NO	IY5TAX	IY6NO	IY6TAX	IY7NO	IY7TAX	USMB	US
Interest Rate	0.5001	0.35007	0.6351	0.44457	0.7823	0.54761	1.073296	1
Interest Limit	6100	0	6100	0	6100	0	0	0

MODEL: Decision Variables	Y5NO	Y5TAX	Y6NO	Y6TAX	Y7NO	Y7TAX	USMB	US
values	12197.56	10004.9	0	0	7797.5201	0	21203.268	7720.844518
units	DM	DM	DM	DM	DM	DM	DM	DM

Objective Function:	max { (Y5NO*IY5NO) + (IY5TAX*Y5TAX)+ (IY6NO*Y6NO) + (IY6TAX*Y6TAX) + (Y7NO*IY7NO) + (IY7TAX*Y7TAX) + (CDTAX*Y6CD) + (CDTAX*Y7CD) }							
Decision Variables	6100	3502.42	0	0	6100	0	22757.38273	7720.844518
Overhead								
Maximize Profit								30478.23 $

FIGURE 11: Sensitivity report corresponding to maximum interest income of USD 30,478.23 is gained when considering certificates of deposit and US municipal funds

Microsoft Excel 14.6 Sensitivity Report
Worksheet: [Assignment 3 - Case 12.2 - International Investment.xlsx]Question 1-g
Report Created: 9/27/2016 4:28:28 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C21	values Y5NO	12197.56049	0	0	1E+30	0.107351066
D21	values Y5TAX	10004.91938	0	0	0.02415018	0.057627598
E21	values Y6NO	0	-0.057627598	0	0.057627598	1E+30
F21	values Y6TAX	0	-0.163477598	0	0.163477598	1E+30
G21	values Y7NO	7797.520133	0	0	1E+30	0.02415018
H21	values Y7TAX	0	-0.106233154	0	0.106233154	1E+30
I21	values USMB	21203.268	0	1.073296	0.026832142	0.064027345
J21	values US	7720.844518	0	1	0.063439347	0.024390015

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F11	6. USMB All 6 Decision Variables	0	-1.073296	0	21203.268	1E+30
F12	7. US All 6 Decision Variables	0	-1	0	7720.844518	1E+30
F7	2. TBB All 6 Decision Variables	30000	0.966016487	30000	1E+30	10004.91938
F8	3. Y5T All 6 Decision Variables	6100	0.2146592	6100	5003.460182	6100
F9	4. Y6T All 6 Decision Variables	0	0	6100	1E+30	6100
F10	5. Y7T All 6 Decision Variables	6100	0.03087074	6100	7826.84843	6100