PRATHAM

IIT BOMBAY STUDENT SATELLITE

Preliminary Design Report

Payload

By

Ashish Goel

Subhasis Das

Abhinav Mittal

Srujan M

Department of Aerospace Engineering,

Indian Institute of Technology, Bombay

April, 2009

Table of Contents

1. Introduction

TEC is the Total Electron Count of the Ionosphere. It refers to the total number of electrons in a cylinder of unit area of cross section extending from the ground station up to our satellite in space. TEC values are one of the most prominent sources of information for understanding the structure and dynamic behavior of the ionosphere. However, since there is a fraction of the ionosphere (about 5%) above the altitude of our satellite, what we wish to measure is known as the Ionosphere Electron Count.

2. Importance of TEC

2.1 Error correction for GPS systems

These days GPS satellites are used on many fronts and their use for position determination has proved to be important and useful for not only navigation but also several other research applications. But in order to achieve a high level of accuracy from GPS (of the order of millimetres), one must correct the carrier phase advance and pseudo range group delay that are caused when GPS signals pass through the ionosphere. After integrating the group and phase delay along the GPS signal path we obtain a range between satellite and receiver that is different from geometrical distance between them. This difference is known as ionospheric error. Error is negative for carrier phase (phase is advanced, measured range is shorter than the geometrical range) and positive for pseudo ranges (i.e. phase is delayed, so measured range is greater than geometrical range).These two are equal in magnitude but opposite in sign.[1]

If we have a dual frequency GPS receiver, we can then deal with this error by taking into account ionosphere's dispersive nature. But if we are using a single frequency GPS receiver then we need some other method for getting rid of this error.

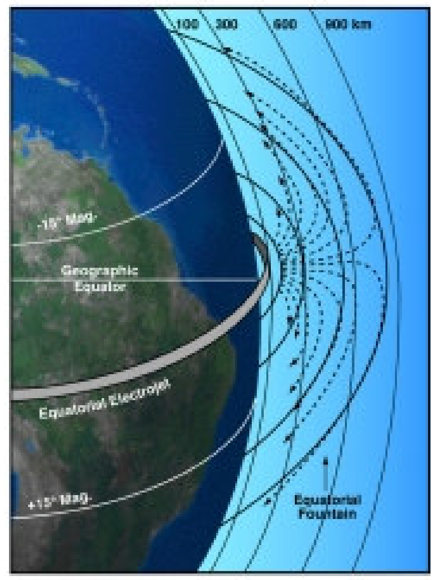
Most GPS companies provide an inbuilt GPS model that predicts TEC values on the basis of previous history sheets. But the fact that we are situated near the equator makes life a bit difficult for us because most of these models are based on data points obtained predominantly from mid latitude regions and have very sparse data of equatorial region. Further, the equatorial region is highly susceptible to fluctuations in TEC values due to various reasons. So these models fail to deliver a satisfactory performance. An increase in data points in this region can be used by GPS manufacturers for correcting this ionospheric error. The currently used global ionospheric models can only model the monthly mean total electron content to about ± 10 percent. These models cannot tell us about the day-to-day variability of the TEC which can be 20 to 25 percent (1 sigma) of the monthly mean value. So, even if the monthly mean TEC is modelled perfectly using one of the global ionospheric models, TEC predictions can be off by 20

[1] The Physics behind the ionospheric delay is explained in section 4.2

to 25 percent. If the bias between the monthly mean TEC and the predictions provided by these models is 10 percent of the monthly mean values, then the performances of these models are considered to be excellent. The combined effect of the error in the monthly mean predictions and the day-to-day variability of the TEC will result in an overall performance of these models to be at about 22-27 percent (RMS) of the ionospheric delay. The performance can only be worse if we take into account the potential effect of a solar or geomagnetic storm during medium or high solar activity times. We wish to do it by supplementing the current database and the ongoing Indian projects for TEC measurement with our TEC data.

2.2 Equatorial Ionization Anomaly (Appleton Anomaly):

EIA, or equatorial ionization anomaly is characterized by a depression in ionization densities (or trough) at the geomagnetic equator and two peaks (crests) on either side of the equator at about 15° magnetic latitudes.



Possible reasons:

It is suggested that the trough exists because plasma produced by photo ionization at great heights over the magnetic equator diffuses downwards and outwards to the north and south leaving depletion at the equator.

Another explanation is that the mutually perpendicular east-west electric field and north-south geomagnetic field give rise to an upward electrodynamic ($\mathbf{E} \times \mathbf{B}$) drift of plasma during the daytime. As the plasma is lifted to greater heights, it diffuses downward along geomagnetic field lines towards higher latitudes under the influence of gravity and pressure gradients and produces the anomaly. The anomaly crests in both hemispheres occur at lower altitudes and become weaker with height. The ratio of the electron density at the crest to the electron density at the trough is a measure of the intensity of the anomaly. The crest-to-trough ratio is maximum near the height of the F2 region peak and decreases both downward and upward.

The EIA is also asymmetric about the geomagnetic equator, caused by field aligned plasma flow due to factors like neutral winds.

An interesting feature in the geographic location of India is that the magnetic equator passes through the bottomside tip of the country and the northern crest of the equatorial ionization anomaly lies in the middle of the country, providing a unique opportunity for making studies on the latitudinal variation equatorial ionization anomaly (EIA) which occurs at 5 – 25 degrees geographic latitude. As Mumbai falls in this region, we can easily get involved with this experiment.

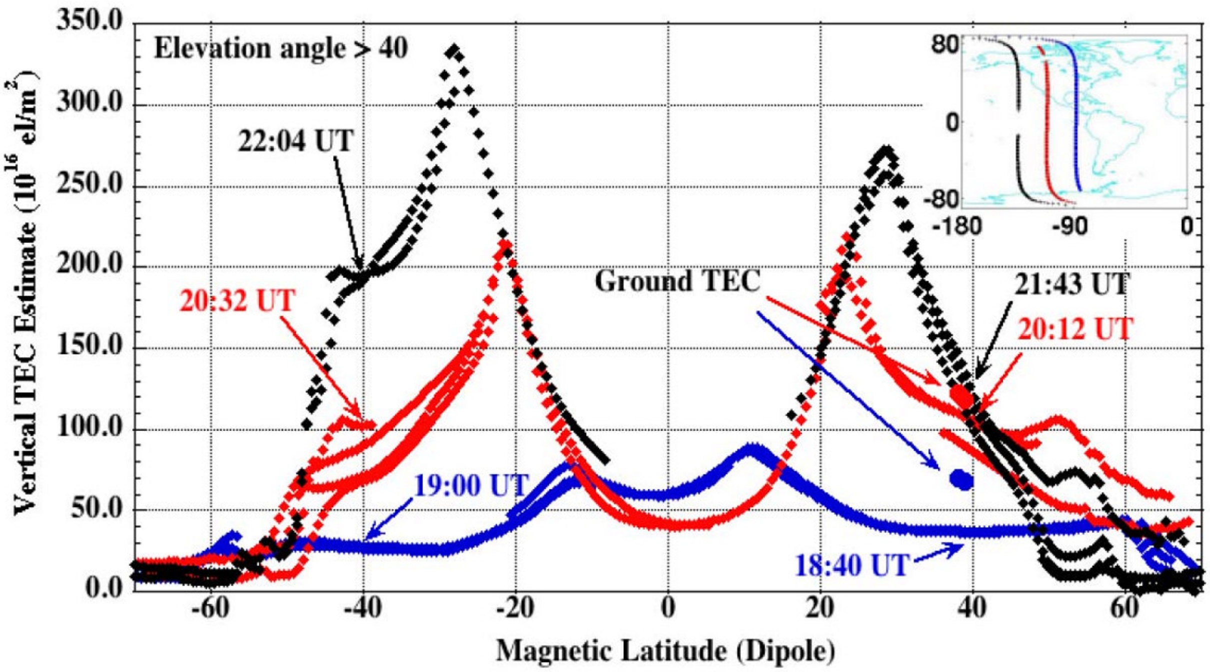


Figure 1 Equatorial Ionization Anomaly

2.3 ESF - Equatorial Spread F

Technically, ESF can be defined as,

"*The spread F/plasma bubble irregularity phenomenon results from vertical coupling process involving upward propagation of atmospheric waves (in the form of tides, gravity- and planetary waves) from the lower atmospheric regions of their origin to the dynamo region in the ionosphere where the electric fields are generated. The development of zonal (eastward) electric field enhancement in the evening hours, (that is, the prereversal electric field, PRE), by the F region dynamo is known to be the most basic requirement for the post sunset F-layer uplift as a precursor to the irregularity development.*"

On certain nights, the spread in the width of the pulses reflected from the F-layer increases abnormally, showing up as a diffuse trace, and this phenomenon is called Spread-F. ESF actually refers to formation of areas in ionosphere that have abnormally low plasma density. These bubbles are found to travel upwards in the ionosphere.

Location:

Usually ESF is not observed at higher latitudes. Its presence is felt at lower latitudes during late evening. It happens between 20 deg South and 20 deg North belt. Usually spread F persists only for few hours and on some occasions continues throughout the night. Sometimes, after the disappearance in the pre-midnight period, spread F reappears in the post-midnight period.

On the days of ESF occurrence, the F-layer near the magnetic equator becomes unstable around evening and as a consequence of this, some irregularities develop in the electric field and plasma densities with a wide range of scale sizes. These irregularities get aligned with the Earth's magnetic field lines forming sometimes severe density depletions called 'holes' or 'bubbles' (These are termed as plasma bubbles) .These plasma bubbles can be observed with the help of Radars operating in the VHF Range.

Link between EIA & ESF:

Studies have shown that ESF is closely linked with EIA.

EIA as we know is intensification of ionization crests in the late afternoon near +/- 20deg latitude. Formation of EIA is due to flow of plasma along a fountain from the equator towards the crest, which is caused due to Electric field generated in the atmosphere. This increase in ionization comes at the expense of depletion in ionization of lower F layer (near the equator). So we get very steep plasma gradients in the equatorial region, which are conducive to Rayleigh Taylor instability and this explains the relationship between EIA & ESF.

ESF is not completely understood:

The variations of ESF with longitude, season, and from day to day are not clearly understood. The altitude of ESF onset where the instability is initiated can vary over an altitude range of more than 100 km.

3. Dynamics of TEC variation

This randomness in the day-to-day variation in TEC may be attributed to the changes in the activity of the sun itself and to the associated changes in the intensity of the incoming radiations, and the zenith angle at which they fall on the Earth's atmosphere, in addition to the changes which take place in the Earth's magnetic field and the equatorial electrojet (EEJ) strength, added to the effects due to the dynamics of the neutral winds.

3.1 Diurnal Variations

A logical explanation for day night variation in TEC can be given as:-

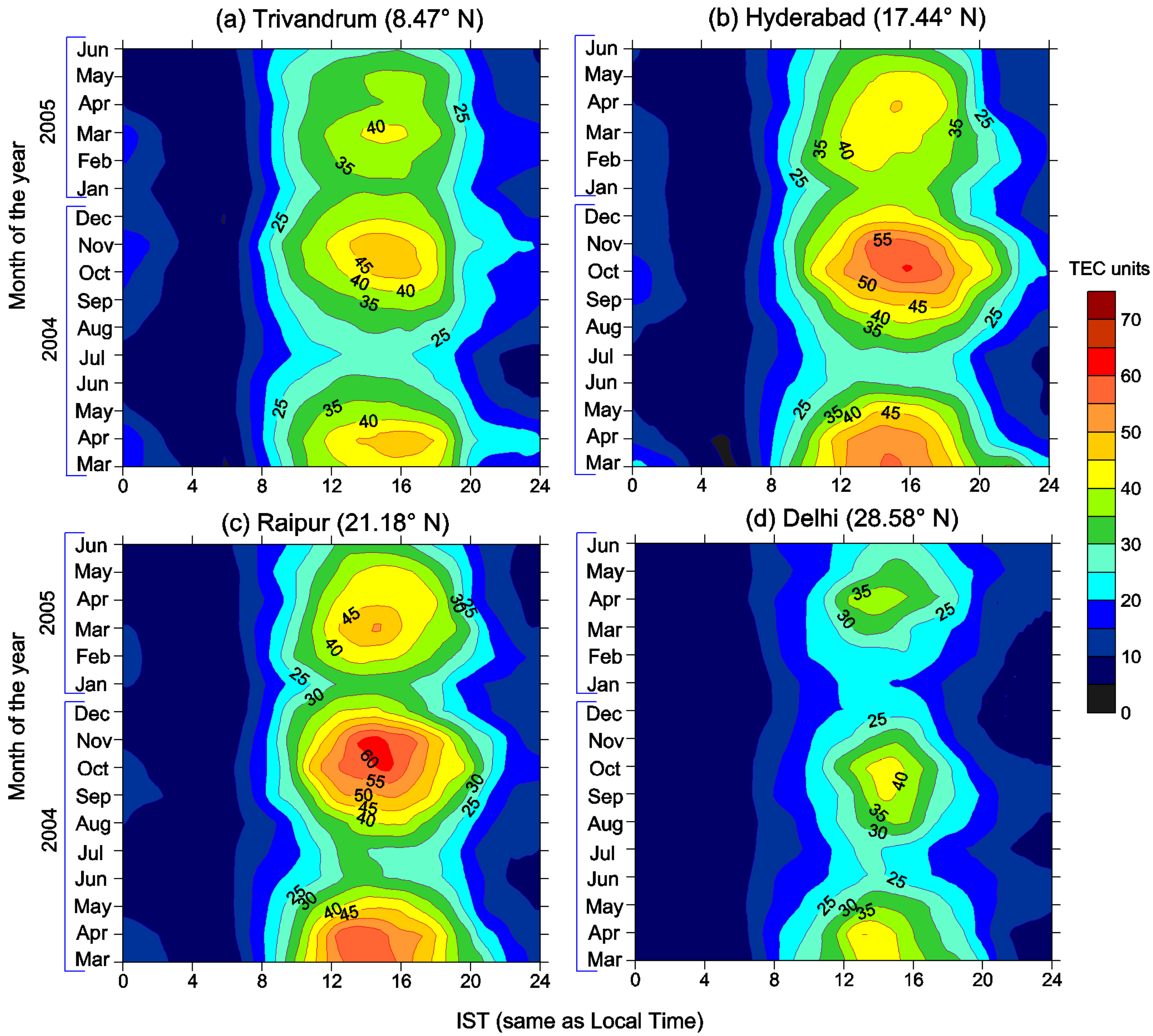
During night time, there is no solar radiation falling on the atmosphere, so there is nothing that is going to ionize the atoms present in the ionosphere. But the opposite process of recombination of ions takes place and so there is a decrease in ion density of atmosphere and hence reduced TEC is observed. During afternoon, when the solar radiation is maximum, ionization increases, overtaking the process of recombination of ions and hence TEC value increases.

3.2 Variation in equatorial regions:

The daily variation in TEC at the EIA region shows its steep increase and reaches its maximum value between 13:00 and 16:00 LT, while at the equator the peak is broad i.e. it occurs for a longer duration of time and occurs around 16:00 LT. A short-lived day minimum occurs between 05:00 to 06:00 LT at all the stations from the equator to the EIA crest region. Beyond the crest region, the day maximum values decrease with the increase in latitude. The day minimum in TEC is flat during most of the nighttime hours, i.e. from 22:00 to 06:00 LT. The diurnal variation in TEC show a minimum to maximum variation of about 5 to 50 TEC units, respectively, at the equator and about 5 to 90 TEC units at the EIA crest region.

3.3 Climatic variations in TEC:

Talking about the seasonal variations, TEC maximizes during the equinox months (equinox is that period of time when sun passes over equator and lengths of day and night become equal , this occurs on 21st March and 22nd September) followed by winter and is minimum during the summer months.



Contour plots of the monthly average diurnal variation of TEC at four stations across India (taken from the results of GPS Aided Geo Augmented Navigation (GAGAN) project of ISRO + AAI)

Having looked at the complicated dynamics of TEC variations, one must realize that the lack of complete understanding of the above phenomena often leads to navigational errors and communication losses. TEC measurements are necessary to improve our understanding of the interaction between the solar wind and the earth's ionosphere, which is very crucial for the functioning of several space borne and ground based systems and also to increase our capability for forecasting space weather events. Our venture will shed some light on some of the complex phenomena occurring in space.

4. Orbit and lifetime

The efficacy of our endeavor depends entirely on the orbit of our satellite. Most of the ionospheric phenomena occur at specific times of the day and we need the satellite to have a pass over the ground stations at those times for us to obtain the relevant TEC values.

A circular orbit with an inclination of approximately 24 degrees would render several passes at different times of the day and these passes would occur at different times of the day over the course of the lifetime of the satellite. However, this will end up mixing temporal and spatial variations since the passes will be nearly perpendicular to the longitudes. Hence, keeping tomography in mind, this orbit is not ideal for our mission.

A polar sun synchronous orbit would only allow us to measure the TEC values at the same local time. Given this limitation, a 2:30 am-pm orbit would be optimum for the satellite as it would allow us to measure the TEC values at a time when the EIA is at its peak.

Literature survey tells us that TEC measurements would require a period of around a couple of months for validation of data. Hence, the satellite would have a minimum lifetime of 3 months. However, the data generated would be comprehensive if the satellite operates for an entire year, so that we can capture the seasonal variations in TEC values.

5. Techniques for measurement of TEC from LEO satellite

The various techniques that can be used to measure TEC from LEO satellites are described below:

1. Measurement of Faraday rotation (our method)
2. Measurement of group delay of received signal
3. Measurement of Doppler shift of received signal due to ionospheric fluctuations
4. Measurement of amplitude scintillations of received signal.

Irrespective of the method used for measurement, current standards require electron content distribution (which will be determined from tomography discussed later) values up to an accuracy of 10-15%. This requires us to determine TEC upto an accuracy of 1TECU and the polarization angle upto an accuracy of 2degrees.

5.1 Measurement of Faraday rotation of polarization angle:

When a linearly polarized radio wave passes through an ionized medium with a magnetic field in the direction of propagation, the plane of polarization rotates. This effect is called Faraday rotation. The relation between the rotation angle and the TEC is given by:

$$\Delta\phi = 4.87\times10^{4}\times f^{-2}\int_{h_1}^{h_2} NB\cos\theta dl ,$$

where N = electron density

B = magnetic field of earth
Θ = angle between the radio wave and line of sight
Δφ = angle of rotation
f = frequency of the wave

Measurement process:

In order to radiate linearly polarized waves from the satellite, we plan to use a monopole antenna. We will be measuring the angle of polarization at the ground station by using a crossed Yagi antenna and measuring the intensities of the signals at the two feeds. The ratio of these intensities will give us the polarization angle.

Measurement of phase angle by AD8302:

We plan to measure relative intensities of the radio waves by an Analog Devices chip AD8302. This IC takes two inputs INPA and INPB and gives a voltage o/p proportional to the level ratio of these signals in dB. From this dB ratio (x) we can calculate the phase angle as:

$$\Delta\phi = \tan^{-1}(10^{\frac{x}{20}})$$

This chip gives ±0.3 dB error for magnitude ratios within ±20 dB.

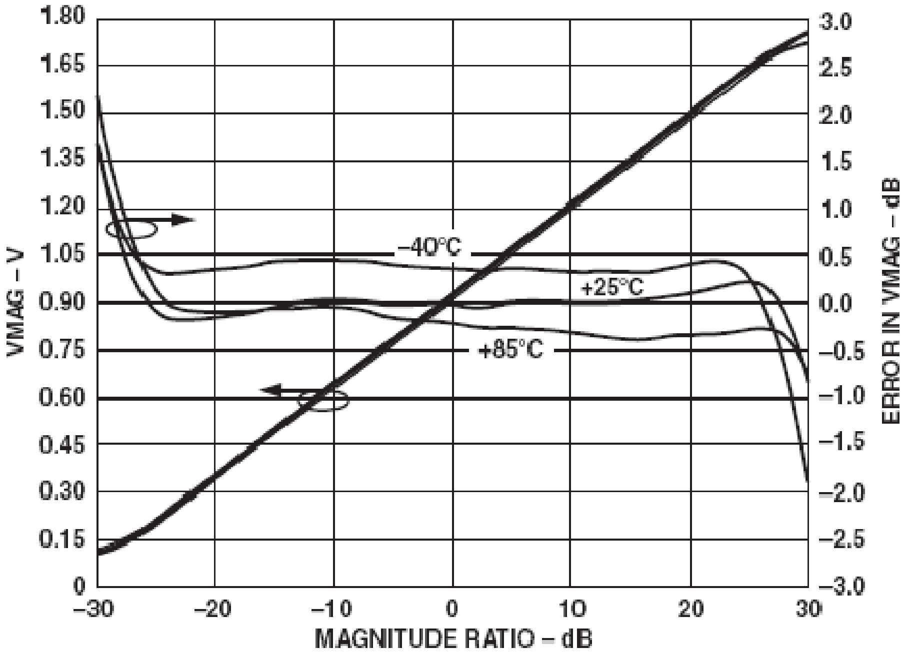


Figure 2: Voltage magnitude o/p and log conformance vs. input level ratio at frequency of 900 MHz, reference level = -30 dB

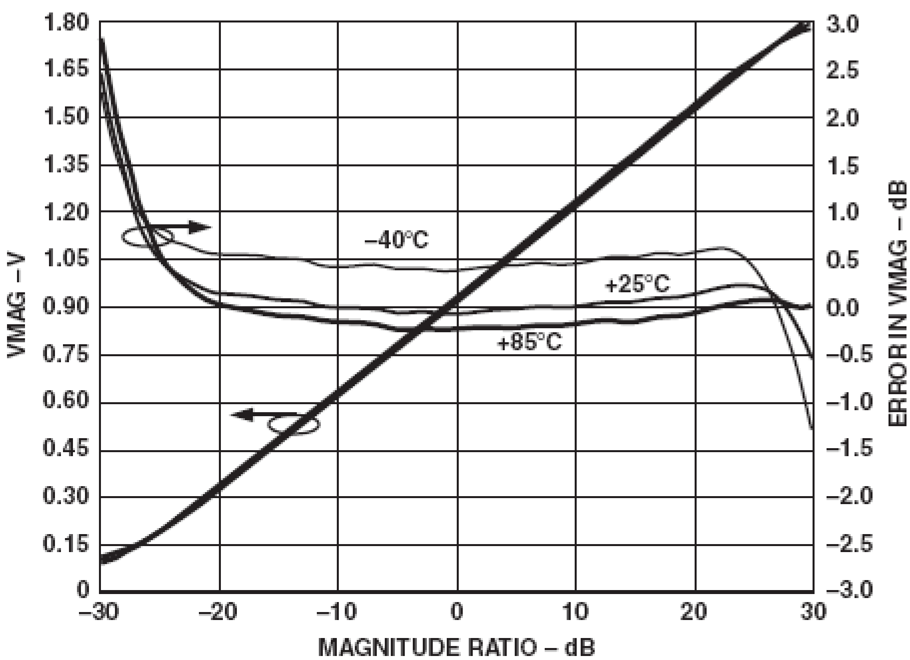


Figure 3: Voltage magnitude o/p and log conformance vs. input level ratio at frequency of 100 MHz, reference level = -30 dB

Thus, the maximum error in angle corresponding to this dB error will be ~ 1° which can give us an SNR of about 100 which will be quite good for measurements.

Resolution of the nπ ambiguity, use of two stations:

We will be using the two-station or the Leighteinjeir method in order to resolve the n-pi ambiguity in the measurement of the polarization angle at the ground stations. This method involves the use of readings at two ground stations simultaneously in order to determine the n-pi ambiguity at both the stations.

Here, we use the approximation that the ionospheric electron distribution is in a thin shell of uniform density. Obviously, this is a very bad approximation, but it is enough to determine the n-pi ambiguity to the nearest integer.

Under this approximation, we can write

$$\left(T_{i,1} + n_1\pi\right)\sin\theta_{i,1} = \left(T_{j,2} + n_2\pi\right)\sin\theta_{j,2} \quad \ldots(3)$$

where $T_{i,1}$ and $T_{j,2}$ are measurements taken at the same location

$\theta_{i,1}$ and $\theta_{j,2}$ are as shown in the figure

n_1 and n_2 are the ambiguities at the two stations

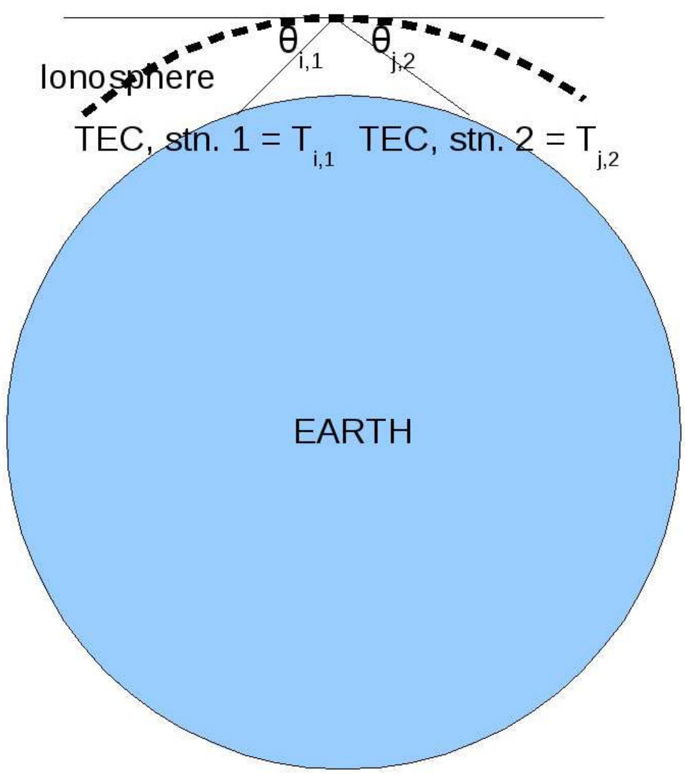


Figure 4 Leighteinjeir method

With several values of $T_{i,1}$ and $T_{j,2}$, we can fit a straight line to equation (3) for all values of i, j and we can then get least squares estimates of n1 and n2.

Simulations

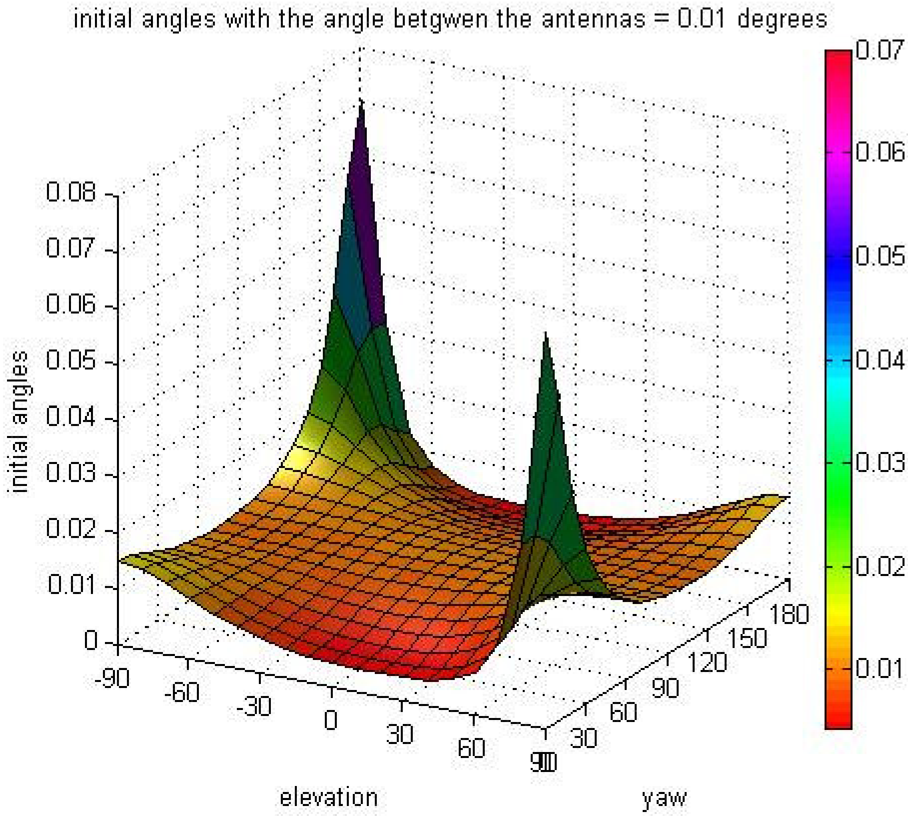
Simulations of this method performed with a standard model of the ionosphere yield values of n1 and n2 which are close to the actual values. Some typical results are tabulated in the following table.

Actual value		Simulated value	
N1	N2	N1	N2
0	1	-0.19	0.87
2	2	2.18	2.08

However, measuring the angle of rotation without any attitude data on ground will be impossible as the signals will have some initial polarization angle which will depend upon the yaw of the satellite. We hence propose to use two monopole antennae operating at two different frequencies. If we keep the initial angle of polarization for the two monopoles same (by keeping their orientation), we can eliminate the effect of the initial polarization angle by taking the difference of the polarization angles measured at the two frequencies at the ground station. To achieve this, we require the yaw angle of the satellite to be 90±5 degrees and the monopoles to remain parallel within 1 degree angle. These values come from the following analysis.

Misalignment of antennae

The initial angle between the two waves depends upon the angle between the antennae and the position of the ground station relative to the antennae. Thus, if the two antennae are not parallel, there will be some initial angle between the waves, which we can’t know unless we know the attitude of the satellite. This initial angle depends upon the yaw angle of the satellite and the elevation of the satellite above the horizon. The relation between them is brought out in the following graph.



Even if we initially place the monopoles within one degree of parallelism, severe distortions can occur in the shape and position of the monopoles at the time of deployment and during launch. As we can see from the graph, the error is minimum if the satellite satellite has a yaw angle of 90 degrees.

- **Polarization impurities and Faraday ellipticization of wave**
  The radio wave which will be received at the ground station will not be perfectly linearly polarized due to mainly two reasons:
  i. Polarization impurity of the monopole antenna:
     The wave emitted by the monopoles aboard the satellite will not be perfectly linearly polarized because the monopoles will have a finite radius and such factors. This will introduce a circularly polarized component into the waves and thus the measurements made at the ground station will be faulty. Considering the requirement of measuring polarization angle at the ground station upto an accuracy of 1degree, we would require the monopoles on the satellite to transmit radio waves with a polarization purity of 99.9%
  ii. Faraday ellipticization of wave:
     When a linearly polarized EM wave passes through a medium with a transverse magnetic field and plane of polarization neither parallel nor perpendicular to the magnetic field, another effect known as Faraday ellipticization also takes place, by which the EM wave becomes elliptically polarized. However, for the frequencies we are considering, this effect will not be so important. For a frequency of 433 MHz, the

minimum length needed to traverse in order to get an axial ratio of even 20dB (very weakly elliptically polarized) is of the order of 10000 km, which is far greater than what we will be experiencing (670 km).

Advantages
The advantages of Faraday Rotation technique are the following:
- Needs comparatively less hardware on board the satellite.
- Measurement is done only on carrier phase polarization angle which is an inherent property of the signal unlike a modulated signal where errors might arise.

Disadvantages
The disadvantages of this process are as follows:
- The signal transmitted from the satellite must have a high degree of polarization purity which is not so easy.
- The final result will not be directly proportional to the TEC but will depend upon the magnetic field along the path as well. Thus this dependence will have to be accounted for by some other way.
- The angle between the antennae will have to be very small.

5.2 Relative Group Delay of two Different Frequency Waves:

When a radio wave passes through the ionosphere, the group velocity of the wave is decreased due to the refractive index of the ionosphere. The relation between the group refractive index and the electron content of the ionosphere is given by:

$n_g = 1 + \frac{40.3 N_e}{f^2}$, where N_e=electron density (in el/m^3),

f=frequency of the wave (in Hz).

Now, the time delay in modulation phase is given by:

$$\Delta t = \frac{1}{c}\int_{h_1}^{h_2} (n-1)dl = \frac{40.3}{cf^2}\int_{h_1}^{h_2} N_e dl$$

Thus this delay is directly proportional to the TEC from the satellite to the ground station. Now, suppose we send two carrier waves with the same signal upon them, the information on one will arrive faster than the other due to the $1/f^2$ dependence of this delay on the frequency. By measuring this delay between the two signals, we can detect the TEC.

Disadvantages:
- The same signal has to be imposed on two different carrier frequencies simultaneously, which is difficult
- Phase measurement at the ground station is not an easy task.

Advantages:
- The obtained reading will be directly proportional to the TEC, which is very much preferable
- There are no stringent conditions on the antennae on board the satellite

5.3 Doppler Shift due to electron density fluctuations:

We know that, phase refractive index,

$$n_p = 1 - \frac{40.3N_e}{f^2}.$$

Thus, we have that the phase of the carrier wave will depend upon the TEC as,

$$\phi = \phi_0 + \Delta\phi,$$

where, $\Delta\phi = \frac{40.3}{cf}\int_{h_1}^{h_2} N_e dl$

But, frequency is simply the time derivative of phase. Thus, if TEC is dependent upon time, then we will have,

$$\begin{aligned} f &= \frac{d\phi}{dt} \\ &= \frac{d\phi_0}{dt} + \frac{40.3}{cf_0}\frac{d(TEC)}{dt} \\ &= f_0 + \frac{40.3}{cf_0}\frac{d(TEC)}{dt} \end{aligned}$$

Thus, there will be a shift in the frequency proportional to the change in the TEC. However, this technique is useful for GEO satellites only. In case of LEO satellites the term $\frac{d(TEC)}{dt}$ consists of two parts:

b. Due to the fast motion of the satellite relative to the ground station,
c. Ionospheric fluctuations.

The typical magnitudes of the former are about 10 TECU/min whereas that of the latter is about 0.1 TECU/min. Thus, in case of LEO satellites the former factor always swamps out the latter. Thus, this method is of use only in case of GEO satellites where the former factor is absent due to the orbit of the satellite.

5.4 Amplitude and Phase Scintillations of Received Signal:

The electron density of the ionosphere is not homogenous but rather it has both fine scale and large scale irregularities. These irregularities cause an EM wave to face a diffraction grating type structure of the ionosphere and as a result, when they emerge from the ionosphere, they have spatial amplitude as well as phase scintillations, just like a diffraction pattern. As the satellite passes over the ground station, these scintillations sweep over the station and as a result, we get some noisy picture. These can then be used to determine the scale of the irregularities.

5.5 Method and Frequencies Chosen

After analyzing all the techniques mentioned above, the Payload team decided to go ahead with Faraday Rotation for measuring the Total Electron Content of the Ionosphere. 150MHz and 437MHz were chosen as the operating frequencies since the expression for the Faraday Rotation angle clearly shows an inverse relationship with the square of the frequency. The following is a summary of the various requirements for this mission, as discussed above.

1) Polarization purity of the signal from the monopoles to be better than 99.9%
2) Determination of position of the satellite at the ground station up to an accuracy of 1km
3) Measurement of signal strength at the ground station with an accuracy better than 0.3dB
4) Maintenance of yaw angle within 90±5 degrees
5) Maintenance of parallelism between the monopoles better than 1 degree

6. Simulations

Here we describe the simulations for determining the value of Faraday rotation of polarization due to a typical ionosphere. We know that the Faraday rotation of a linearly polarized wave propagating through the ionosphere is given by:

$$\Delta\varphi = 4.72\times 10^{4} f^{-2} B\cos\theta \int_{h_1}^{h_2} N dl$$

(where B $\cos\theta$=magnetic field component in the direction of propagation of the wave,

N = electron density,

f=frequency of the wave) ⋯ (1)

To model this rotation, we assumed a standard model of the ionosphere available from http://ccmc.gsfc.nasa.gov/modelweb/models/iri_vitmo.php.

The magnetic field values were obtained from http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp.

We assumed the magnetic field to be constant in magnitude and direction at all the places. This is not a good approximation but can give us a good idea about the order of magnitudes of the values. For example, the values of magnetic field components at 300 km over Mumbai are (32674.2 nT, -506.3 nT, 16230.9 nT) and those at 10° to the north are (29040.5 nT, 410.4 nT, 28548.4 nT) while those at 10° to the south are (33812.2 nT, -1509.7 nT, 2230.3 nT) where the components are in order (north, east, vertical).

We assumed the vertical profile of electron density also to be the same over all the places. This gave us the following graphs of rotation angles vs. elevation angle and angle with the east direction respectively.

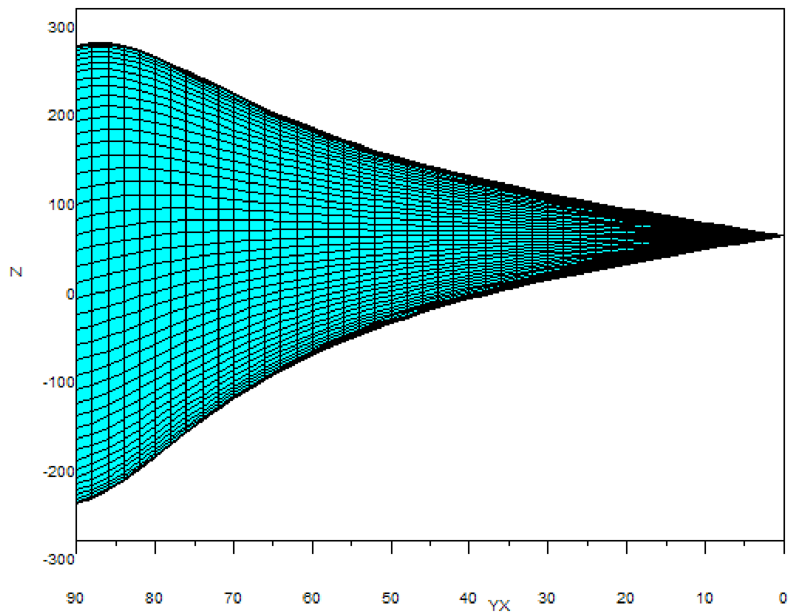


Figure 5 Constant phi plots (Phi contours)

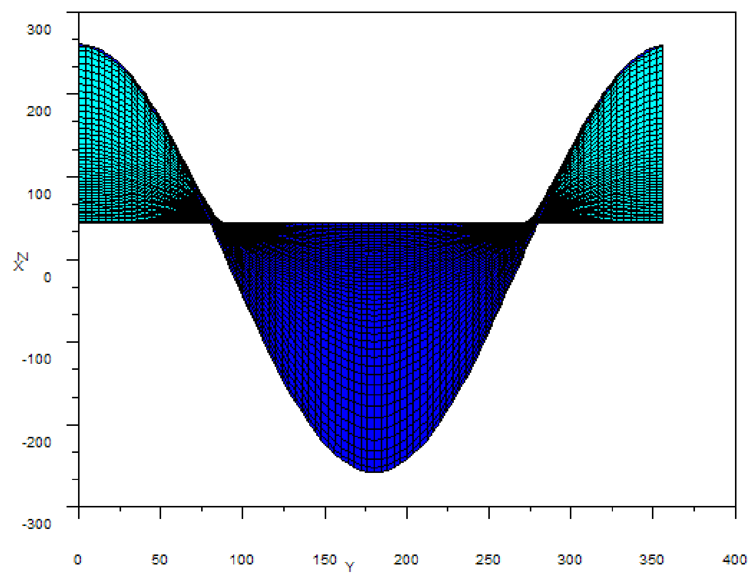


Figure 6 Constant theta plots

Taking the variation with height and latitude into consideration, the following plot is obtained.

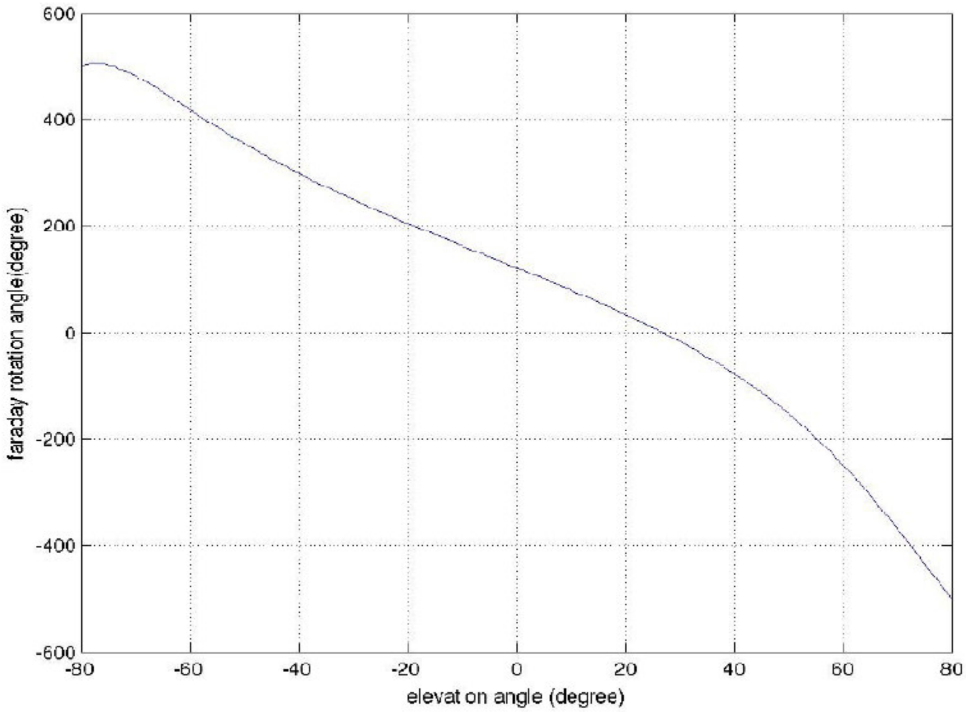


Figure 7: Plot at constant azimuth taking into consideration the variable electron density completely

7. Ionospheric Tomography

7.1 Introduction:

By Faraday rotation method, we can measure the TEC (Total Electron Content) at various elevation angles of the satellite. This gives us information about the integrals of electron density in various directions from the IIT Bombay ground station. From these integrals, we can derive the values of the electron densities at various positions of the ionosphere. This technique is called ionospheric tomography.

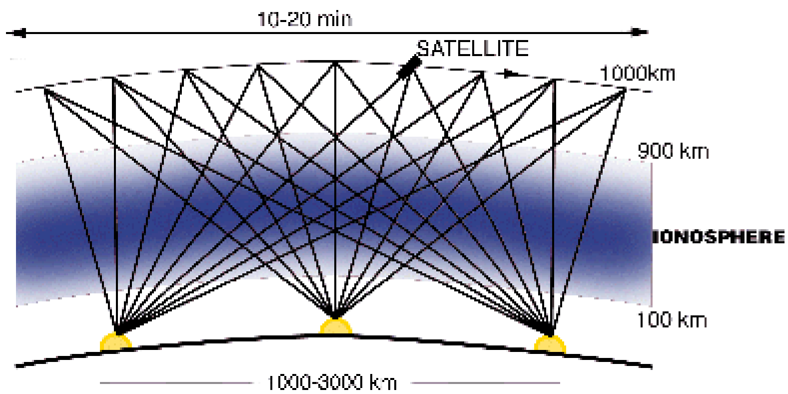


Figure 8: Geometry of the situation

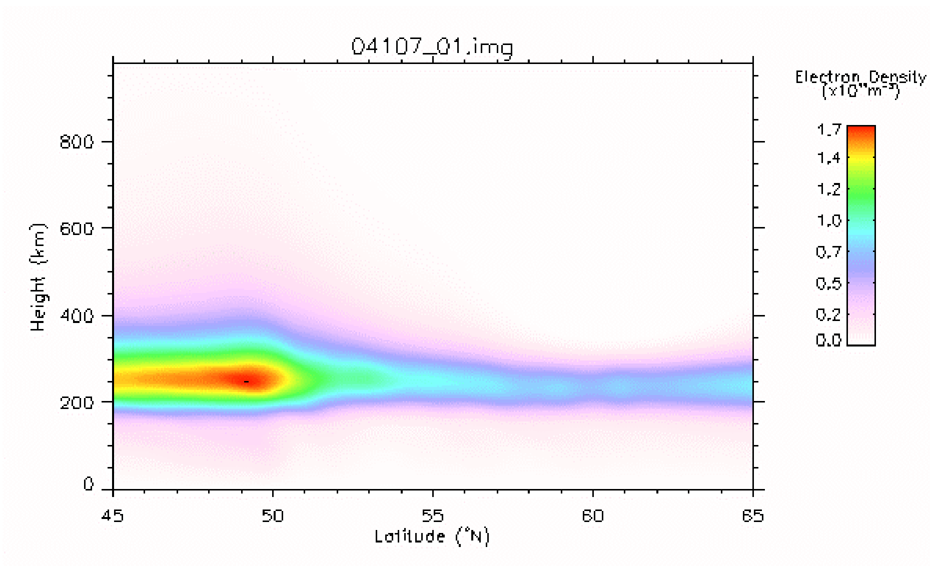


Figure 9: Tomographic image

7.2 Our technique

We will be setting up ground stations at various centers over India, preferably over the same longitude (since we wish to avoid the mixing up of temporal and spatial variations).

Algorithms:

Various algorithms are followed in tomography. These are summarized below:

TRANSFORM METHODS:

The procedure for these methods is:

1) The derived values are transformed to a different domain (usually the frequency domain) through some transform (usually Fourier Transform).
2) Then some relation between the transformed variable and the transform of the required variable is used to determine the transform of the actual variable.
3) Upon applying an appropriate inverse transform, the required variable is generated.

ADVANTAGE:

Continuity of the output variable comes automatically as the inverse Fourier transform of the obtained transform is usually continuous.

DISADVANTAGES:

1) The transforms for various geometries of the measurements can lead to very different transforms for generating the required variable. This is the reason why we could not use this method in our case, as we were unable to come up with an appropriate transform for this special case
2) These methods are inherently continuous. But in case of a computer, we cannot get any continuous function. Thus, there are always some discretization errors in these methods.

ALGEBRAIC METHODS:

The procedure for these methods is:

1) The domain of the required function is divided up into pixels, i.e. discrete zones where the function is assumed to be constant.
2) Then the integrals can be expressed as a linear combination of these pixel values.
3) Solving these systems of linear equations, we get a solution which is a picture of the required function.

Mathematically, let the column vector consisting of the values of the electron density at various pixels be $[x_i]$. Also, let the column vector consisting of the components of the integrated electron density values at various elevation angles and at various stations be $[y_j]$. Then, the relation between [x] and [y] is given by:

$$y_i = A_{ij} x_j$$

$$\text{where } A_{ij} = \text{ length intercepted at i}^{th} \text{ pixel by the j}^{th} \text{ ray}$$

Now the problem reduces to inverting *y* to get *x*. However, the matrix A is severely rank deficient owing to the geometry of the problem and thus the problem is ill-defined. Hence we turn to the following approach to solve the problem.

We assume that an a-priori electron density vector x_0 is known and use that in order to get the actual value of the electron density viz. x. This a-priori electron density can be obtained from ionospheric modeling websites. In our case, we obtained it from http://ccmc.gsfc.nasa.gov/modelweb/models/iri_vitmo.php. We seek to minimize the objective function

$$f(x) = \|Ax - y\|^2 + \alpha \|x - x_0\|^2$$

$$= \frac{1}{2} x^T \left(2A^T A + 2\alpha I\right) x - \left(2y^T A + 2\alpha x_0^T\right) x$$

where α is a variable parameter expressing our relative confidence

in the tec data and the assumed data

We also place the constraints on x that the neighbouring pixels don't differ by too much (assumed to be greater than 2 times the corresponding difference in the base picture). This translates to

$$A_{in} x \le b_{in} \quad ...(6)$$

where A_{in} and b_{in} are some matrices

Furthermore we also confine x to be within some bounds. Specifically, we say that,

$$\alpha x_{0,i} \le x_i \le \beta x_{0,i}$$

where α and β are some constants

Here, they are chosen to be 0.5 and 2.5 respectively.

Simulation Procedure:

The simulation was carried on a part of the ionosphere above a certain longitude, spanning certain fixed ranges of latitude and altitude. This two-dimensional space is divided into a grid consisting of equally spaced radial lines, at 1^0 and circular arcs, at 10 km. Further, five ground stations were considered, each of which has a fixed field of view. Assuming the satellite is at a fixed orbital height, we calculate the TEC values received by the ground stations along a large number of paths. In order to do so, the length matrix is first calculated in the following manner.

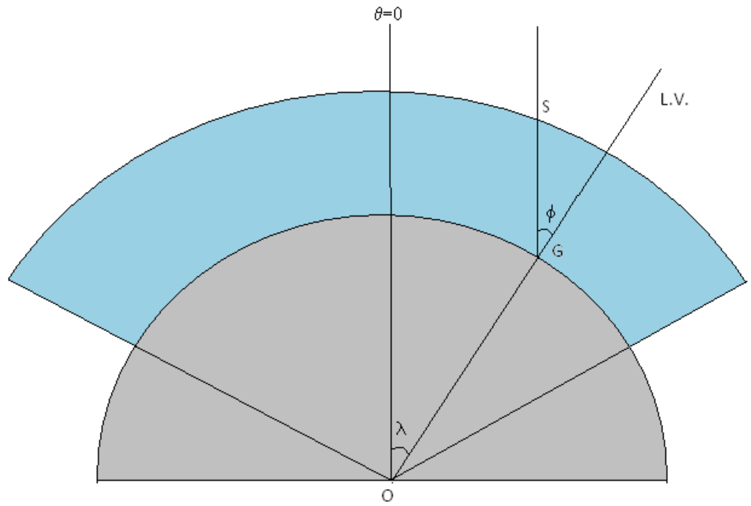


Fig. – O – origin, G – ground station, S – satellite, L.V. – local vertical

We work in a two-dimensional polar co-ordinate system, with the centre of the earth as origin and the radial line at the equator as the pole. To obtain the $(i,j)^{th}$ entry of L, i.e. the length of the intercept made by the i^{th} ray in the j^{th} cell of the grid, we proceed as follows.

The ray is represented by an equation of the form

$$r\sin\theta = mr\cos\theta + R(m\cos\lambda - \sin\lambda) \qquad - (2)$$

where R = radius of the earth

λ =latitude of the ground station

m = slope of the ray = $\tan(\lambda+\phi)$, where ϕ is the angle between the ray and the local vertical

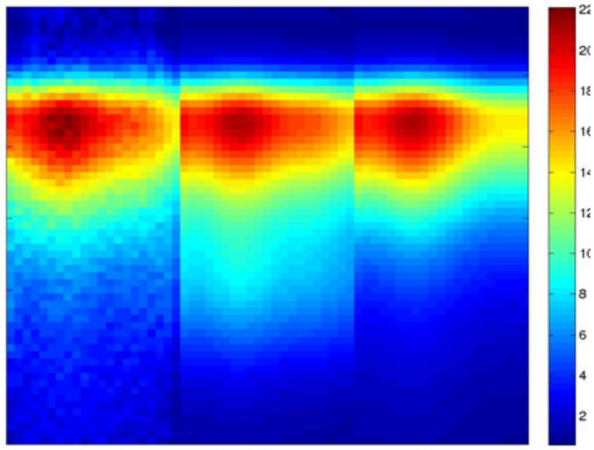
Each cell in the grid is represented by its bounding curves at r_1, θ_1 and r_2, θ_2.

We solve the ray, represented by equation (2), with each of the bounding curves of a particular cell, and determine if the ray does intersect the cell. If so, we calculate the distance between the two points of intersection and enter this as L(i,j). Else we enter zero. Thus the length matrix, L is constructed.

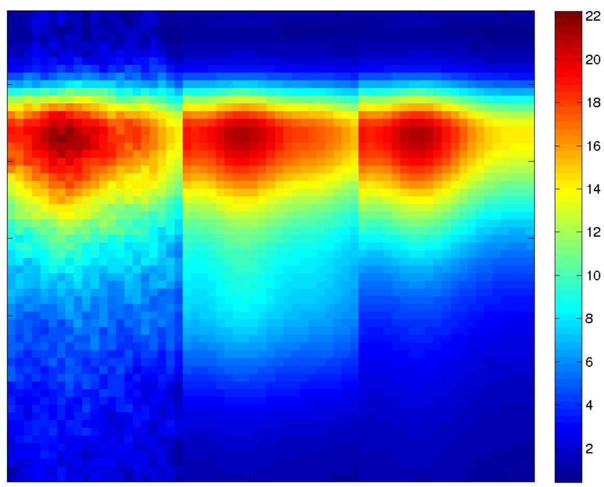
We then obtain the TEC vector, T from equation (1).

Now, this problem reduces to a quadratic programming problem, which we solve by the program ILOG CPLEX package. The results are shown below.

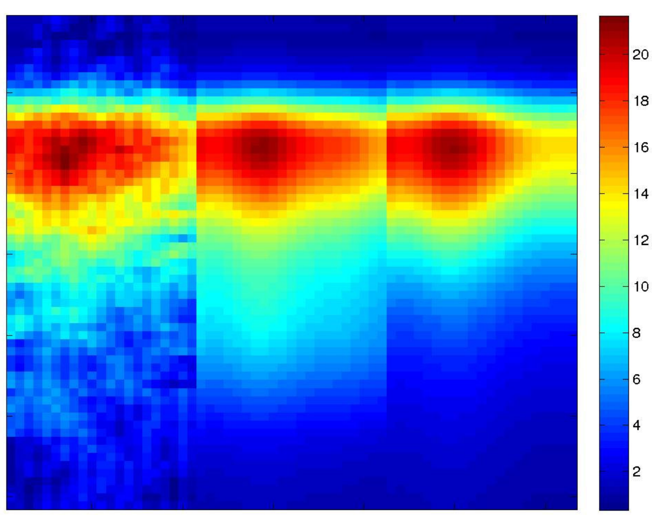
Assuming 5 ground stations each with 3 degrees latitude interval between them and no noise in theTEC data:



Assuming 2.5% normally distributed noise in the TEC data



Assuming 10% normally distributed noise in the TEC data



In each of the above pictures, the figures in order from left are

a) The reconstructed electron density profile

b) The actual electron density profile used to generate the TEC values

c) The base profile i.e. x_0.

We introduced an extra peak in the base value to get the actual electron density profile. It can be seen that the reconstructed profile gets worse as the noise increases.