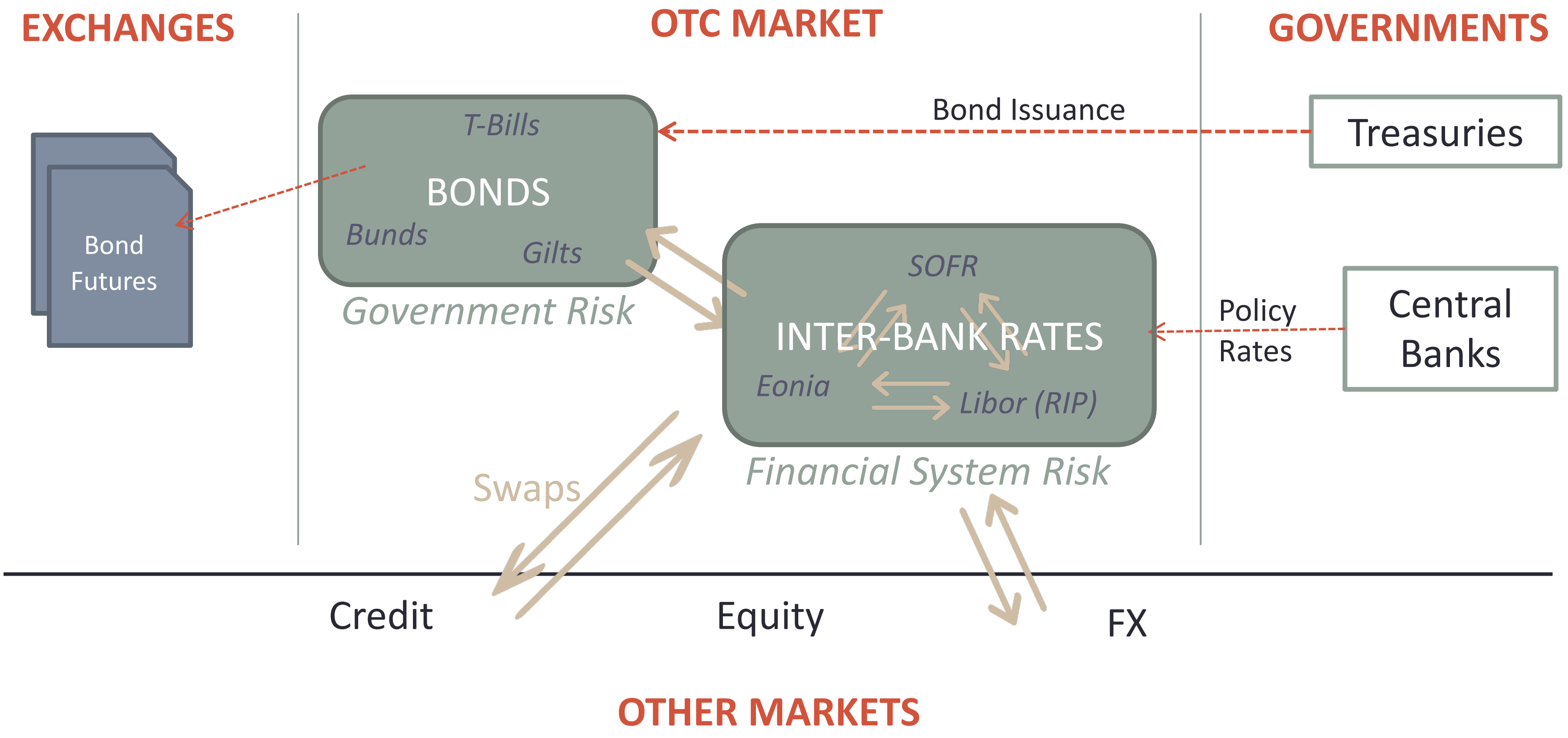
CMF
EDUCATIONAL PROGRAM

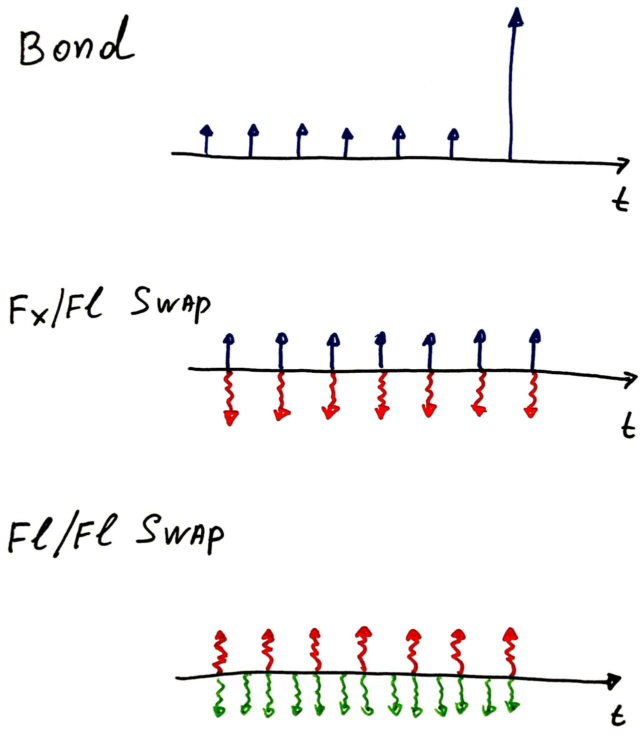
Basics of Interest Rate Markets

Ivan Saroka, BNP Paribas

Interest Rate Markets Structure



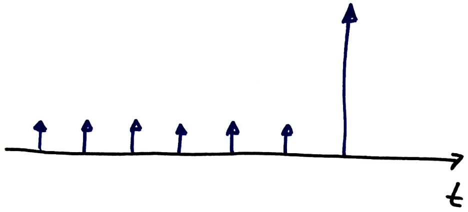
Cashflows of Swaps



Swap Floating Leg Reference Rates

- Libor (Being Discontinued)
  Unsecured term inter-bank lending rate
- Overnight Rate
  Compounded to get a term payment
- Rates in different currencies
  (Cross-currency swaps)

Bonds: Quotes and Valuation



D_i - discount factor to time period t_i.

Bonds

Present value on a bond:

$$PV_B = \sum_i c_i D_i = \sum_i c_i e^{-Y t_i}$$

Where Y is the is the Yield to maturity.

Bond quotes are Prices and Yields

RDSALN 6 3/8 12/15/38 (vs <Enter Security> 11) Actions 12) Setti

No Criteria Applied View All Quotes Date Range 2 Days 95) Buy

Time	Src	Dlr	B Px / A Px	B YTW / A YTW	B Sprd / A Sprd	B Sz X A Sz
09:34	INV	MEUR	135.074 / 136.891	3.92 / 3.82	103.32 / 93.28	250M X 143M
09:34	INV	SWST	133.391 / 135.482	4.02 / 3.90	113.00 / 101.00	174M X 11M
09:34	INV	CT2P	135.219 / 136.008	3.91 / 3.87	102.57 / 98.25	
09:34	INV	CTGI	135.219 / 136.008	3.91 / 3.87	102.57 / 98.25	
09:34	INV	BNP	133.918 / 134.816	3.99 / 3.94	110.03 / 105.03	100M X 100M
09:33	INV	HSBC	133.713 / 135.485	4.00 / 3.90	111.00 / 101.00	1MM X 1MM
09:33	INV	HSIP	133.713 / 135.485	4.00 / 3.90	111.21 / 101.21	
09:33	INV	JPM	134.453 / 136.567	3.96 / 3.84	106.90 / 94.89	1MM X 1MM
09:33	INV	JPM	134.453 / 136.567	3.96 / 3.84	106.95 / 95.10	1MM X 1MM
09:33	INV	JPS2	134.453 / 136.567	3.96 / 3.84	106.95 / 95.10	1MM X 1MM
09:31	INV	UBS	134.478 / 137.237	3.96 / 3.80	106.81 / 91.35	100M X 100M
07:00	INV	BACR				
06:17	INV	WRET				
05:46	INV	EDFM	134.089 / 136.964	3.98 / 3.81	109.04 / 92.88	275M X 375M
05:27	INV	SMBC	109.560 / 110.885	5.60 / 5.50	270.00 / 260.00	
05:00	INV	CFAL	134.225 / 135.413	3.97 / 3.90	108.26 / 101.62	1 X 1
03:59	INV	BMLX	133.910 / 134.798	3.99 / 3.94	110.19 / 105.24	

Swaps: Quotes and Valuation

Swaps

$$PV_S = \underbrace{K \sum_i D_i}_{\text{Fixed Leg}} - \underbrace{\sum_i L_i D_i}_{\text{Floating Leg}}$$

Where L_i are forwards of the underlying reference rate. K is called "coupon" or "fixed leg rate" or "strike rate". It can be freely agreed by participants. On the market participants quote "par swap rate" - that is the coupon that makes PV = 0.

$$\text{Floating Leg} = ZeroPV - ParFixedLeg = S \sum_i D_i$$

Where S is the current observed par swap rate. So we have

$$PV_S = (K - S) \sum_i D_i$$

Basically strike rate minus asset rate. Easy to do write payoff of a swaption:

$$\max[(K - S), 0] \sum_i D_i$$

Swap quotes are Par Rates

200<Go> to view in Launchpad

97) Settings 98) Output 100) Feedback

ICAP Intercapital - USD Swaps ICAP

ICAP US Dollar (ICAU) -> Current Monitor (GDCO 45 3)

USD Swap Rates vs 3 Month Libor Day Count Ann Act/360

vs. LIBOR	Ask	Bid	Time
1) 1 Year	0.334	0.304	12:44
2) 2 Year	0.397	0.367	12:59
3) 3 Year	0.497	0.467	12:59
4) 4 Year	0.660	0.630	12:59
5) 5 Year	0.868	0.838	12:59
6) 6 Year	1.091	1.061	12:59
7) 7 Year	1.303	1.273	12:59
8) 8 Year	1.491	1.461	12:59
9) 9 Year	1.658	1.628	12:59
10) 10 Year	1.809	1.779	12:59
11) 12 Year	2.059	2.029	12:59
12) 15 Year	2.310	2.280	12:58
13) 20 Year	2.513	2.483	12:59
14) 25 Year	2.613	2.583	12:59
15) 30 Year	2.676	2.646	12:59
16) 40 Year	2.709	2.679	12:59
17) 50 Year	2.696	2.666	12:59

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 73
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 2

Bonds Yields v Swap Rates

How to compare?

How to compare bond yield Y to swap rate S? Can we take same maturities? No, because cashflow structure is different, bonds have a large payment at the end and swaps don't. We need some sort of "duration"...
Better simply use first derivative with respect to rate (yield)! It is called "DV01"

Bonds

$$\mathbf{DV01}_B = -\frac{\partial PV_B}{\partial Y} = \sum_i t_i c_i e^{-Y t_i}$$

Swaps

$$\mathbf{DV01}_S = -\frac{\partial PV_S}{\partial S} = -\frac{\partial}{\partial S}(K - S)\sum_i D_i = \sum_i D_i$$

Exercise 1

We have a 7 year bond paying annual coupon of 2.5%, currently trading at 102.4. A 7 year Fixed-Float swap is quoted at 1.78% par rate. Discount curve is given below.

a. Calculate Bond yield and DV01.
b. Calculate Swap DV01 and thus the hedge ratio.

Discount curve

Term	Zero Rate
1Y	0.44
2Y	0.56
3Y	0.89
4Y	1.01
5Y	1.12
6Y	1.35
7Y	1.41
8Y	1.45
9Y	1.47
10Y	1.49

Swap spread

Swap spread is the difference between the swap rate (the rate of the fixed leg of a swap) and the yield on the government bond with a similar maturity.

$$\text{Swap Rate Spread} = \text{Swap Rate} - \text{Yield on Government Bond} \tag{2}$$

Swap spread for a particular European country can be either positive or negative depending on whether the government is considered more or less solvent than the European banking sector. For example Germany has a negative Swap-Spread and Greece a positive one.

Compared to the Swap rate itself Swap spread is usually much less volatile. On a short time horizon, unless there is a significant country-specific event, bond yields are primarily driven by the swap rate, and the spread can sometimes even be considered constant. Another way of thinking about it is that Swap rate is the first factor, and Spread in the second factor.

Figure 1. Yield curves for Swaps and Government bonds

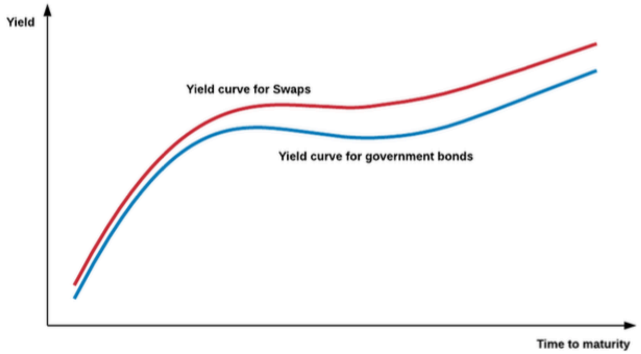
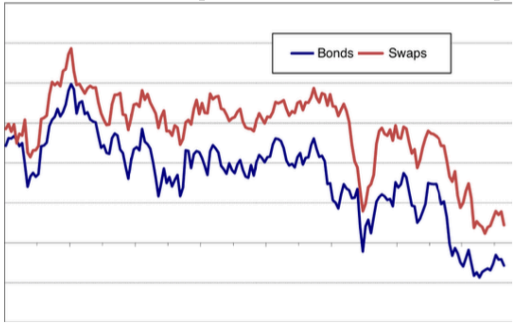


Figure 2. Time series of Swap Rate and Bond Yield of equal maturity



Modelling Interest Rates

When modelling interest rates and spreads, especially on short time horizons (under 1 year), it became standard practice to use simple linear random walks, instead of exponential ones. Practice proved that interest rates can be negative, and on short horizons one does not need to worry about mean-reversion. Moreover, stochastic processes that have normal distributions are easy to work with. We can easily add them and subtracted them from one another.

$$A_t = S_t + X_t \tag{3}$$

We will assume that both, the Swap Rate and the Swap Spread follow a linear brownian motion with no drift:

$$dS_t = \sigma_S dW_t$$
$$dX_t = \sigma_X dZ_t$$

where $dW dZ = 0$.

Hence $S_T \sim N(S_t, \sigma_S^2(T-t))$ and $X_T \sim N(X_t, \sigma_X^2(T-t))$ are independant and normally distributed.

Exercice 2.1:

a) Compute volatility of σ_A of process A_t.

b) Compute correlation between A_t and S_t.

Options on Interest Rates

Interest rate options are priced using the Bachelier formula. Under the risk-neutral measure, rate is assumed for follow a process with no drift:

$$dS_t = \sigma dW_t \tag{9}$$

Hence at expiry time T we simply have

$$S_T = S_t + \sigma W_T \quad \sim \quad N(S_t, \sigma^2(T-t)) \tag{10}$$

S_T is distributed normally with mean S_t and variance $\sigma^2(T-t)$. Further we will write $\tau = T - t$ to mean time to expiry.

Price of a call option is expectation under risk neutral measure of the payoff, for a call option this would be

$$\begin{aligned} Call(S_t, K, \sigma, \tau) &= \mathbb{E}_t[(S_T - K)^+] && (11) \\ &= \mathbb{E}_t[S_T \mathbb{I}_{S_T > K}] - \mathbb{E}_t[K \mathbb{I}_{S_T > K}] && (12) \\ &= \mathbb{E}_t[S_T \mathbb{I}_{S_T > K}] - K\mathbb{Q}_t[S_T > K] && (13) \end{aligned}$$

where $\mathbb{I}_A$ is indicator function (equal to 1 if condition is true, and 0 if false); $\mathbb{Q}$ is simply the probability under the risk neutral measure; and we use the fact that $\mathbb{E}[\mathbb{I}_A] = \mathbb{Q}[A]$.

Using the pdf function for the normal distribution we have

$$\mathbb{E}_t[S_T \mathbb{I}_{S_T > K}] = \frac{1}{\sqrt{2\pi^2\tau}} \int_K^\infty x e^{-\frac{(x-S_t)^2}{2\pi^2\tau}} dx \tag{14}$$

and

$$\mathbb{Q}_t[S_T > K] = \frac{1}{\sqrt{2\pi^2\tau}} \int_K^\infty e^{-\frac{(x-S_t)^2}{2\pi^2\tau}} dx \tag{15}$$

Options on Interest Rates - exercises

Exercice 2.2:

Compute integrals in (14) and (15) and substitute into (13) to derive a formula for the call option for the Bachelier model:

$$Call(S_t, K, \sigma, \tau) = (S_t - K)N(h) + \frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}e^{h^2/2} \tag{16}$$

where

$$h = \frac{S_t - K}{\sigma\sqrt{\tau}}$$

Exercice 2.3:

Use Put - Call parity to derive the formula for the put option:

$$Put(S_t, K, \sigma, \tau) = (K - S_t)N(-h) + \frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}e^{h^2/2} \tag{17}$$

Congratulations! You now know the most useful formula you'll ever use when pricing any kind of interest-rate optionality. When to use: short horizons, when vol is the most important factor.

Exercice 2.4:

By differentiating (16) and (17) with respect to S compute the Delta for Call and Put.

Exercice 2.5:

Implement a function in Python for the Bachelier option pricing formula. Create a plot for option price and delta. Compare it to the Black-Sholes for $r = 0$.

Data exercises

Exercice 3

a) From intraday data provided estimate σ_S and σ_X – daily volatilities of the Swap Rate and Swap Spread.

b) Test whether the Swap Rate and Swap Spread are indeed linear brownian motions. Try this on different time horizons.

c) Test our assumption that $dSdZ = 0$. Use significance tests for correlation coefficient. Try this on different time horizons.

Data for this exercise is to be provided soon….