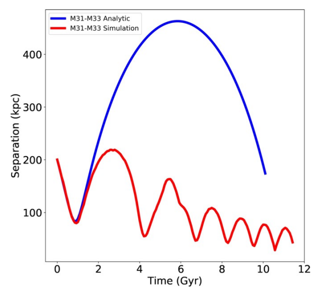
Dynamical Friction

The transfer of energy from the orbital motion of a satellite into random motions of the particles in the medium through which the satellite is moving (e.g. a dark matter halo or a stellar halo).

This causes the satellite's orbit to decay.

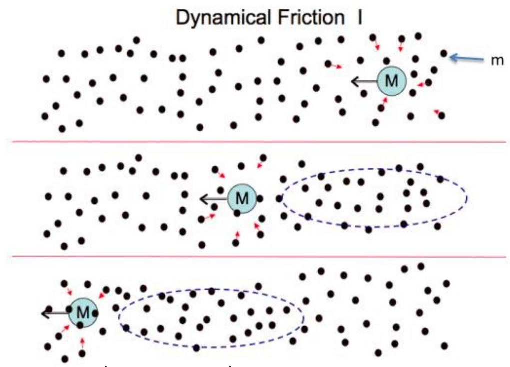


This Lab: We need to fix our orbit code!

Section 7 in Sparke & Gallagher

1

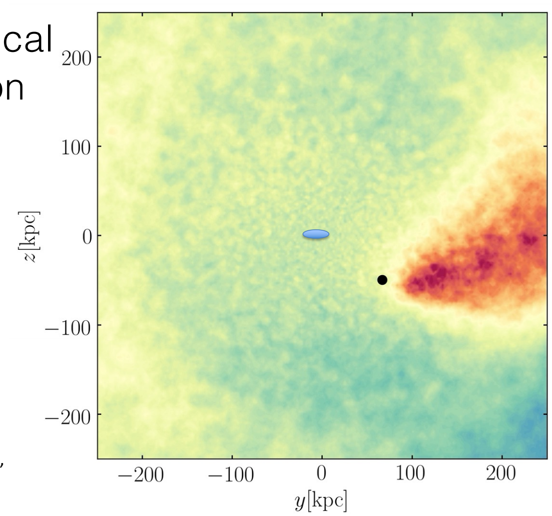
Dynamical Friction --- Wake



This Over Density then Tugs BACK on M

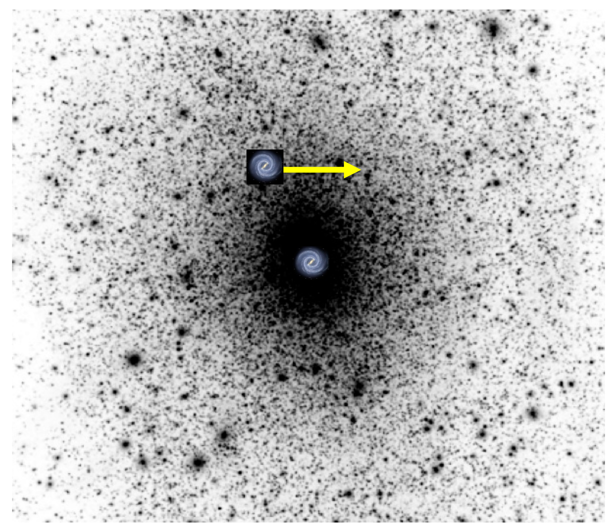
2

Dynamical Friction



Garavito-Camargo, Besla+2019,2020

3



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Encounter with a Single Star / DM particle

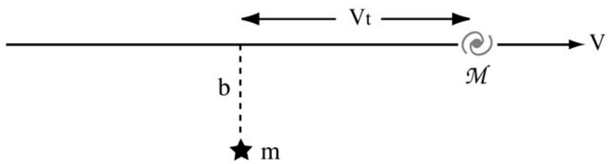

Fig 7.4 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

A galaxy of mass $\mathcal{M}$ moves with speed **V** past a stationary star (or clump of dark matter) of mass **m** in the host halo, a distance **b** from its path.

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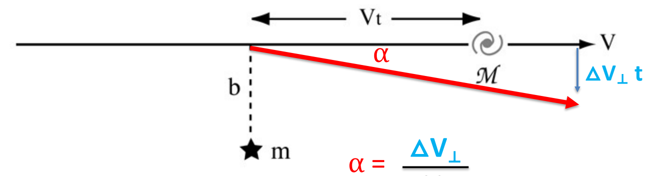
Encounter with a Single Star / DM particle



$$\alpha = \frac{\Delta V_\perp}{V}$$

$$F_\perp = \mathcal{M}\,(\Delta V_\perp / \Delta t)$$

Impulse Approximation: $\Delta V_\perp = \mathcal{M}^{-1} \int F_\perp dt$

Where : $F_\perp = \dfrac{G\,\mathcal{M}\,m}{R^2}\,\dfrac{\mathbf{b}}{R}$

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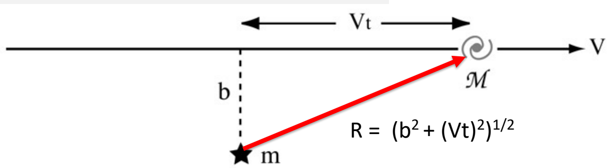
Encounter with a Single Star / DM particle



Galaxy is deflected by:

$$\Delta V_\perp = \mathcal{M}^{-1} \int \frac{G\,\mathcal{M}\,m \quad b\,dt}{(b^2 + (Vt)^2)^{3/2}}$$

$$= G\,m\,b \left. \frac{t}{b^2(b^2 + (Vt)^2)^{1/2}} \right|_{-\infty}^{\infty} \quad \text{X 2}$$

b << Vt so drop b in the brackets

$$= \frac{2\,G\,m}{b\,V}$$

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Encounter with a Single Star / DM particle

The star/DM Particle ALSO receives a kick, so the change in energy of the system:

$$\Delta\, KE_\perp = \tfrac{1}{2}\,\mathcal{M}\,\Delta V_\perp{}^2{}_{\text{from } m} + \tfrac{1}{2}\,m\,\Delta V_\perp{}^2{}_{\text{from } \mathcal{M}}$$

$$= \tfrac{1}{2}\,\mathcal{M} \left(\frac{2Gm}{b\,V}\right)^2 + \tfrac{1}{2}\,m \left(\frac{2G\mathcal{M}}{b\,V}\right)^2$$

Note that since $m \ll \mathcal{M}$, the term on the RIGHT dominates

$$= \frac{2G^2\,m\,\mathcal{M}(\mathcal{M} + m)}{b^2\,V^2}$$

The DM particle / Star is acquiring MOST of the energy. Since energy is conserved, that energy must come from the motion of the satellite ALONG its orbit (Parallel, denoted by **II**)

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Encounter with a Single Star / DM particle

Determining the change in speed of the satellite **along** its orbit: $\Delta V_{||}$

Initial Kinetic Energy = Total Energy After the Encounter

$$\tfrac{1}{2}\mathcal{M}V^2 = \Delta KE_{\perp} + \Delta KE_{||\,\text{satellite}} + \Delta KE_{||\,\text{particle}}$$

$$= \Delta KE_{\perp} + \tfrac{1}{2}\mathcal{M}\,(V + \Delta V_{||\,\text{from } m})^2 + \tfrac{1}{2} m(\Delta V_{||\,\text{from } \mathcal{M}})^2$$

Recall, $\Delta V_{\perp\,\text{from } \mathcal{M}} = \mathcal{M}/m\ \Delta V_{\perp\,\text{from } m}$

So : $\Delta V_{||\,\text{from } \mathcal{M}} = \mathcal{M}/m\ \Delta V_{||\,\text{from } m}$

$$\cancel{\tfrac{1}{2}\mathcal{M}V^2} = \Delta KE_{\perp} + \tfrac{1}{2}\mathcal{M}\,(\cancel{V^2} + 2V\Delta V_{||} + \Delta V_{||}^2) + \tfrac{1}{2} m(\mathcal{M}/m)\Delta V_{||}^{\,2}$$

Assuming $\Delta V_{||} << V$, then we can drop $\Delta V_{||}^2$ terms

$$\Delta KE_{\perp} = -\tfrac{1}{2}\mathcal{M}\,(2V\Delta V_{||})$$

$$\Delta V_{||} = -\frac{\Delta KE_{\perp}}{\mathcal{M}V} = \frac{-2G^2 m(\mathcal{M}+m)}{b^2V^3}$$

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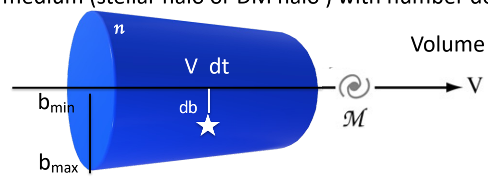
NEXT: (weak) Encounters with **MANY** Stars / DM particles

Consider a satellite galaxy moving through a cylindrical volume of a medium (stellar halo or DM halo) with number density $n = \rho / m$



Volume = 2π b V dt

Integrate $\Delta V_{||}$ over **b** to determine the total perturbation to the velocity along the orbit from n weak encounters

$$\Delta V_{||\,\text{TOTAL}} = \int_{b_{min}}^{b_{max}} \Delta V_{||\,\text{per encounter}}\, n\, 2\pi V\, dt\, b\, db$$

$$= \frac{-2G^2(\mathcal{M}+m)\,m\,n\,2\pi V\,dt}{V^3}\int_{b_{min}}^{b_{max}}\frac{b\,db}{b^2}$$

$ln(\Lambda)$

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Weak Encounters with **MANY** Stars / DM particles

Coulomb Logarithm:

$$ln(\Lambda) = ln\,(b_{max}/ b_{min})$$

b_{max} = Current Separation between satellite and COM of the host galaxy

b_{min} = r_s Radius for a "Strong Encounter"

Strong Encounters: If the change in potential energy is comparable to the initial kinetic energy of the satellite

$$\frac{G(M+m)}{r} \gtrsim \frac{V^2}{2}$$

$$r_s \equiv \frac{2G(M+m)}{V^2}$$

We need our separation, b, to be larger than r_s or we invalidate our energy assumptions (that change in potential energy is small)

For the DF calculation we will take V = V_{circ} of the DM halo

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Weak Encounters with **MANY** Stars / DM particles

Dynamical Friction: the corresponding deceleration needed to achieve a change in velocity $\Delta V_{||}$ along the direction of motion over some dt

$\mathcal{M} + m \sim \mathcal{M}$ $\rho = m\,n$ density of the halo

$$a_{DF} = a_{||} = \frac{\Delta V_{||}}{dt} = \frac{-2G^2(\mathcal{M}+m)\,m\,n\,2\pi V\,ln(\Lambda)}{V^3}$$

$$= \frac{-4\pi G^2\mathcal{M}\rho\ ln(\Lambda)}{V^2}$$

1. If V of satellite is low, a_{DF} increases
2. Total mass of the halo doesn't matter, it's the density that does.
   - If the density increases (more encounters), the deceleration is stronger
3. A more massive Satellite ($\mathcal{M}$) slows down faster than a lighter one (major encounters will decay faster than minor encounters) (MW and M31 vs. M33 and M31), F = $\mathcal{M}$ a_{DF}

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Weak Encounters within a HALO

We ignored the fact that stars/DM particles in the host also have some internal velocity distribution (we assumed the initial speed of the halo particles was 0).

$$a_{DF} = -2\, G^2\, \mathcal{M}\, m\, \boldsymbol{n}\, 2\, \boldsymbol{\pi}\, \ln(\boldsymbol{\Lambda}) \int f(v)\, \mathbf{v} / |\mathbf{v}|^3$$

$$f(v) = \frac{n_o}{(2\pi\sigma^2)^{3/2}} exp(-\frac{v^2}{2\sigma^2})$$ Isotropic, Maxwellian velocity distribution function

$$\frac{dv}{dt} = -\frac{4\pi G^2 M \rho}{v^2} \ln\Lambda \left[erf(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \frac{\mathbf{v}_x}{v}$$

$X = v/(\sqrt{2}\sigma)$ and erf is the error function. Consider an isothermal sphere with a flat rotation curve Vc and velocity dispersion $\sigma = v_c/\sqrt{2}$, so $X = 1$

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}$$

$$\boxed{\mathbf{a} = -0.428 \frac{G M_{\text{sat}} \ln(\Lambda)}{r^2} \frac{\mathbf{v}}{v}}$$

Dynamical Friction for a satellite moving through a density distribution that follows the profile for an Isothermal Sphere.

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Orbit Decay Time

Angular momentum of the orbit is changing owing to dynamical friction $\frac{dL}{dt} = \frac{dv}{dt} r = -0.428 ln\Lambda \frac{GM\text{sat}}{r}$

Assuming a *circular* orbit, the angular momentum is : L = r Vc

$$\frac{dL}{dt} = Vc \frac{dr}{dt} = \frac{-0.428\, \ln(\boldsymbol{\Lambda})\, G\, Msat}{r}$$

$$\int_{r_{initial}}^{0} r\, dr = \frac{-0.428\, \ln(\boldsymbol{\Lambda})\, G\, Msat}{Vc} \int_0^{t_{fric}} dt$$

$$t_{fric} = \frac{1.17\ r_i^2\ Vc}{\ln(\boldsymbol{\Lambda})\ G\ Msat} \qquad t_{fric} = \frac{264\text{Gyr}}{ln\Lambda} \left(\frac{r_i}{2\text{kpc}}\right)^2 \left(\frac{v_c}{250\text{km/s}}\right) \left(\frac{10^6 M_\odot}{M}\right)$$

Equation above is normalized For Globular clusters in M31:

$\ln(\boldsymbol{\Lambda}) \sim 10$ $t_{fric} \sim 26$ Gyr.

If t_{fric} = 13.8 Gyr, then the equation gives r_i = 500 pc: globular clusters < 500 pc of the M31 galactic center should be destroyed 17

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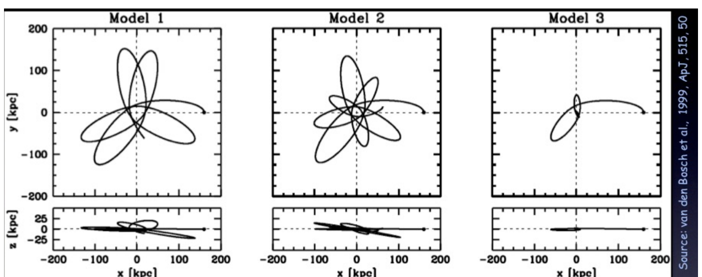
Dynamical Friction & Orbital Decay



Model 1: $M_S/M_h = 2 \times 10^{-4}$
Model 2: $M_S/M_h = 2 \times 10^{-3}$
Model 3: $M_S/M_h = 2 \times 10^{-2}$

- All three orbits have initial eccentricity e=0.8
- Orbits of more massive subjects decay faster
- No (obvious) orbit circularization

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