

Hubble Parameter as a function of time

$$\dot{R}^2(t) = \frac{8\pi G}{3} \rho(t) R^2(t) - Kc^2$$

Friedmann's First Equation

$$H(t)^2 \left[1 - (\Omega_m + \Omega_{rad} + \Omega_\Lambda) \right] = -\frac{Kc^2}{R^2}$$

$$\Omega(t) = \Omega_m(t) + \Omega_{rad}(t) + \Omega_\Lambda(t)$$

$$H(t)^2 (1 - \Omega(t)) R^2 = -Kc^2 = H_o^2 (1 - \Omega_0) R_0^2$$

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Density Parameters as a function of time

$$\Omega(t) = \Omega_m(t) + \Omega_{rad}(t) + \Omega_\Lambda(t)$$

$$\Omega_m(t) = \Omega_{m0} (1+z)^3 \frac{H_o^2}{H(t)^2}$$

Baryons, Dark Matter

$$\Omega_{rad}(t) = \Omega_{rad0} (1+z)^4 \frac{H_o^2}{H(t)^2}$$

Photons, Neutrinos

$$\Omega_\Lambda(t) = \Omega_{\Lambda0} \frac{H_o^2}{H(t)^2}$$

Dark Energy

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Hubble Parameter as a function of time

$$H(t)^2 = H_o^2 \left[\Omega_{m,o} (1+z)^3 + \Omega_{rad,o} (1+z)^4 + \Omega_{\Lambda,o} + (1 - \Omega_o) (1+z)^2 \right]$$

Where $1 - \Omega_0 = \Omega_k$

Curvature Density Parameter

This equation describes the fractional rate of expansion of the universe as a function of time. Where now every density parameter is defined in terms of their present day values.

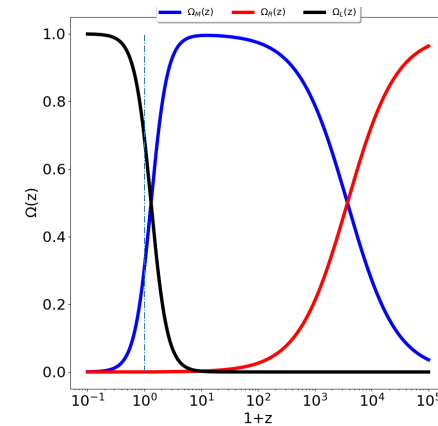
Benchmark Cosmology:

2015 Planck results (Table 4 column 2)

$$\Omega_{m0} = 0.308 \pm 0.012 \quad \Omega_{\Lambda0} = 0.692 \pm 0.012 \quad \Omega_{rad0} = 8.24 \times 10^{-5} \quad H_o = 67.81 \pm 0.92$$

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In Class Lab 10 Discussion



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In Class Lab 10 Discussion

Matter-Radiation EqualitySetting $\Omega_m(t) = \Omega_{rad}(t)$

$$\Omega_{m0}(1+z)^3 \frac{H_0^2}{H(t)^2} = \Omega_{rad0}(1+z)^4 \frac{H_0^2}{H^2} \quad (1+z) = \frac{\Omega_{m0}}{\Omega_{rad0}} = 0.308/8.24e-5 \sim 3700$$

(0.05 Myr after the big bang)

Era of Dark Energy

At some point the Cosmological Constant started to dominate (but it was pretty much negligible before then).

$$(1+z) = \frac{\Omega_\Lambda}{\Omega_m} = (0.698/0.308)^{1/3} = 1.3 \quad (2)$$

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Proper Distance:Recall: $r = R(t)u$

Robertson Walker Metric

$$ds^2 = (cdt)^2 - R(t)^2 \left[\left(\frac{du}{\sqrt{1-ku^2}} \right)^2 + (u d\theta)^2 + (u \sin\theta d\phi)^2 \right]$$

Light rays travel paths with $ds^2 = 0$, called null geodesics. $c^2 dt^2 = d\ell^2$ Light rays travel on radial paths, where $(\theta = 0, \phi = 0)$ $cdt = R(t) \frac{du}{\sqrt{1-ku^2}}$

Proper Distance:

$$c \int_{t_e}^{t_0} \frac{dt}{R(t)} = \int_{u_e}^{u_0} \frac{du}{\sqrt{1-ku^2}} \quad (\text{Ruler Distance - distance to an object TODAY})$$

$$\text{Soln to RHS} = \begin{cases} |\kappa|^{-1/2} \sinh^{-1} \sqrt{|\kappa|} u & \text{if } \kappa < 0 \text{ (a negatively curved 'hyperbolic' universe)} \\ u & \text{if } \kappa = 0 \text{ (a spatially flat universe)} \\ |\kappa|^{-1/2} \sin^{-1} \sqrt{|\kappa|} u & \text{if } \kappa > 0 \text{ (a positively curved 'spherical' universe)} \end{cases}$$

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Look Back Time

Ryden Chapter 6

Because light travels at a finite speed, we see a younger cosmos as we look toward more distant galaxies at higher redshift.

If you observe a galaxy at redshift z , at what time (t_e) did those photons leave on their journey towards us?

$$R(t) = \frac{1}{1+z} = \lambda_e / \lambda_{obs}$$

$$1+z = \frac{1}{R(t)}$$

$$H(t) = \frac{\dot{R}}{R}$$

$$\frac{dz}{dt} = -\frac{1}{R(t)^2} \frac{dR}{dt} = -\frac{1}{R(t)} H(z) = -(1+z)H(z)$$

$$-\int_{t_0}^{t_e} dt = t_0 - t_e = \int_0^z \frac{1}{H(z)(1+z')} dz' = t_L$$

Look Back Time (Gyr ago) [inverse of Hubble Parameter]

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Comoving Radial Distance

$$c \int_{t_e}^{t_0} \frac{dt}{R(t)} = \int_{u_e}^{u_0} \frac{du}{\sqrt{1-ku^2}}$$

Re-write the LHS of this equation

$$\text{Recall: } \frac{dz}{dt} = -(1+z)H(z)$$

$$= \int_0^z \frac{c}{R(t)(1+z)H(z)} dz' \quad (\text{replacing } dt)$$

$$R(t) = \frac{1}{1+z}$$

$$D_C = c \int_0^{z_e} \frac{dz'}{H(z')} \quad \text{Comoving Radial Distance:}$$

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Proper Distance in terms of D_c

Proper Distance
at any z_0 :
(Ruler Distance, $K=0$)

$$cdt = R(t)D_c = \frac{D_c}{1+z_0}$$

Today, $z_0 = 0$ And $R(0) = 1$

$$cdt = R(t_0)u = D_c$$

So the Proper Distance to an object from us TODAY IS the SAME as its Comoving Distance TODAY.
This is the **line of sight distance** to a galaxy at a given redshift at the present day.

i.e. This is the distance you would put
into Hubble's Law

$$v = H(t_0)R(t_0)u = H(t_0)D_c$$

$$D_C = c \int_0^{z_e} \frac{dz'}{H(z')}$$

This is true regardless of
the observer's redshift

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Horizon Distance: Size of the Observable Universe

- The Size of the observable universe is the proper distance traveled by a photon over the age of the universe.

Horizon Distance today = Comoving Radial Distance

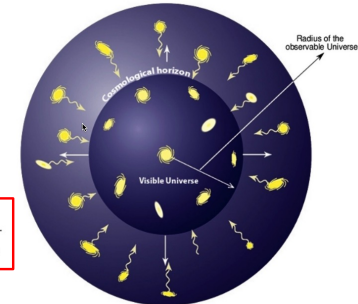
$$d_H = cdt = R(t_0)u = D_c$$

$$D_C = c \int_0^z \frac{dz'}{H(z')} \quad z \text{ is large}$$

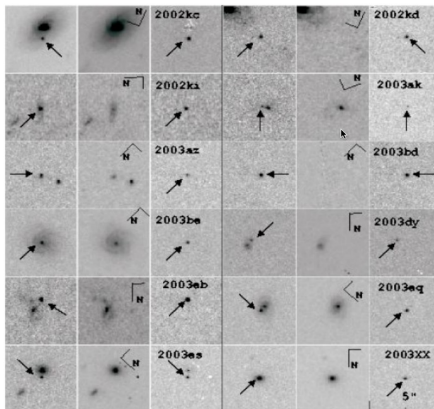
At an arbitrary redshift, however:

$$d_H(t_{obs}) = c\Delta t = R(t_{obs})D_c = \frac{D_c}{1+z_{obs}}$$

Proper Distance at an arbitrary observer distance



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Supernovae
in distant
galaxies
found by
HST

How do we
measure distances
to standard candles
in an expanding
universe???

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Luminosity Distance $F = \frac{L}{4\pi d^2}$

Ryden Chapter 7

In an expanding universe, far away objects appear dimmer:

$$L_e = \frac{h\nu_e}{\Delta t_e} \quad L_o = \frac{h\nu_o}{\Delta t_o} \quad L_e \text{ is the intrinsic luminosity of the source}$$

$$L_o = L_e \frac{\nu_o}{\nu_e} \frac{\Delta t_e}{\Delta t_o} = L_e R(t_e) R(t_o)$$

$$R(t) = \frac{1}{1+z} = \lambda_e / \lambda_{obs} \quad \text{From redshift calculation}$$

$$\frac{\nu_o}{\nu_e} = R(t_e) \quad \frac{\Delta t_e}{R(t_e)} = \frac{\Delta t_o}{R(t_o)}$$

d is the Proper Distance between us and the source (Comoving Distance, D_C)

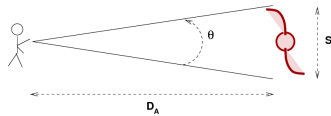
$$F = \frac{L_e}{4\pi D_C^2} \frac{1}{(1+z)^2} \frac{1}{(1+z)} = \frac{L_e}{4\pi d_L^2}$$

$$d_L = (1+z)D_C$$

Luminosity Distance: how far an object of known luminosity L would have to be in Euclidean space so that we measure a total flux F .

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Angular Diameter Distance



D_A is the distance to the source, such that it subtends the same angle it would have in Euclidean Space

$$D_A \equiv \frac{S}{\theta}$$

If $K = 0$

$$\theta [\text{rad}] = \frac{S}{R(t_e)u_e} \equiv \frac{S}{D_A}$$

D_A is the Proper Distance at the time of emission – so that you see the correct angle

$$D_A = R(t_e)u_e = \frac{R(t_0)u_e}{1+z} = \frac{D_C}{(1+z)} = \frac{D_L}{(1+z)^2} = \frac{S}{\theta}$$

So, the size of a galaxy (or equivalently, the separation between two galaxies) that subtends angle Theta is:

$$S = \theta D_A = \frac{D_C}{1+z} \theta$$