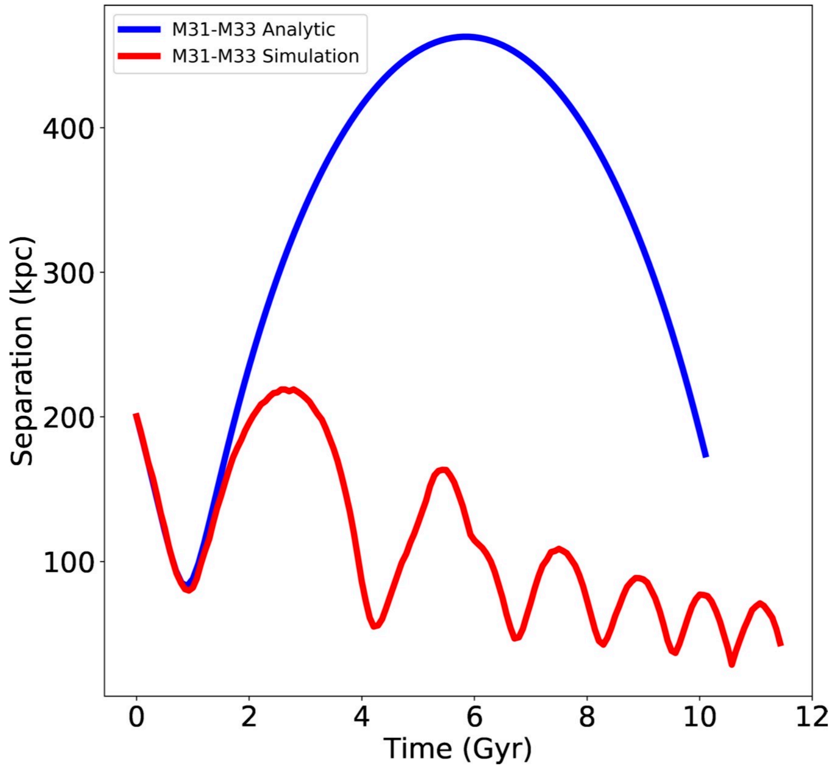
Dynamical Friction

The transfer of energy from the orbital motion of a satellite into random motions of the particles in the medium through which the satellite is moving (e.g. a dark matter halo or a stellar halo).

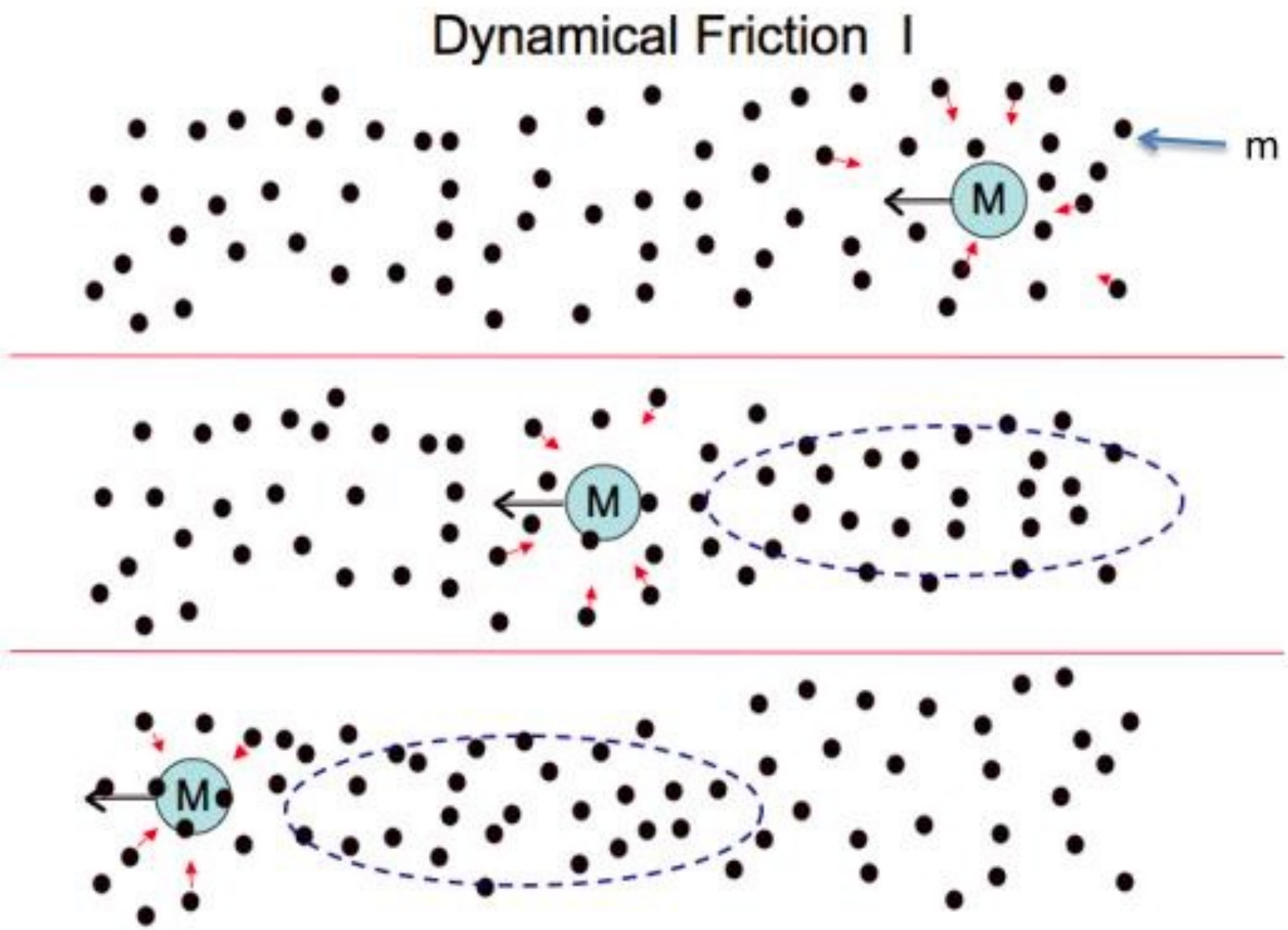
This causes the satellite's orbit to decay.



This Lab: We need to fix our orbit code!

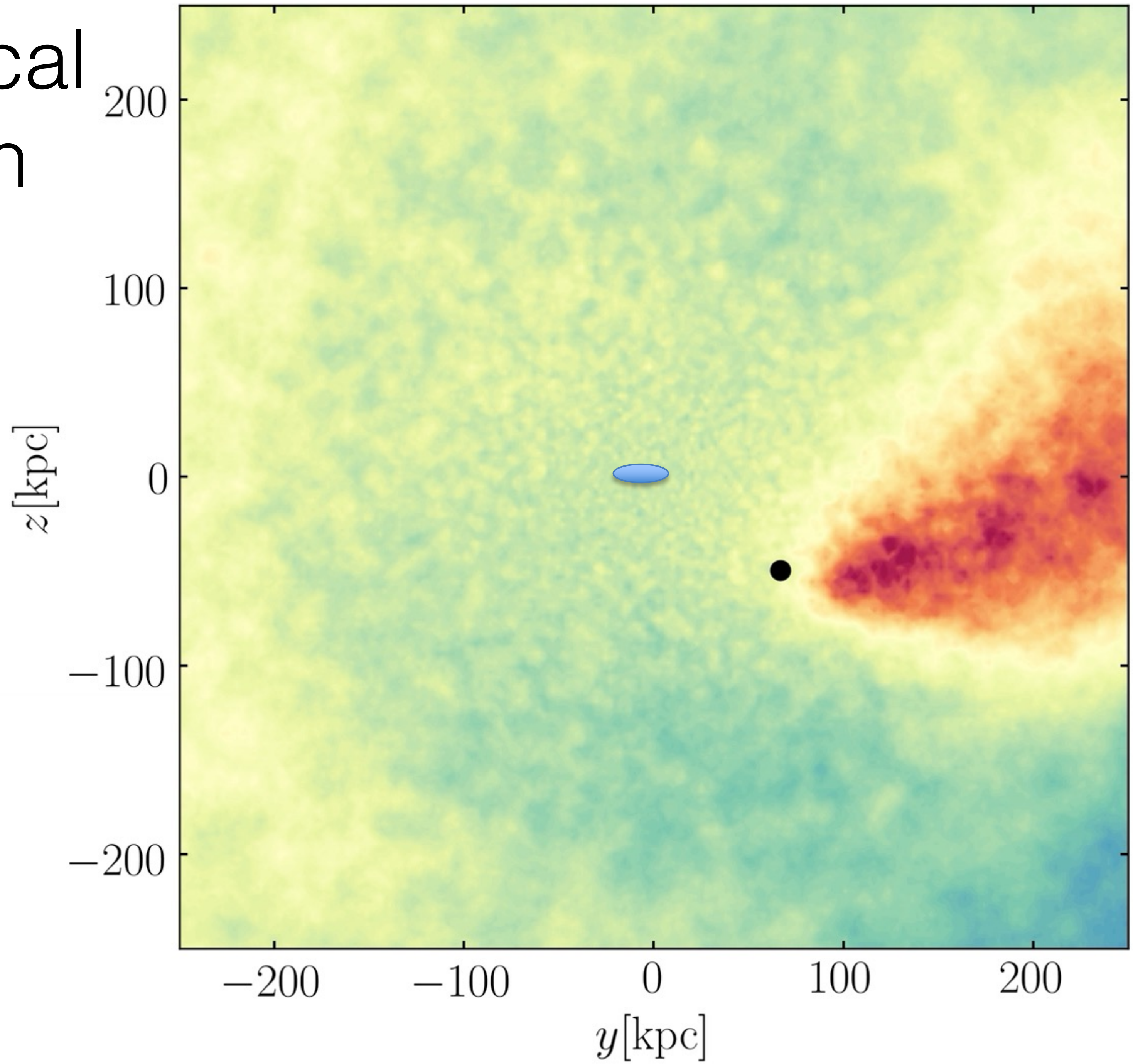
Section 7 in Sparke & Gallagher

Dynamical Friction --- Wake

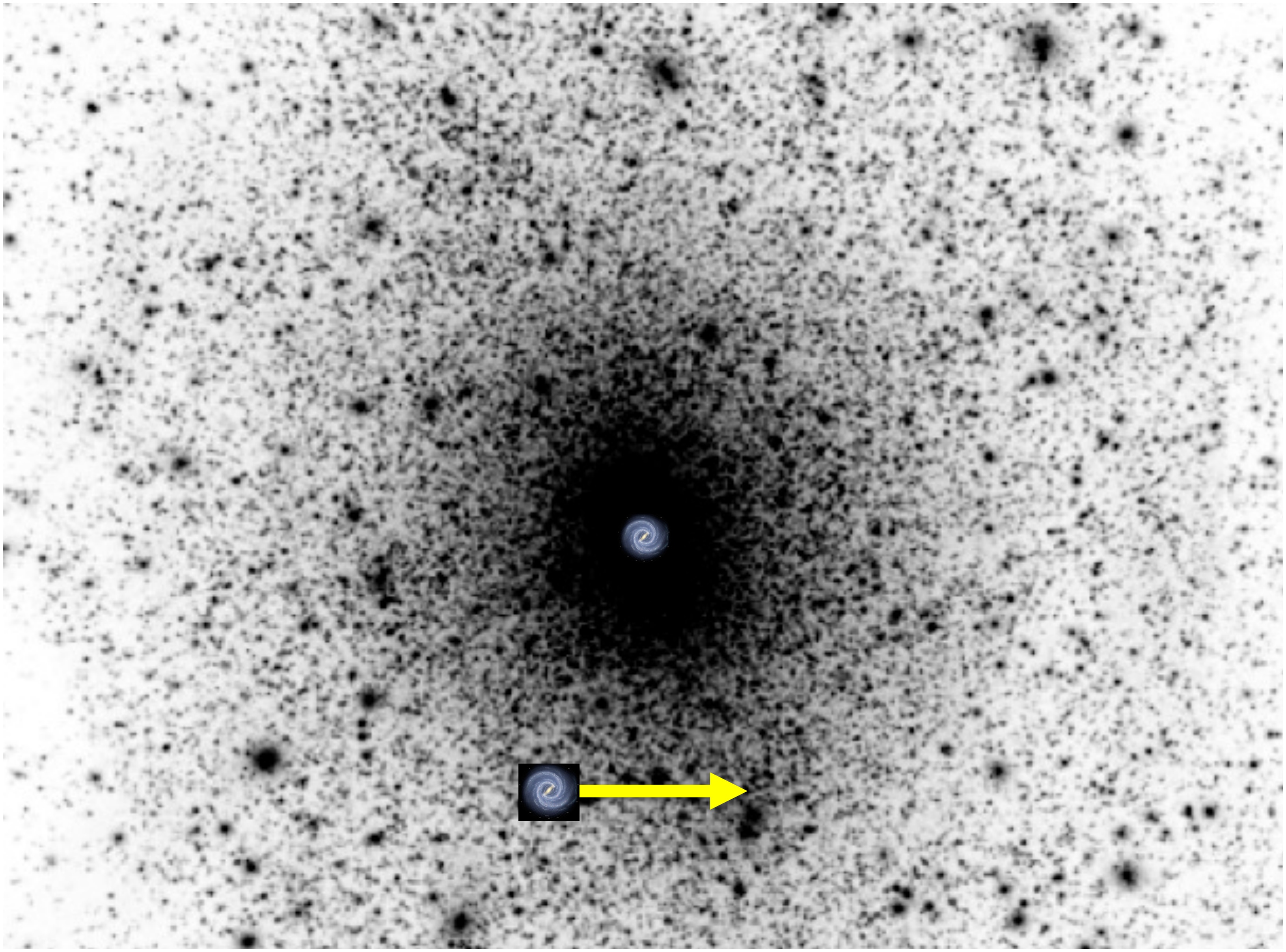


This Over Density then Tugs BACK on M

Dynamical Friction



Garavito-Camargo, Besla+2019,2020



We want to compute how the orbital speed of the galaxy will change owing to dynamical friction.

Encounter with a Single Star / DM particle

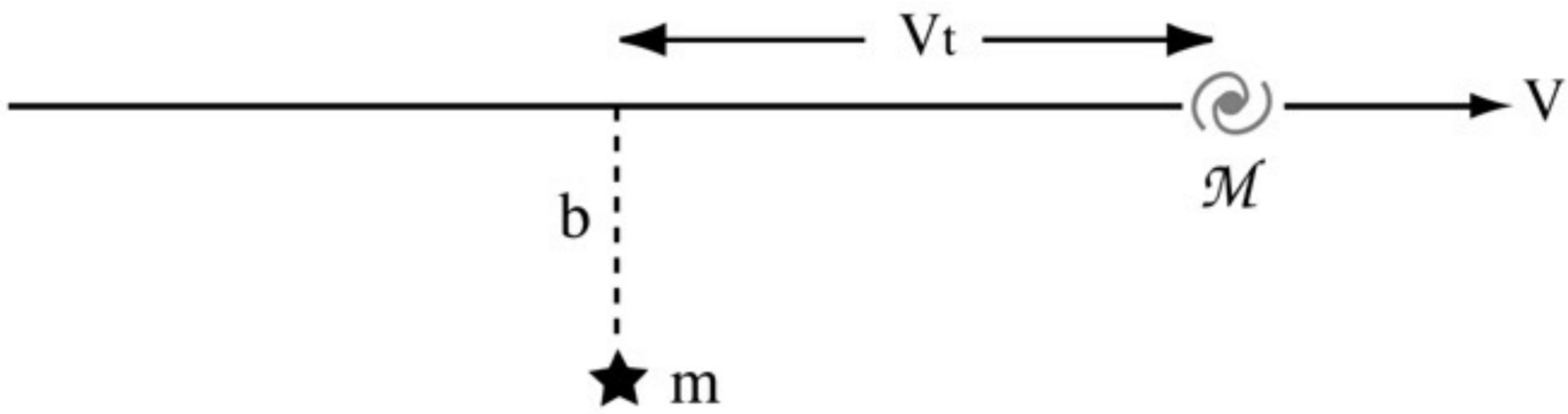
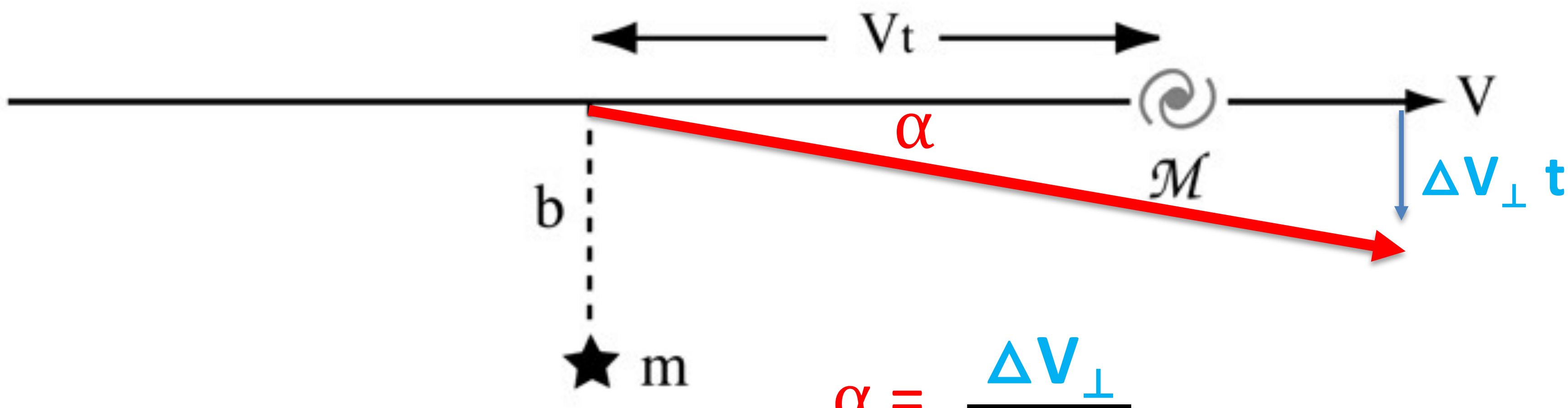


Fig 7.4 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

A galaxy of mass $\mathcal{M}$ moves with speed **V** past a stationary star (or clump of dark matter) of mass **m** in the host halo, a distance **b** from its path.

Encounter with a Single Star / DM particle



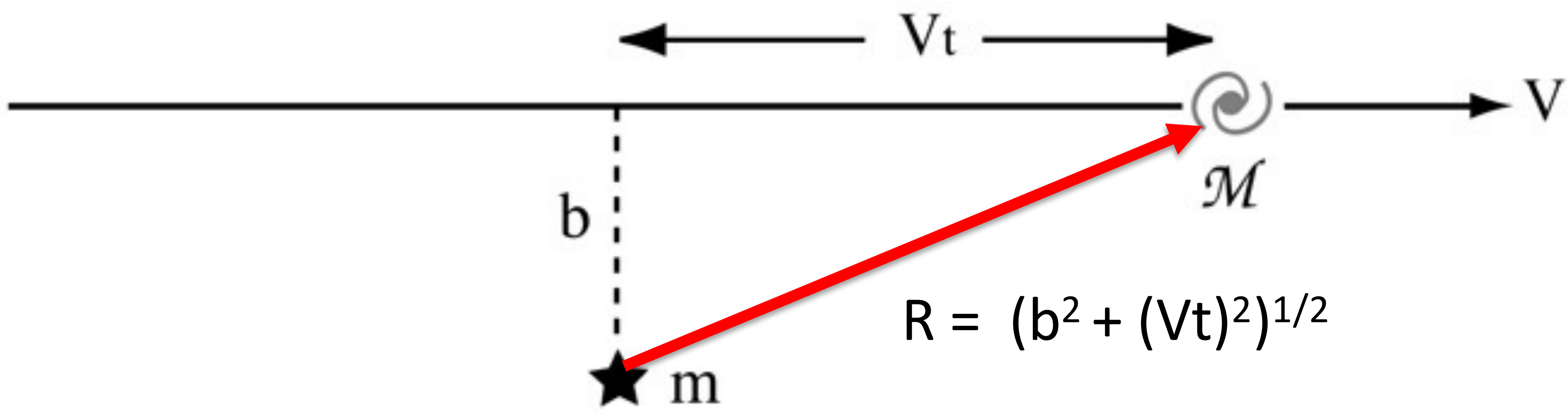
$$\alpha = \frac{\Delta V_\perp}{V}$$

$$F_\perp = \mathcal{M}\,(\Delta V_\perp / \Delta t)$$

Impulse Approximation: $\Delta V_\perp = \mathcal{M}^{-1} \int F_\perp dt$

Where : $F_\perp = \dfrac{G\,\mathcal{M}\,m}{R^2}\,\dfrac{\mathbf{b}}{R}$

Encounter with a Single Star / DM particle



Galaxy is deflected by:

$$\Delta V_\perp = \mathcal{M}^{-1} \int \frac{G\,\mathcal{M}\,m \quad b\,dt}{(b^2 + (Vt)^2)^{3/2}}$$

$$= G\,m\,b \left. \frac{t}{b^2 (b^2 + (Vt)^2)^{1/2}} \right|_{-\infty}^{\infty}$$

X 2

b << Vt so drop b in the brackets

$$= \frac{2\,G\,m}{b\,V}$$

Encounter with a Single Star / DM particle

The star/DM Particle ALSO receives a kick, so the change in energy of the system:

$$\Delta\, KE_{\perp} = \tfrac{1}{2}\,\mathcal{M}\,\Delta V_{\perp}{}^{2}{}_{\text{from m}} + \tfrac{1}{2}\,m\,\Delta V_{\perp}{}^{2}{}_{\text{from } \mathcal{M}}$$

Conservation of Momentum: $m\,\Delta V_{\perp\,\text{from}\,\mathcal{M}} = \mathcal{M}\,\Delta\, V_{\perp\,\text{from m}}$

$$= \tfrac{1}{2}\,\mathcal{M}\left(\frac{2Gm}{b\,V}\right)^{2} + \tfrac{1}{2}\,m\left(\frac{2G\mathcal{M}}{b\,V}\right)^{2}$$

Note that since $m \ll \mathcal{M}$, the term on the RIGHT dominates

$$= \frac{2G^{2}\, m\, \mathcal{M}(\mathcal{M} + m)}{b^{2}\, V^{2}}$$

The DM particle / Star is acquiring MOST of the energy. Since energy is conserved, that energy must come from the motion of the satellite ALONG its orbit (Parallel, denoted by **II**)

Encounter with a Single Star / DM particle

Determining the change in speed of the satellite **along** its orbit: $\Delta V_{||}$

Initial Kinetic Energy = Total Energy After the Encounter

$$\tfrac{1}{2}\mathcal{M}V^2 = \Delta KE_{\perp} + \Delta KE_{||\,\text{satellite}} + \Delta KE_{||\,\text{particle}}$$

$$= \Delta KE_{\perp} + \tfrac{1}{2}\mathcal{M}\left(V + \Delta V_{||\,\text{from } m}\right)^2 + \tfrac{1}{2}m\left(\Delta V_{||\,\text{from } \mathcal{M}}\right)^2$$

Recall, $\Delta V_{\perp\,\text{from } \mathcal{M}} = \mathcal{M}/m\,\Delta V_{\perp\,\text{from } m}$

So : $\Delta V_{||\,\text{from } \mathcal{M}} = \mathcal{M}/m\,\Delta V_{||\,\text{from } m}$

$$\cancel{\tfrac{1}{2}\mathcal{M}V^2} = \Delta KE_{\perp} + \tfrac{1}{2}\mathcal{M}\left(\cancel{V^2} + 2V\Delta V_{||} + \Delta V_{||}^2\right) + \tfrac{1}{2}m(\mathcal{M}/m)\Delta V_{||}^{\,2}$$

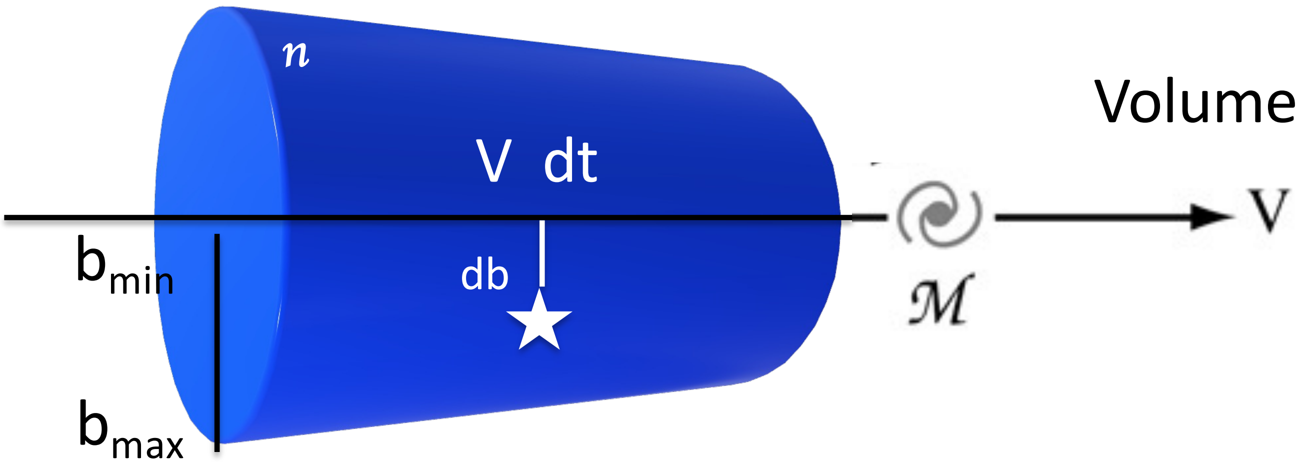
Assuming $\Delta V_{||} << V$, then we can drop $\Delta V_{||}^2$ terms

$$\Delta KE_{\perp} = -\tfrac{1}{2}\mathcal{M}\left(2V\Delta V_{||}\right)$$

$$\Delta V_{||} = -\frac{\Delta KE_{\perp}}{\mathcal{M}V} = -\frac{2G^2 m(\mathcal{M}+m)}{b^2V^3}$$

NEXT: (weak) Encounters with **MANY** Stars / DM particles

Consider a satellite galaxy moving through a cylindrical volume of a medium (stellar halo or DM halo) with number density $n = \rho / m$



Volume = $2\pi\, b\, V\, dt$

Integrate ΔV_{II} over **b** to determine the total perturbation to the velocity along the orbit from n weak encounters

$$\Delta V_{II\ TOTAL} = \int_{b_{min}}^{b_{max}} \Delta V_{II\ \text{per encounter}}\, n\, 2\, \pi\, V\, dt\, b\, db$$

$$= \frac{-\,2\, G^2 (\mathcal{M} + m)\, m\, n\, 2\, \pi\, V\, dt}{V^3} \int_{b_{min}}^{b_{max}} \frac{b\, db}{b^2}$$

$\ln(\Lambda)$

Weak Encounters with **MANY** Stars / DM particles

Coulomb Logarithm:

$$\ln(\Lambda) = \ln\,(b_{max} / b_{min})$$

b_{max} = Current Separation between satellite and COM of the host galaxy

b_{min} = r_s Radius for a "Strong Encounter"

Strong Encounters: If the change in potential energy is comparable to the initial kinetic energy of the satellite

$$\frac{G(M+m)}{r} \gtrsim \frac{V^2}{2}$$

$$r_s \equiv \frac{2G(M+m)}{V^2}$$

We need our separation, b, to be larger than r_s or we invalidate our energy assumptions (that change in potential energy is small)

For the DF calculation we will take V = V_{circ} of the DM halo

Weak Encounters with **MANY** Stars / DM particles

Dynamical Friction: the corresponding deceleration needed to achieve a change in velocity ΔV_{II} along the direction of motion over some dt

$\rho = m\,n$ density of the halo

$\mathcal{M} + m \sim \mathcal{M}$

$$\mathbf{a}_{DF} = \mathbf{a}_{II} = \frac{\Delta \mathbf{V}_{II}}{dt} = \frac{-2\, G^2 (\mathcal{M} + m)\, m\, n\, 2\, \boldsymbol{\pi}\, \mathbf{V}\, \ln(\boldsymbol{\Lambda})}{|V|^3}$$

$$= \frac{-\,4\,\boldsymbol{\pi}\; G^2\, \mathcal{M}\, \rho\; \ln(\boldsymbol{\Lambda})}{|V|^2}\;\frac{\mathbf{V}}{|V|}$$

1. If V of satellite is low, a_{DF} increases
2. Total mass of the halo doesn't matter, it's the density that does.
   - If the density increases (more encounters), the deceleration is stronger
3. A more massive Satellite ($\mathcal{M}$) slows down faster than a lighter one (major encounters will decay faster than minor encounters) (MW and M31 vs. M33 and M31), $F = \mathcal{M}\, a_{DF}$

Weak Encounters within a HALO

We ignored the fact that stars/DM particles in the host also have some internal velocity distribution (we assumed the initial speed of the halo particles was 0).

$$a_{DF} = -2\, G^2\, \mathcal{M}\, m\, n\, 2\, \boldsymbol{\pi}\, ln(\boldsymbol{\Lambda}) \int f\,(v)\, \mathbf{v}\, /\, |\mathbf{v}|^3$$

$$f(v) = \frac{n_o}{(2\pi\sigma^2)^{3/2}} exp(-\frac{v^2}{2\sigma^2})$$

Isotropic, Maxwellian velocity distribution function

$$\mathbf{a}_{\mathrm{DF},\mathbf{x}} = \frac{dv}{dt} = -\frac{4\pi G^2 M \rho}{v^2} \ln\Lambda \left[erf(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \frac{\mathbf{v}_x}{v}$$

$X = v/(\sqrt{2}\sigma)$ and erf is the error function. Consider an isothermal sphere with a flat rotation curve Vc and velocity dispersion $\sigma = v_c/\sqrt{2}$, so $X = 1$

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}$$

$$\boxed{\mathbf{a} = -0.428 \frac{G M_{\mathrm{sat}} \ln(\Lambda)}{r^2} \frac{\mathbf{v}}{v}}$$

Dynamical Friction for a satellite moving through a density distribution that follows the profile for an Isothermal Sphere.