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Analysis of Binary Search using recursion.

$$T(N) = C + T\left(\frac{N}{2}\right) \rightarrow (1)$$

$$T\left(\frac{N}{2}\right) = C + T\left(\frac{N}{4}\right) \rightarrow (2)$$

Substitute (2) in (2).

$$T(N) = T\left(\frac{N}{4}\right) + 2C \rightarrow (3)$$

$$T\left(\frac{N}{4}\right) = C + T\left(\frac{N}{8}\right) \rightarrow (4)$$

Substitute (4) in (3)

$$T(N) = T\left(\frac{N}{8}\right) + 3C \rightarrow (5)$$

⋮

Pattern identified; →

$$T(N) = T\left(\frac{N}{2^i}\right) + iC.$$

At some point, as $\frac{N}{2^i}$ diminishes, we reach only 1 element.

$$T\left(\frac{N}{2^i}\right) = T(1)$$

$$T(N) = T\left(\frac{N}{2^i}\right) + ic \rightarrow (6)$$

$$T\left(\frac{N}{2^i}\right) = T(1)$$

$$\frac{N}{2^i} = 1$$

$$N = 2^i$$

Take log on both side.

$$\log_2 N = i(\log_2 2) \quad \log_2 2 \rightarrow 1$$

$$\log_2 N = i \rightarrow (7)$$

Substitute (7) in (6)

$$T(N) = T\left(\frac{N}{2^{\log_2 N}}\right) + c \cdot \log_2 N \quad // \; 2^{\log_2 N} \rightarrow N$$

$$T(N) = T\left(\frac{\cancel{N}}{\cancel{N}}\right) + c \cdot \log_2 N$$

$$T(N) = T(1) + c \cdot \log_2 N.$$

$$T(N) = \underbrace{K.}_{} + c \cdot \log_2 N \Rightarrow O(\log_2 N)$$

even if 1 element exist, same constant time is involved to search element.

ignore all constant, to calculate worst case ...