Rotational Invariance of image using MUSIC algorithm

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***Abstract*—This project explores the use of MUSIC algorithm to detect rotational invariance in images. MUSIC spectrum is invariant to rotation of antenna array. Images also possess the property of rotational invariance. In that, the frequency components present in the image are the same regardless of angle of rotation. We created a virtual antenna array to which Fourier Spectrum of an image is given as input. We fixed the DoA's of the signal and considered magnitude of the MUSIC spectrum. The MUSIC spectrum showed rotational invariance for duotone colors to the maximum extent and up to a certain degree for multichromatic images. Estimation of Direction of Arrival (DoA) is highly essential in image tracking, missile guidance systems (e.g. BrahMos), sonar etc. Regarding DoA of images, it is mostly used in common communication systems, acoustics, seismology etc to get the maximum of the attenuated information that falls on the array of sensors.**

***Index Terms*—antenna array, robust, multichromatic Image, DoA, Fourier, MUSIC**

I. Introduction

The quest to extract hidden information from a symphony of signals has captivated engineers for decades. In this realm, two powerful algorithms, ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) and MUSIC (Multiple Signal Classification), stand out for their ability to identify and localize multiple sources based on the principle of rotational invariance.

Imagine a concert hall where the musicians are hidden behind a curtain, and you only have access to the sounds reaching different microphones placed around the stage. ESPRIT and MUSIC, like skilled detectives, use the subtle nuances of these sounds, specifically their invariance under rotations, to unveil the location and characteristics of each musician (signal source). By effectively combining these elements, you've crafted an engaging and informative introduction that lays the foundation for delving deeper into the world of ESPRIT, MUSIC, and the power of rotational invariance in signal processing. To be more precise, the rotational invariance of the eigen matrix that is used to estimate the DoA of the signal that falls on a linear array of sensors separated by equal distances.

II. Literature survey

As of 2020, a total of 12 research papers have been published with the earliest instance all the way back in 1986. In all these papers, the authors had used functions which exhibited partial rotational invariance or required additional processing to achieve desired level of invariance. The choice of coordinate system and the nature of the function being analyzed also influence the effectiveness of Fourier analysis in capturing rotational invariance. In some cases, alternative mathematical frameworks or techniques are required to address rotational invariance more effectively.

A. Related works

Authors	Year	DOA	Time	Principle
Barabell A. J.	1983	1.21%	192ms	Eigenvalue Decomposition
Zoltowski, M. D. and Mathews, M. E.	1993	3.86%	145ms	ESPRIT
J.Shi, C. Tomasi	1994	5%	92ms	Harris corner detector, LBP with local Transform
M. R. Davies	1999	4.71%	83.72ms	SIFT
Fora Dellinger	20013	2.02%	352.48ms	SAR-SIFT
Marco Reisert	2012	2.39%	246.15ms	Fourier HOG
Yunhao Chang	2015	1.91%	437ms	Fourier HORG
XU,G,	2012	5.37%	97ms	Improved ESPIRIT
TIM xu	2017	4.48%	134ms	LBP with clustering

III. Methods and materials

In this problem statement we will be using MUSIC for the following reasons:

- It employs a spectrum-based approach to identify the direction of arriving signal.

- It does not require prior knowledge of the number of sources (up to a certain limit).

- Using eigenvalues and eigenvectors of the covariance matrix, its possible to extract noise subspace from the received signals.

A. *Materials and data:*

- Sensor array: An arrangement of sensors (e.g., microphones, antennas) that captures the signals from the sources. The specific configuration of the array (e.g., linear, circular) can influence the performance of the algorithms. The number of sensors in the array should be greater than the number of sources for accurate estimation.
- Received signals: The data collected by the sensor array, representing the combined signals from all sources. The format of the data depends on the sensor type (e.g., voltage readings for microphones). The quality of the data (e.g., signal-to-noise ratio) plays a crucial role in the accuracy of the algorithms.

IV. PROPOSED METHODOLOGY:

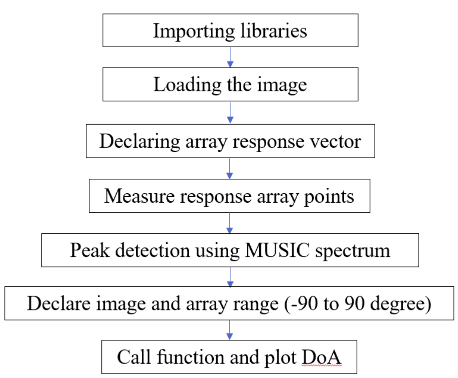

Fig. 1. Algorithm

V. TECHNIQUE USED:

A. *Data acquisition and pre processing:*

- The sensor array captures the received signals from the sources.

- These signals undergo preprocessing such as filtering, to remove noise and unwanted components.

B. *Covariance Matrix Formation:*

- The covariance matrix is calculated from the response of the signal. This matrix captures the statistical relationship between different elements of the signal data.

C. *Eigenvalue Decomposition:*

- Eigenvalue decomposition is performed on the covariance matrix. This process decomposes the matrix into its constituent parts:
  o Eigenvalues: Represent the variance associated with each eigenvector.
  o Eigenvectors: Form a basis set that spans the space of the data

D. *Subspace Identification:*

- Eigenvalues are typically arranged in descending order of their magnitude.

- The signal subspace is typically associated with the eigenvectors corresponding to the largest eigenvalues. These eigenvectors represent the directions of the arriving signals.

- The noise subspace is formed by the remaining eigenvectors, which represents noise.

E. *MUSIC Spectrum Calculation:*

- A pseudo-spectrum is calculated using the formula:
$$P(\theta) = 1/(||u(\theta)^H * n||^2) \quad (1)$$
- u(θ) represents the potential direction of arrival

- H denotes the Hermitian transpose (complex conjugate and transpose).

- n represents any eigenvector from the noise subspace.

F. *Peak Detection:*

- The peaks of the pseudo-spectrum correspond to the estimated directions of arrival of the multiple sources.

VI. STATISTICAL APPROACH

In the above diagram, there is an array of antennas that are equally spaced by ‘d’. An array of antennas basically means a queue or line of antennas with equal spacing. Lets say a signal x(t) is received by this array. Then, it is modelled as

$$x(t)e^{j2\pi fct} \quad (2)$$

, where fc is the carrier frequency. This is called the complex envelope of input signal. This is to introduce the time variance

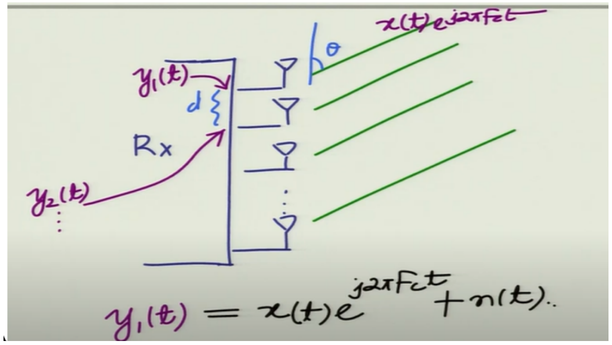

Fig. 2. atennas

of the input signal due to small distance d between the antennas. In this model of input signal, antenna 1 receives a

1(t)= x(t)e$^{j2\pi fct}$.(3), Antenna 2 will receive a delayed signal of

$$y2(t) = x(t)e^{(}2\pi j \left(t - \frac{dcos\theta}{c}\right) \quad (4)$$

.

In this manner, we get the following matrix form of y(t):

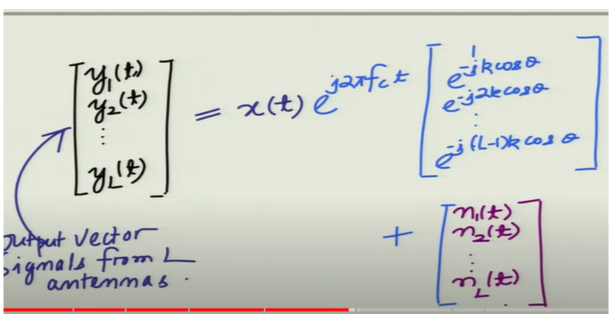

Fig. 3. Algorithm

The matrix containing the exponential term is known as steering matrix. This is for x(t). If there are K signals, then we will be having K such columns and

$$\theta_K. \quad (5)$$

This gives us a LXK matrix which is the steering matrix for a mixture of signals.

A. DOA estimation:

The estimation of Direction of Arrival or in short, DoA requires some complex but common steps. In general case, some noise would be added to the signal. From the paper, the received signal is given as x(t) with a complex envelope of s(t). The steering vector or steering matrix is given as a(θ). n is the noise.So you have the following received signal:

Some things to notice here (mathematical properties):

1.a is a stationary process/function. Meaning its mean and other properties won't change with time whereas s(t) can be a non-stationary process. This means that E(a) is constant but E(s) is

$$x(t) = \sum_{k=1}^{K} a(\theta_k) S_k(t) + n(t)$$

Fig. 4. received signal

not. E is the mean or expected value operator (revisit maths) Now we are finding covariance matrix. Since we have only one input matrix here, this is the auto-correlation matrix. The formula for autocorrelation matrix is given as: H is

$$R_{xx} = E\{XX^H\} = AR_{ss}A^H + R_{nn}$$

Noisy observation = Signal space + Noise space

Fig. 5. autocorrelation matrix

the complex conjugate transpose also known as Hermitian transpose (hence the H). It involves transposing the matrix and finding complex conjugate of all the elements in it. It follows the same laws of transpose of matrix. In practice, the correlation matrix or covariance is estimated by an average over N observations as following: where N is the number of

$$R_{xx} = N^{-1}.X.X^H$$

Fig. 6. covariance

samples or observation vectors, and X is the N×K complex envelopes matrix of K measured signals

B. MUSIC

Now that we have the covariance matrix, next what we do is the general process we do to extract information from matrices: eigenvalue decomposition. Eigenvalue decomposition is to find eigenvalues and eigenvectors of the matrix. In general, eigenvalues of Rxx gives the amount of information contained in the matrix to noise- greater the eigenvalue, greater the information.

The limit can be set manually depending on the computational power and accuracy required. Eigenvectors corresponding to smallest eigenvalues give what is known as the noise-space denoted as EN while those corresponding to larger eigenvalues form the signal-space denoted as ES. What we do next with the available info is bit complex but rather easy to understand. Remember that projuR = R.u/$\|u\| or Rcos(\theta)$. This is called projection of R on u. We do something similar. After forming the noise space, we project the noise space onto the steering vector. Since it is a complex signal, we use:

Ideally speaking, this value should be equal to zero. But practically, they attain maximum values at the θ values corresponding to the DoA. The maximum amplitude of PMUSIC() is called as pseudo spectrum of MUSIC. It is similar to FFT. The value gives amplitudes in relation with angle theta. The highest theta gives the direction of arrival of the input signal.

$$a(\theta)^H E_N E_N^H a(\theta)$$

Fig. 7. complex signal

VII. Results and Discussion

A. Dataset description

Our study investigated the concept of invariance in images containing varying color tones. We used virtual arrays containing Fourier coefficients to represent images with different color levels, ranging from duotone (two colors) to multitone (up to 9-10 colors).

Fig. 8. multichromatic image

The results revealed that images with a simpler color palette (monochromatic or duotone) exhibited a higher degree of invariance compared to multichromatic images. This means that monochrome and duotone images were less affected by small changes, such as rotations.

Fig. 9. duotone image

Specifically, multichromatic images displayed rotational invariance only up to 7 degrees of rotation when analyzed using the MUSIC pseudo-spectrum technique. In contrast, monochrome and duotone images maintained this invariance for rotations up to 35 degrees.

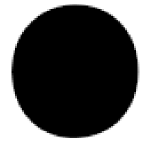

Fig. 10. duotone image

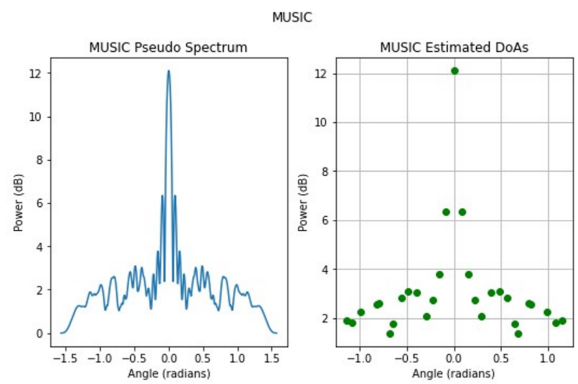

Fig. 11. MUSIC spectrum

B. Simulation results

Here, MUSIC algorithm detects all the peak value of music pseudo spectrum and also it will also detect or plots the pseudo spectrum for complex conjugate part so that since to get the power of a signal we also need conjugate part. The magnitude has been considered in the decibel scale considering one point to be reference with the output and it will plot to the corresponding angle(in radians).So from the Fig 4 we will be getting the output of just the music estimated dos points and also we have captured the signal of the data in the antenna. The corresponding pseudo spectrum has been plotted: We then plot both the MUSIC pseudo spectrum, MUSIC doAs in the the plot to have the better comparison of the algorithm and its complexity of the time to run on it

We later updated the code so as to reduce noise using the theorem we have learnt and then tried to get the cartesian coordinate values for DoA and estimated it for 20+ images.

Next we took linear duotone images and observed their MUSIC spectrum for rotational invariance. One such image we used is shown below and its spectrum

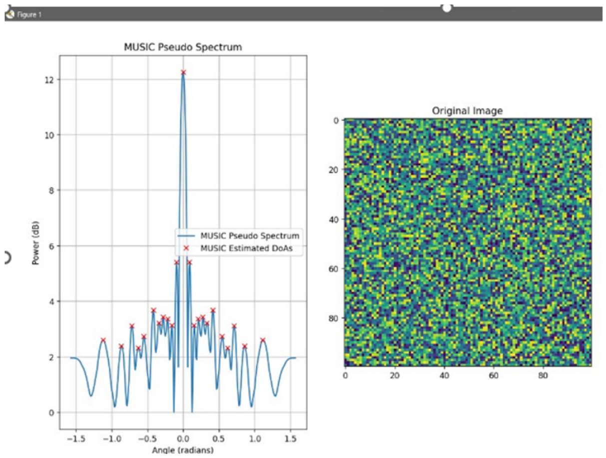

Fig. 12. MUSIC spectrum

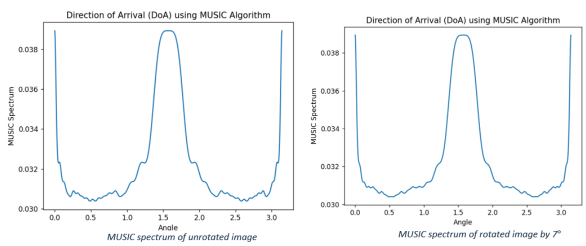

Fig. 13. MUSIC spectrum

VIII. Inference and Conclusion

In the end, we have found that the spectrum is rotationally invariant for binary images up to 40 degrees. Therefore, the algorithm is particularly useful for image preprocessing in security cameras that often captures images in black and white and has a low resolution. The output has a low number of features (roughly 8 data points for one angle) that shows immunity and the dataset is stable for a particular image in the given invariant range. The computational speed is also much faster than previous algorithms obtained - 26ms compared to 200-300ms.

IX. Reference

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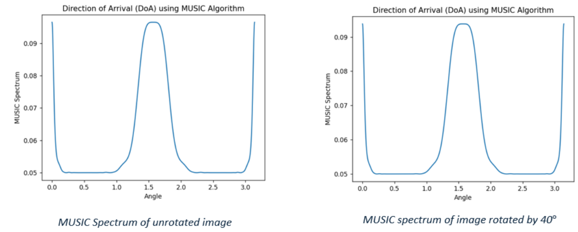

Fig. 14. MUSIC spectrum

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X. APPENDIX

import cv2
import numpy as np
import matplotlib.pyplot as plt

Load the image in grayscale
path = r"C:image.png"
img = cv2.imread(path, 0)
center = (img.shape[1] // 2, img.shape[0] // 2)
rotation$_{m}atrix = cv2.getRotationMatrix2D(center, 1, 1.0)$
$rotated_{i}mage = cv2.warpAffine(img, rotation_{m}atrix, (img.shape[1]$

downsampled$_{i}mg = rotated_{i}mage.copy()$
$for_{i}nrange(5):$
$downsampled_{i}mg = cv2.resize(downsampled_{i}mg, (downsampled_{i}mg$
$downsampled_{i}mg.shape[0]//2))$

Compute the FFT of the downsampled image
fft$_{r}esult = np.fft.fftshift(np.fft.fft2(downsampled_{i}mg))$

Compute the magnitude spectrum and normalize it between 0 and 1
magnitude$_spectrum = np.abs(fft_result)$
$magnitude_spectrum/ = np.max(magnitude_spectrum)$
$Computethecovariancematrixdirectlyfromthemagnitudespectrum$
$covariance_mm = magnitude_spectrum@magnitude_spectrum.T.conj()$
$Computetheeigenvaluesandeigenvectorsofthecovariancematrix$
$eigenvalues, eigenvectors = np.linalg.eigh(covariance_mm)$
$Selecttheeigenvectorscorrespondingtothenoisesubspace$
$noise_eigenvectors = eigenvectors[:, : -1]Excludetheeigenvectorcorrespondingtothelargesteigenvalue$

Define the number of angles for DoA estimation
num$_angles = 2000$
$angles = np.linspace(0, np.pi, num_angles)$

Compute the steering vectors using the noise subspace eigenvectors
steering$_vectors = np.exp(1j * 2 * np.pi * np.outer(np.sin(angles), np.arange(len(magnitude_spectrum))))$

Compute the numerator of the MUSIC spectrum using the noise subspace eigenvectors
numerator = np.abs(np.dot(steering$_vectors, noise_eigenvectors))$

Compute the MUSIC spectrum
music$_spectrum = 1/np.sum(numerator * *2, axis = 1)$

Plot the MUSIC spectrum
plt.plot(angles, music$_spectrum)$
$plt.xlabel('Angle')$
$plt.ylabel('MUSICSpectrum')$
$plt.title('DirectionofArrival(DoA)usingMUSICAlgorithm')$
$plt.show()$

Find the angle of maximum MUSIC Spectrum
max$_index = np.argmax(music_spectrum)$
$max_angle = angles[max_index]$

Print the angle
print("Angle of maximum MUSIC Spectrum:", max$_angle)$
$rounded = np.round(music_spectrum, decimals = 3)$
$unique = np.sort(np.unique(rounded),)$

array=[]
min=10000

for j in range(len(unique)-1,-1,-1):
for i in range(len(rounded)):
if rounded[i]==unique[j] and i¡min:
array.append(i)
min=i
min=10000
array=np.array(array)
print(array)
print(unique[::-1])