COMS 3110: Homework 1
Due: June 29^{th}, 11:59pm
Total Points: 60

Late submission policy. Late submissions will not be accepted. If your assignment is late, you will receive 0 points.

Submission format. Homework solutions will have to be typed. You can use word, LaTeX, or any other type-setting tool to type your solution. Your submission file should be in pdf format. Do **NOT** submit a photocopy of handwritten homework except for diagrams that can be hand-drawn and scanned. We reserve the right **NOT** to grade homework that does not follow the formatting requirements. Name your submission file: `<Your-net-id>-3110-hw1.pdf`. For instance, if your netid is `asterix`, then your submission file will be named `asterix-3110-hw1.pdf`. Each student must hand in their own assignment. If you discussed the homework or solutions with others, a list of collaborators must be included with each submission. Each of the collaborators has to write the solutions in their own words (copies are not allowed).

General Requirements

- When proofs are required, do your best to make them both clear and rigorous. Even when proofs are not required, you should justify your answers and explain your work.
- When asked to present a construction, you should show the correctness of the construction.
- When concluding the big-O of a function, unless otherwise indicated, you should give the simplest form of the function (e.g. $O(n)$ rather than $O(2n+1)$).
- When giving the big-O of a function, give the tightest applicable bound (e.g. $2n \in O(n)$, not $2n \in O(n^{100})$)

Some Useful (in)equalities

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $2^{\log_2 n} = n$, $a^{\log_b n} = n^{\log_b a}$, $n^{n/2} \leq n! \leq n^n$, $\log x^a = a \log x$
- $\log(a \times b) = \log a + \log b$, $\log(a/b) = \log a - \log b$
- $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n-1)}{r-1}$
- $1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^n} = 2(1 - \frac{1}{2^{n+1}})$
- $1 + 2 + 4 + ... + 2^n = 2^{n+1} - 1$

1. Prove or disprove the following.

   (a)
   $$(3n^2 + 1)(\frac{4n^3 + n}{2}) \in O(n^5)$$

   (b)
   $$2^{2^{n+2}} \in O(2^{2^{n+1}})$$

   (c) For $a > 1, b > 1$,
   $$O(\log_a(n)) = O(\log_b(n))$$

2. Derive the runtime of the following code as a function of n and determine its Big-O upper bound. You must show the derivation of the end result. Simply stating the final answer without any derivation steps will result in zero points.

(a)

```
a = 0;
for i in the range [1, n] {
    for j in the range [1, i] {
        for k in the range [1, i+j] {
            a++;
        }
    }
}
```

(b)

```
i = 1
while i < n {
    for j in range [1, 300] {
        /* constant number of primitve operations */
    }
    i = i * 5
}
```

(c) You may assume a naive implementation of $pow(a, b)$ that runs in $O(b)$ time.

```
i = pow(2,n)
while i >= 1 {
  i = i / 2
  for j in range [1, 2*i] {
    /* constant number of primitve operations */
  }
}
```