From Amdahl's Law to Big Data: *A Story of* Computer Sciences and Technology

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The Journey of Supercomputing

- The Background of Parallel Processing
  - Speedup
  - Sources of overhead
- The Laws of Scalable Computing
  - The Amdahl's law
  - The Gustafson's law
  - The Sun-Ni's law
- Impacts and Discussions

Performance Evaluation

(Improving performance is the goal)

- Performance Measurement
  - Metric, Parameter
- Performance Prediction
  - Model, Application-Resource
- Performance Diagnose/Optimization
  - Post-execution, Algorithm improvement, Architecture improvement, State-of-the-art, Scheduling, Resource management/Scheduling

Performance of Parallel Processing

Models of Speedup

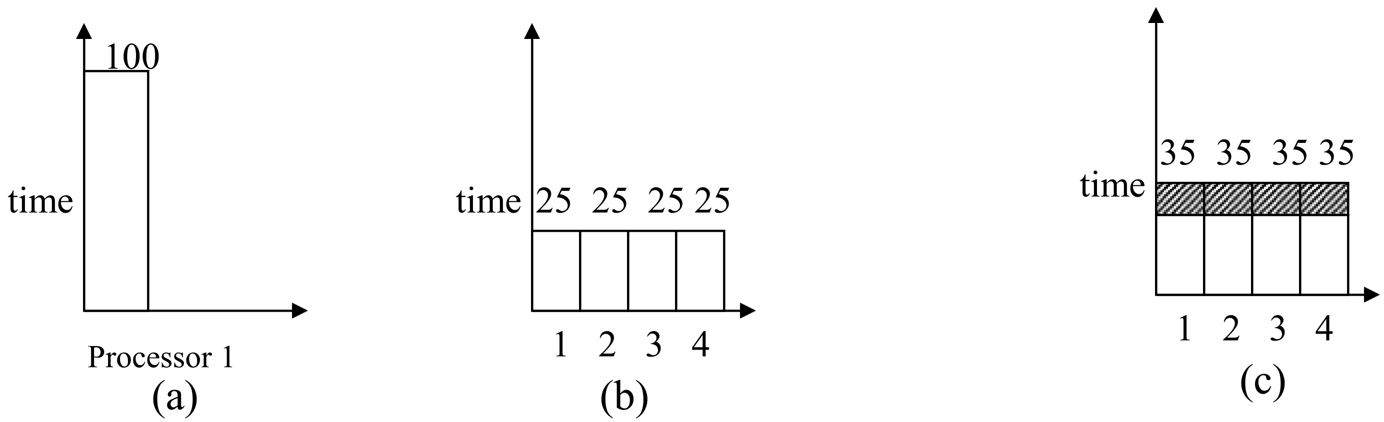
- Speedup
  - Ts = time for the best serial algorithm
  - Tp= time for parallel algorithm using p processors

$$S_p = \frac{T_s}{T_p}$$

- Simple enough, but also unexpected complex

$$S_p = \frac{\text{Uniprocessor Execution Time}}{\text{Parallel Execution Time}}$$

Example



$$S_p = \frac{100}{25} = 4.0,$$

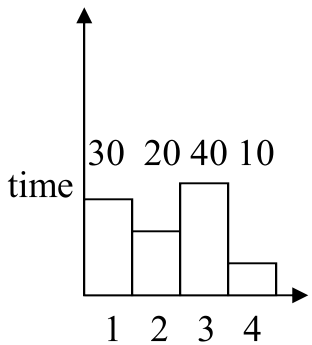
perfect parallelization

$$S_p = \frac{100}{35} = 2.85,$$

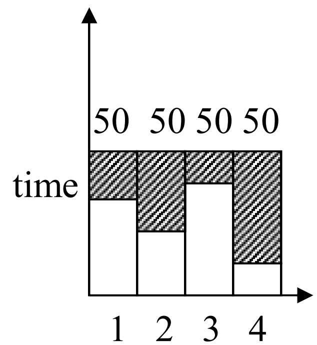
perfect load balancing

but synch cost is 10

Example (cont.)



(d)



(e)

$$S_p = \frac{100}{40} = 2.5,$$

no synch

but load imbalance

$$S_p = \frac{100}{50} = 2.0,$$

load imbalance

and synch cost

What Is "Good" Speedup?

- *Linear* speedup:

$$S_p = p$$

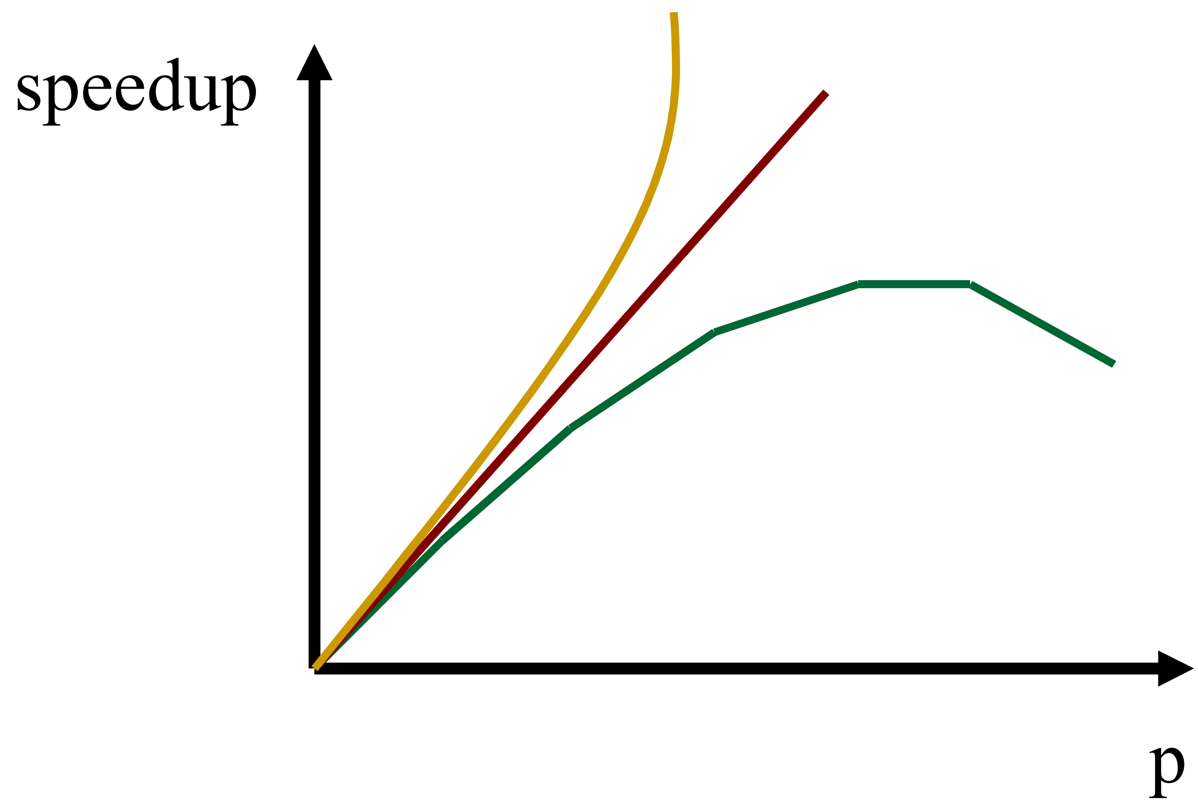
- *Superlinear* speedup

$$S_p > p$$

- *Sub-linear* speedup:

$$S_p < p$$

Speedup



Sources of Parallel Overheads

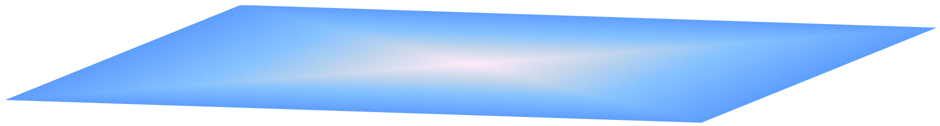
- Interprocessor communication
- Load imbalance
- Synchronization
- Extra computation

Causes of Superlinear Speedup

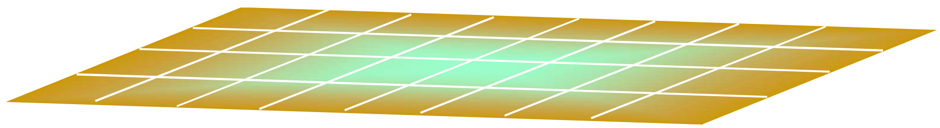
- Cache size increased
- Overhead reduced
- Latency hidden
- Randomized algorithms
- Mathematical inefficiency of the serial algorithm
- Higher memory access cost in sequential processing

• X.H. Sun, and J. Zhu, "Performance Considerations of Shared Virtual Memory Machines," *IEEE Trans. on Parallel and Distributed Systems*, Nov. 1995

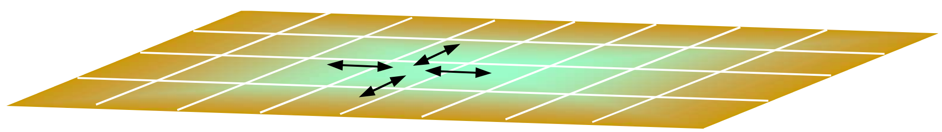
Degradations of Parallel Processing



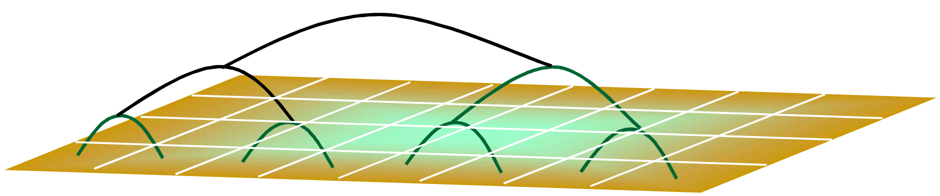
Unbalanced Workload



Communication Delay



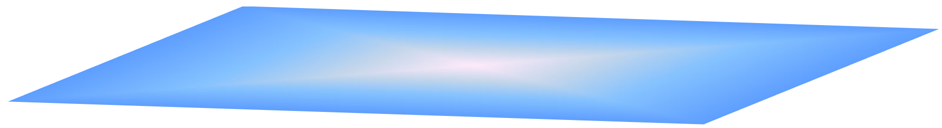
Overhead Increases with the Ensemble Size



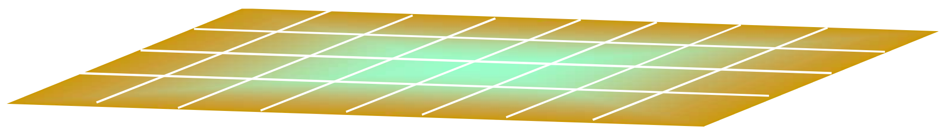
Overheads

- *communication*
- *Load imbalance*
- *Synchronization*
- *Extra computation*

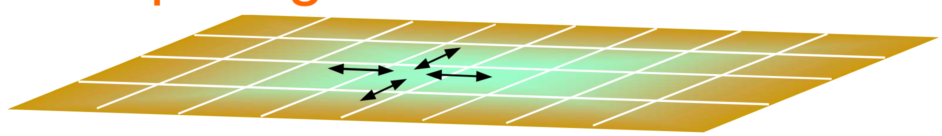
Degradations of Distributed Computing



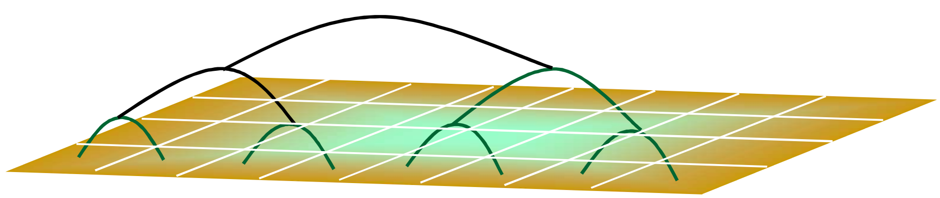
Unbalanced Computing Power and Workload



Shared Computing and Communication Resource

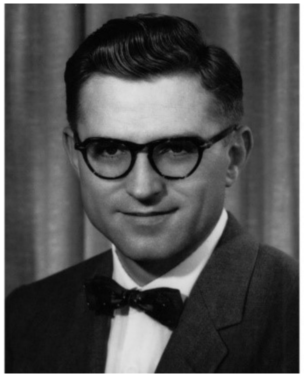


Uncertainty, Heterogeneity, and Overhead Increases with the Ensemble Size



Principals of Architecture Design

- Make common case fast (90/10 Rule)
- Amdahl's Law
  - Law of diminishing returns
- Speedup
  - Achieved performance improvement over original



Gene Amdahl

$$\text{Speedup Overall} = \frac{\text{speed new}}{\text{speed old}} = \frac{\text{execution time old}}{\text{execution time new}}$$

Here performance is measured in **Speed**

Amdahl's Law

Execution time of any code has two portions

Portion I: not affected by enhancement
Portion II: affected by enhancement

$$\text{execution time}_{old} = \text{execution time}_{p1} + \text{execution time}_{p2}$$

α is % of original code that cannot benefit from enhancement

As p -> infinity, execution time$_{new}$ -> α * execution time$_{old}$

$$\text{execution time}_{new} = \underbrace{(\alpha) * \text{execution time}_{old}}_{\text{Execution time}_{p1}} + \underbrace{(1-\alpha) * \frac{\text{execution time}_{old}}{p}}_{\text{Execution time}_{p2}}$$

p is speedup factor of old/new execution times for portion II

Amdahl's Law for Parallel Processing (1967)

- Let α= fraction of program (algorithm) that is serial and cannot be parallelized. For instance:
  - Loop initialization
  - Reading/writing to a single disk
  - Procedure call overhead
- Parallel run time is given by

$$\text{execution time}_{\text{new}} = (\alpha) * \text{execution time}_{\text{old}} + (1-\alpha) * \frac{\text{execution time}_{\text{old}}}{p}$$

$$T_p = (\alpha + \frac{1-\alpha}{p}) \bullet T_s$$

Gene M Amdahl, "Validity of the single processor approach to achieving large scale computing capabilities," AFIPS spring joint computer conference, 1967

Amdahl's Law

- Amdahl's law gives a limit on speedup in terms of α

$$S_p = \frac{T_s}{T_p} = \frac{T_s}{\alpha T_s + \frac{(1-\alpha)T_s}{p}} = \frac{1}{\alpha + \frac{1-\alpha}{p}}$$

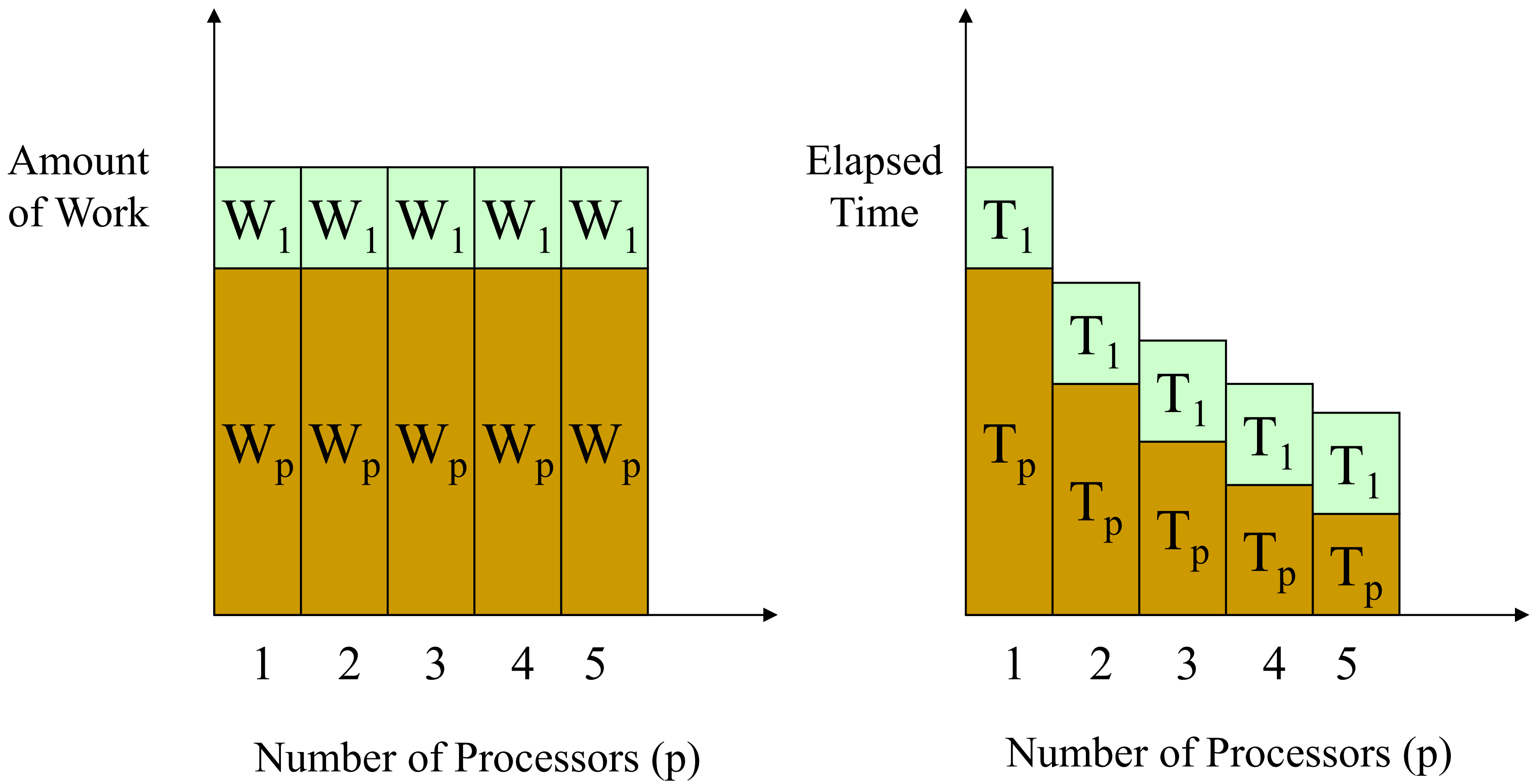
- If we assume that the serial fraction is fixed, then the speedup for infinite processors is limited by 1/α

$$\lim_{p->\infty} S_p = \frac{1}{\alpha}$$

- For example, **if α=10%,** then the maximum speedup is **10,** even if we use an infinite number of processors

Amdahl Law

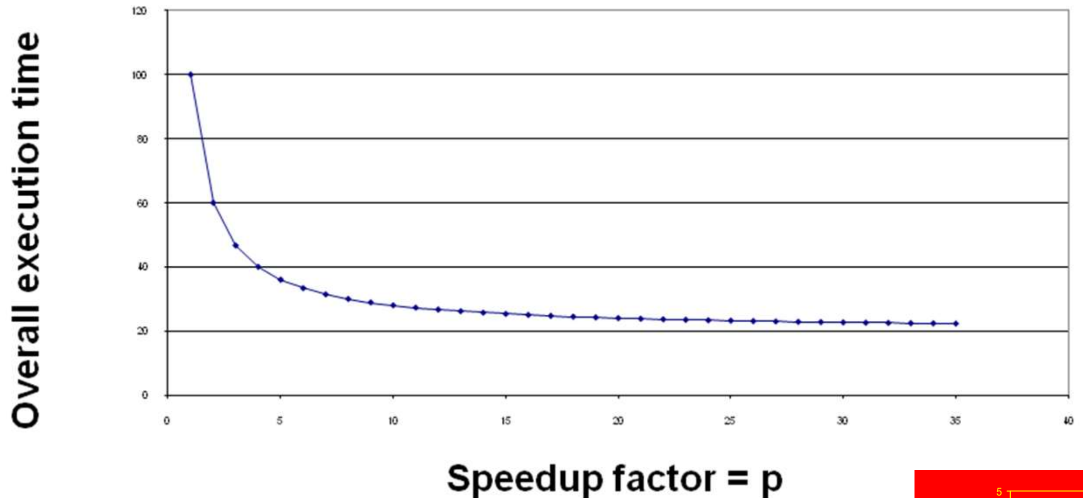
- The sequential part becomes the dominate factor quickly



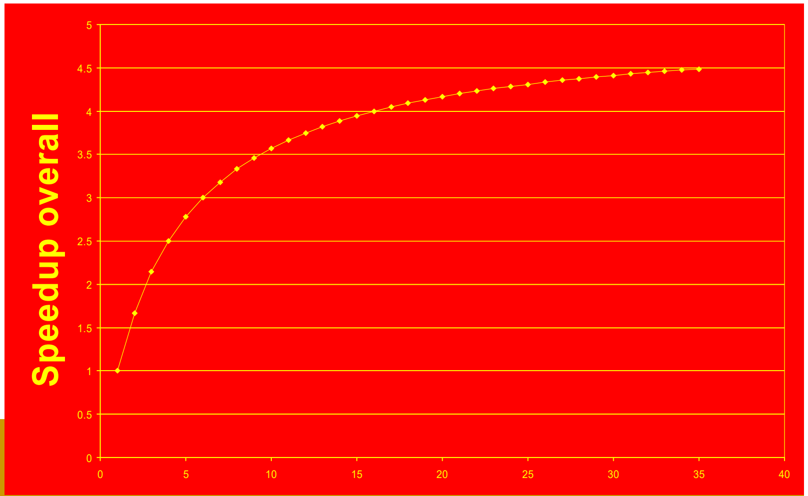
Amdahl's Law

$$\text{execution time}_{\text{new}} = (\alpha) * \text{execution time}_{\text{old}} + (1-\alpha) * \frac{\text{execution time}_{\text{old}}}{p}$$

Example: alpha = 20%



$$\text{Speedup}_{\text{overall}} = \frac{\text{execution time}_{\text{old}}}{\text{execution time}_{\text{new}}} = \frac{1}{(\alpha) + \frac{1-\alpha}{p}}$$



Amdahl's Law with Overhead

- To include overhead will be even worse
- The overhead includes parallelism and interaction overheads

$$Speedup_{FS} = \frac{T_1}{\alpha T_1 + \frac{(1-\alpha)T_1}{p} + T_{overhead}} \rightarrow \frac{1}{\alpha + \frac{T_{overhead}}{T_1}} \ as\ p \rightarrow \infty$$

Amdahl's law: argument against massively parallel systems

Example

- Enhanced mode is used 50% of resulting execution time$_{new}$
- Portions of code using enhanced mode improve performance by factor 10x
- What is speedup for fast mode?

First find 1-α (% of code affected by enhancement)

$$\text{execution time}_{new} = \underbrace{(.5) * \text{execution time}_{new}}_{\text{not enhanced}} + \underbrace{(.5) * \text{execution time}_{new}}_{\text{enhanced}}$$

$$(\alpha) * \text{execution time}_{old} = \frac{(1-\alpha) * \text{execution time}_{old}}{10}$$

$$1-\alpha = .909091$$

$$\text{Speedup}_{overall} = \frac{1}{(\alpha) + \frac{1-\alpha}{n}} = \frac{1}{(1-.9) + \frac{.9}{10}} = 5.3$$

Example

- Enhancement: Parallel processing with 20 computing nodes
- Portions of code containing computations run 20x faster in parallel processing mode.
- What % of original code must be parallelizable to achieve $\text{speedup}_{\text{overall}} = 2$?

$$\text{Speedup}_{\text{overall}} = \frac{\text{execution time}_{\text{old}}}{\text{execution time}_{\text{new}}} = \frac{1}{(\alpha) + \frac{1-\alpha}{n}}$$

$$2 = \frac{1}{(\alpha) + \frac{1-\alpha}{20}} \qquad 1 - \alpha = .5263$$

History back to 1988

All have up to 8 processors, citing Amdahl's law,

$$\lim_{p \to \infty} Speedup_{Amdahl} = \frac{1}{\alpha}$$

IBM 7030 Stretch

IBM 7950 Harvest

Cray X-MP
Fastest computer 1983-1985

Cray Y-MP

Gene Amdahl

Summit: the World Fastest Computer

- 148.6 petaflops (187.66 petaflop theoretical peak)
- 2,282,544 IBM Power 9 core
- 2,090,880 Nvidia Volta GV100 core
- Power efficiency 11.324gigaflop/watt

Bombshell: *Gustafson, etc. Got Speedup of more than 1,000 on Three Applications*

- On a 1024-processor nCUBE parallel computer
- For three applications: wave mechanics, fluid dynamics, and structural analysis.
- Introduced the concept of **Scalable Computing,** *problem size increases with the machine size*

John L. Gustafson, Gary R. Montry, and Robert E. Benner, "Development of Parallel Methods for a 1024-Processor Hypercube," SIAM Journal on Scientific and Statistical Computing, Vol. 9, No.4, 1988 (submitted 3/10/1988, accepted 3/25/1988, appeared April 1988)

John Gustafson, "Reevaluation of Amdahl's Law," Communications of the ACM, Vol. 31, No. 5, May 1988.

Reevaluate Amdahl's Law

- **Amdahl's Law** is designed for technology improvement, but has been widely used to against parallel processing in terms of reducing execution time

- **But**: large computers are not (only) designed for solving existing problem faster, they are designed for solving otherwise unsolvable large problems

- The introduction of **scalable computing,** where *problem size increases with the machine size*

- Fixed-Time Speedup (Gustafson, 88)
  - Emphasis on work finished in a fixed time
  - Problem size is **scaled** from W to W'
  - W': Work finished within the fixed time with parallel processing

John L. Gustafson

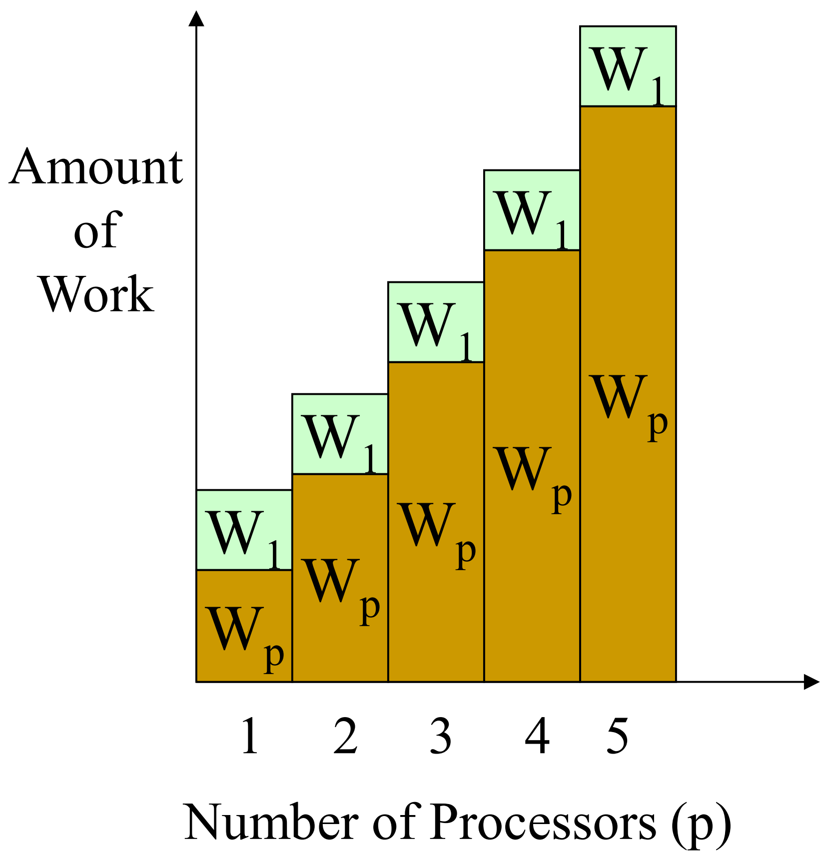
$$S'_p = \frac{\text{Uniprocessor Time of Solving } W'}{\text{Parallel Time of Solving } W'}$$

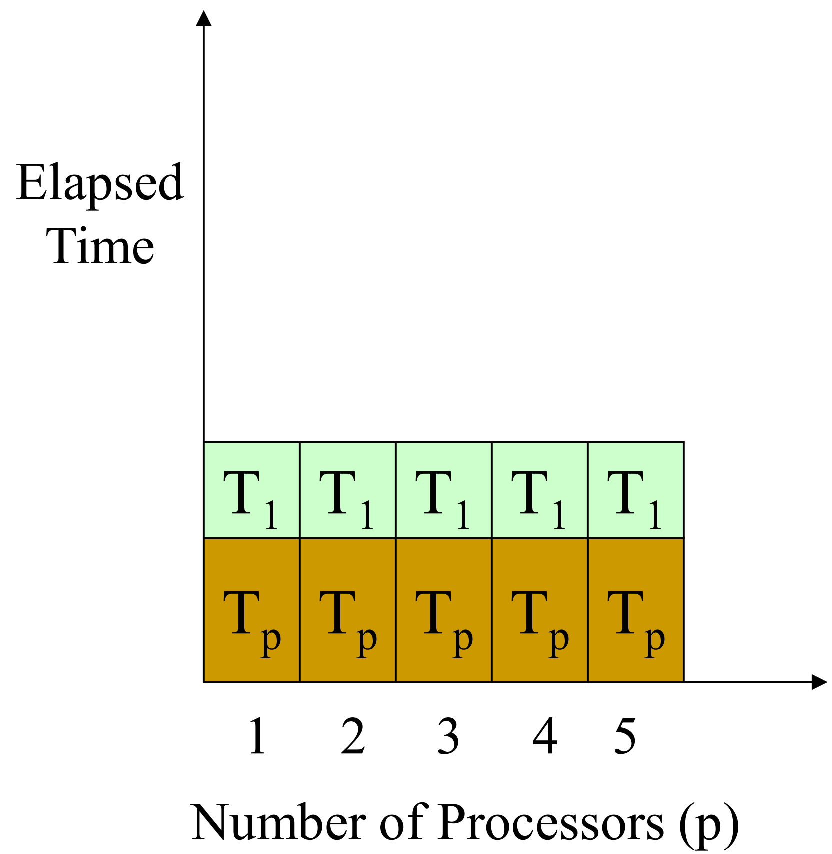
$$= \frac{\text{Uniprocessor Time of Solving } W'}{\text{Uniprocessor Time of Solving } W}$$

$$= \frac{W'}{W}$$

Fixed-Time Speedup (Gustafson)

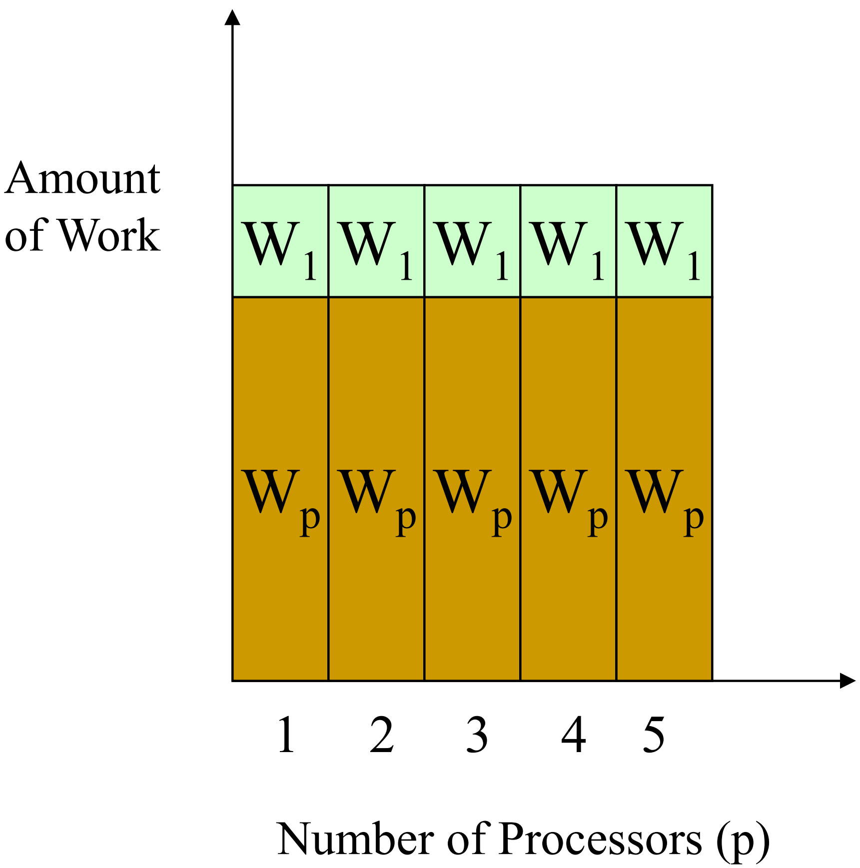
- Solving a larger application within the time limit

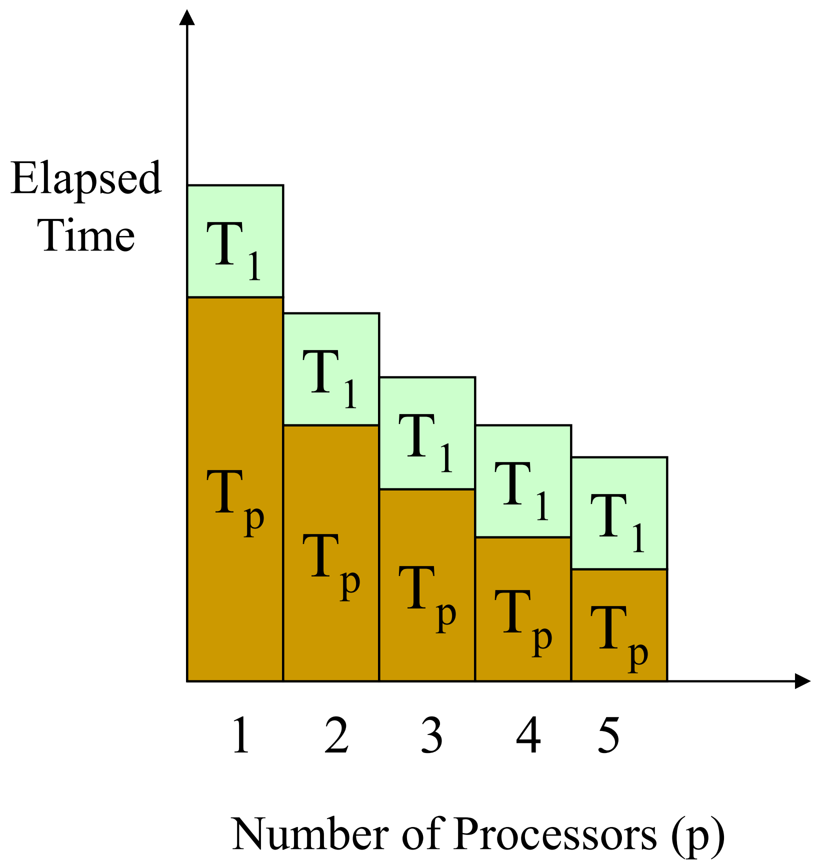




Reexam Amdahl Law (Fixed-Size Speedup)

- It is on time reduction for solving a fixed problem (size)





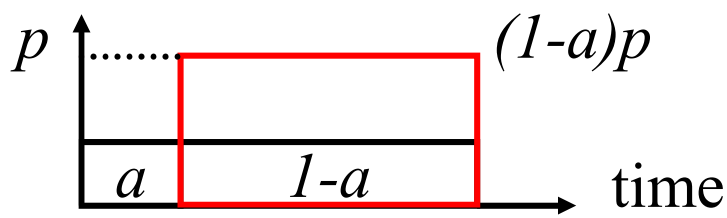
- Amdahl’s law (Fixed-Size Speedup)
  - Emphasis on turnaround time
  - Problem size, ***W***, is fixed

$$S_p = \frac{\text{Uniprocessor Execution Time}}{\text{Parallel Execution Time}}$$

$$S_p = \frac{\text{Uniprocessor Time of Solving } W}{\text{Parallel Time of Solving } W}$$

Gustafson's Law (Without Overhead)

- Under **Gustafson's Law** the parallel processing part is changing with the number of processors, *p,* and problem size
- Linear speedup



$$\alpha = \frac{t_s}{t_s + t_p}$$

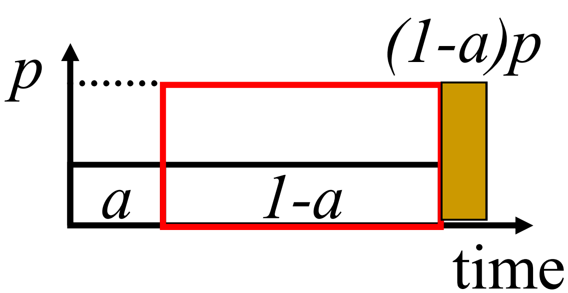
$$Speedup_{FT} = \frac{Work(p)}{Work(1)} = \frac{\alpha W + (1-\alpha)pW}{W} = \alpha + (1-\alpha)p$$

If α=0.1

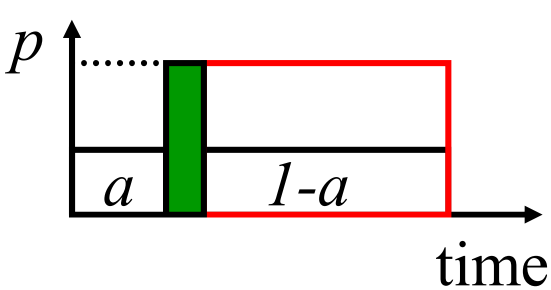
$$Speedup_{FT} = \alpha + (1-\alpha)p = 0.1 + 0.9p$$

Gustafson's Law (With Overhead)

- In practice, the **overhead** can increase with P, and **limit** the scalability



$$Speedup_{FT} = \frac{Work(p)}{Work(1)} = \frac{\alpha + (1-\alpha)p}{1 + T_{overhead} / T_1}$$



$$Speedup_{FT} = \frac{Work(p)}{Work(1)} = \alpha + (1 - \alpha - \frac{T_{overhead}}{T_1})p$$

But: ***Gustafson's Applications are not Scalable***

- Most applications cannot get more than 1,000 speedup on a 1024-processor nCUBE parallel computer

Parallel Processing overhead

- Even the three applications are not **Scalable** (increase *problem size further does not help)*

Why?

Memory Constrained Scaling:

Sun and Ni's Law

- **Scaling is limited by memory space** (disk will increase overhead significantly), e.g. fixed memory capacity/usage per processor
  - (ex) N-body problem
- Problem size is scaled from W to W*, W* is the work executed under memory limitation
- The relation between memory & computing requirement is determined by the underlying algorithm/program
- **Memory-scaling function**

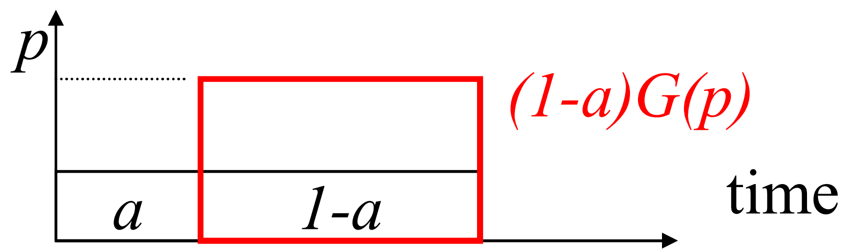
$$W^* = G(p * M)$$

X.H. Sun, and L. Ni , *"Scalable Problems and Memory-Bounded Speedup,"* *Journal of Parallel and Distributed Computing*, Vol. 19, pp.27-37, Sept. 1993 (**SC90**).

Sun & Ni's Law

存储受限理论

Xian-He Sun



Lionel M. Ni

$$Speedup_{MB} = \frac{Work(p)/Time(p)}{Work(1)/Time(1)} = \frac{\alpha + (1-\alpha)G(p)}{\alpha + (1-\alpha)G(p)/p}$$

Assuming $\alpha = 0.1$, the problem needs $2n^3$ computation and $3n^2$ memory

Then $G(p) = G(\mathrm{p}) = p^{\frac{3}{2}}$, and

$$Speedup_{MB} = \left(0.1 + 0.9 \times p^{\frac{3}{2}}\right) / \left(0.1 + (0.9 \times p^{\frac{3}{2}})/p\right)$$

Memory-Bounded Speedup 存储受限理论

(Sun & Ni, 90)

- Emphasis on work finished under current physical limitation
  - Problem size is scaled from W to W^*
  - W^*: Work executed under memory limitation with parallel processing

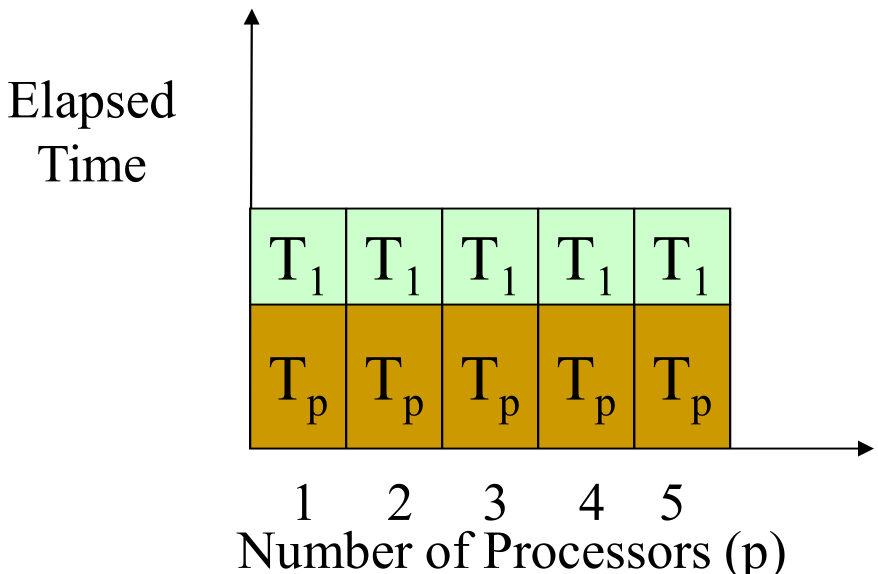
$$S_p^* = \frac{\text{Uniprocessor Time of Solving } W^*}{\text{Parallel Time of Solving } W^*}$$

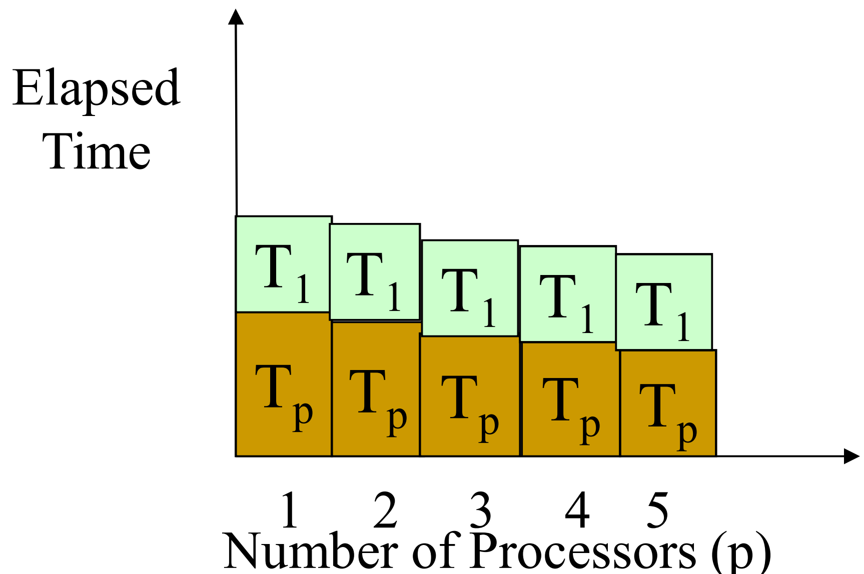
Memory-Bounded Speedup (Sun & Ni)

- In practice, memory-bounded performs better than fixed-time but both hard to achieve linear speedup

$$Speedup_{MB} = \frac{Work(p)/Time(p)}{Work(1)/Time(1)} = \frac{\alpha + (1-\alpha)G(p)}{\alpha + \frac{(1-\alpha)G(p)}{p} + overhead(p, G(p))}$$

$$Speedup_{FT} = \frac{Work(p)}{Work(1)} = \alpha + (1 - \alpha - \frac{T_{overhead}}{T_1})p$$





Example of calculating Sun & Ni's Law

$$Speedup_{MB} = \frac{Work(p)/Time(p)}{Work(1)/Time(1)} = \frac{\alpha + (1-\alpha)G(p)}{\alpha + (1-\alpha)G(p)/p}$$

- Example:
  - Assuming that the problem needs $2n^3$ computation and $3n^2$ memory requirement
  - $\alpha = 0.6$
  - $p = 8$
  - $G(p) = G(8) = 8^{\frac{3}{2}} \approx 22.63$
  - $Speedup_{MB} = (0.6 + 0.4 \times 22.63)/(0.6 + (0.4 \times 22.63)/8) \approx 5.576$

Rethinking of Speedup

- Speedup

$$S_p = \frac{Uniprocessor\ Execution\ Time}{Parallel\ Execution\ Time}$$

- It is only the true speedup if problem size is fixed, but now we have scalable computing
- Generalized speedup

$$S_p = \frac{\text{Parallel Speed}}{\text{Sequential Speed}}$$

X.H. Sun, and J. Gustafson, *"Toward A Better Parallel Performance Metric," Parallel Computing*, Vol. 17, pp.1093-1109, Dec. 1991.

Performance Example

- Performance is measured in **Speed**
- But since they work on the same of work, Speedup equals Time Reduction
- My car (X) travels a distance of 1 in one hour
  - Time between start and completion of event is 1 hour
    - Execution time
- Your car (Y) travels a distance of 1 in two hours
- Intuition: my car is twice as fast as your car

Performance Example

- My car (X) travels a distance of 1 in one hour
  - Time between start and completion of event is 1 hour
    - Execution time
- Your car (Y) travels a distance of 1 in two hours
- Intuition: my car is twice as fast as your car
- Intuition assumes performance = speed

Is intuition correct?

$$\frac{\text{execution time of your car (Y)}}{\text{execution time of my car (X)}} = n$$

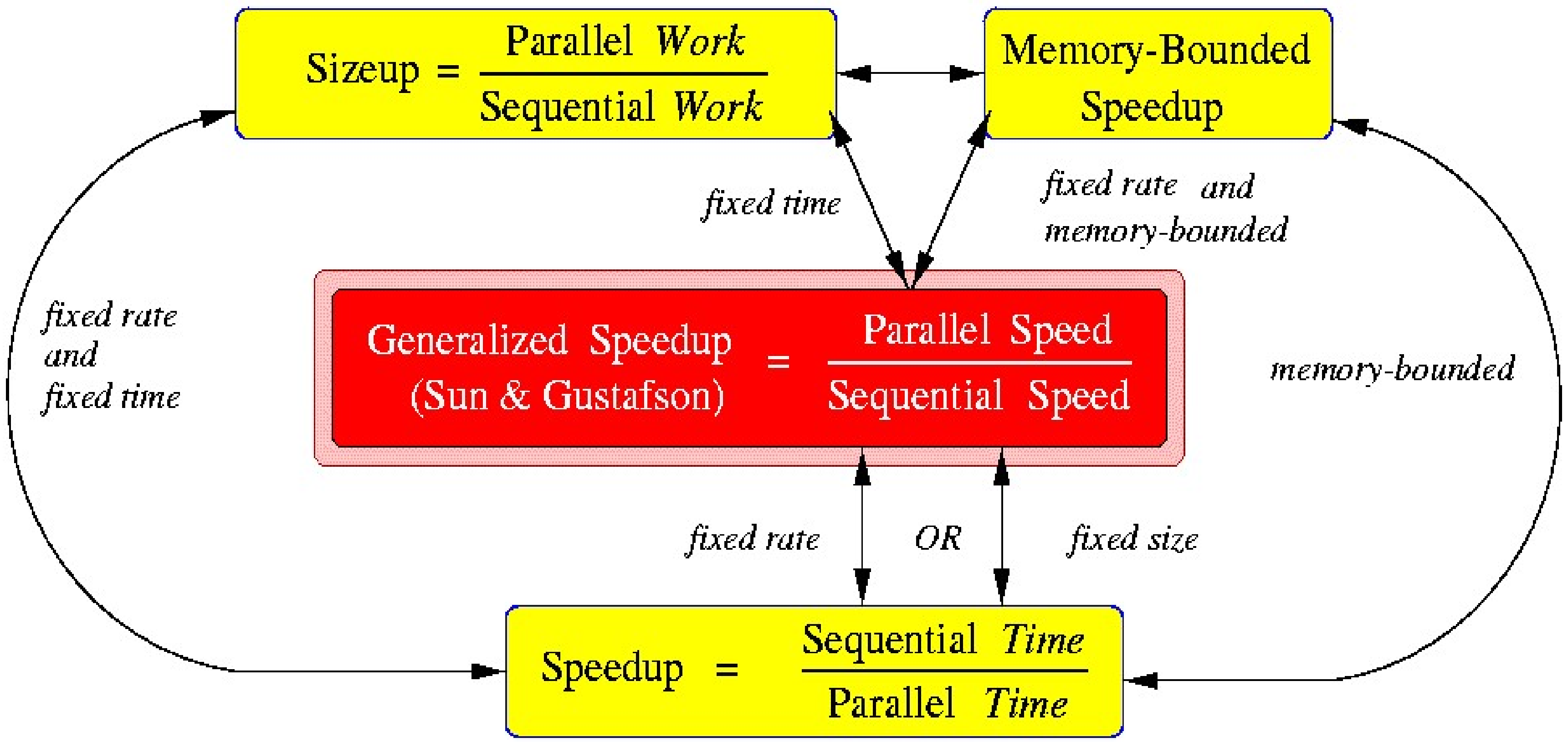
Implies X performs n times better than Y

$$\text{performance} = \text{speed} = \frac{1}{\text{execution time}}$$

$$n = \frac{\text{execution time of Y}}{\text{execution time of X}} = \frac{1/\text{speed of Y}}{1/\text{speed of X}} = \frac{\text{speed of X}}{\text{speed of Y}} = \frac{\text{performance of X}}{\text{performance of Y}}$$

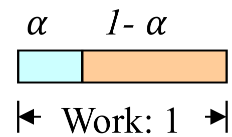
Speed is one measure of performance, throughput is another

Models of Speedup

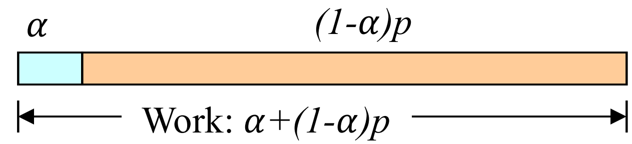


The Three Laws

- Tacit assumption of Amdahl's law
  - Problem size is fixed
  - Speedup emphasizes on time reduction

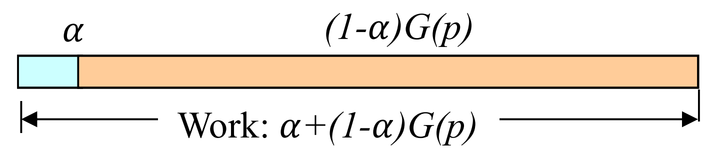


- Gustafson's Law, 1988
  - Fixed-time speedup model



$$Speedup_{fixed-time} = \frac{Sequential\ Time\ of\ Solving\ Scaled\ Workload}{Parallel\ Time\ of\ Solving\ Scaled\ Workload}$$

$$= \alpha + (1-\alpha)p$$



- Sun and Ni's law, 1990
  - Memory-bounded speedup model

$$Speedup_{memory-bound} = \frac{Sequential\ Time\ of\ Solving\ Scaled\ Workload}{Parallel\ Time\ of\ Solving\ Scaled\ Workload}$$

$$= \frac{\alpha + (1-\alpha)G(p)}{\alpha + (1-\alpha)\,G(p)/p}$$

X.-H. Sun, and L. Ni, "Another View of Parallel Speedup," Proc. of IEEE Supercomputing'90, NY, NY, Nov.12--Nov.16, 1990.

The Three Laws: and their impact

I can improve Amdahl's law

- **Amdahl's law** (1967) shows the inherent limitation of parallel processing
- **Gustafson's law** (scalable computing, 1988) shows there is no inherent limitation for scalable parallel computing, exce engineering issues

I have a huge memory

- **Sun-Ni's law** (memory-bounded, 1990) shows memory (data) is the constraint of scalable computing (**the** engineering issue)

- The **Memory-Wall Problem** (1994) shows memory-bound is a general performance issue for computing, not just for parallel computing

William Wulf, Sally Mckee, "Hitting the memory wall: implications of the obvious," ACM SIGARCH Computer Architecture News Homepage archive, Vol. 23 Issue 1, March 1995

Impact of Scalable Computing