A Partition Method for Parallel Processing (simple example)

$Ax=d$, if A is a tridiagonal matrix with order 4

$$A=\begin{bmatrix} b_0 & c_0 & & \\ a_1 & b_1 & c_1 & \\ & a_2 & b_2 & c_2 \\ & & a_3 & b_3 \end{bmatrix}$$, assume the number of processors $=2$

so we have $n = p \cdot m$

$$\underset{4}{n} = \underset{2}{p} \cdot \underset{2}{m}$$

how to partition A into submatrices ?

$$A = \tilde{A} + \Delta A = \begin{bmatrix} A_0 & c_1 \\ a_2 & A_1 \end{bmatrix} = \begin{bmatrix} A_0 & \\ & A_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $A_0 = \begin{bmatrix} b_0 & c_0 \\ a_1 & b_1 \end{bmatrix}$

$A_1 = \begin{bmatrix} b_2 & c_2 \\ a_3 & b_3 \end{bmatrix}$

$$\Delta A = [a_2 e_2, c_1 e_1] \begin{bmatrix} e_1^T \\ e_2^T \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & c_1 \\ a_2 & 0 \\ 0 & 0 \end{bmatrix}}_{V} \cdot \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{E^T} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

both V and E are $n \times 2(p-1)$ matrices

Thus we have $A = \tilde{A} + VE^T$