uNiFTswap Formula and Impermanent Loss

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1. Introduction

We got a very useful question about uNiFTswap Formula and Impermanent Loss, so I'm going to write up a detailed answer and publish it.

uNiFTswap is not running on the mainnet, so please be careful.

uNiFTswap has the following feature.

- ERC20token DEX, which guarantees the upper limit of impermanent loss with lower impermanent loss than normal DEX in some sections.
- Decentralized protocol to swap NFTs which has less liquidity.

The feature of NFT-to-NFT decentralized swap is attractive in that it provides a new way to buy & sell NFT, expand the NFT community, distribute NFTs, connect NFT communities, make NFT gaming more attractive, and open up NFT DeFi through an automated NFT pricing process.

First of all, let's review uNiFTswap. uNiFTswap allows you to put in a certain amount of tokens and get back a pair of tokens of the same amount. The difference between values of tokens is adjusted by adding or subtracting adjustment tokens(e.g. DAI, MIM, WAVAX). The calculation formula is basically similar to

$$X * Y = K$$

The fact that the same amount of tokens can be transferred in and out means that tokens can be transferred in and out without using a decimal point, which means that it is possible to swap NFTs which can be converted to ERC20 without decimals of real world. In addition, based on the mechanism of virtual price and difference determination, NFTs with low liquidity can be traded with pseudo-liquidity, which means that swapping between NFTs can work well. The following video uses this feature to actually SWAP the NFT on the Avalanche FUJI C-Chain test net.

https://youtu.be/E-3xuMozd1A

In this article, we will look at the formula for computing the impermanent loss of uNiFTswap. However, before the detail, we will briefly summarize the advantages of the impermanent loss of uNiFTswap.

The impermanent loss mechanism of uNiFTswap is basically the same as that of DEX like Uniswap, but in uNiFTswap, there is a mechanism to suppress the impermanent loss in some sections.

As explained below, uNiFTswap's impermanent loss mechanism is important for the ecosystem of NFTs with unstable values, as it provides a minimum guarantee to the liquidity provider. If the simple Uniswap's impermanent loss mechanism is applied, a value crash of one of the NFTs will reduce the total assets to zero. However, even for NFTs, though a lot of NFTs are prone to crash, there is a great brake about the impermanent loss when one of NFT in pair crashs, as explained below, it can provide liquidity provider with safety net.

The core part of uNiFTswap only deals with ERC20 tokens, so it is better to trade ERC20 tokens that are prone to crash on uNiFTswap. In Uniswap, tokens that are not stable in value run the risk of having their total assets rapidly go to zero as they are provided as liquidity. However, the uNiFTswap mechanism already had a minimum guarantee against asset loss risk built into the formula. This feature can increase the number of people who are willing to provide liquidity for tokens in their infancy. Even for extremely reliable tokens such as ETH and BTC, people who are worried that the price might drop drastically but want to provide liquidity might want to adopt the above mechanism of uNiFTswap. Considering the fact that it is difficult to function without a proper virtual price and the fact that the liquiditiy provider fee is calculated differently from ordinary DEX like Uniswap, uNiFTswap cannot be simply compared to Uniswap, but it has the potential to attract a lot of liquidity providers in the future.

Now, we would like to carefully explain the relationship between uNiFTswap's formula and impermanent loss.

2. uNiFTswap formula

The formula for uNiFTswap are as follows

x: Amount of token0
y: Amount of token1
v: Virtual price
X: Virtual price of token0 (=x * v)
Y: Virtual price of token1 (=y * v)
N_x : Amount of token2 (when X > Y)

N_y : Amount of token2 (when Y > X)

$$((X + Y) / 2)^2 = K$$

$$N_x * N_y = 0$$
$$(X + N_x) * (Y + N_y) = K$$

Another rules

・The absolute value of the difference between K and X and the absolute value of the difference between K and Y must be the same.

・K is constant unless liquidity changes.

・When x and y are compared, N on the small side becomes 0.

In order to understand the mechanism of impermanent loss in uNiFTswap, it is essential to understand the uNiFTswap formula. Here is a brief explanation of the formula.

uNiFtswap uses a formula similar to Uniswap's x*y=k, but it is slightly different from Uniswap.

uNiFTswap is a DEX for ERC20token. Note that in uNiFTswap, all NFTs are converted 1:1 to ERC20tokens. uNiFtswap only deals with converted ERC20tokens. Please understand that these ERC20tokens will be converted back to NFTs the moment they are swapped or anytime.

Here is an explanation of the Formula. First, x is the amount of NFTs, not the price. y is also the amount of NFTs. It is not a price.

Virtual price is a tentative value for calculation per NFT. Virtual price is determined by the Pool creator.

X is x multiplied by the virtual price, which is the total virtual price of x in the pool, and Y is the total virtual price of y as well.

Now, we can find the value of K by $((X + Y) / 2)^2 = K$. For the moment, consider K to be constant. If you know how Uniswap works, you may have thought that K changes when liquidity is provided or when there is a swap. The mechanism is slightly different in uNiFTswap. In uNiFTswap, K changes only with the provision of liquidity, different from Uniswap. All the liquidity fees collected each swap are paid as the adjustment token, and the fee part is not incorporated into the liquidity. For the sake of clarity, please understand that the value of K is determined by $((X + Y) / 2)^2 = K$, and K is constant.

In uNiFTswap, the amount of NFTs exchanged in a single swap is always 1:1: if you insert 1 NFT, it will be exchanged for 1 NFT, and the difference of value will be adjusted by the adjustment token (token2). If you put in 2 NFTs, you will get 2 NFTs. The difference of value is adjusted by adjustment token(token2). It is important to remember that the same amount of NFTs will be returned as the NFTs you put in to understand uNiFTswap formula.

$$(X + N_x) * (Y + N_y) = K$$

Look at this formula again, where N_x or N_y is the amount of adjustment token(token2), which in the video example is DAI. N_x or N_y must be zero. In the video example, it is DAI, and either N_x or N_y is zero. For example, if X is 4800 and Y is 5200, then N_x is 0. Compare x and y. N on the small side must be 0. This feature is very important to understand the impermanent loss, so please keep it in mind. Now we know that we have to find the remaining N_y, which is the amount of adjustment tokens (token2) that should be in the pool. For example, if X is 4800, Y is 5200, virtual price is 100, and K is 25,000,000, then N_y is approximately 8.333.... Thus, about 8.333... DAIs have to be in the pool. In this way, the uNiFTswap formula effectively calculates the difference between value of NFTs and automatically calculates the value of NFTs.

What we can see is that, by calculation, the X side is 4800 and the Y side is 5200+8.333... That's it. The liquidity provider can withdraw liquidity according to the percentage of liquidity he has put in. If the liquidity provider had provided 50% of the liquidity in this example, it would have been able to withdraw the value of X side 2400, Y side 2600+4.166... (with liquidity fee in reserve3). In reality, we need to divide the virtual price for NFTs, so we can extract 24 NFTs for x, 26 NFTs for y, and 4.166 adjustment tokens... (with liquidity fee in reserve3).

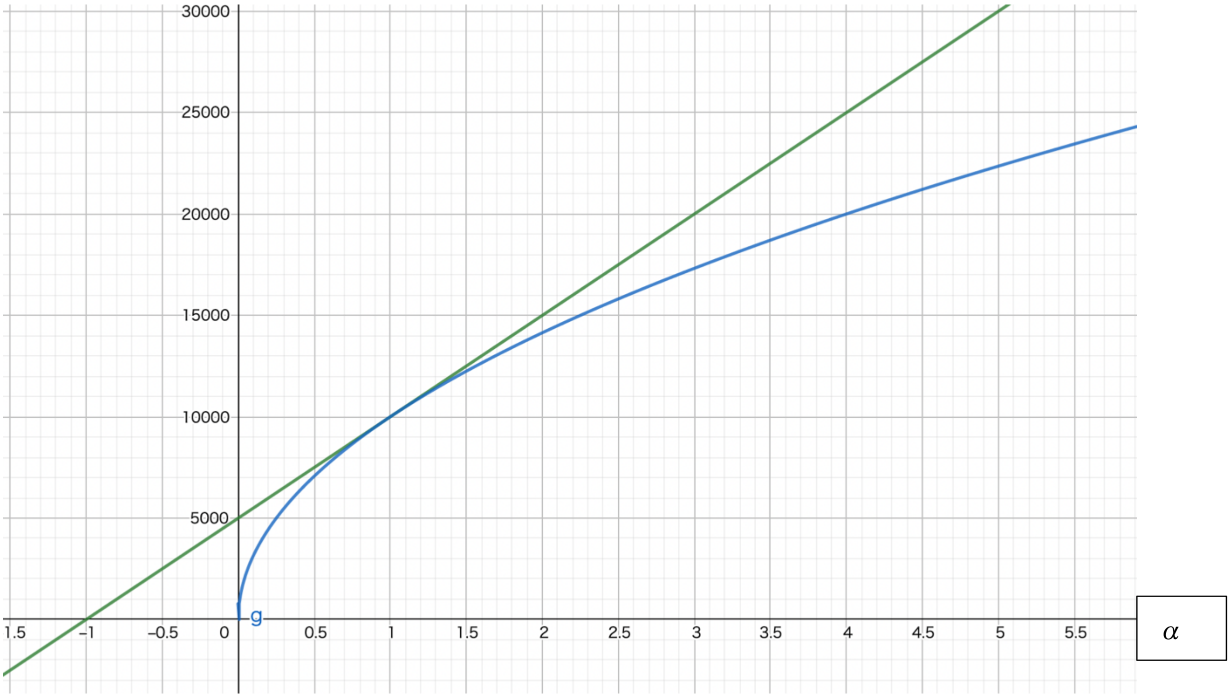
The mechanism of this liquidity withdrawal itself is not much different from uniswap.

3. Impermanent loss about Uniswap

If you understand uNiFTswap and ordinary inpermanent loss, it is easy to understand how impermanent loss works in uNiFTswap. First, let's review the impermanent loss in Uniswap.

To simplify the explanation, consider that 1ETH = 1DAI (very cheap lol). Also, consider that 1DAI=1USD and the value of DAI is always constant.

First, look at the graph, with the price volatility of ETH on the horizontal α axis (e.g., if the price of ETH increases 10x from the initial point, α is 10) and the value of your total assets (unit is USD) on the vertical axis. The green curve is when you were holding and the blue is when you provided liquidity to Uniswap, not uNiFTswap.



Let's say you have 5,000 ETH and 5,000 DAI, not in a pool. Your current total asset value will be 10,000USD. If the value of ETH were to quadruple, the value of your total assets would be $5{,}000 * 4 + 5{,}000 = 25{,}000USD$. Even if the value of ETH were to go to zero, your total asset value of 10,000 USD would be 5,000 USD. The formula about green curve is the following.

$$f(\alpha) = 5000 * (\alpha + 1)$$

If you put 5,000 ETH and 5,000 DAI into the Uniswap pool, it will not be the same as when you hold it. Since the amount of ETH and DAI in the pool fluctuates according to x*y=k, the value of the total asset is

$$g(\alpha) = 5000 * 2\sqrt{\alpha}$$

If the value of ETH quadruples, the value of your total assets will change according to the following formula. If the value of ETH were to quadruple, the value of your total assets would be

$$5000 * 2\sqrt{4} = 20{,}000 \text{ USD}$$

If the value of ETH goes to zero, your total asset value of 10,000USD will be reduced to

$$5000 * 2\sqrt{0} = 0 \text{ USD}$$

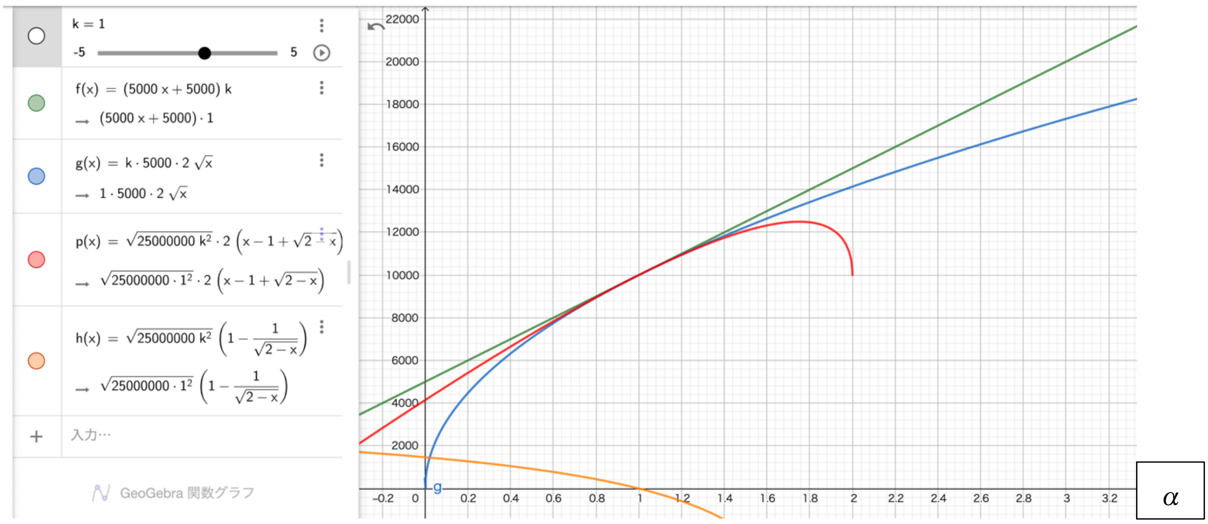
You can see that the value of your total assets decreases when you are in uniswap compared to when you are in hold. This is impermanent loss. This is a commonly known fact.

Thus, if you put it in the pool, you will inevitably incur losses, but the liquidity provider provides liquidity to the pool because they get a liquidity fee.

4. Impermanent loss about uNiFTswap (α >1)

This section describes the impermanent loss of uNiFTswap.

First of all, let's look at the graph of the price change of total assets when we put the assets in uNiFTswap, when each amount of pair token is same. In this case, uNiFTswap has basically the same blue curve as uniswap(when α >1), but when the horizontal axis goes from 0 to 1(when α <1), the curve becomes red. You can see that the impermanent loss is suppressed. The explanation is as follows.



The function in the graph uses x, but all x in the graph should be read as α .

Note that all of the equations in this article are in USD. The units on the left side are USD, and the units on the right side are also USD.

First, assume that we have DAI.n and ETH.n. DAI.n and ETH.n are wrapped NFTs of DAI and ETH, respectively. DAI.n is an NFT containing 100 DAI, and ETH.n is an NFT containing 100 ETH. Note that again, 1DAI=1USD, and when $\alpha=1$, 1ETH=1DAI.

This time, we will put 50DAI.n and 50ETH.n into the pool. In other words, we have 5000 ETH and 5000 DAI in the pool. In the graph, you can see that at the first position ($\alpha=1$), we have 10000USD worth of assets(5000 ETH and 5000 DAI).

By the way, the core part of uNiFTswap does not deal with NFTs at all. uNiFTswap never create NFT version of DAI or ETH. It only deals with ERC20 tokens, so there is no need to wrap DAI or ETH in NFTs. However, since uNiFTswap is a protocol that is supposed to handle NFTs for the user, we will use DAI.n and ETH.n as NFTs for the explanation since it is easier to understand.

This time, X is DAI.n and Y is ETH.n.

In this case, we define the virtual price per NFT as 100, and the adjustment token (token2, N_x or N_y) is also defined as DAI. Although virtual price 100 never means $100, it is useful to think of it as meaning $100 anyway, for ease of understanding.

To understand the impermanent loss of uNiFTswap, it is essential to understand the formula.

$$(X + N_x) * (Y + N_y) = K$$

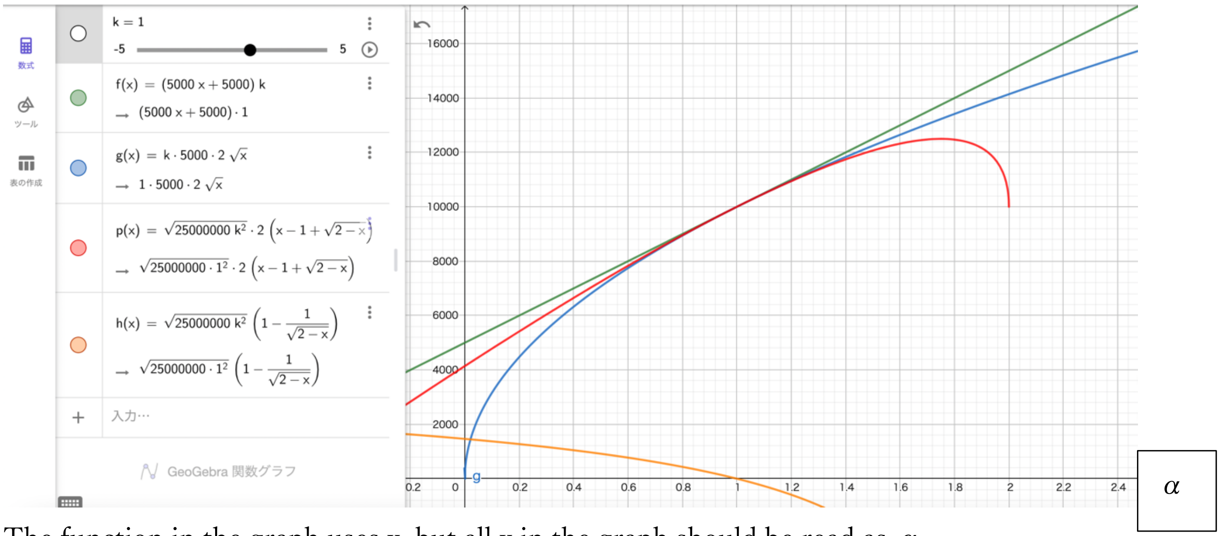
This is the formula of uNiFTswap. The important thing here is that, as explained before, comparing X and Y, the smaller N will be 0 and the larger N will be calculated as the amount of the adjustment token which have to be in the pool.

This time, X is DAI.n and Y is ETH.n.

Assuming this formula, we need to distinguish between two cases: when X>Y and when X<Y. In this case, when X>Y, that is, the value of ETH.n is increasing because there is less Y in the pool, then $\alpha>1$. In this case, N_y is zero, so we have to calculate

$$(X + N_x) * (Y) = K$$

X is DAI.n, and N_x is also DAI.n contains 100 DAI, but the virtual price is also 100, so 1 DAI.n is properly calculated to be worth 100 USD. If this is the case, then the X side represents the amount of DAI, and the Y side represents the amount of ETH. Therefore, when X>Y, the function of uNiFTswap formula is the same as UNISWAP. Therefore, when $\alpha>1$, the graph will have the same blue curve as uniswap.



The function in the graph uses x, but all x in the graph should be read as α.

5. Impermanent loss about uNiFTswap ($\alpha < 1$)

Definition.

α : Price volatility of ETH (same avobe)

c : The difference between the current total virtual price of one of the NFTs and the total virtual price of a small amount of the NFT supplied for swapping (*small amount* × *virtual price*)

p(α): Total asset value in uNiFTswap's pool.

h(α): The difference between the amount of one NFT at α, and the number obtained by adding the total amount of both NFTs in the pool and dividing by 2, then multiplied by the virtual price.

K : $((X + Y) / 2)^2$

The change of ETH price in $\alpha > 1$ was intuitive and easy to understand. On the other hand, what about the case when $\alpha < 1$?

In this case, when X<Y, that is, when there are more Y in the pool and the value of Y(ETH.n) is decreasing, $\alpha < 1$. In this case, N_x is zero, so we have to calculate

$$(X) * (Y + N_y) = K \qquad ...(1)$$

We are now going to find the function p(α) that determines the value of total assets when DAI.n and ETH.n are in uNiFTswap and the value of ETH is decreasing ($\alpha < 1$).

Since the value of the total assets in the pool is the sum of the value of DAI.n(token0), the value of ETH.n(token1), and the quantity (=value) of the adjustment token (token2 = DAI) in the pool, p(α) is following formula.

$$p(\alpha) = X + \alpha Y + N_y \qquad \text{... (2)}$$

X is multiplied by 1 because USD is the standard of value, but it is omitted. If we can express this in terms of only α and the constant K, we can find the function to easily know the value of total asset.

What we need to remember here is how uNiFTswap works. uNiFTswap was that if you put in the same amount of NFTs, you would get the same amount of NFTs back, and the difference would be adjusted by the adjustment token. At the initial stage of supplying liquidity (α =1), there is few adjustment token in the pool yet. It is almost same as there is no adjustment token, so at the initial stage, it will be.

$$p(1) = X + Y$$

When several swaps are made and the amount of NFTs and adjustment tokens in the pool changes, the value of X will change from the initial value of X. In the case of α <1, the value of X should have decreased by only $h(\alpha)$ Therefore, the value of X should be

$$X = \sqrt{K} - h(\alpha) \qquad \text{... (3)}$$

and Y should have increased by $h(\alpha)$ Since Y should have increased by

$$Y = \sqrt{K} + h(\alpha) \qquad \text{... (4)}$$

Then, assuming that (1), (3), and (4) are true, N_y is like this.

$$N_y = \frac{K}{\sqrt{K} - h(\alpha)} - \left(\sqrt{K} + h(\alpha)\right) = \frac{(h(\alpha))^2}{\sqrt{K} - h(\alpha)}$$

From the above, it follows that p(α)

$$p(\alpha) = \sqrt{K} - h(\alpha) + \alpha\left(\sqrt{K} + h(\alpha)\right) + \frac{(h(\alpha))^2}{\sqrt{K} - h(\alpha)} \qquad \text{...(5)}$$

This is the first step in the process. We will proceed on the assumption that this is what we want.

Let me change the subject a bit. If you put a small amount of ETH.n multiplied by the virtual price (=c), and you know how much DAI.n you will receive and how much DAI you will pay as an adjustment token, then you can determine the current value of ETH. Note that there is more ETH.n in the pool, so if you do a swap with ETH.n in it, you will have to add the corresponding adjustment token. According to (1), the current value of Yside that should be at α is

$$\frac{K}{X}$$

and according to (1), the value of Yside that should be in the pool when a small amount of ETH.n multiplied by the virtual price (=c) is

$$\frac{K}{X-c}$$

So, if we put a small amount of ETH.n multiplied by the virtual price (=c), the value of Yside that we need to put in the pool is

$$\frac{K}{X-c}-\frac{K}{X}$$

and the amount of adjustment token that needs to be put in if c is put in can be found by subtracting the c that needs to be put in, which is

$$\frac{K}{X-c}-\frac{K}{X}-c$$

The amount and value of NFTs to be inserted in the uNiFTswap is the same as the amount and value of NFTs to be emitted. In this case, considering that DAI=USD, DAI.n=100USD, and virtual price=100, the value of DAI.n emitted is c. The value of the quantity of ETH.n put in is the value of the DAI.n emitted minus the DAI(adjustment token) put in, so

$$c-\left(\frac{K}{X-c}-\frac{K}{X}-c\right)$$

$$=2c-\left(\frac{K}{X-c}-\frac{K}{X}\right) \qquad \ldots(6)$$

Note that this value of ETH.n is the value of a small amount of ETH.n and is the value per (c/virtual price). Therefore, the value of ETH.n per unit is

$$\left(2c-\left(\frac{K}{X-c}-\frac{K}{X}\right)\right)\times\frac{virtual\ price}{c}$$

If we divide this by the value per unit of the first ETH.n, we can calculate the price fluctuation rate α. In this case, considering that ETH=DAI=USD and the unit of value is USD, the value of the first ETH.n is the virtual price equivalent of 100, so α is

$$\alpha=\left(2c-\left(\frac{K}{X-c}-\frac{K}{X}\right)\right)\times\frac{virtual\ price}{c}\div 100$$

Calculating this, we get

$$\alpha=2-\frac{K}{X(X-c)}$$

...(7)

In this case (7), if c is as close to zero as possible, the price impact of the swap does not need to be considered, so we make c as close to 0 as possible. In this way, we get

$$\lim_{c\to 0}\left(2-\frac{K}{X(X-c)}\right)=2-\frac{K}{X^2}=\alpha$$

The result is Calculating this, we get

$$X=\pm\sqrt{\frac{k}{2-\alpha}}$$

Because of the way uNiFTswap works, X>0, we have

$$X=\sqrt{\frac{k}{2-\alpha}}$$

As mentioned above, from (3).

$$h(\alpha) = \sqrt{K} - \sqrt{\frac{k}{2-\alpha}} \qquad \dots (8)$$

The result is From (5) and (8), substituting h(α) into p(α), we get

$$p(\alpha) = 2\sqrt{k}\left(\alpha - 1 + \sqrt{2-\alpha}\right)$$

We were able to express p(α) using only K and α .

When $\alpha < 1$, and K=25,000,000, the value of the total assets in the pool draws the following red curve, as shown in the graph posted earlier. Since the value of ETH is effectively $\alpha > 0$, the range of the red curve is $0 < \alpha < 1$.

From the figure below, we can see that uNiFTswap has a lower impermanent loss than uniswap during the value reduction phase ($0 < \alpha < 1$). In particular, if we look at the $\alpha = 0$ part, it is easy to see that when $\alpha = 1$, the value of total assets is 10000, when $\alpha = 0$, the value of total assets when held is 5000, when in uniswap it is 0, and when in uNiFTswap it is roughly 4000. If you put it in uNiFTswap, the value of your total assets will only go down by roughly 40%, not to zero.

α

The function in the graph uses x, but all x in the graph should be read as α .

K depends on the total amount of NFTs in the pool and the value of the virtual price. The percentage of impermanent loss at $0 < \alpha < 1$ does not change even if liquidity is not 50DAI.n and ETH.n. In other words, he conclusion that the value of total assets is roughly 40% remains the same though this calculation is for the liquidity provider who add DAI.n and ETH.n at 1:1.

When α =0, the value of total assets will be 41.414... However, when α =0, the value of total assets becomes 41.414...%. This means that

$$\frac{p(0)}{p(1)}$$

This can be obtained by calculating Calculating this, we get

$$\frac{p(0)}{p(1)} = \sqrt{2} - 1 \approx 0.41421356$$

In summary, in this example, when $\alpha > 1$, the curve is the same as Uniswap, and when $\alpha < 1$, the curve is less impermanent loss than Uniswap. The reason for this is that uNiFTswap divides the case according to whether XY is larger or smaller when calculating the adjustment token. During the decrease in value phase, Uniswap will only increase the quantity of low value tokens in the pool, while uNiFTswap will increase the number of low value tokens and constant value adjustment tokens, preventing the pool from being filled with low value tokens. In other words, the DAI acts as a brake on the decline in total assets.

The fact that impermanent losses can be kept low means that there will be fewer opportunities to trade during the decline situation($X < Y$) in value. This is a disadvantage for traders.

However, for $X > Y$, it works the same as Uniswap. Therefore, the trading opportunities do not decrease in this interval($X > Y$), and the fee corresponding to the trading opportunities also comes in. If you want to increase the intervals where the price moves the same as uniswap, you can change the virtual price and the base currency pair to make your wish come true (although more research is needed for this).

Rather, NFTs themselves are not stable in price and always carry the risk of crashing. The above brakes are considered necessary to prevent NFT holders from hesitating to provide liquidity for fear of crash risks.

In addition, the following spread sheet shows the record of swapping 50 NFTs on the test net to reduce the ratio of NFTs in the pool from 50:50 to 99:1. Please refer to it.
https://docs.google.com/spreadsheets/d/1jCxDibE-S5Jv4FGZr4k9wZPexiDPyWSiP6JGpUAzcX8/edit#gid=0

6. Notes on the features of uNiFTswap

In this case, since the quantity of NFTs was set to be the same at the beginning, a case separation occurred at $\alpha=1$, and different curves were adopted after $\alpha=1$. However, it should be noted that if the liquidity of NFTs is made different at the stage when liquidity is first supplied, the timing at which the curve is changed will be at boundary between X>Y and Y<X. The same can be said for latecomer liquidity providers. Since most latecomer liquidity providers have to provide liquidity when the amount of NFTs in the pool is not the same as pair token, the timing of the curve change will still be at boundary between X>Y and Y<X .

Of course not be when $\alpha=1$ for first liquidity provider who provide liquidity at 1:1 ratio. However even then, we can compare the amount of XY present in the pool, and if there is more X(DAI.n), we can adopt the uniswap curve, and if there is more Y, we can calculate p(α) as described above to find the curve.

7. Impermanent loss according to virtual price

One of the features of uNiFTswap is the ability to change the amount of trades that can be processed according to virtual price. For example, it is possible to increase the amount of trades that can be processed if the impermanent loss is somewhat large, or to reduce the amount of trades that can be processed instead of reducing the impermanent loss.

For example, consider a pair of DAI and JPYC, where JPYC is a Japanese stable coin. Let's say 1USD=1DAI and initially 120JPYC=1DAI.

In the Uniswap example, when 120JPYC = 1DAI, consider the case where 120JPYC and 1DAI are put into a pool. Suppose that when the value of JPYC increases by 10%, the amount of DAI in the pool is $\sqrt{\frac{11}{10}} \approx 1.048808848\,DAI$ If the value of JPYC increases by 10%, the amount of DAI which you have to put in is $0.048808848\,DAI$ This means that when the value of JPYC increases by 10%, $0.048808848\,DAI$ is needed.

In the uNiFTswap example, consider a pool of 120JPYC and 1DAI. when the virtual price is the same as the value of the DAI, it will behave the same as Uniswap when the value of the JPYC is increasing, as mentioned above. In other words, when the value of JPYC increases by

10%, the amount of DAI in the pool is $\sqrt{\frac{11}{10}} \approx 1.048808848\ DAI$. Therefore, when the value of JPYC increases by 10%, the amount of DAI which you have to put in is $0.048808848\ DAI$ This shows that when the value of JPYC increases by 10%, $0.048808848\ DAI$ is needed.

Next, let's consider the example of uNiFTswap, when 120JPYC and 1DAI are put into a pool and the virtual price is set to 200. If the value of JPYC increases by 10%, the amount of DAI in the pool will be 1.025290226140DAI. The amount of DAI required will be about 0.025290226140 DAI. The amount that can be processed is about 0.518 times less. Of course, since the rate of impermanent loss is reduced instead of the amount that can be processed, there is a trade-off here.

On the other hand, in the uNiFTswap example, let's consider what happens when we initially put 120 JPYC and 1 DAI into the pool and set the virtual price to 10. When the value of JPYC increases by 10%, the amount of DAI in the pool will be about 1.3050252531 DAI. The amount of DAI that can be processed is about 6.249 times larger than when virtual price = 1, which means that more transactions can be processed. Of course, the amount of trades that can be processed increases, but so does the amount of impermanent loss.

8. supplement

The above is an example of ETH, DAI and JPYC for the sake of clarity, but the curve is expected to remain the same for other token pairs and other NFT pairs. Also, even if the adjustment token is another token such as MIM, WAVAX, it will work. The logical consequence itself is the same. However, this is an argument based on the assumption that one of the pair and the adjustment token are stabled. In particular, further discussion is needed on how the behavior will be when the adjustment token fluctuates.