uNiFTswap's answer to Impermanent loss

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1. Introduction

We got a very useful question about Impermanent loss, so I'm going to write up a detailed answer and publish it.

uNiFTswap is not running on the mainnet, so please be careful.

uNiFTswap has the following feature.

- ERC20token DEX, which guarantees the upper limit of impermanent loss with lower impermanent loss than normal DEX in some sections.
- NFTs should also be able to swap in DEX.

The NFT-to-NFT swap feature is attractive in that it provides a way to expand the NFT community, a new way to distribute NFTs, connect NFT communities, promote NFT gaming, and open up NFT DeFi through an automated NFT pricing process.

First of all, let's review uNiFTswap. uNiFTswap allows you to put in a certain amount of tokens and get back a pair of tokens of the same amount. The difference is adjusted by adding or subtracting USDC or other adjustment tokens. The calculation formula is basically

$$X * Y = K$$

The fact that the same amount of tokens can be transferred in and out means that tokens can be transferred in and out without using a decimal point, which means that it is possible to swap NFTs. In addition, based on the mechanism of virtual price and difference determination, NFTs with low liquidity can be traded with pseudo-liquidity, which means that swapping between NFTs can work.

The following video uses this feature to actually SWAP the NFT on a test net.

https://youtu.be/WREaEjDzi6M

In this article, we will look at the formula for computing the impermanent loss of uNiFTswap. We will briefly summarize the characteristics of the impermanent loss of uNiFTswap.

The impermanent loss mechanism of uNiFTswap is basically the same as that of Uniswap, but there is a mechanism to suppress the impermanent loss in some sections.

As explained below, uNiFTswap's impermanent loss mechanism is critical to the ecosystem of NFTs with unstable values, as it provides a minimum guarantee of the liquidity provider's percentage of asset loss. If the simple Uniswap's impermanent loss mechanism is applied, a value crash of one of the NFTs will reduce the total assets to zero. However, even for NFTs that are prone to crash, if there is a brake about the impermanent loss, as explained below, it can provide liquidity with more security.

The core part of uNiFTswap only deals with ERC20 tokens, so it is better to trade ERC20 tokens that are prone to crash on uNiFTswap. In Uniswap, tokens that are not stable in value run the risk of having their total assets rapidly go to zero as they are provided as liquidity. However, the uNiFTswap mechanism already had a minimum guarantee against asset loss risk built into the formula. This feature can increase the number of people who are willing to provide liquidity for tokens in their infancy. Even for extremely reliable tokens such as ETH and BTC, people who are worried that the price might drop drastically but want to provide liquidity might want to adopt the above mechanism of uNiFTswap. Considering the fact that it is difficult to function without a proper virtual price and the fact that the liquiditiy provider fee is calculated differently from ordinary DEX, uNiFTswap cannot be simply compared to Uniswap, but it has the potential to be a promising liquidity provider in the future.

The uNiFTswpa allows the formation of pools that vary the amount of trades that can be processed. This is an ancillary feature. It is easier to understand if you refer to the examples in the back of the article, but here is a quick summary of this feature: uNiFTswap allows you to change the amount of transactions that is necessary to reflect the current market price of NFT or tokens, or the percentage of impermanent loss, even if the amount of tokens in the pool remains the same. By taking advantage of this feature, for example, it is possible to form a pool with a larger virtual price for those who are afraid of taking on the risk of impermanent loss. On the other hand, for a pair of dollar-yen stable coins, it would be possible to use uNiFTswap with a smaller virtual price to handle a large volume of transactions. If you are willing to sacrifice a bit of impermanent loss, uNiFTswap can be used to handle large volumes of inter-currency trades. The problem with the current DEX is that it is difficult to handle large volumes of trades without sufficient liquidity. uNiFTswap can alleviate some of this problem and provide a future where large volumes of trades can be realized. (It should be noted, however, that in the end, UniswapV3 and curveV2 using Oracle are likely to be more appropriate for large scale trading, as they can manipulate liquidity more flexibly. I look forward to the various issues raised in this regard.)

Now, we will carefully explain the relationship between uNiFTswap's formula and impermanent loss.

2. uNiFTswap formula

The formulas for uNiFTswap are as follows

x: Amount of token0
y: Amount of token1
v: Virtual price
X: Virtual price of token0 (=x * v)
Y: Virtual price of token1 (=y * v)
N_x : Amount of token2 (when X > Y)
N_y : Amount of token2 (when Y > X)

$((X + Y) / 2)^2 = K$

$N_x * N_y = 0$
$(X + N_x) * (Y + N_y) = K$

In order to understand the mechanism of impermanent loss in uNiFTswap, it is essential to understand the uNiFTswap formula. Here is a brief explanation of the uNiFtswap formula.

uNiFtswap uses a formula based on Uniswap's x*y=k, but it is slightly different from Uniswap.

uNiFTswap is a DEX for ERC20token. Note that in uNiFTswap, all NFTs are converted 1:1 to ERC20tokens. uNiFtswap only deals with converted ERC20tokens. Please understand that these ERC20tokens will be converted back to NFTs the moment they leave the uNiFTswap.

Here is an explanation of the Formula. First, x is the amount of NFTs, not the price. y is also the amount of NFTs. It is not a price.

Virtual price is a calculated value per NFT. These are determined by the Pool creator.
X is x multiplied by the virtual price, which is the total virtual price of x in the pool, and Y is the total virtual price of y as well.

Now, we can find the value of K by $((X + Y) / 2)^2 = K$. For the moment, consider K to be constant. If you know how Uniswap works, you may have thought that K changes when liquidity is provided or when there is a swap. The mechanism is slightly different in uNiFTswap. In uNiFTswap, K changes with the provision of liquidity, just as in Uniswap. On the other hand, you can create a pool that changes K, or you can leave K unchanged at swap time, by using a different function to calculate fee. However, for the sake of clarity, please understand that the value of K is determined by $((X + Y) / 2)^2 = K$, and K is constant.

In uNiFTswap, the amount of NFTs exchanged in a single swap is always 1:1: if you insert 1 NFT, it will be exchanged for 1 NFT, and the difference of value will be adjusted by the adjustment token (token2). If you put in 2 NFTs, you will get 2 NFTs. Adjust the difference with adjustment token(token2). It is important to remember that the same amount of NFTs will be returned as the NFTs you put in.

$$(X + N_x) * (Y + N_y) = K$$

Look at this formula again, where N_x or N_y is the adjustment token(token2), which in the video example is USDC. In the video example, it is USDC, and either N_x or N_y is zero. For example, if X is 4800 and Y is 5200, then N_x is 0. This feature is very important to understand the impermanent loss, so please keep it in mind. Now we know that we have to find the remaining N_y, which is the amount of adjustment tokens (token2) that should be in the pool. For example, if X is 4800, Y is 5200, virtual price is 100, and K is 25,000,000, then N_y is approximately 8.333.... According to the formulas, we need about 8.333.... Thus, we have calculated that we need about 8.333... USDCs in the pool. In this way, the uNiFTswap formula effectively calculates the difference between NFTs and automatically calculates the value of NFTs.

What we can see is that, by calculation, the X side is 4800 and the Y side is 5200+8.333... That's it. The liquidity provider can withdraw liquidity according to the percentage of liquidity he has put in. If the liquidity provider had provided 50% of the liquidity in this example, it would have been able to withdraw the value of X side 2400, Y side 2600+4.166... In reality, we need to divide the virtual price for NFTs, so we can extract 24 NFTs for x, 26 NFTs for y, and 4.166 adjustment tokens...

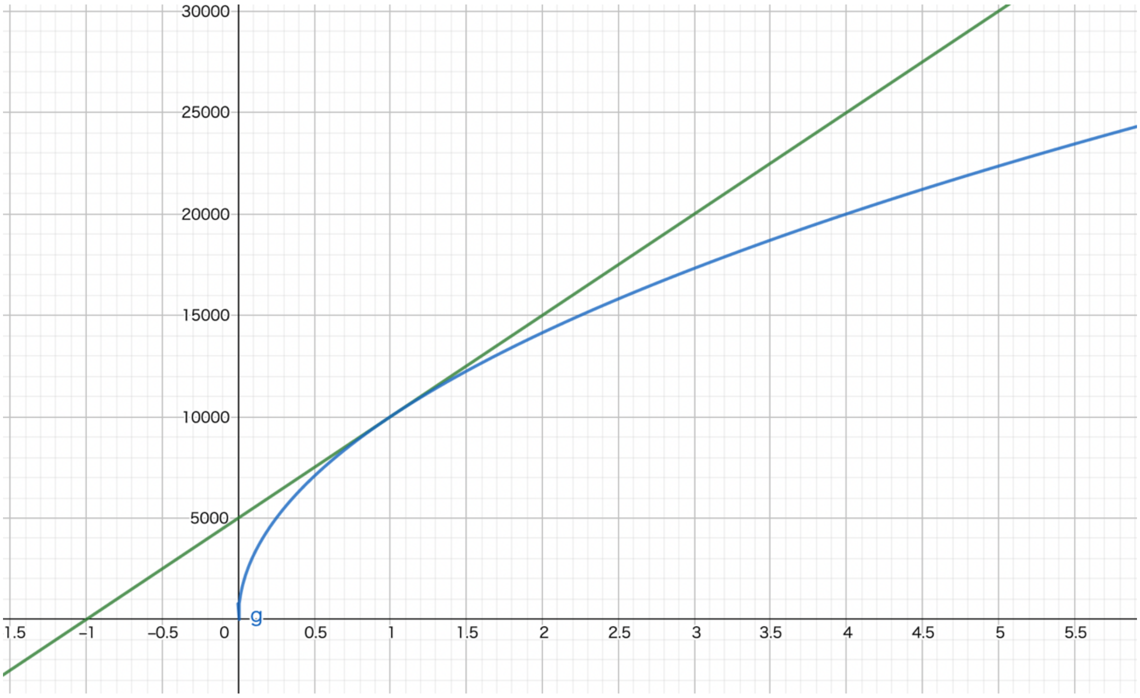
The mechanism of this liquidity withdrawal itself is not much different from uniswap.

3. Impermanent loss about Uniswap

With this as a premise, it is easy to understand how impermanent loss works. First, let's review the impermanent loss in Uniswap.

To simplify the explanation, consider that 1ETH = 1USDC (very cheap lol). Also, consider that 1USDC=1USD and the value of USDC is always constant.

First, look at the graph, with the price volatility of ETH (α) on the horizontal axis (e.g., if the price of ETH increases 10x from the initial point, α is 10) and the value of your total assets (USD) on the vertical axis. The green curve is when you were holding and the blue is when you provided liquidity to uniswap.



Let's say you have 5,000 ETH and 5,000 USDC, not in a pool. Your current total asset value will be 10,000USD. If the value of ETH were to quadruple, the value of your total assets would be $5{,}000 * 4 + 5{,}000 = 25{,}000USD$. If the value of ETH were to go to zero, your total asset value of 10,000 USD would be 5,000 USD. The formula about green curve is the following.

$$f(\alpha) = 5000 * (\alpha + 1)$$

If you put 5,000 ETH and 5,000 USDC into the Uniswap pool, it will not be the same as when you hold it. Since the amount of ETH and USDC in the pool fluctuates according to x*y=k, the value of the total asset is

$$g(\alpha) = 5000 * 2\sqrt{\alpha}$$

If the value of ETH quadruples, the value of your total assets will change according to the following formula. If the value of ETH were to quadruple, the value of your total assets would be

$$5000 * 2\sqrt{4} = 20{,}000 \text{ USD}$$

If the value of ETH goes to zero, your total asset value of 10,000USD will be reduced to

$$5000 * 2\sqrt{0} = 0 \text{ USD}$$

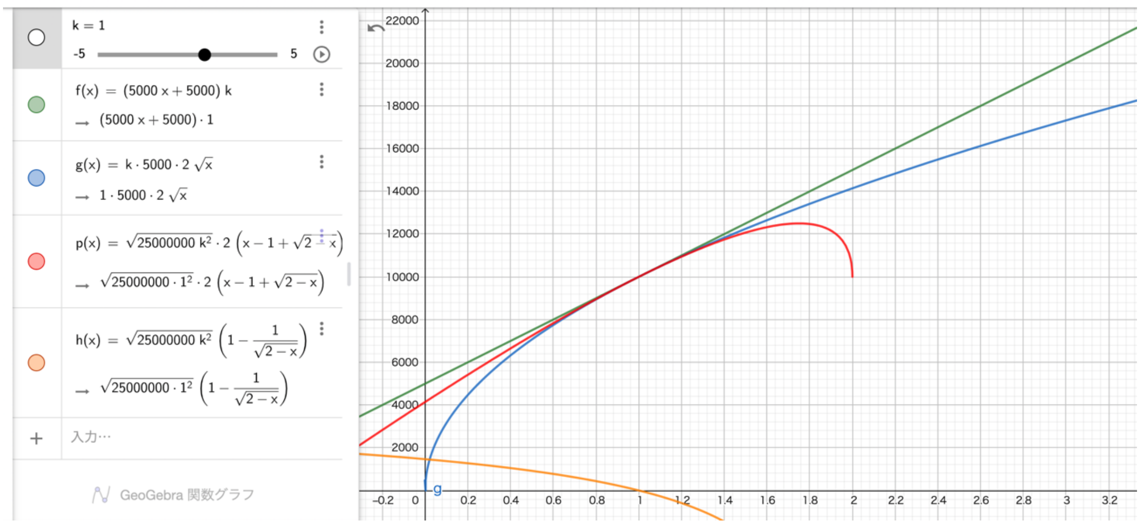
You can see that the value of your total assets decreases when you are in uniswap compared to when you are in hold. This is impermanent loss. This is a commonly known fact, and the graphs and formulas can be more easily understood by referring to the following sites.
https://academy.binance.com/ja/articles/impermanent-loss-explained

Thus, if you put it in the pool, you will inevitably incur losses, but the liquidity provider provides liquidity to the pool because they get a liquidity fee.

4. Impermanent loss about uNiFTswap (α >1)

This section describes the impermanent loss of uNiFTswap.

First of all, let's look at the graph of the price change of total assets when we put the assets in uNiFTswap. In this case, uNiFTswap has basically the same blue curve as uniswap, but when the horizontal axis goes from 0 to 1, the curve becomes red. You can see that the impermanent loss is suppressed. The explanation is as follows.



Note that all of the equations in this article are in USD. The units on the left side are USD, and the units on the right side are also USD. First, assume that we have USDC.n and ETH.n. USDC.n and ETH.n are wrapped NFTs of USDC and ETH, respectively. USDC.n is an NFT containing 100 USDC, and ETH.n is an NFT containing 100 ETH. Note that again, 1USDC=1USD, and when $\alpha=1$, 1ETH=1USDC. This time, we will put 50USDC.n and 50ETH.n into the pool. In other words, we have 5000 ETH and 5000 USDC in the pool. In the graph, you can see that at the first position ($\alpha=1$), we have 10000USD worth of assets. (By the way, the core part of uNiFTswap does not deal with NFTs at all, it only deals with ERC20 tokens, so there is no need to wrap USDC or ETH in NFTs. However, since uNiFTswap is a protocol that is supposed to handle NFTs for the user, we will use USDC.n and ETH.n as NFTs for the explanation since it is easier to understand.)

This time, X is USDC.n and Y is ETH.n.

In this case, we define the virtual price per NFT as 100, and the adjustment token (token2, N_x or N_y) is also defined as USDC.

To understand the impermanent loss of uNiFTswap, it is essential to understand the formula.

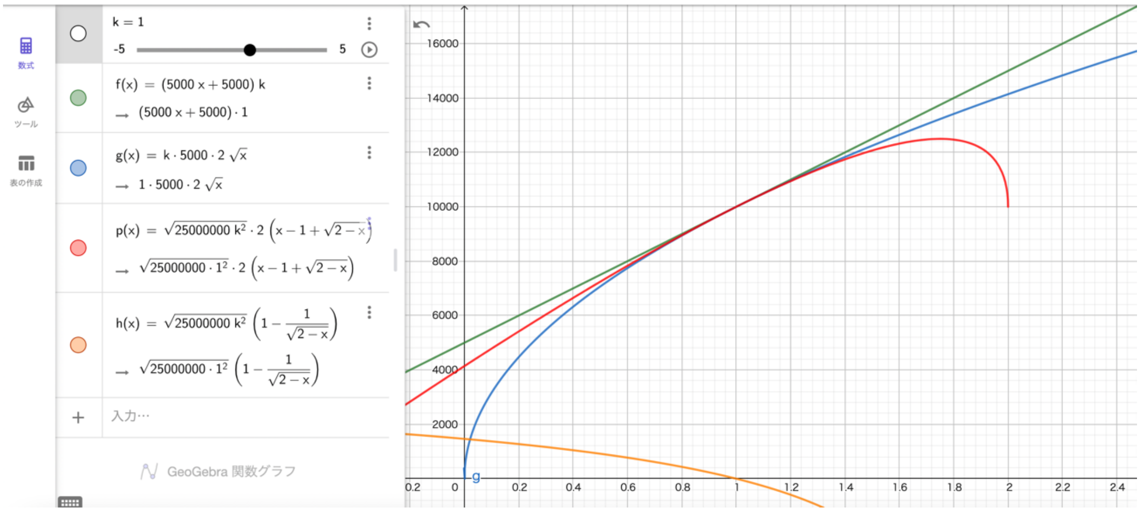
$$(X + N_x) * (Y + N_y) = K$$

This is the formula of uNiFTswap. The important thing here is that, as explained before, comparing X and Y, the smaller N will be 0 and the larger N will be calculated as the amount of the adjustment token.

Assuming this formula, we need to distinguish between two cases: when X>Y and when X<Y. In this case, when X>Y, that is, the value of ETH.n is increasing because there is less Y in the pool, then $\alpha>1$. In this case, N_y is zero, so we have to calculate

$$(X + N_x) * (Y) = K$$

X is USDC.n, and N_x is also USDC. USDC.n contains 100 USDC, but the virtual price is also 100, so 1 USDC.n is properly calculated to be worth 100 USD. If this is the case, then the X side represents the amount of USDC, and the Y side represents the amount of ETH. Therefore, when X>Y, the function of FORMULA is the same as UNISWAP. Therefore, when $\alpha>1$, the graph will have the same blue curve as uniswap.



5. Impermanent loss about uNiFTswap (α <1)

Definition.

α : Price volatility of ETH

c : The difference between the current total virtual price of one of the NFTs and the total virtual price of a small amount of the NFT supplied for swapping (*small amount* × *virtual price*)

p(α): Total asset value in uNiFTswap's pool.

h(α): The number obtained by adding the total amount of both NFTs in the pool and dividing by 2, and the difference between the amount of one NFT at α and the other, multiplied by the virtual price.

K : $((X + Y) / 2)^2$

The change in α > 1 was intuitive and easy to understand. On the other hand, what about the case when α <1 ? In this case, when X<Y, that is, when there are more Y in the pool and the value of Y(ETH.n) is decreasing, α <1. In this case, N_x is zero, so we have to calculate

$$(X) * (Y + N_y) = K \qquad ...(1)$$

We are now going to find the function p(α) that determines the value of total assets when 50USDC.n and 50ETH.n are placed in uNiFTswap and the value of ETH is decreasing (α <1).

Since the value of the total assets in the pool is the sum of the value of USDC.n(token0), the value of ETH.n(token1), and the quantity (=value) of the adjustment token (token2 =

USDC) in the pool

$$p(\alpha) = X + \alpha Y + N_y \qquad \ldots (2)$$

(In this function, X is multiplied by 1 because USD is the standard of value, but it is omitted.) If we can express this in terms of only α and the constant K, we can find the function.

What we need to remember here is how uNiFTswap works. uNiFTswap was that if you put in the same amount of NFTs, you would get the same amount of NFTs back, and the difference would be adjusted by the adjustment token. At the initial stage of supplying liquidity (α =1), there is no adjustment token in the pool yet, so at the initial stage, it will be.

$$p(1) = X + Y$$

When several swaps are made and the amount of NFTs and adjustment tokens in the pool changes, the value of X will change from the initial value of X. In the case of α <1, the value of X should have decreased by only $h(\alpha)$ Therefore, the value of X should be

$$X = \sqrt{K} - h(\alpha) \qquad \ldots (3)$$

and Y should have increased by $h(\alpha)$ Since Y should have increased by

$$Y = \sqrt{K} + h(\alpha) \qquad \ldots (4)$$

It becomes Incidentally, assuming that (1), (3), and (4) are true, N_y

$$N_y = \frac{K}{\sqrt{K}-h(\alpha)} - \left(\sqrt{K} + h(\alpha)\right) = \frac{(h(\alpha))^2}{\sqrt{K}-h(\alpha)}$$

From the above, p(α) becomes From the above, it follows that p(α)

$$p(\alpha) = \sqrt{K} - h(\alpha) + \alpha\left(\sqrt{K} + h(\alpha)\right) + \frac{(h(\alpha))^2}{\sqrt{K}-h(\alpha)} \qquad \ldots (5)$$

This is the first step in the process. We will proceed on the assumption that this is what we want.

Let me change the subject a bit. If you put a small amount of ETH.n multiplied by the virtual price (=c), and you know how much USDC.n you will receive and how much USDC you will pay as an adjustment token, you can determine the current value of ETH. Note that there is more ETH.n in the pool, so if you do a swap with ETH.n in it, you will have to add the corresponding adjustment token. According to (1), the current value of Yside that should be at α is

$$\frac{K}{X}$$

and according to (1), the value of Yside that should be in the pool when a small amount of ETH.n multiplied by the virtual price (=c) is

$$\frac{K}{X-c}$$

So, if we put a small amount of ETH.n multiplied by the virtual price (=c), the value of Yside that we need to put in the pool is

$$\frac{K}{X-c}-\frac{K}{X}$$

and the amount of adjustment token that needs to be put in if c is put in can be found by subtracting the c that needs to be put in, which is

$$\frac{K}{X-c}-\frac{K}{X}-c$$

The amount and value of NFTs to be inserted in the uNiFTswap is the same as the amount and value of NFTs to be emitted. In this case, considering that USDC=USD, USDC.n=100USD, and virtual price=100, the value of USDC.n emitted is c. The value of the quantity of ETH.n put in is the value of the USDC.n emitted minus the USDC(adjustment token) put in, so

$$c-\left(\frac{K}{X-c}-\frac{K}{X}-c\right)$$

$$=2c-\left(\frac{K}{X-c}-\frac{K}{X}\right) \qquad \ldots⑥.$$

Note that this value of ETH.n is the value of a small amount of ETH.n and is the value per (c/virtual price). Therefore, the value of ETH.n per unit is

$$\left(2c-\left(\frac{K}{X-c}-\frac{K}{X}\right)\right)\times\frac{virtual\ price}{c}$$

If we divide this by the value per unit of the first ETH.n, we can calculate the price fluctuation rate α. In this case, considering that ETH=USDC=USD and the unit of value is USD, the value of the first ETH.n is the virtual price equivalent of 100, so α is

$$\alpha = \left(2c - \left(\frac{K}{X-c} - \frac{K}{X}\right)\right) \times \frac{virtual\ price}{c} \div 100$$

Calculating this, we get

$$\alpha = 2 - \frac{K}{X(X-c)}$$

...7.

In this case (7), if c is as close to zero as possible, the price impact of the swap does not need to be considered, so we make c as close to 0 as possible. In this way, we get

$$\lim_{c \to 0}\left(2 - \frac{K}{X(X-c)}\right) = 2 - \frac{K}{X^2} = \alpha$$

The result is Calculating this, we get

$$X = \pm\sqrt{\frac{k}{2-\alpha}}$$

Because of the way uNiFTswap works, X>0, we have

$$X = \sqrt{\frac{k}{2-\alpha}}$$

As mentioned above, from (3).

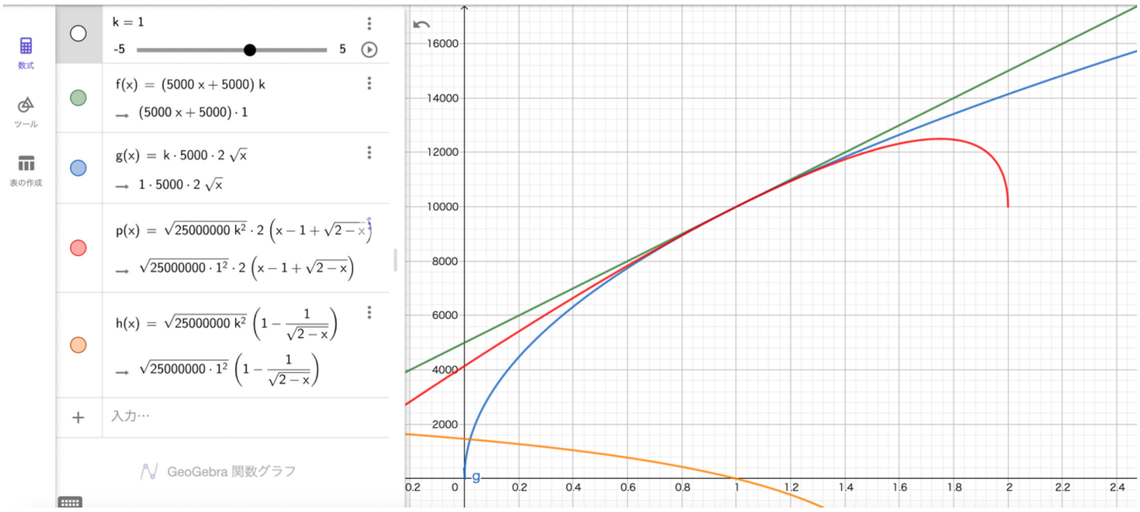
$$h(\alpha) = \sqrt{K} - \sqrt{\frac{k}{2-\alpha}} \qquad \ldots (8)$$

The result is From ⑤ and ⑧, substituting h(α) into p(α), we get

$$p(\alpha) = 2\sqrt{k}\left(\alpha - 1 + \sqrt{2-\alpha}\right)$$

We were able to express p(α) using only K and α.
When $\alpha < 1$, and K=25,000,000, the value of the total assets in the pool draws the following red curve, as shown in the graph posted earlier. Since the value of ETH is effectively $\alpha > 0$, the range of the red curve is $0 < \alpha < 1$.

From the figure below, we can see that uNiFTswap has a lower impermanent loss than uniswap during the value reduction phase ($0<\alpha<1$). In particular, if we look at the $\alpha=0$ part, it is easy to see that when $\alpha=1$, the value of total assets is 10000, when $\alpha=0$, the value of total assets when held is 5000, when in uniswap it is 0, and when in uNiFTswap it is roughly 4000. If you put it in uNiFTswap, the value of your total assets will only go down by roughly 40%, not to zero.



Incidentally, if additional liquidity is provided, the following movements will occur.
https://twitter.com/uNiFTswap/status/1453327716103561218?s=20

K depends on the total amount of NFTs in the pool and the value of the virtual price. Here, we can see that the percentage of impermanent loss at $0<\alpha<1$ does not change even if we supply additional liquidity. In other words, we can see from the above movie that the conclusion that the value of total assets is roughly 40% remains the same even if we vary the liquidity.

This consequence means that the percentage of impermanent loss remains the same even if a liquidity provider joins the pool of uNiFTswap later.

When $\alpha=0$, the value of total assets will be 41.414... However, when $\alpha=0$, the value of total assets becomes 41.414...%. This means that

$$\frac{p(0)}{p(1)}$$

This can be obtained by calculating Calculating this, we get

$$\frac{p(0)}{p(1)} = \sqrt{2} - 1 \approx 0.41421356$$

In other words, in this example, no matter what the value of K is at the beginning, the impermanent loss can be suppressed to about 40%.

In summary, in this example, when $\alpha > 1$, the curve is the same as Uniswap, and when $\alpha < 1$, the curve is less impermanent loss than Uniswap. The reason for this is that uNiFTswap divides the case according to whether XY is larger or smaller when calculating the adjustment token. During the decrease in value phase, Uniswap will only increase the quantity of low value tokens in the pool, while uNiFTswap will increase the number of low value tokens and constant value tokens, preventing the pool from being filled with low value tokens. In other words, the USDC acts as a brake on the decline in total assets.

In addition, the following spread sheet shows the record of swapping 50 NFTs on the test net to reduce the ratio of NFTs in the pool from 50:50 to 100:0. Please refer to it.
https://docs.google.com/spreadsheets/d/1jCxDibE-S5Jv4FGZr4k9wZPexiDPyWSiP6JGpUAzcX8/edit#gid=0

6. Notes on the features of uNiFTswap

In this case, because the quantity of NFTs was set to be the same at the beginning, a case separation occurred at $\alpha = 1$, and different curves were adopted after $\alpha = 1$. However, it should be noted that if the quantity of NFTs is made different at the stage when liquidity is first supplied, and the necessary adjustment token is added, the timing at which the curve is changed will not be at $\alpha = 1$. The same can be said for latecomer liquidity providers. Since most latecomer liquidity providers have to provide liquidity when the amount of NFTs in the pool is not the same, the timing of the curve change will still not be when $\alpha = 1$.

But even then, we can compare the amount of XY present in the pool, and if there is more X(USDC.n), we can adopt the uniswap curve, and if there is more Y, we can calculate p(α) as described above to find the curve.

7. Impermanent loss about uNiFTswap when virtual price is changed.

One of the features of uNiFTswap is the ability to change the amount of trades that can be processed. For example, it is possible to increase the amount of trades that can be processed even if the impermanent loss is somewhat large, or to reduce the amount of trades that can be processed instead of reducing the impermanent loss.

For example, consider a pair of USDC and JPYC, where JPYC is a Japanese stable coin. Let's say 1USD=1USDC and initially 120JPYC=1USDC.

For example, in the Uniswap example, when 120JPYC = 1USDC, consider the case where 120JPYC and 1USDC are put into a pool. Suppose that when the value of JPYC increases by 10%, the amount of USDC in the pool is $\sqrt{\frac{11}{10}} \approx 1.048808848\,USDC$ If the value of JPYC increases by 10%, the amount of USDC which you have to put in is $0.048808848\,USDC$ This means that when the value of JPYC increases by 10%, $0.048808848\,USDC$ is needed.

In the uNiFTswap example, consider a pool of 120JPYC and 1USDC. when the virtual price is the same as the value of the USDC, it will behave the same as Uniswap when the value of the JPYC is increasing, as mentioned above. In other words, when the value of JPYC increases by 10%, the amount of USDC in the pool is $\sqrt{\frac{11}{10}} \approx 1.048808848\,USDC$ Therefore, when the value of JPYC increases by 10%, the amount of USDC which you have to put in is $0.048808848\,USDC$ This shows that when the value of JPYC increases by 10%, $0.048808848\,USDC$ is needed.

Next, let's consider the example of uNiFTswap, when 120JPYC and 1USDC are put into a pool and the virtual price is set to 2. If the value of JPYC increases by 10%, the amount of USDC in the pool will be 1.025290226140USDC. The amount of USDC required will be about 0.025290226140 USDC. The amount that can be processed is about 0.518 times less.

However, since the rate of impermanent loss is reduced instead of the amount that can be processed, there is a trade-off here.

On the other hand, in the uNiFTswap example, let's consider what happens when we initially put 120 JPYC and 1 USDC into the pool and set the virtual price to 0.1. When the value of JPYC increases by 10%, the amount of USDC in the pool will be about 1.3050252531 USDC. The amount of USDC that can be processed is about 6.249 times larger than when virtual price = 1, which means that more transactions can be processed. Of course, the amount of trades that can be processed increases, but so does the amount of impermanent loss.

You can see from the following video that the percentage of impermanent loss changes as the virtual price changes: https://twitter.com/uNiFTswap/status/1456601301391872007?s=20 Note that the light blue curve is only applied when the horizontal axis is greater than or equal to 1.

8. supplement

The above is an example of ETH, USDC and JPYC for the sake of clarity, but the curve is expected to remain the same for other token pairs and other NFT pairs. Also, even if the adjustment token is another token such as USDT or MATIC, if we can assume and think that it is a stable token, then the logical consequence is the same. However, this is an argument based on the assumption that one of the pair and the adjustment token are stabled. In particular, further discussion is needed on how the behavior will be when the adjustment token fluctuates.