An Improved Approximation Algorithm for Multiway Cut
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Interim report for CS 6150

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1 Understanding

Isolation Heuristic - In isolation heuristic, if there are K terminals, in each iteration we attach one terminal to the source and all the other terminals to the sink and then run a max-flow and and find the min-cut. Let the edge set after each iteration be E_i. Now the lowest K-1 cuts are taken and the union of them gives the multi-way cut. This algorithm gives an approximation of $2(1 - \frac{1}{k})$. Proof:

- Run the isolation heuristic and get the set of edges. Let them be A.
- Lets say E^* is the optimal edge set for the multiway cut with K terminals. Then it means if E^* is removed then there will be K disjoint connected graphs$(V_1, V_2, \cdots, V_k)$, each having one terminal respectively
- Lets say $E^* = \sum i = 1^k E_i^*$ and each E_i^* represents the edges removed to disconnect V_i from the rest.
- Lets say $\delta(V_i)$ gives the set of all outgoing edges from V_i.
- Now we can say that, $w(E_i) \leq w(\delta(V_i))$ because, both of isolates the terminal i from the rest and we know E_i is the mincut. So $w(E_i)$ cannot be greater than $w(\delta(V_i))$
- Lets says there is m edges between V_i and V_{i+1}. Then both $\delta(V_i)$ and $\delta(V_{i+1})$ will include those m edges.
- $2w(E^*) = \sum_{i=1} k\delta(V_i)$. Its 2 times since each edges is double counted.
- We know $w(A) \leq (1 - \frac{1}{k})(\sum_{i=1}(E_i^*))$. This is because, A is the union of first K-1 smallest set. In this expression, we are adding up all the K values. Since only $k - 1$

values are taken, we are multiplying it with $1 - \frac{1}{k}$ and since it the union of all these set, it will be equal to or less than the summation of indiviual sets.

$$w(A) \leq (1 - \frac{1}{k})(\sum_{i=1}(E_i^*))$$
$$le(1 - \frac{1}{k})(\sum_{i=1}\delta(V_i)$$
$$le2(1 - \frac{1}{k})w(E^*)$$
$$le2(1 - \frac{1}{k})OPT$$