

# Multi-Objective RL With Multi-Agent Bidding

Anonymous authors

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**Keywords:** Multi-Objective RL, Multi-Agent RL

## Summary

The summary appears on the cover page. Although it can be identical to the abstract, it does not have to be. One might choose to omit the stated contributions in the Summary, given that they will be stated in the box below. The original abstract may also be extended to two paragraphs. The authors should ensure that the contents of the cover page fit entirely on a single page. The cover page does **not** count towards the 8–12 page limit.

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## Contribution(s)

1. Provide a succinct but precise list of the contribution(s) of the paper. Use contextual notes to avoid implications of contributions more significant than intended and to clarify and situate the contribution relative to prior work (see the examples below). If there is no additional context, enter “None”. Try to keep each contribution to a single sentence, although multiple sentences are allowed when necessary. If using complete sentences, include punctuation. If using a single sentence fragment, you may omit the concluding period. A single contribution can be sufficient, and there is no limit on the number of contributions. Submissions will be judged mostly on the contributions claimed on their cover pages and the evidence provided to support them. Major contributions should not be claimed in the main text if they do not appear on the cover page. Overclaiming can lead to a submission being rejected, so it is important to have well-scoped contribution statements on the cover page.

**Context:** None

2. The submission template for submissions to RLJ/RLC 2025

**Context:** Built from previous RLC/RLJ, ICLR, and TMLR submission templates

3. *[Example of one contribution and corresponding contextual note for the paper “Policy gradient methods for reinforcement learning with function approximation” (?).]*

This paper presents an expression for the policy gradient when using function approximation to represent the action-value function.

**Context:** Prior work established expressions for the policy gradient without function approximation (?).

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## Abstract

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## 2 1 Introduction

## 3 2 A Multi-Agent Bidding Approach for Multi-Objective RL

4 **Definition 1** (MO-MDP). *A multi-objective Markov decision process (MO-MDP) with  $m \in \mathbb{Z}_{>0}$*   
 5 *objectives is specified by a tuple  $\mathcal{M} = (S, A, T, R, \mu_0)$ , where*

- 6 •  *$S$  is the set of states,*
- 7 •  *$A$  is the set of actions,*
- 8 •  *$T : S \times A \rightarrow \mathbb{D}(S)$  is the transition function mapping a state-action pair to a distribution over*  
 9 *states,*
- 10 •  *$R : S \times A \times S \rightarrow \mathbb{R}^m$  is the reward function with each output component corresponding to the*  
 11 *different objectives, and*
- 12 •  *$\mu_0 \in \mathbb{D}(S)$  is the initial state distribution.*

13 A policy in an MO-MDP  $\mathcal{M}$  is a function  $\pi : (S \times A)^* \times S \rightarrow \mathbb{D}(A)$  that maps a history of  
 14 state-action pairs and the current state to a distribution over actions.

15 **Definition 2** (MAB-MDP). *Let  $\mathcal{M} = (S, A, T, R, \mu_0)$  be an MO-MDP with  $m$  objectives and*  
 16 *let  $b \in \mathbb{Z}_{>0}$  be the bid upper bound. Also, define  $M = \{1, \dots, m\}$  be indices of the  $m$  agents*  
 17 *corresponding to the  $m$  objectives along with  $\perp$  representing a null agent. Lastly, let  $B = \{0, \dots, b\}$*   
 18 *be the range of bids and  $\rho > 0$  be the bid penalty factor. We define the multi-agent bidding Markov*  
 19 *decision process (MAB-MDP) as a tuple  $\mathcal{B}_{\mathcal{M}} = (\hat{S}, \hat{A}, \hat{T}, P, \hat{R}, \hat{\mu}_0)$  where*

- 20 •  *$\hat{S} = M \times S$  is the new state space augmented with the index of the agent that won the previous*  
 21 *round of bidding,*
- 22 •  *$\hat{A} = A^m \times B^m$  represents the action space of the  $m$  agents in which each agent selects an action*  
 23 *from  $A$  and a bid from  $B$ ,*
- 24 •  *$\hat{T} : \hat{S} \times \hat{A} \rightarrow \mathbb{D}(\hat{S})$  is the new transition function defined as,*

$$\hat{T}((\_, s), (\mathbf{a}, \mathbf{b})) := \frac{1}{|B_{\max}|} \sum_{i \in B_{\max}} (T(s, a_i), i)$$

25 where  $B_{\max} := \{i \mid b_i = \max\{b_1, \dots, b_m\}\}$  is the set of agents with maximal bids. The tuple  
 26  $(T(s, a_i), i)$  represents the distribution over  $\hat{S}$  induced by the original transition function  $T$  such  
 27 that the second component is fixed, and the weighted sum represents taking the weighted sums of  
 28 the distributions over  $\hat{S}$ .

- 29 •  *$P : \hat{A} \times M \rightarrow \mathbb{R}^m$  is the bidding penalty for the  $m$  agents and the second component is the index*  
 30 *of the agent that won the bidding.*

- 31 •  $\hat{R} : \hat{S} \times \hat{A} \times \hat{S} \rightarrow \mathbb{R}^m$  is the reward function for the  $m$  agents with

$$\hat{R}_k((\_, s_0), (\mathbf{a}, \mathbf{b}), (i, s)) := R_k(s_0, a_i, s) - P_k((\mathbf{a}, \mathbf{b}), i)$$

32 where  $i \in M$  is the index of the agent that won the bid and chose the action.

- 33 •  $\hat{\mu}_0 := (\mu_0, 1)$  is the initial state distribution over  $\hat{S}$  induced by  $\mu_0$  and the second component is  
 34 fixed to be 1 without loss of generality.

35 Given an MAB-MDP  $\mathcal{B}_{\mathcal{M}}$ , a policy for each agent indexed by  $i \in \{1, \dots, m\}$  takes a similar form:  
 36  $\pi_i : (\hat{S} \times \hat{A})^* \times \hat{S} \rightarrow \hat{A}$ . Intuitively, a state  $(i, s) \in \hat{S}$  encodes the agent that won the bidding and  
 37 chose the action to reach  $s$  in the previous step. At each step, each of the agents choose an action and  
 38 a bid, and an action amongst the set of highest bidders is chosen uniformly at random. The reward  
 39 function includes a penalty term that captures the desired bidding mechanism.

## 40 References