Module-4	
SAMPLING AND QUANTIZATION: Introduction, Why Digitize Analog Sources?, The Low pass Sampling process Pulse Amplitude Modulation. Time Division Multiplexing, Pulse-Position Modulation, Generation of PPM Waves, Detection of PPM Waves.**(7.1 – 7.7 in Text)**	**L1, L2,L3**

THE TRANSITION FROM ANALOG TO DIGITAL

INTRODUCTION

- In continuous-wave (CW) modulation, some parameter of a sinusoidal carrier wave is varied continuously in accordance with the message signal. Ex: voice
- In the first step from analog to digital, an analog source is sampled at discrete times. The resulting analog samples are then transmitted by means of analog pulse modulation.
- In the second step from analog to digital, an analog source is not only sampled at discrete times but the samples themselves are also quantized to discrete levels.
- Historically, the conversion from an analog information source, such as voice or video, to a digital representation and subsequent transmission, was often implemented as a single step.

WHY DIGITIZE ANALOG SOURCES?

- **Digital systems are less sensitive to noise than analog**. For long transmission lengths, the signal may be regenerated effectively error-free at different points along the path, and the original signal transmitted over the remaining length.
- **With digital systems, it is easier to integrate different services**, for example, video and the accompanying soundtrack, into the same transmission scheme.
- **The transmission scheme can be relatively independent of the source**. For example, a digital transmission scheme that transmits voice at 10 kbps could also be used to transmit computer data at 10 kbps.
- **Circuitry for handling digital signals is easier to repeat** and digital circuits are less sensitive to physical effects such as vibration and temperature.
- **Digital signals are simpler to characterize** and typically do not have the same amplitude range and variability as analog signals. This makes the associated hardware easier to design.
- Various media sharing strategies, known as **multiplexing techniques**, are more easily implemented with digital transmission strategies.
- There are techniques for **removing redundancy** from a digital transmission, so as to minimize the amount of information that has to be transmitted.
- There are techniques for adding controlled redundancy to a digital transmission, such that errors that occur during transmission may be corrected at the receiver.
- Digital techniques make it easier to specify complex standards that may be shared on a worldwide basis.

- Other channel compensations techniques, such as equalization, especially adaptive versions, are easier to implement with digital transmission techniques.

THE SAMPLING PROCESS

Through use of the sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.

State and prove Sampling theorem:

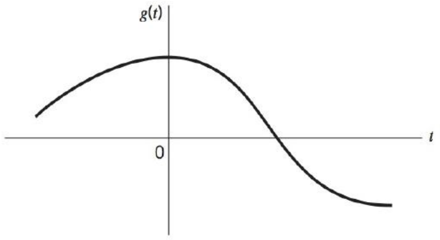
Statement:

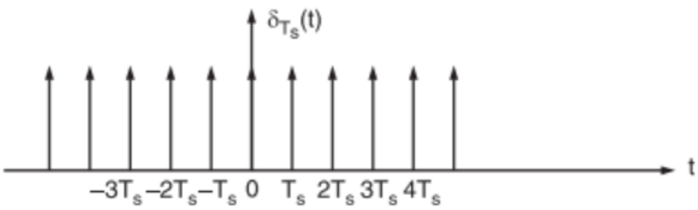
1. *A band-limited signal of finite energy, which only has frequency components less than W Hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds.*
2. *A band-limited signal of finite energy, which only has frequency components less than W Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2W samples per second.*

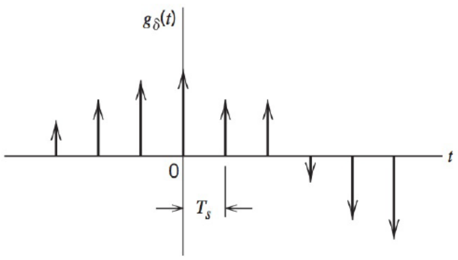
   - Nyquist rate $= 2W,$ samples/ sec
   - Nyquist interval = $\frac{1}{2W}$ seconds

Proof using Ideal (instantaneous or impulse) Sampling:

Consider an arbitrary signal $g(t)$ of finite energy, which is specified for all time.







Instantaneous sampling is carried out using a periodic sequence of impulses, separated by time period T_s.

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$\delta_{T_s}(t)$ is called 'ideal sampling function'.

The ideal sampled signal is given by $g_\delta(t) = g(t)\, \delta_{T_s}(t)$

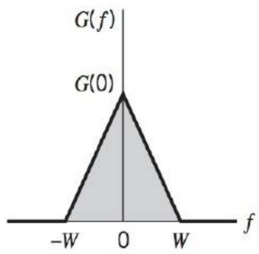
$$g_\delta(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$\boxed{g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)} \qquad (1)$$

T_s = sampling period; $f_s = \frac{1}{T_s}$ = sampling rate

<u>Frequency domain representation of ideal sampling signal:</u>

Suppose that the signal $g(t)$ is *strictly band-limited*, with no frequency components higher than W Hertz. Then, $G(f)$ is zero for $|f| \geq W$.



Suppose that we choose the sampling period $T_s = 1/2W$.

The Fourier transform of ideal sampling function is given by

$$\delta_{f_s}(f) \rightleftharpoons \delta_{T_s}(t)$$

$$\delta_{f_s}(f) = F\{\delta_{T_s}(t)\}$$

$$\delta_{f_s}(f) = F\left\{\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right\}$$

$$\boxed{\delta_{f_s}(f) = f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s)} \qquad (2)$$

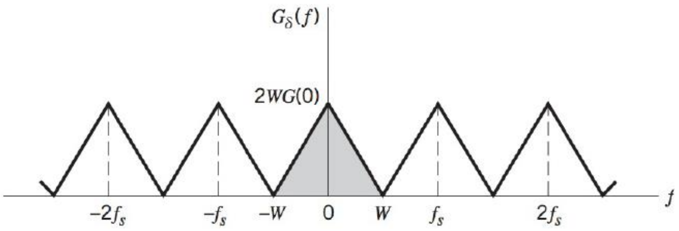
Then, the spectrum of ideally sampled signal is obtained as shown below:

$$G_\delta(f) = F\{g_\delta(t)\} = F\{g(t)\, \delta_{T_s}(t)\}$$

$$G_\delta(f) = G(f) * \delta_{f_s}(f)$$

$$G_\delta(f) = G(f) * f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s)$$

$$\boxed{G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)} \quad (3)$$



So, **when a signal is ideally sampled in time-domain, it's spectrum repeats periodically**.

$G(f)$ = the Fourier transform of the original signal $g(t)$

$G_\delta(f)$ = the Fourier transform of the original signal $g_\delta(t)$

Another Approach:

$$G_\delta(f) = F\{g_\delta(t)\}$$

$$G_\delta(f) = F\left\{\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)\right\}$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, F\{\delta(t - nT_s)\}$$

$$\boxed{G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n f T_s}} \quad (4)$$

This relation is called the *discrete-time Fourier transform.*

It may be viewed as a complex Fourier series representation of the periodic frequency function $G_\delta(f)$, with the sequence of samples $\{g(nT_s)\}$ defining the coefficients of the expansion.

<u>Reconstruction of $g(t)$ from its samples:</u>

From equation (3), we can write that

$$G_\delta(f) = f_s G(f) + \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - mf_s)$$

$$G_\delta(f) = f_s G(f), \qquad -W \le f \le W$$

$$G_\delta(f) = 2W\ \ G(f), \qquad -W \le f \le W$$

Therefore, $G(f) = \frac{1}{2W} \quad G_{\delta}(f), \qquad -W \le f \le W \qquad (5)$

Now, using equation (4) in (5), we get

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi n f T_s} \qquad -W \le f \le W$$

Putting $T_s = 1/2W$, we get

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-\frac{j\pi n f}{W}}, \qquad -W \le f \le W$$

The inverse Fourier transform defining $g(t)$ in terms of $G(f)$, we get

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$$

$$g(t) = \int_{-W}^{W} \left[\frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-\frac{j\pi n f}{W}}\right] e^{j2\pi f t}\, df$$

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \int_{-W}^{W} \left[e^{-\frac{j2\pi n f}{2W}}\right] e^{j2\pi f t}\, df$$

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \int_{-W}^{W} \left[e^{j2\pi f\left(t-\frac{n}{2W}\right)}\right] df$$

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \left[\frac{e^{j2\pi f\left(t-\frac{n}{2W}\right)}}{j2\pi\left(t-\frac{n}{2W}\right)}\right]_{-W}^{W}$$

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \left[\frac{e^{j2\pi W\left(\frac{2Wt-n}{2W}\right)}}{j2\pi\left(\frac{2Wt-n}{2W}\right)} - \frac{e^{-j2\pi W\left(\frac{2Wt-n}{2W}\right)}}{j2\pi\left(\frac{2Wt-n}{2W}\right)}\right]$$

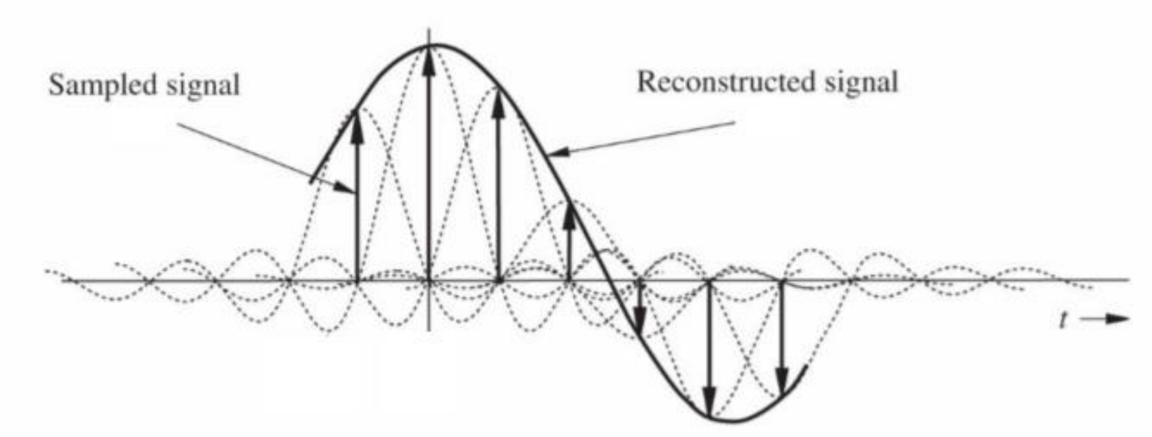
$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{2W}{\pi(2Wt-n)} \left[\frac{e^{j\pi(2Wt-n)} - e^{-j\pi(2Wt-n)}}{2j}\right]$$

$$g(t) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{2W}{\pi(2Wt-n)} \sin[\pi(2Wt-n)]$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin[\pi(2Wt-n)]}{\pi(2Wt-n)}$$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt-n)}$$

This equation provides an *interpolation formula* for reconstructing the original signal $g(t)$ from the sequence of sample values $\left\{g\left(\frac{n}{2W}\right)\right\}$, with the sine function $\text{sinc}(2Wt)$ playing the role of an *interpolation function*.



Aliasing

The derivation of the sampling theorem is based on the assumption that the signal *g(t)* is strictly band limited. In practice, however, an information-bearing signal is *not* strictly band limited. This results in some degree of under-sampling. Consequently, some *aliasing* is produced by the sampling process.

“Aliasing refers to the phenomenon of a high frequency component in the spectrum of the signal seemingly taking on the identity of a lower frequency in the spectrum of its sampled version”.

The aliased spectrum shown by the solid curve in Figure below.

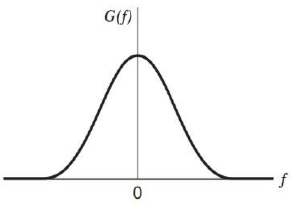


FIGURE Spectrum of a signal,

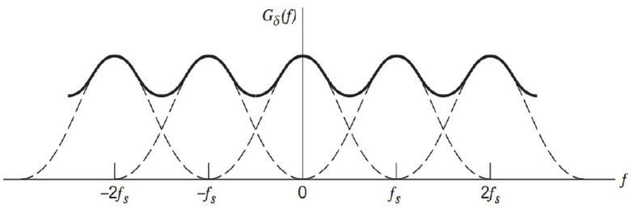
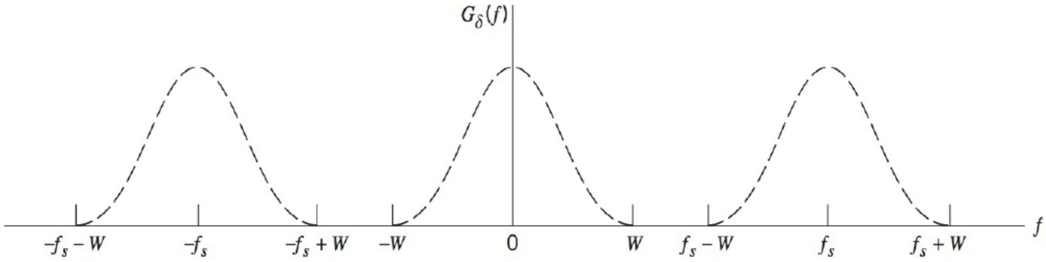


FIGURE spectrum of an undersampled version of the signal exhibiting the aliasing phenomenon.

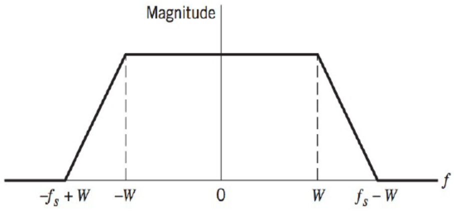
To avoid (combat) the effects of aliasing in practice, we may use two corrective measures, as described here:

1. Prior to sampling, a low-pass *pre-alias filter* is used to attenuate those high-frequency components of the signal that are not essential.
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.

$$\boldsymbol{f_s > 2W}, \quad \text{samples/sec}$$



The use of a sampling rate higher than the Nyquist rate also has the beneficial effect of easing the design of the *reconstruction filter* used to recover the original signal from its sampled version.



The design of the reconstruction filter may be specified as follows :

- The reconstruction filter is low-pass with a passband extending from $-W$ to W.
- The filter has a transition band extending (for positive frequencies) from W to $(f_s - W)$, where f_s is the sampling rate.