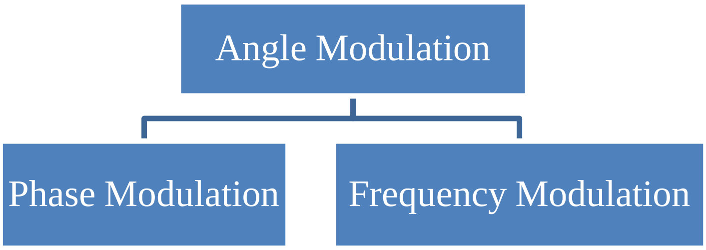
ANGLE MODULATION



<u>*Angle modulation*</u> is the one in which the angle of the carrier wave is varied according to the baseband signal.
An important feature of angle modulation is that it can provide better discrimination against noise and interference than amplitude modulation.

Let $m(t)$ = information-bearing message signal
$c(t)$ = carrier signal $= A_c \cos 2\pi f_c t$

Then, angle modulated wave is given by

$$\boxed{s(t) = A_c \cos \theta_i(t)} \quad \text{volts}$$

Where $\theta_i(t) = 2\pi f_i t$ = instantaneous phase angle, varies from '0' to '2π' radians.
$\theta_i(t)$ changes proportional to the baseband signal, $m(t)$.

Phase Modulation (PM):

Phase modulation (PM) *is that form of angle modulation in which the instantaneous angle* $\theta_i(t)$ is varied linearly with the message signal as shown by

$$\boxed{\theta_i(t) = \theta_c + k_p\, m(t)}$$

$\theta_c = 2\pi f_c t$ = phase angle *the unmodulated* carrier when $m(t) = 0$

Now, the phase modulated signal is given by

$$\boxed{s(t) = A_c \cos\left(2\pi f_c t + k_p\, m(t)\right)}$$

k_p = constant = *phase sensitivity* of the modulator, expressed in radians per volt

Frequency Modulation (FM):

Frequency modulation (FM) *is that form of angle modulation in which the instantaneous frequency* $f_i(t)$ *is varied linearly with the message signal* $m(t)$*, as shown by*

$$\boxed{f_i(t) = f_c + k_f\, m(t)}$$

f_c = the frequency of the unmodulated carrier, and
k_f = constant = the *frequency sensitivity* of the modulator, expressed in Hertz per volt

Then, angle modulated wave is given by

$$s(t) = A_c \cos \theta_i(t)$$

Where $\theta_i(t) = 2\pi f_i t$

Over an interval from 't' to '$t + \Delta t$', change in phase angle is

$$\theta_i(t + \Delta t) - \theta_i(t) = 2\pi \ f_{\Delta t}(t) \ \ \Delta t$$

$$f_{\Delta t}(t) = \frac{1}{2\pi} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{\Delta t}$$

As $\Delta t \to 0$, $f_{\Delta t}(t) = f_i(t)$, instantaneous frequency

The instantaneous frequency,

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \lim_{\Delta t \to 0} \left[\frac{1}{2\pi} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{\Delta t}\right]$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\,\theta_i(t)}{dt}$$

$$d\,\theta_i(t) = 2\pi\, f_i(t)\, dt$$

$$d\,\theta_i(t) = 2\pi\, (f_c + \Delta f)\, dt$$

$$d\,\theta_i(t) = 2\pi \left(f_c + k_f\, m(t)\right)\, dt$$

$$d\,\theta_i(t) = 2\pi f_c\, dt + 2\pi\, k_f\, m(t) dt$$

Integrating with respect to 't',

$$\theta_i(t) = 2\pi f_c t + 2\pi\, k_f \int_0^t m(\tau) d\tau$$

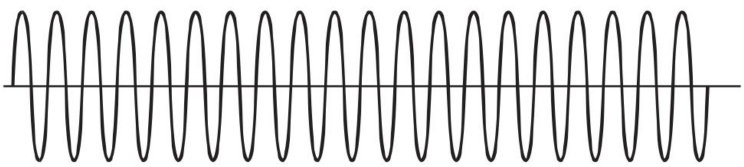
Now, the frequency modulated signal is given by

$$\boxed{\boldsymbol{s(t) = A_c \cos\left[2\pi f_c t + 2\pi\, k_f \int_0^t m(\tau) d\tau\right]}}$$

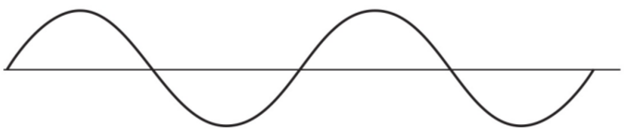
Waveforms:

Example 1:

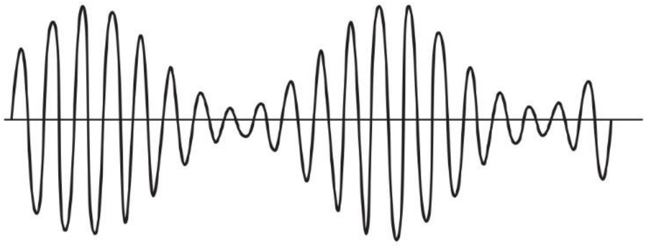
Carrier signal:



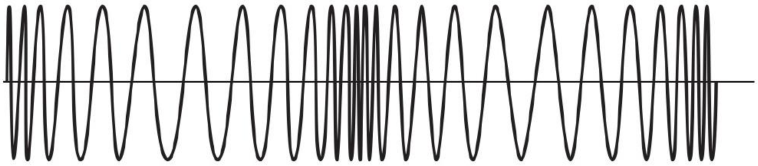
Message signal:



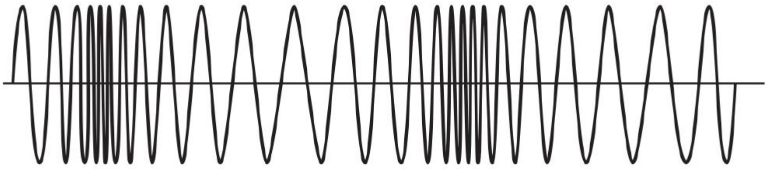
AM signal:



FM signal:

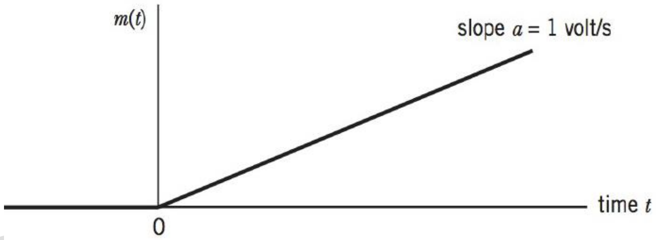


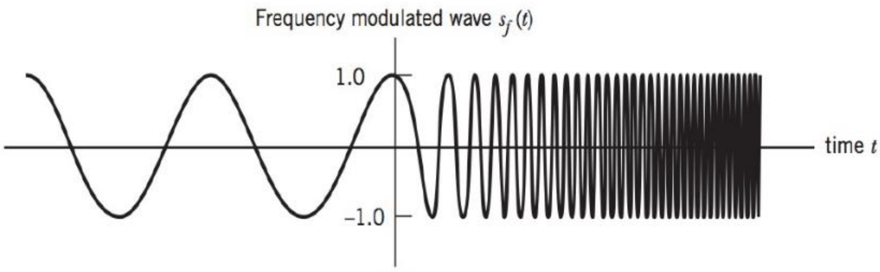
PM signal:

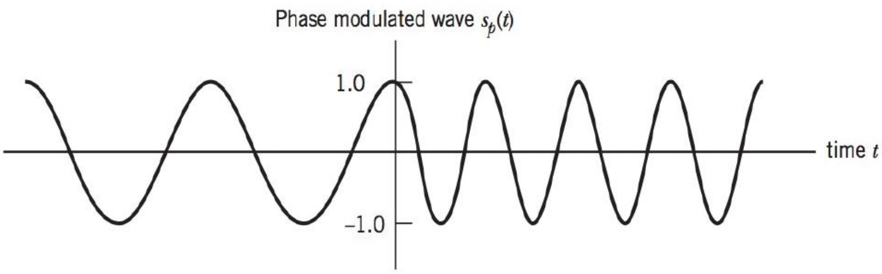


Example 2:

Message signal:







Properties of FM signal

1. **Constant transmitter power:**

   The average transmitted power of angle-modulated waves is a constant

$P_{av} = \frac{1}{2} A_c^2$ where it is assumed that the load resistance is 1 ohm.

2. **Nonlinear modulation**

   Consider a message signal $m(t)$ consist of two different components ie

$$m(t) = m_1(t) + m_2(t)$$

Let $s(t)$, $s_1(t)$, and $s_2(t)$ denote the FM waves produced by $m(t)$, $m_1(t)$, and $m_2(t)$, respectively. Then,

$$s_1(t) = A_c \cos\left[2\pi f_c t \ + \ k_f \ \int_0^t m_1(\tau)d\tau\right]$$

$$s_2(t) = A_c \cos\left[2\pi f_c t \ + \ k_f \ \int_0^t m_2(\tau)d\tau\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t \ + \ k_f \ \int_0^t m(\tau)d\tau\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t \ + \ k_f \ \int_0^t [m_1(t) + m_2(t)]d\tau\right]$$

$$s(t) \neq s_1(t) + s_2(t)$$

   So, FM is a non-linear process.

3. **Irregularity of zero crossings**
4. **Difficulty of visualization of message signal**

   In AM, we see the message waveform as the envelope of the modulated wave. This is not so in angle modulation.

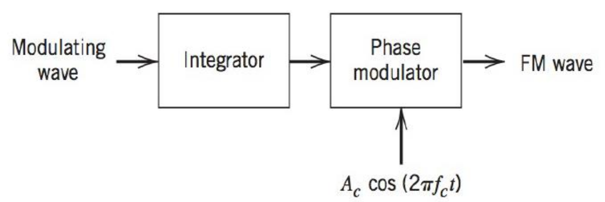
5. **Trade-Off of Increased Transmission Bandwidth for Improved Noise Performance**

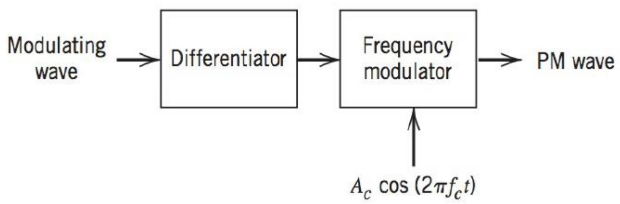
   An important advantage of angle modulation over amplitude modulation is the realization of improved noise performance. This advantage is attributed to the fact that the transmission of a message signal by modulating the angle of a sinusoidal carrier wave is less sensitive to the presence of additive noise than transmission by modulating the amplitude of the carrier.

Relationship between PM and FM

$$s_{PM}(t) = A_c \cos\left(2\pi f_c t + k_p\, m(t)\right)$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t \ + \ k_f \ \int_0^t m(\tau)d\tau\right]$$





FREQUENCY MODULATION (FM)

Definition: *"FM is the process by which frequency of the sinusoidal carrier signal is varied in accordance with the instantaneous value of modulating signal".*

The FM signal, $s(t)$ is a nonlinear function of the modulating signal $m(t)$, which makes frequency modulation a *nonlinear modulation process.*

The angle modulated wave is given by

$$s(t) = A_c \cos \theta_i(t)$$

The instantaneous frequency of the FM signal can be obtained from

$$f_i(t) = \frac{1}{2\pi} \frac{d\,\theta_i(t)}{dt}$$

$$d\,\theta_i(t) = 2\pi\, f_i(t)\, dt$$

$$d\,\theta_i(t) = 2\pi \left(f_c + 2\pi k_f\, m(t)\right) dt$$

$$d\,\theta_i(t) = 2\pi f_c\, dt + 2\pi k_f\, m(t) dt$$

Integrating with respect to 't',

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$$

Now, the frequency modulated signal is given by

$$\boxed{\boldsymbol{s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]}}$$

Consider a sinusoidal modulating signal defined by

$$m(t) = A_m \cos(2\pi f_m t)$$

Then, the FM signal becomes

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi\, k_f A_m \int_0^t \cos(2\pi f_m \tau)\, d\tau\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi\, \Delta f \int_0^t \cos(2\pi f_m \tau)\, d\tau\right]$$

Where $\Delta f = k_f A_m$ = frequency deviation

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi\, \Delta f\, \frac{\sin(2\pi f_m t)}{2\pi f_m}\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)\right]$$

$$\boxed{\boldsymbol{s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]}}$$

Where $\beta = \frac{\Delta f}{f_m} = \frac{frequency\ deviation}{modulating\ frequency}$

β is called '**modulation index**'.

Depending on the value of the modulation index β, we may distinguish two cases of frequency modulation:

- *Narrow-band* FM, for which β is small compared to one radian.
- *Wide-hand* FM, for which β is large compared to one radian.

1. for highly efficient and complex hardware