PRINCIPLES OF COMMUNICATION SYSTEMS

1. INTRODUCTION

Communication is the process of conveying information from source to destination.
A communication system consists of transmitter, receiver and medium.

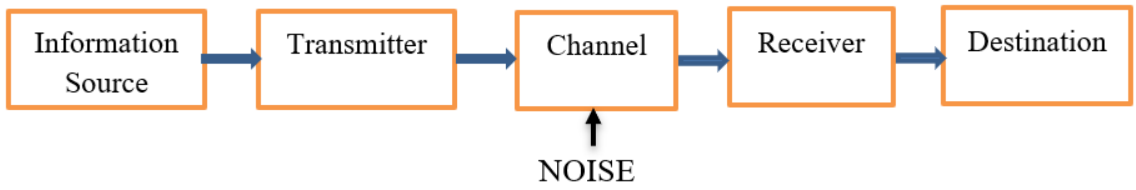


Figure 1.1 Basic block diagram of communication system

Information source: The source generates message that contains information to be transmitted. Common examples of information are: voice, music, pictures, videos, or data.

Transmitter: Transmitter processes the information provided by the source into a form that is suitable for transmitting over the *channel.* Amplification and modulation are the mandatory signal.

Channel: The channel is the medium of transmission. The transmission medium may be telephone cable, co-axial cable, optical fiber, or free space.

Receiver: Receiver is a system that processes the received signal to extract the information. The receiver uses demodulator (or detector)

Destination or sink: It represents an end user.

- The purpose of a communication system is to convey *information* through a medium or communication channel separating the transmitter from the receiver.
- The information is often represented as a baseband signal, that is, a signal whose spectrum extends from 0 to W Hz, some maximum frequency.
- Proper utilization of the communication channel often requires a shift of the range of baseband frequencies into other frequency ranges suitable for transmission, and a corresponding shift back to the original frequency range after reception.
- A shift of the range of frequencies in a signal is accomplished by using *modulation.*

Modulation

Definition:

Modulation is a process by which some characteristics of a carrier is varied in accordance with a modulating wave (signal).

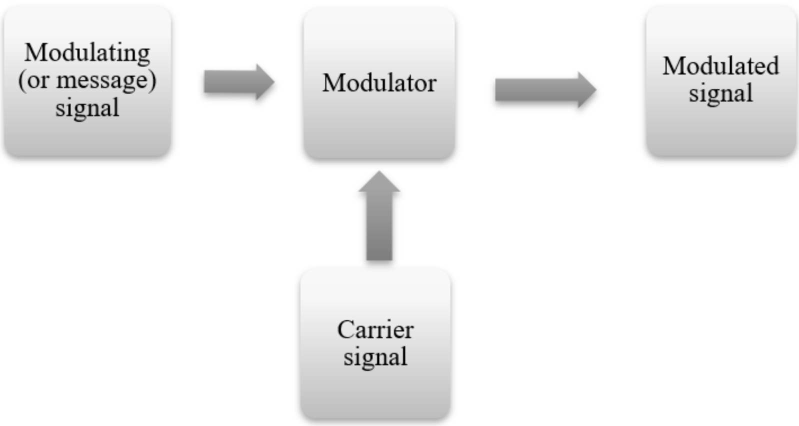


Figure 1.2 Block diagram of modulation process

- *Continuous-wave modulation* is one which uses a *sinusoidal signal as carrier.*
- The baseband signal is referred to as the *modulating signal*, and the result of the modulation process is referred to as the *modulated signal.*
- Modulation is performed at the transmitting end of the communication system. At the receiving end of the system, we usually require the original baseband signal to be restored. This is accomplished by using a process known as *demodulation,* which is the reverse of the modulation process.

Need for modulation:

1. <u>*To reduce size of antenna*</u>

   The dimension of an antenna is given by $L = \lambda/2$

   Where $\lambda =$ wavelength of the signal to be transmitted $= \frac{C}{f}$

Since the modulation increases the frequency, λ and length of the antenna can be reduced by modulation.

For Example:

At $f = 1\ KHz$, length of an antenna, $L = \lambda/2 = \frac{C}{2f} = \frac{3\times10^8}{2\times1\times10^3} = 150$ Km

At $f = 1\ MHz$, length of an antenna, $L = \lambda/2 = \frac{C}{2f} = \frac{3\times10^8}{2\times1\times10^6} = 150$ metres

At $f = 1\ GHz$, length of an antenna, $L = \lambda/2 = \frac{C}{2f} = \frac{3\times10^8}{2\times1\times10^9} = 15$ cm

Thus, modulation reduces size of antenna needed for transmission/ reception.

2. *To multiplex signals for transmission through common channel*

The process of combining several signals for simultaneous transmission on a single channel is called multiplexing. To use a channel to transmit the different base band signals (information) at the same time, it becomes necessary to translate different signals so as to make them occupy different frequency slots or bands so that they do not interfere. This is accomplished by using carrier of different frequencies.

3. *To reduce effect of noise*

Some of the frequencies are severely affected by noise. In such cases, the information signal can be modulated to new frequency range where the effect of noise is minimum. For example, some of the frequencies are absorbed oxygen and water vapour in the atmosphere. These frequencies can be avoided using modulation.

4. *To avoid mixing of signals:*

Generally, the signals are mixed with external interfering signals during transmission. This can be avoided by changing the carrier frequency signal through modulation.

5. *To avoid mixing of signals:*

The signals can be modulated to radio frequency range or microwave range which avoids the necessity of wires or cables for transmission over long distances.

Classification of Modulation techniques

Continuous-wave modulation: The carrier signal is a sinusoidal signal of high frequency. Based on the modulation signal, this can be further classified as:

1. Analog modulation: The modulating signal is analog in nature
2. Digital modulation: The modulating signal is digital.

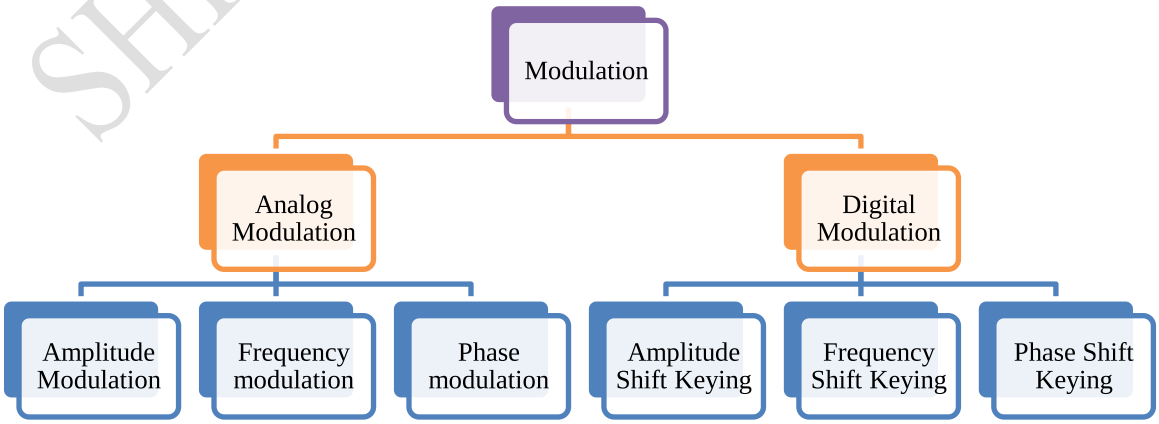


Figure 1.2 Classification of modulation methods

Analog Modulation:

1. Amplitude Modulation (AM)
2. Frequency Modulation (FM)
3. Phase Modulation (PM)

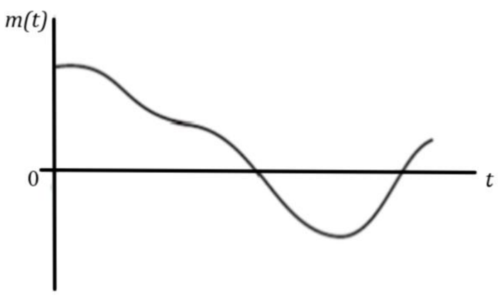
2. AMPLITUDE MODULATION (AM)

Definition:

Amplitude Modulation is defined as a process in which amplitude of the carrier wave is varied linearly with the message signal. That is amplitude of the carrier is made proportional to the instantaneous values (amplitude) of the modulating signal.

Message Signal, $m(t)$: It is a baseband signal, generally, bandlimited using a low pass filter of cut-off frequency, W Hz. Let $M(f)$ be the spectrum of $m(t)$.

$$m(t) \xleftrightarrow{\text{FT}} M(f)$$



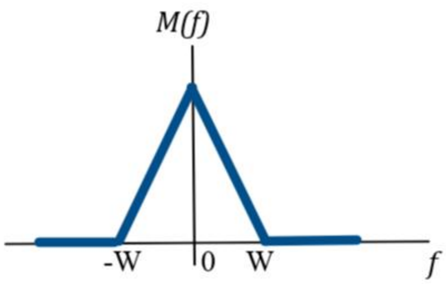


Figure 2.1 Message signal and it's spectrum

Carrier Signal, c(t): It is a sinusoidal signal of high frequency, f_c and amplitude A_c.

$$c(t) = A_c \cos(2\pi f_c t)$$

The spectrum of the carrier signal is

$$C(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

Conditions:

- $f_c > W$
- $A_c \gg |m(t)|_{max}$

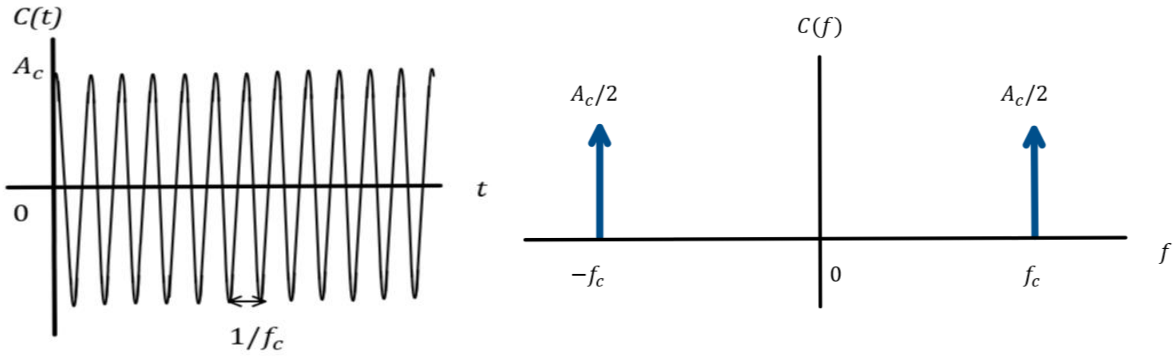


Figure 2.2 Carrier signal and it's spectrum

The **standard Amplitude Modulation** (AM) signal, s(t) is given by

$$\boxed{s(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)}$$

where k_a is a constant, called the *'amplitude sensitivity'* of the modulator.

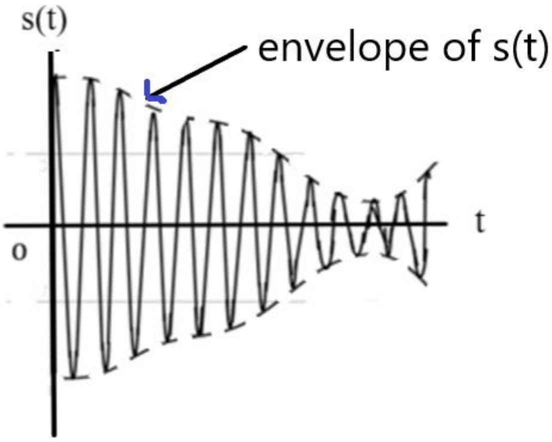


Figure 2.3 Amplitude Modulated signal

The envelope of the AM wave s(t) is given by

$$\boldsymbol{a(t) = A_c[1 + \mathrm{k_a m(t)}]}$$

$$S(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)$$

$$S(t) = A_c \cos(2\pi f_c t) + A_c \mathrm{k_a m(t)}\cos(2\pi f_c t)$$

The frequency domain representation of s(t) is given by the Fourier Transform ie

$$\boxed{S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c k_a}{2}[\mathrm{M}(f - f_c) + \mathrm{M}(f + f_c)]}$$

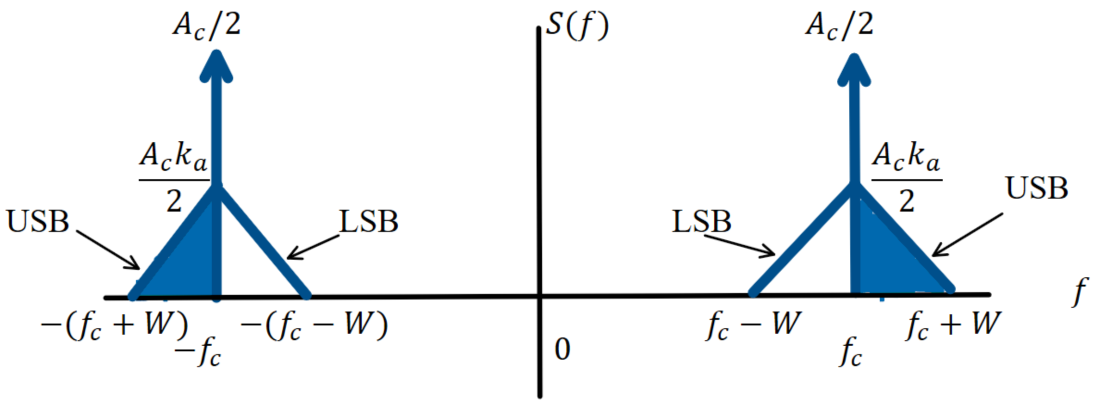


Figure 2.4 Spectrum of AM signal

The bandwidth of AM signal is given by

$$B = (f_c + W) - (f_c - W)$$

$$\boldsymbol{B = 2W}$$

Bandwidth of AM = 2 × maximum frequency of message signal

NOTE:

Amplitude Modulated Signal		
Carrier	f_c	A_c
Upper sideband (USB)	f_c to $(f_c + W)$	$A_c k_a$
Lower sideband (USB)	$(f_c - W)$ to f_c	$A_c k_a$

Depth of modulation (modulation index or modulation factor)

The modulation factor is defined by

$$\boxed{\mu \triangleq |\mathrm{k_a m(t)}|_{max}}$$

Percentage of modulation = $|\mathrm{k_a m(t)}|_{max} \times 100\%$

For distortion-less modulation, the condition is $|\mathrm{k_a m(t)}|_{max} \leq 1$

Case (i): *<u>under modulation</u>*

- $|\mathrm{k_a m(t)}|_{max} < 1$ or $\mu < 1$
- Depth of modulation is less than 100%
- No distortion occurs
- Envelope detector can be used to extract information

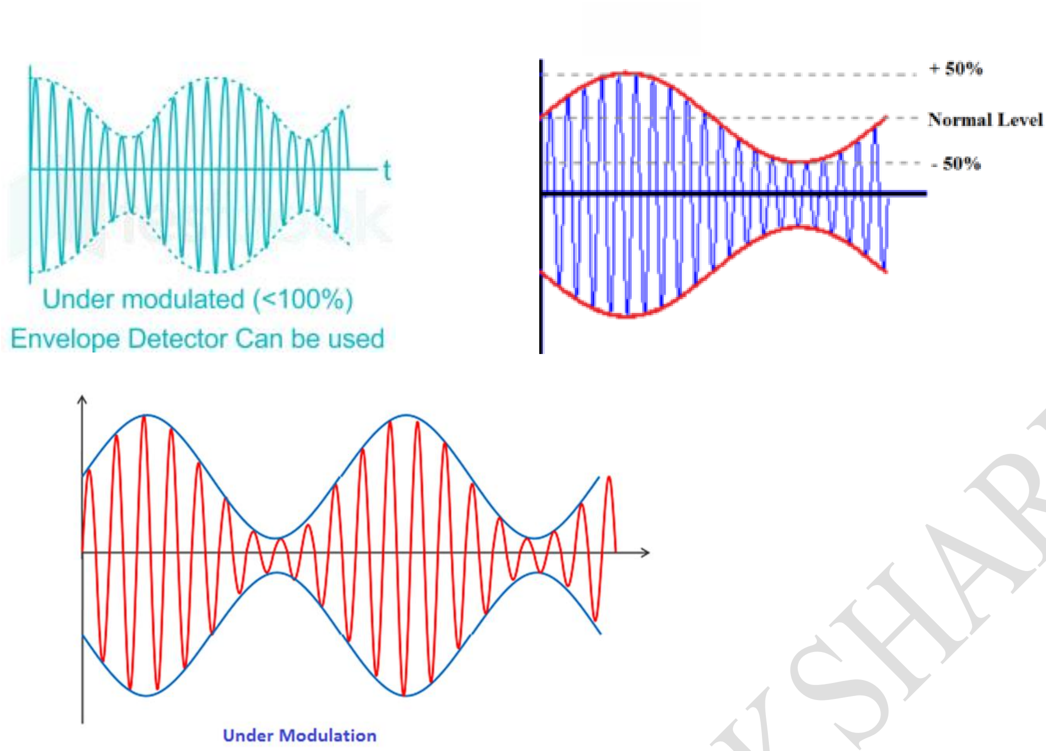


Figure 2.5 Different examples of under modulation

Case (ii): <u>*100% modulation*</u>

- $|k_a m(t)|_{max} = 1$ or $\mu = 1$
- Depth of modulation is equal to 100%
- No distortion occurs
- Envelope detector can be used to extract information

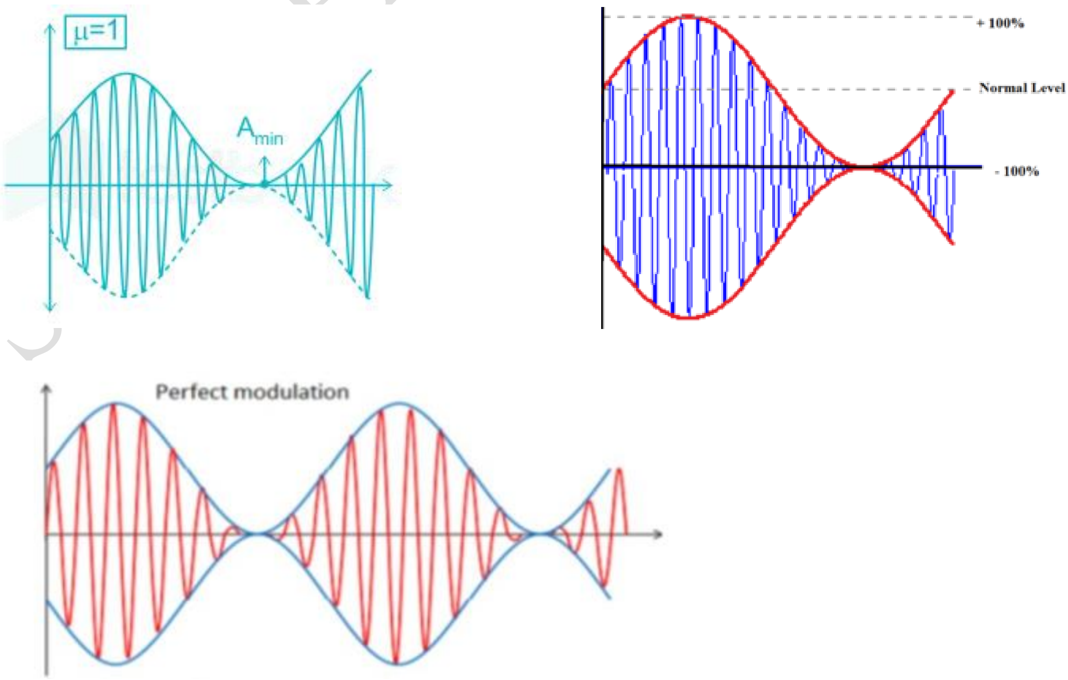


Figure 2.6 Different examples of right modulation

Case (iii): *over modulation*

- Condition: $|k_a m(t)|_{max} > 1$ or $\mu > 1$
- Depth of modulation is more than 100%
- distortion occurs ie carrier undergoes phase reversal
- Envelope detector cannot be used

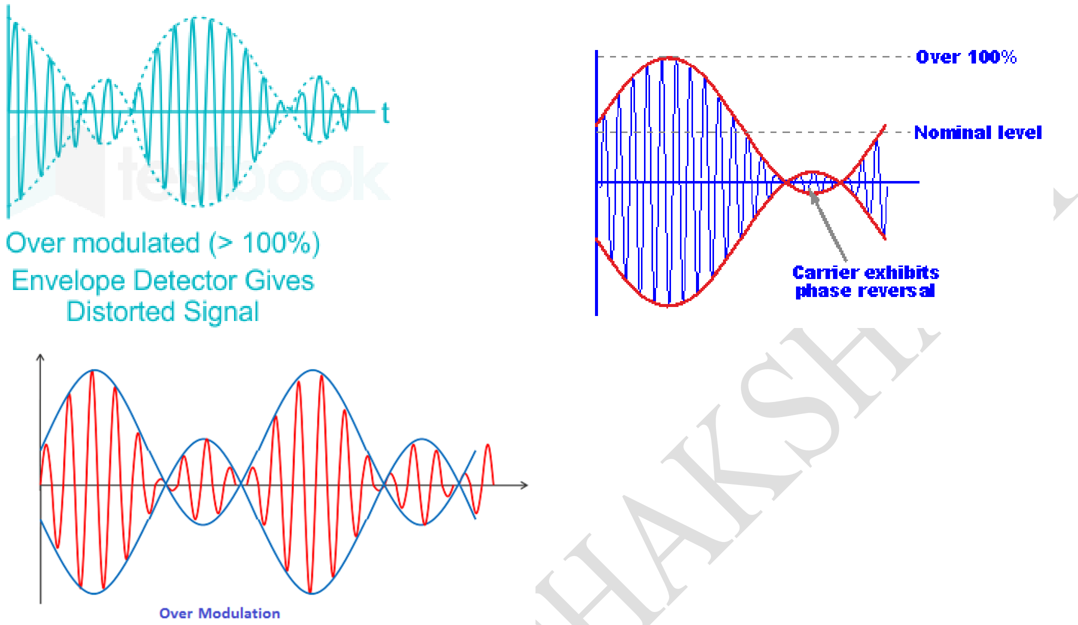


Figure 2.7 Different examples of right modulation

Single tone Amplitude Modulation

(modulation by a sinusoidal signal)

Consider a modulating wave m(t) that consists of a single tone (or frequency component)

ie $m(t) = A_m \cos(2\pi f_m t)$

The standard form of AM wave is defined by

$$S(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$S(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$$\boxed{\boldsymbol{S(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)}}$$

where μ is the modulation factor (or modulation index) and it is a dimensionless quantity.

$$S(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$$S(t) = A_c\cos(2\pi f_c t) + A_c\mu\cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$\boxed{\boldsymbol{S(t) = A_c\cos(2\pi f_c t) + \frac{A_c\mu}{2}\cos[2\pi(f_c + f_m)t] + \frac{A_c\mu}{2}\cos[2\pi(f_c - f_m)t]}}$$

Let $S(f)$ be the Fourier transform of $S(t)$.

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c\mu}{4}\left\{\delta[f - (f_c + f_m)] + \delta[f + (f_c + f_m)]\right\} + \frac{A_c\mu}{4}\left\{\delta[f - (f_c - f_m)] + \delta[f + (f_c - f_m)]\right\}$$

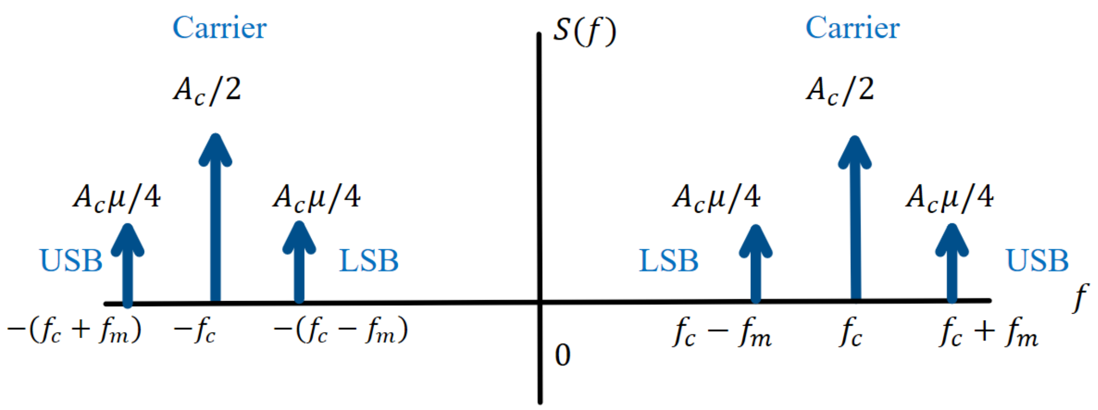


Figure 2.8 Line Spectrum of single tone AM signal

Bandwidth of single tone AM signal is given by

$$B = (f_c + f_m) - (f_c - f_m)$$

$$\boldsymbol{B = 2f_m}$$

Bandwidth is equal to twice the maximum frequency of modulating signal

Frequency components in AM	Frequency	Amplitude
Carrier: $\boldsymbol{A_c \cos(2\pi f_c t)}$	f_c	A_c
Upper Side Band (USB): $\boldsymbol{\frac{A_c\mu}{2}\cos[2\pi(f_c + f_m)t]}$	$f_c + f_m$	$\frac{A_c\mu}{2}$
Lower Side Band (LSB): $\boldsymbol{\frac{A_c\mu}{2}\cos[2\pi(f_c - f_m)t]}$	$f_c - f_m$	$\frac{A_c\mu}{2}$

Depth of modulation of single-tone modulation
(modulation index or modulation factor)

The modulation factor is defined by $\mu \triangleq |\mathrm{k_a} \mathrm{m(t)}|_{max}$

For single-tone modulation, $m(t) = A_m \cos(2\pi f_m t)$

$\mu \triangleq |\mathrm{k_a m(t)}|_{max} = |\mathrm{k_a} A_m \cos(2\pi f_m t)|_{max} = \mathrm{k_a} A_m$

$$\boxed{\mu = \mathrm{k_a} A_m}$$

Percentage of modulation index = $\% \mu = k_a A_m \times 100$

For distortion-less modulation, the condition is $\mu \leq 1$

Case (i): *under modulation* ($\mu < 1$) (less than 100%)

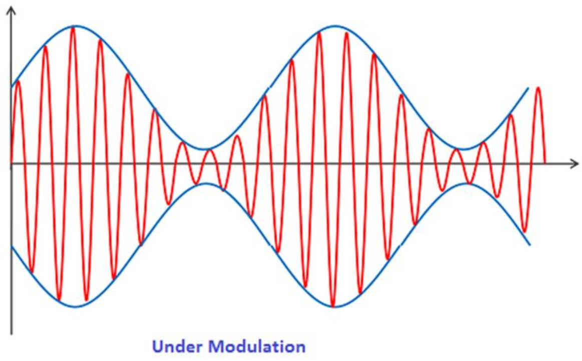


Figure 2.9 Single-tone under modulation

Case (ii): *100% modulation* ($\mu = 1$) (equal to 100%)

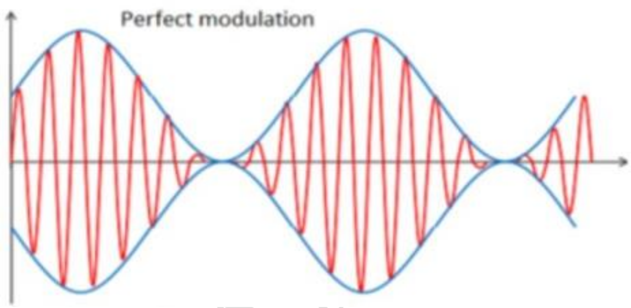


Figure 2.10 Single-tone full modulation

Case (iii): *over modulation* ($\mu > 1$) (more than 100%)

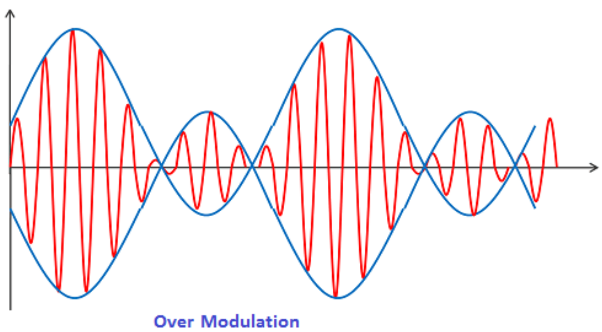


Figure 2.11 Single-tone full modulation

Derive an expression for modulation index of AM signal

Consider a single-tone modulation signal given by

$$S(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

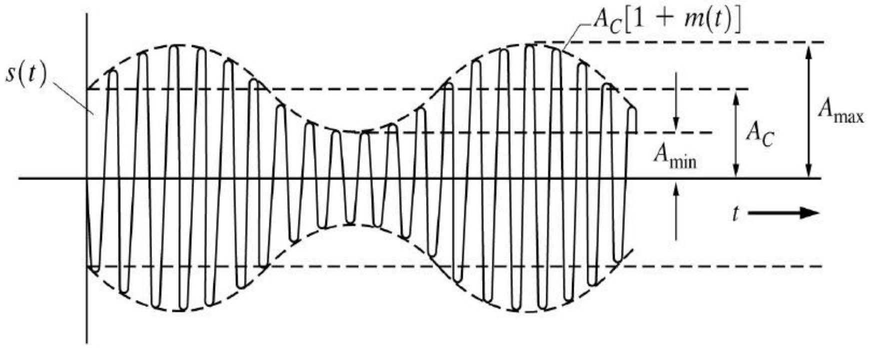


Figure 2.12 Single-tone AM signal

A_{max} = Maximum value of envelope of s(t) = $|A_c[1+\mu\cos(2\pi f_m t)]|_{max}$

$A_{max} = A_c[1+\ \mu]$

A_{min} = Minimum value of envelope of s(t) = $|A_c[1+\mu\cos(2\pi f_m t)]|_{min}$

$A_{min} = A_c[1-\ \mu]$

$$\frac{A_{max}}{A_{min}} = \frac{A_c[1+\mu]}{A_c[1-\mu]}$$

$$\frac{A_{max}}{A_{min}} = \frac{1+\mu}{1-\mu}$$

$$A_{max}\,(1\ -\ \mu) = A_{min}\,(1+\ \mu)$$

$$A_{max} - \mu A_{max} = A_{min} + \mu A_{min}$$

$$A_{max} - A_{min} = \mu A_{max} + \mu A_{min}$$

$$A_{max} - A_{min} = \mu(A_{max} + A_{min})$$

$$\boxed{\boldsymbol{\mu = \frac{A_{max}-A_{min}}{A_{max}+A_{min}}}}$$

Power relation of Amplitude Modulation (AM)

The single-tone AM signal is given by

$$S(t) = A_c\cos(2\pi f_c t) + \frac{A_c\mu}{2}\cos[2\pi(f_c+f_m)t] + \frac{A_c\mu}{2}\cos[2\pi(f_c-f_m)t]$$

The AM signal contains 3 components:

- Carrier component: $A_c\cos(2\pi f_c t)$
- Upper sideband (USB): $\frac{A_c\mu}{2}\cos[2\pi(f_c+f_m)t]$
- Lower sideband (LSB): $\frac{A_c\mu}{2}\cos[2\pi(f_c-f_m)t]$

The total average power of AM signal is

$$P_t = P_c + P_{USB} + P_{LSB}$$

P_c = power contained in carrier signal = = $\dfrac{\left(A_c/\sqrt{2}\right)^2}{R} = \dfrac{A_c^2}{2R}$

P_{USB} = power contained in USB = $\dfrac{\left(\mu A_c/2\sqrt{2}\right)^2}{R} = \dfrac{A_c^2\mu^2}{8R}$

P_{LSB} = power contained in LSB = $\dfrac{\left(\mu A_c/2\sqrt{2}\right)^2}{R} = \dfrac{A_c^2\mu^2}{8R}$

P_{SB} = power contained in sidebands = $P_{USB} + P_{LSB} = \dfrac{A_c^2\mu^2}{8R} + \dfrac{A_c^2\mu^2}{8R}$

$P_{SB} = \dfrac{A_c^2\mu^2}{4R} = \dfrac{A_c^2}{2R}\dfrac{\mu^2}{2} = P_c\dfrac{\mu^2}{2}$

Total power contained in AM signal = $P_t = P_c + P_{SB}$

$P_t = P_c + P_c\dfrac{\mu^2}{2}$

$$\boxed{P_t = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

Current relation of Amplitude Modulation (AM)

The power content of an AM signal is given by

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

Since $P_t = {I_t}^2 R$ and $P_c = {I_c}^2 R$,

$${I_t}^2 R = {I_c}^2 R\left[1 + \frac{\mu^2}{2}\right]$$

$${I_t}^2 = {I_c}^2\left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{I_t = I_c\sqrt{1 + \frac{\mu^2}{2}}}$$

Efficiency of standard Amplitude Modulation (AM)

The efficiency, η is defined as the ratio of sideband power to total power content in AM signal ie

$$\eta = \frac{sideband\ power}{total\ power}$$

$$\eta = \frac{P_{SB}}{P_t}$$

$$\eta = \frac{P_c\frac{\mu^2}{2}}{P_c\left[1+\frac{\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}}{1+\frac{\mu^2}{2}} = \frac{\frac{\mu^2}{2}}{\frac{2+\mu^2}{2}}$$

$$\eta = \frac{\mu^2}{2+\mu^2}$$

$$\boxed{\text{Percentage of efficiency, } \%\,\eta = \frac{\mu^2}{2+\mu^2} \times 100}$$

Modulation by many sinusoidal message signals:(Multi-tone modulation)

The standard form of AM wave is defined by

$$S(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)$$

In multi-tone modulation, modulating signal consists of more than one frequency component whereas in single-tone modulation, it consists of only one frequency component.

Consider three modulating signals $A_1\cos(2\pi f_1 t)$, $A_2\cos(2\pi f_2 t)$ and $A_3\cos(2\pi f_3 t)$ with frequencies $f_1 < f_2 < f_3$ and amplitudes $A_1 > A_2 > A_3$ respectively. Then

$$m(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t) + A_3\cos(2\pi f_3 t)$$

and

$$S(t) = A_c\{1 + \mathrm{k_a}[A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t) + A_3\cos(2\pi f_3 t)]\}\cos(2\pi f_c t)$$

$$S(t) = A_c\{1 + \mathrm{k_a}A_1\cos(2\pi f_1 t) + \mathrm{k_a}A_2\cos(2\pi f_2 t) + \mathrm{k_a}A_3\cos(2\pi f_3 t)\}\cos(2\pi f_c t)$$

$$\boxed{\boldsymbol{S(t) = A_c[1 + \mu_1\cos(2\pi f_1 t) + \mu_2\cos(2\pi f_2 t) + \mu_3\cos(2\pi f_3 t)]\cos(2\pi f_c t)}}$$

Where modulation indices μ_1, μ_2 and μ_3 respectively are defined by

$$\mu_1 \triangleq \mathrm{k_a}A_1$$

$$\mu_2 \triangleq \mathrm{k_a}A_2 \quad \text{and}$$

$$\mu_3 \triangleq \mathrm{k_a}A_3$$

$$\begin{aligned} s(t) = A_c\cos(2\pi f_c t) &+ A_c\mu_1\cos(2\pi f_1 t)\cos(2\pi f_c t) \\ &+ A_c\mu_2\cos(2\pi f_2 t)\cos(2\pi f_c t) \\ &+ A_c\mu_3\cos(2\pi f_3 t)\cos(2\pi f_c t) \end{aligned}$$

$$\boxed{\begin{aligned} \boldsymbol{s(t)} = \boldsymbol{A_c\cos(2\pi f_c t)} &+ \frac{\boldsymbol{A_c\mu_1}}{\boldsymbol{2}}\boldsymbol{\cos[2\pi(f_c - f_1)t]} + \frac{\boldsymbol{A_c\mu_1}}{\boldsymbol{2}}\boldsymbol{\cos[2\pi(f_c + f_1)t]} \\ &+ \frac{\boldsymbol{A_c\mu_2}}{\boldsymbol{2}}\boldsymbol{\cos[2\pi(f_c - f_2)t]} + \frac{\boldsymbol{A_c\mu_2}}{\boldsymbol{2}}\boldsymbol{\cos[2\pi(f_c + f_2)t]} \\ &+ \frac{\boldsymbol{A_c\mu_3}}{\boldsymbol{2}}\boldsymbol{\cos[2\pi(f_c - f_3)t]} + \frac{\boldsymbol{A_c\mu_3}}{\boldsymbol{2}}\boldsymbol{\cos[2\pi(f_c + f_3)t]} \end{aligned}}$$

The line spectrum of $s(t)$ is given by

$$\boldsymbol{S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]}$$

$$\boldsymbol{+\frac{A_c\mu_1}{4}\,\delta[f - (f_c - f_1)] + \frac{A_c\mu_1}{4}\,\delta[f + (f_c - f_1)]}$$

$$\boldsymbol{+\frac{A_c\mu_1}{4}\,\delta[f - (f_c + f_1)] + \frac{A_c\mu_1}{4}\,\delta[f + (f_c + f_1)]}$$

$$+\frac{A_c\mu_2}{4}\,\delta[f-(f_c-f_2)]+\frac{A_c\mu_2}{4}\,\delta[f+(f_c-f_2)]$$
$$+\frac{A_c\mu_2}{4}\,\delta[f-(f_c+f_2)]+\frac{A_c\mu_2}{4}\,\delta[f+(f_c+f_2)]$$
$$+\frac{A_c\mu_3}{4}\,\delta[f-(f_c-f_3)]+\frac{A_c\mu_3}{4}\,\delta[f+(f_c-f_3)]$$
$$+\frac{A_c\mu_3}{4}\,\delta[f-(f_c+f_3)]+\frac{A_c\mu_3}{4}\,\delta[f+(f_c+f_3)]$$

Sketch of spectrum of s(t):

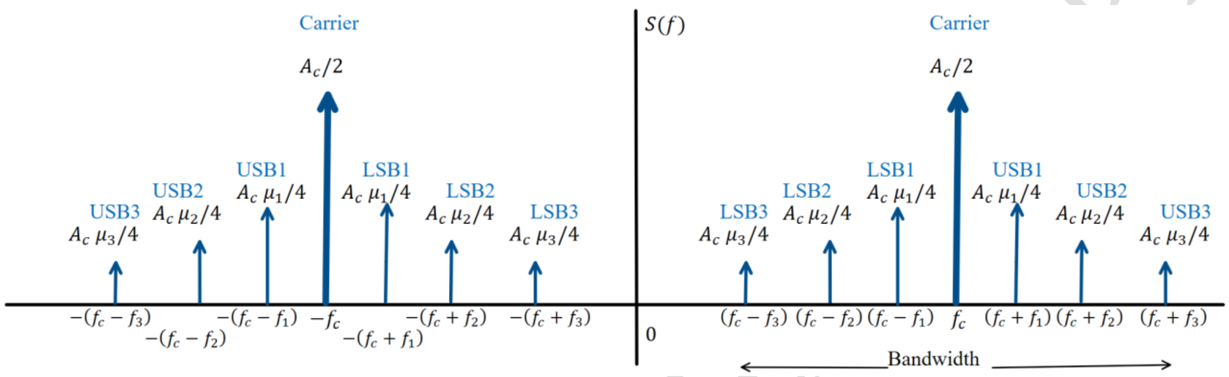


Figure 2.13 Line spectrum of multi-tone signal

Bandwidth = B = $(f_c+f_3)-(f_c-f_3)$

B = $\boldsymbol{2f_3}$

Total power content of the s(t) is

$$P_t = P_c + P_{USB1} + P_{LSB1} + P_{USB2} + P_{LSB2} + P_{USB3} + P_{LSB3}$$
$$P_c\left[1+\frac{\mu^2}{2}\right] = \frac{A_c^2}{2R}+\frac{A_c^2\mu_1^2}{8R}+\frac{A_c^2\mu_1^2}{8R}+\frac{A_c^2\mu_2^2}{8R}+\frac{A_c^2\mu_2^2}{8R}+\frac{A_c^2\mu_3^2}{8R}+\frac{A_c^2\mu_3^2}{8R}$$
$$P_c+P_c\frac{\mu_t^2}{2} = P_c+P_c\frac{\mu_1^2}{4}+P_c\frac{\mu_1^2}{4}+P_c\frac{\mu_2^2}{4}+P_c\frac{\mu_2^2}{4}+P_c\frac{\mu_3^2}{4}+P_c\frac{\mu_3^2}{4}$$
$$1+\frac{\mu_t^2}{2} = 1+\frac{\mu_1^2}{4}+\frac{\mu_1^2}{4}+\frac{\mu_2^2}{4}+\frac{\mu_2^2}{4}+\frac{\mu_3^2}{4}+\frac{\mu_3^2}{4}$$
$$\frac{\mu_t^2}{2} = \frac{\mu_1^2}{4}+\frac{\mu_1^2}{4}+\frac{\mu_2^2}{4}+\frac{\mu_2^2}{4}+\frac{\mu_3^2}{4}+\frac{\mu_3^2}{4}$$
$$\frac{\mu_t^2}{2} = 2\,\frac{\mu_1^2}{4}+2\,\frac{\mu_2^2}{4}+2\,\frac{\mu_3^2}{4} \quad = \frac{\mu_1^2}{2}+\frac{\mu_2^2}{2}+\frac{\mu_3^2}{2}$$
$$\mu_t^2 = \mu_1^2+\mu_2^2+\mu_3^2$$

The overall modulation index is

$$\boxed{\boldsymbol{\mu_t = \sqrt{\mu_1^2+\mu_2^2+\mu_3^2}}}$$

AM Generation: Switching modulator

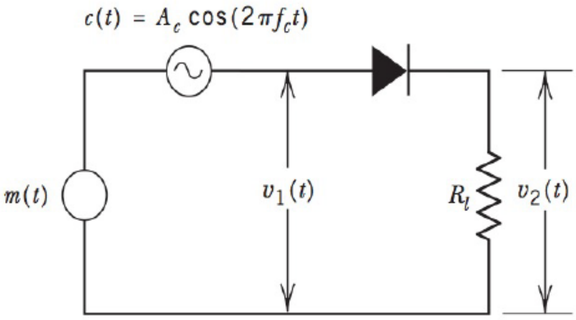


Figure 2.14 Switching Modulator

Assumptions:

1. The carrier wave c(t) applied to the diode is large in amplitude ie $A_C \gg |m(t)|$
2. A diode is an ideal switch:

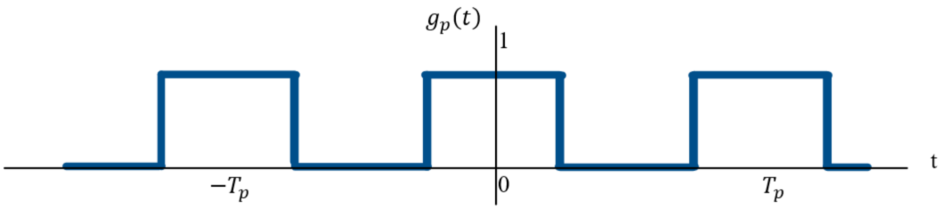


Figure 2.15 Periodic pulse train

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]$$

3. The transfer characteristic of the diode-load resistor combination is a *piecewise-linear characteristic*

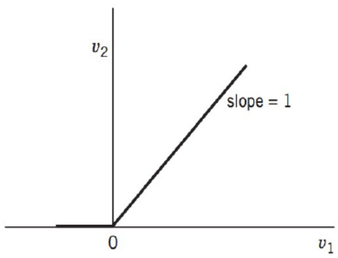


Figure 2.16 piecewise-linear characteristic of diode switch

Working:

Input to diode = $v_1(t) = m(t) + A_C\cos(2\pi f_c t)$

Output of diode = $v_2(t) = \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$

$$v_2(t) = v_1(t) g_p(t)$$

$$v_2(t) = [m(t) + A_C \cos(2\pi f_c t)] \left[\frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)] \right]$$

$$v_2(t) = [m(t) + A_C \cos(2\pi f_c t)] \left[\frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) + odd\ hormonics \right]$$

$$v_2(t) = \frac{1}{2} m(t) + \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{A_C \cos(2\pi f_c t)} + \frac{\mathbf{2}}{\boldsymbol{\pi}} \boldsymbol{m(t) \cos(2\pi f_c t)}$$
$$+ \frac{2}{\pi} A_C \cos^2(2\pi f_c t) + unwanted\ terms$$

$$v_2(t) = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{A_C \cos(2\pi f_c t)} + \frac{\mathbf{2}}{\boldsymbol{\pi}} \boldsymbol{m(t) \cos(2\pi f_c t)} + \frac{1}{2} m(t)$$
$$+ \frac{2}{2\pi} A_C [1 + \cos(4\pi f_c t)] + unwanted\ terms$$

<u>Fourier Transform to $v_2(t)$:</u> Let $M(f)$ be the spectrum of $m(t)$.

$$m(t) \overset{FT}{\longleftrightarrow} M(f)$$

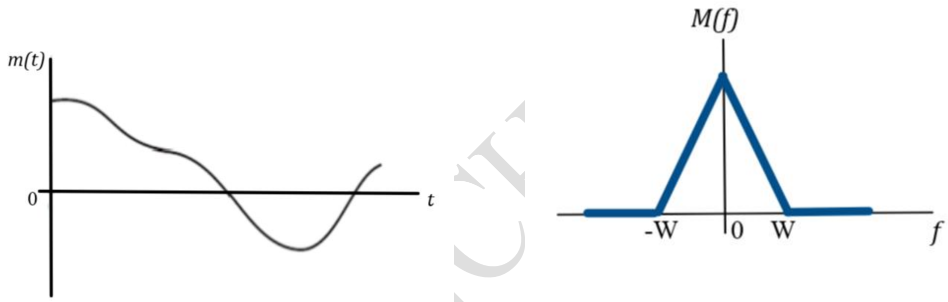


Figure 2.17 Message signal & its spectrum

Then,

$$V_2(f) = \frac{1}{2}\frac{A_C}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{2}{2\pi}\left[M(f - f_c) + M(f + f_c)\right]$$
$$+ \frac{1}{2} M(f) + \frac{A_C}{\pi}\delta(f) + unwanted\ terms$$

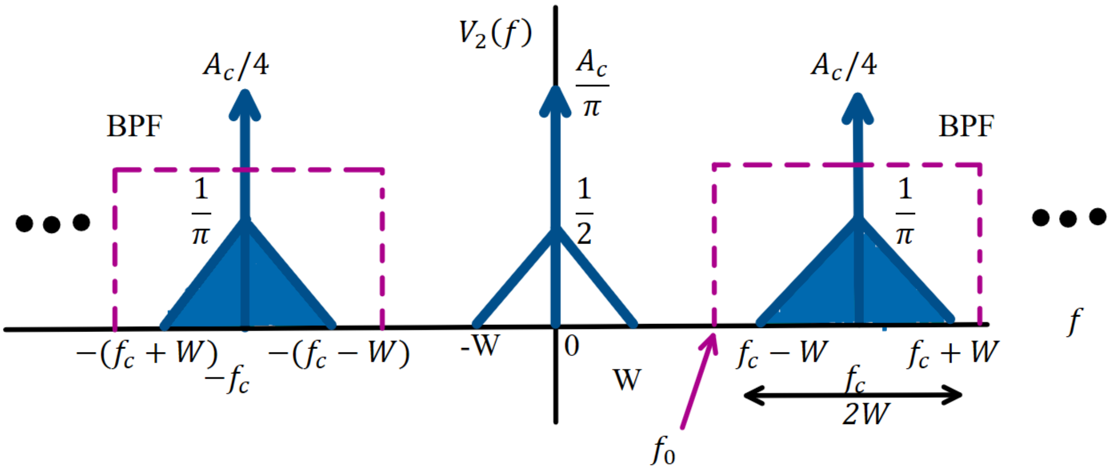


Figure 2.18 Spectrum of $v_2(t)$

The required AM signal can be extracted using a bandpass filter having the following specifications:

Mid-band frequency = f_c
Bandwidth = $2W$
Cut-off frequency (f_0): $W < f_0 < (f_c - W)$
Condition for distortion-less modulation: $(f_c - W) > W$ or $f_c > 2W$

The output of bandpass filter is an AM signal, given by

$$s(t) = \frac{1}{2} A_C \cos(2\pi f_c t) + \frac{2}{\pi} m(t) \cos(2\pi f_c t)$$

$$\boldsymbol{s(t) = \frac{1}{2} A_C \left[1 + \frac{4}{\pi A_c} m(t)\right] \cos(2\pi f_c t)}$$

Where $k_a = \frac{4}{\pi A_C}$

ENVELOPE DETECTOR

Envelope detector is an AM demodulator that is used to recover the original modulating wave from the incoming modulated wave.

Ideally, an envelope detector produces an output signal that follows the envelope of the input waveform exactly; hence the name.

The envelope detector is a simple and cost-effective device that is used in almost all commercial AM radio receivers.

Conditions:

1. the AM wave has to be narrow-band ie $f_c \gg W$
2. the percentage modulation must be less than 100 percent ie $\mu < 1$

Circuit diagram:

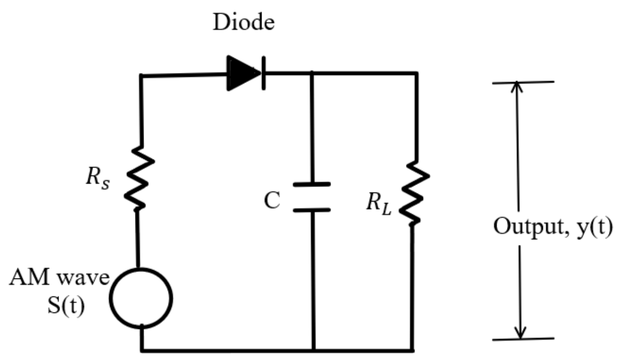


Figure 2.19 Envelope Detector

Working:

The envelope detector consists of a diode and an RC filter. The input to the envelope detector is an AM wave.

$$s(t) = A_C[1 + k_a\, m(t)]\cos(2\pi f_c t)$$

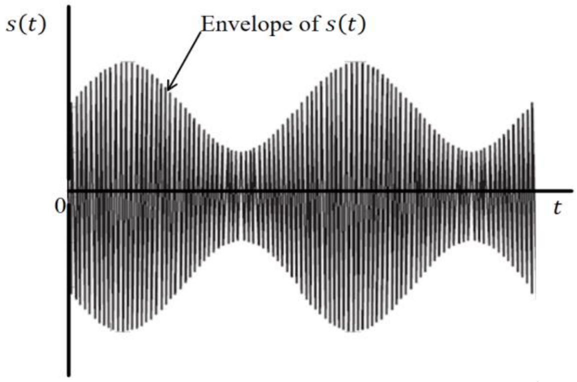


Figure 2.20 Single-tone AM signal

On the positive half-cycle of the input signal, the diode is forward biased and the capacitor charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reverse biased and the capacitor discharges slowly through the load resistor R_L. The discharging process continues until the next positive half-cycle.

When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

The output is the envelope of $s(t)$.

$$a(t) = A_C[1 + k_a\, m(t)] = A_C + A_C k_a\, m(t)$$

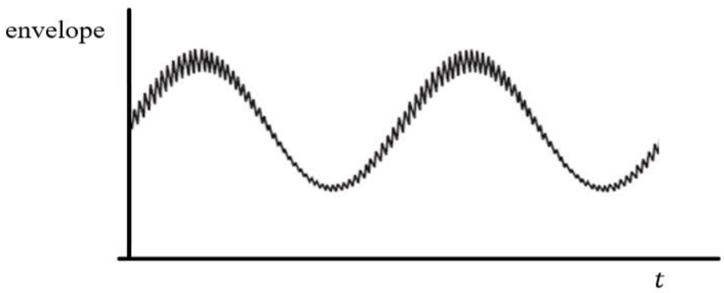


Figure 2.21 Envelope of an AM signal

A_C is the dc component in $a(t)$ which can be removed. Then, the output obtained is proportional to message signal ie

$$y(t) = A_C k_a\, m(t)$$

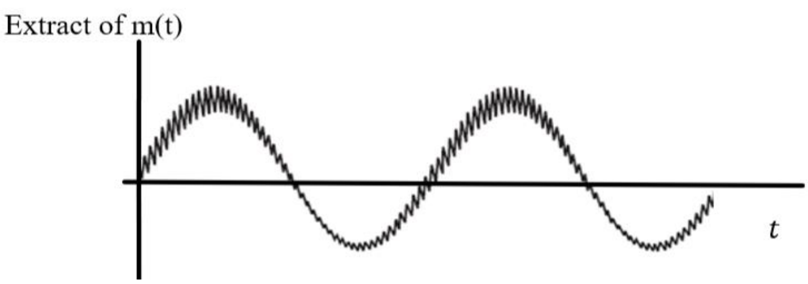


Figure 2.21 Extract of message signal using envelope detector

<u>Design:</u>

$R_s =$ Internal resistance of AM generator
C = capacitance of the charging capacitor
$r_f =$ forward resistance of diode

Charging time constant , R_s C must be short compared with the carrier period $\frac{1}{f_c}$, ie

$$\boxed{(\boldsymbol{r_f} + \boldsymbol{R_s})\mathbf{C} \ll \frac{\mathbf{1}}{\boldsymbol{f_c}}}$$

Discharging time constant , R_L C should satisfy the condition:

$$\boxed{\frac{\mathbf{1}}{\boldsymbol{f_c}} \ll \mathbf{R_L\,C} \ll \frac{\mathbf{1}}{\boldsymbol{W}}}$$

where ‘W’ is the message bandwidth.

LIMITATIONS OF AMPLITUDE MODULATION

1. *Amplitude modulation is wasteful of power*:
The carrier wave *c(t)* is completely independent of the information-bearing signal of baseband signal *m(t)***.** So, transmission of the unmodulated carrier wave in AM is a waste of power
2. *Amplitude modulation is wasteful of bandwidth*
The amplitude modulation is wasteful of bandwidth as it requires a transmission bandwidth equal to twice the message bandwidth**.**

MODIFICATIONS OF AMPLITUDE MODULATION

To overcome the limitations of standard AM, three modified forms of amplitude modulation techniques are proposed:

1. *Double sideband-suppressed carrier (DSB-SC) modulation*, in which the transmitted wave consists of only the upper and lower sidebands**.** Transmitted power is saved here through the suppression of the carrier wave, but the channel bandwidth requirement is the same as before (i.e., twice the message bandwidth)**.**
2. *Vestigial sideband (VSB) modulation*, in which one sideband is passed almost completely and just a trace, or *vestige*, of the other sideband is retained. The required channel bandwidth is therefore in excess of the message bandwidth by an amount equal to the width of the vestigial sideband.
3. *Single sideband (SSB) modulation***,** in which the modulated wave consists only of the upper sideband or the lower sideband.