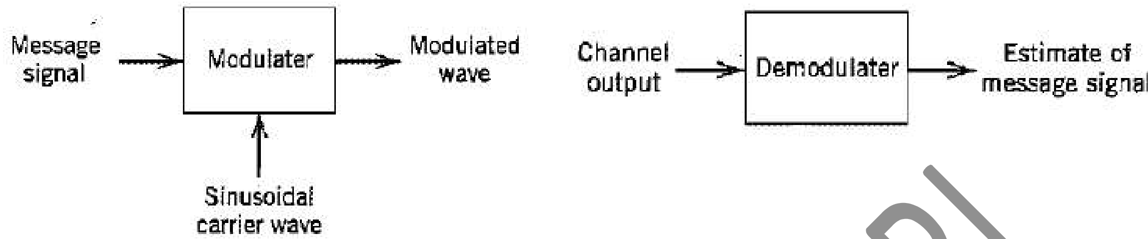
Module-1: Amplitude Modulation (AM) Techniques

Introduction

Modulation is a process of varying one of the characteristics (amplitude, frequency or phase) of high frequency sinusoidal (the carrier) signal in accordance with the instantaneous values of the modulating (the information) signal.



The high frequency carrier signal is mathematically represented by: $\boldsymbol{c(t) = A_c \cos(2\pi f_c t)}$

where c(t)--instantaneous values of the cosine wave

A_c --its maximum value (amplitude)

f_c --carrier frequency

Any of the three characteristics or parameters of the carrier can be varied by the modulating (message) signal, giving rise to amplitude, frequency or phase modulation respectively.

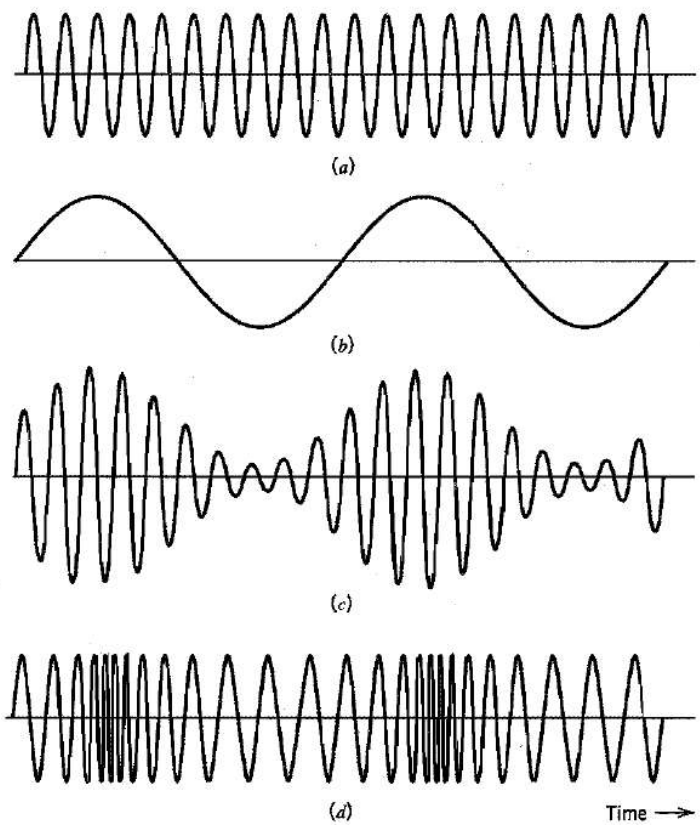


FIGURE Illustrating AM and FM signals produced by a single tone. (*a*) Carrier wave. (*b*) Sinusoidal modulating signal. (*c*) Amplitude-modulated signal. (*d*) Frequency-modulated signal.

Need for modulation:

1. Practicability of antenna

In the audio frequency range, for efficient radiation and reception, the transmitting and receiving antennas must have sizes comparable to the wavelength of the frequency of the signal used. It is calculated using the relation f λ = c . The wavelength is 75 meters at 1MHz in the broadcast band, but at 1 KHz, the wavelength turns out to be 300 Kilometers. A practical antenna for this value of wavelength is unimaginable and impossible.

2. Modulation for ease of radiation

For efficient radiation of electromagnetic waves, the antenna dimension required is of the order of 4 to 2. It is possible to construct practical antennas only by increasing the frequency of the base band signal.

3. Modulation for multiplexing

The process of combining several signals for simultaneous transmission on a single channel is called multiplexing. In order to use a channel to transmit the different base band signals (information) at the same time, it becomes necessary to translate different signals so as to make them occupy different frequency slots or bands so that they do not interfere. This is a accomplished by using carrier of different frequencies.

4. Narrow banding:

Suppose that we want to transmit audio signal ranging from 50 - 104 Hz using suitable antenna. The ratio of highest to lowest frequency is 200. Therefore an antenna suitable for use at one end of the frequency range would be entirely too short or too long for the other end. Suppose that the audio spectrum is translated so that it occupies the range from 50+106 to 104+106 Hz. Then the ratio of highest to lowest frequency becomes 1.01. Thus the process of frequency translation is useful to change wideband signals to narrow band signals. At lower frequencies, the effects of flicker noise and burst noise are severe.

Amplitude modulation (Standard AM)

Definition:

Amplitude Modulation is defined as a process in which amplitude of the carrier wave is varied linearly with the message signal. That is amplitude of the carrier is made proportional to the instantaneous values (amplitude) of the modulating signal.

Consider a sinusoidal carrier wave c(t) defined by $c(t) = A_c \cos(2\pi f_c t)$. Let us assume that phase of the carrier wave is zero because the carrier source always independent of message source.

Time-domain description:

If m(t) is the information signal and $c(t) = A_c \cos(2\pi f_c t)$ is the carrier, the amplitude of the carrier signal is varied proportional to the m(t).

The standard form of AM wave is defined by

$$\boldsymbol{S(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)}$$

where k_a is a constant, called the *'amplitude sensitivity'* of the modulator.

The envelope of the AM wave s(t) is given by

$$\boldsymbol{a(t) = A_c[1 + \mathrm{k_a m(t)}]}$$

$$S(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + A_c\, \mathrm{k_a m(t)} \cos(2\pi f_c t)$$

The first term of S(t) is the un-modulated carrier & the second term is the Double side band suppressed carrier (DSBSC) signal.

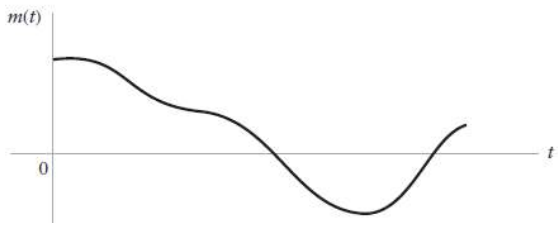


Fig: Modulating signal

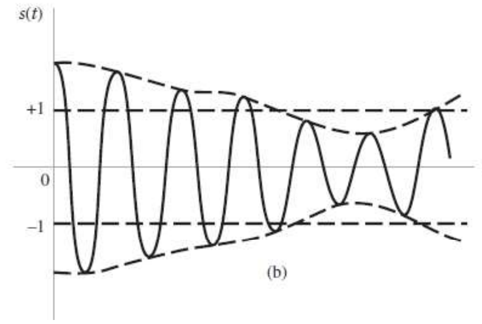


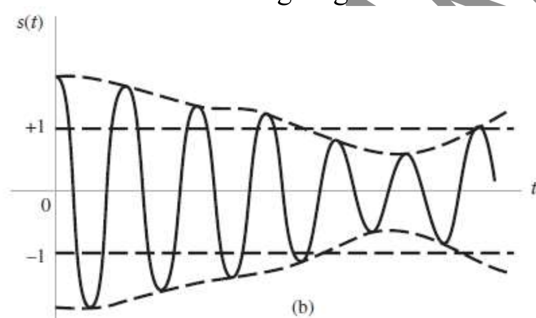
Fig. AM signal

Percentage modulation: It is defined by

% modulation = $|\mathrm{k_a m(t)}|_{max} \times 100\%$

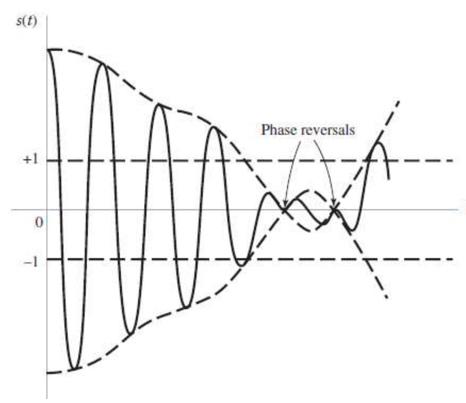
<u>Case (i) :</u> $|\mathbf{k_a m(t)}| \leq$ **1 for all 't'** (under modulation)

Under this condition, $[1 + \mathrm{k_a m(t)}]$ is always positive. The percentage of modulation is less than or equal to 100%. It is also known as 'under modulation'. In this case, AM wave bears one-to-one correspondence with that of message signal.



<u>Case (ii) :</u> $|\mathbf{k_a m(t)}| > 1$ **for some 't'** (over modulation)

Under this condition, $[1 + \mathrm{k_a m(t)}]$ is positive or negative. The percentage of modulation is excess of 100%. It is also known as 'over modulation'. In this case, modulated wave suffers from envelope distortion.



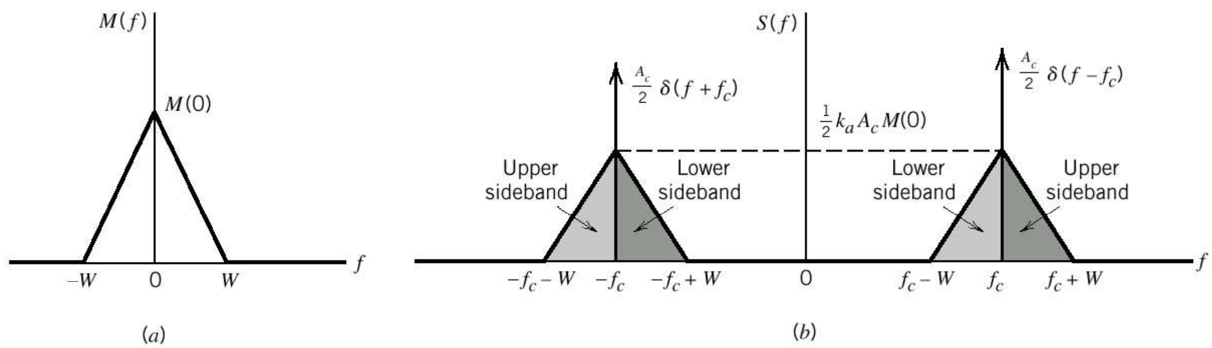
Frequency-domain description:

Consider a message signal m(t), band-limited to 'W' Hz. Let S(f) denote the Fourier transform of s(t) and M(f) denote the Fourier transform of m(t). The standard AM wave, s(t) as a function of time is given by

$$S(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)$$

$$S(t) = A_c \cos(2\pi f_c t) + A_c \, k_a m(t)\cos(2\pi f_c t)$$
$$\mathbf{S(f)} = \frac{A_c}{2} \;[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2} \;[M(f - f_c) + M(f + f_c)]$$

For $f_c > W$, the spectrum of AM is as follows:



The spectrum consists of:

1. Two delta functions weighted by the factor $\frac{A_c}{2}$
2. Lower side band(LSB): for positive frequencies, $(f_c - W) \le f \le f_c$
   for negative frequencies: $-f_c \le -f \le (-f_c + W)$
3. Upper side band(USB): for positive frequencies, $f_c \le f \le (f_c + W)$
   for negative frequencies: $(-f_c - W) \le -f \le -f_c$

The <u>transmission bandwidth</u> of AM wave is twice the message bandwidth.

$$B = (f_c + W) - (f_c - W)$$

B = 2W = Twice the bandwidth of message signal, Hz

<u>Single tone AM wave</u> <u>(modulation by a sinusoidal signal)</u>:

Consider a modulating wave m(t) that consists of a single tone or frequency component ie

$$m(t) = A_m \cos(2\pi f_m t)$$

The sinusoidal carrier wave is given by $c(t) = A_c \cos(2\pi f_c t)$. The standard form of AM wave is defined by $S(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$

$$S(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)]\cos(2\pi f_c t)$$
$$\boldsymbol{S(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)}$$

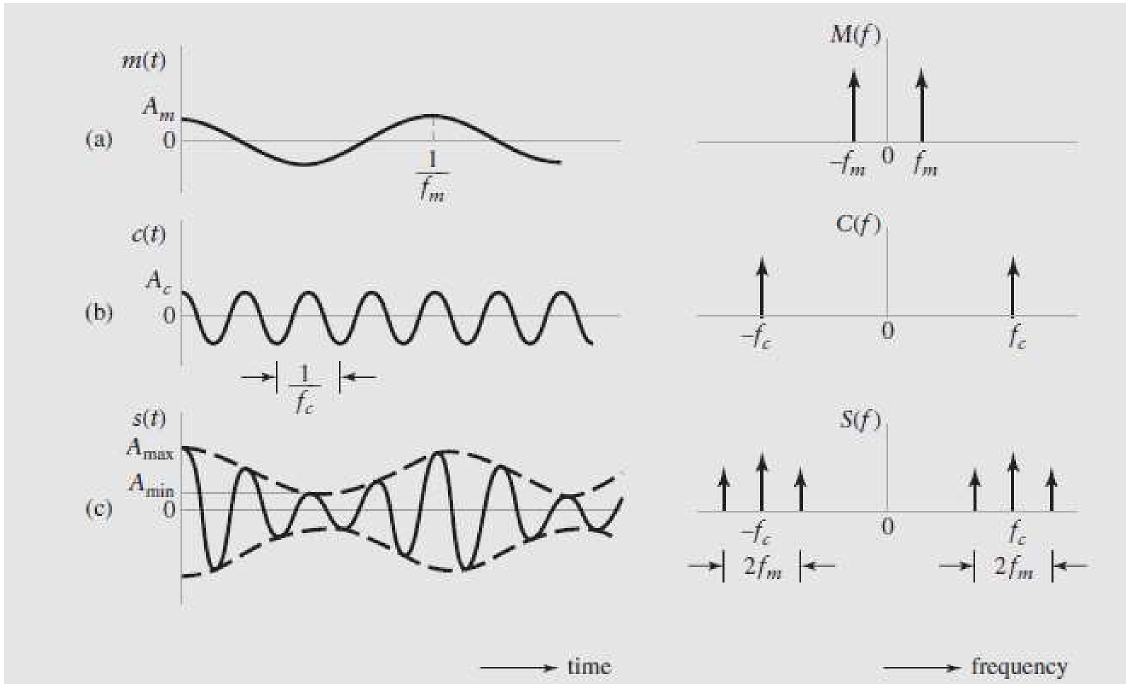
where μ is the modulation factor and it is a dimensionless quantity

$$S(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$
$$= A_c\cos(2\pi f_c t) + A_c\mu\cos(2\pi f_m t)\cos(2\pi f_c t)$$
$$\boldsymbol{S(t) = A_c\cos(2\pi f_c t) + \frac{A_c\mu}{2}\cos[2\pi(f_c + f_m)t] + \frac{A_c\mu}{2}\cos[2\pi(f_c - f_m)t]}$$

Let S(f) be the Fourier transform of s(t). Then

$$\mathbf{S(f)} = \frac{A_c}{2}\;[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c\mu}{4}\;\{\delta[f - (f_c + f_m)] + \delta[f + (f_c + f_m)]\}$$
$$+ \frac{A_c\mu}{4}\;\{\delta[f - (f_c - f_m)] + \delta[f + (f_c - f_m)]\}$$

- Carrier component: $A_c\cos(2\pi f_c t)$
- Upper sideband: $\frac{A_c\mu}{2}\cos[2\pi(f_c + f_m)t]$
- Lower sideband: $\frac{A_c\mu}{2}\cos[2\pi(f_c - f_m)t]$



Depth of modulation (modulation index or modulation factor), μ

The modulation factor is defined by $\mu = |k_a m(t)|_{max}$

Percentage of modulation = $|k_a m(t)|_{max} \times 100\%$

For single tone modulation, $m(t) = A_m \cos(2\pi f_m t)$

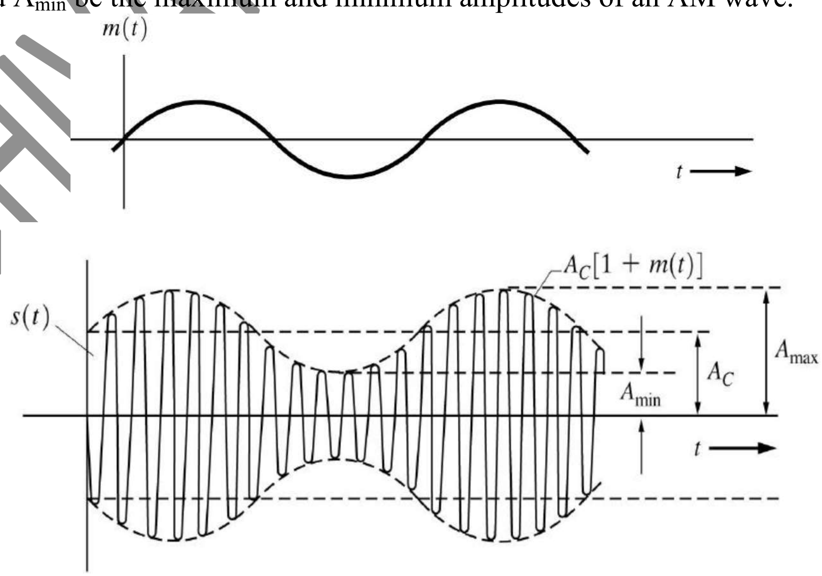
$\mu = |k_a A_m \cos(2\pi f_m t)|_{max}$ $\boldsymbol{\mu = k_a A_m}$

For single tone modulation, modulation index is also defined as the ratio of amplitude of modulating signal to amplitude of sinusoidal carrier.

$$\mu = \frac{A_m}{A_c}$$

Consider $S(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$

Let A_{max} and A_{min} be the maximum and minimum amplitudes of an AM wave.



A_{max} = maximum of s(t) = $|A_c[1 + k_a m(t)|_{max} = A_c[1 + \mu]$

A_{min} = minimum of s(t) = $|A_c[1 + k_a m(t)|_{min} = A_c[1-\ \mu]$

$$\frac{A_{max}}{A_{min}} = \frac{A_c[1+\mu]}{A_c[1-\mu]}$$

$$\boldsymbol{\mu = \frac{A_{max}-A_{min}}{A_{max}+A_{min}}}$$

Trapezoidal display of AM wave:

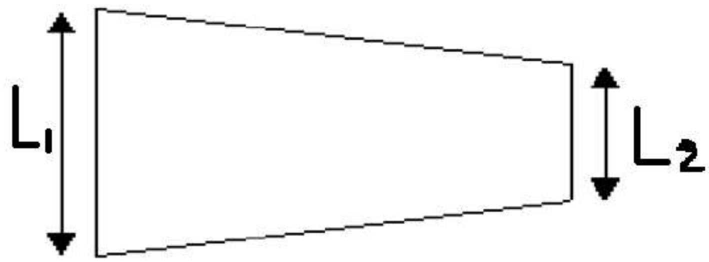


figure - trapezoid presentation of AM signal

$L_1 = 2\ (A_c + A_m)$ and $L_2 = 2\ (A_c - A_m)$

$$A_m = \frac{L_1-L_2}{4} \quad \text{and} \quad A_c = \frac{L_1+L_2}{4}$$

Therefore, $\boldsymbol{\mu = \frac{A_m}{A_c} = \frac{L_1-L_2}{L_1+L_2}}$

Power content of AM wave:

An AM wave contains 3 components:

- Carrier component: $A_c\cos(2\pi f_c t)$
- Upper sideband: $\frac{A_c\mu}{2}\cos[2\pi(f_c + f_m)t]$
- Lower sideband: $\frac{A_c\mu}{2}\cos[2\pi(f_c - f_m)t]$

The total average power of AM wave is

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$P_c = \frac{\left(A_c/\sqrt{2}\right)^2}{R} = \frac{A_c^2}{2R}; \quad P_{USB} = \frac{\left(\mu A_c/2\sqrt{2}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}; \quad P_{LSB} = \frac{\left(\mu A_c/2\sqrt{2}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} = \frac{A_c^2}{2R}\left(1 + \frac{\mu^2}{2}\right)$$

$$\mathbf{P_t = P_c\left(1 + \frac{\mu^2}{2}\right)}$$

NOTE:

- Carrier power = $\frac{A_c^2}{2R}$ = P_c
- USB power = LSB power = $\frac{\mu^2 A_c^2}{8R} = \frac{\mu^2 P_c}{4}$
- Side band power = $\frac{\mu^2 A_c^2}{4R} = \frac{\mu^2 P_c}{2}$

Efficiency($\boldsymbol{\eta}$): The percentage efficiency is as the ratio of sideband power to total power of AM wave ie

$$\% \, \eta = \frac{P_{SB}}{P_t} \times 100$$

$$\eta = \frac{P_U + P_L}{P_t} = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{\text{Pc}\left(1+\frac{\mu^2}{2}\right)} = \frac{\frac{A_c^2}{2R}\left(\frac{\mu^2}{4}+\frac{\mu^2}{4}\right)}{\text{Pc}\left(1+\frac{\mu^2}{2}\right)} = \frac{\frac{\mu^2}{2}}{1+\frac{\mu^2}{2}}$$

$$\eta = \frac{\mu^2}{2+\mu^2}$$

$$\boldsymbol{\% \, \eta = \frac{\mu^2}{2+\mu^2} \times 100}$$

For $\mu = 1$, $\eta = \eta_{max} = 33.33\ \%$

Modulation by many sinusoidal message signals:(Multi-tone modulation)

In multi-tone modulation modulating signal consists of more than one frequency component where as in single-tone modulation modulating signal consists of only one frequency component.

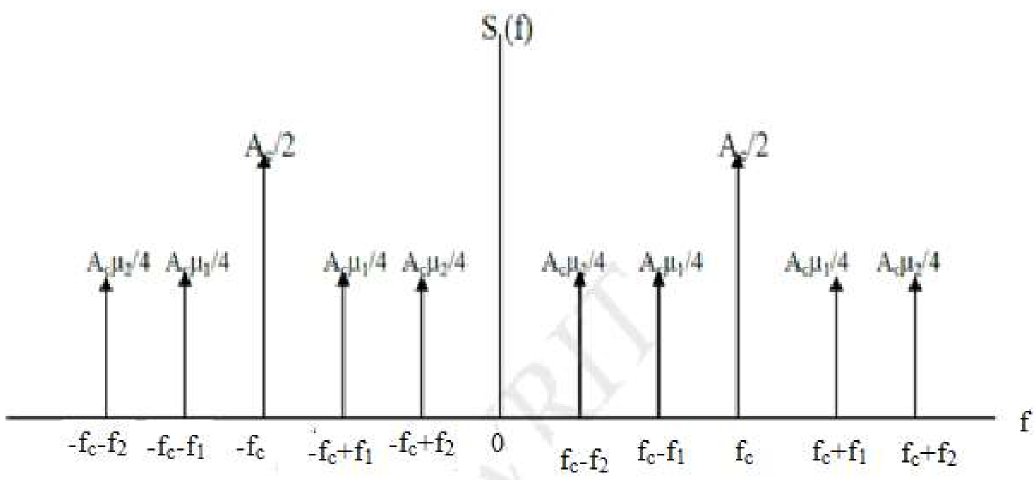
Consider three modulating signals $A_1 \cos(2\pi f_1 t)$, $A_2 \cos(2\pi f_2 t)$ and $A_3 \cos(2\pi f_3 t)$ with modulation indices μ_1, μ_2 and μ_3 respectively. Then resultant AM wave is given by

$$\boldsymbol{S(t) = A_c[1 + \mu_1 \cos(2\pi f_1 t) + \mu_2 \cos(2\pi f_2 t) + \mu_3 \cos(2\pi f_3 t)]\cos(2\pi f_c t)}$$

where $\mu_1 = k_a A_1$, $\mu_2 = k_a A_2$ and $\mu_3 = k_a A_3$

$$\boldsymbol{S(t) = A_c \cos(2\pi f_c t)}$$
$$\boldsymbol{+ \left(A_c\mu_1/2\right)\cos(2\pi(f_c - f_1)t) + \left(A_c\mu_1/2\right)\cos(2\pi(f_c + f_1)t)}$$
$$\boldsymbol{+ \left(A_c\mu_2/2\right)\cos(2\pi(f_c - f_2)t) + \left(A_c\mu_2/2\right)\cos(2\pi(f_c + f_2)t)}$$
$$\boldsymbol{+ \left(A_c\mu_3/2\right)\cos(2\pi(f_c - f_3)t) + \left(A_c\mu_3/2\right)\cos(2\pi(f_c + f_3)t)}$$

The overall modulation index is $\boldsymbol{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}}$



Bandwidth = 2 W = 2 f_2.

Advantages of Amplitude modulation:-

- Generation and detection of AM signals are very easy

- It is very cheap to build, due to this reason it I most commonly used in AM radio broad casting

Disadvantages of Amplitude of modulation:-

- Amplitude modulation is wasteful of power
- Amplitude modulation is wasteful of band width

Application of Amplitude modulation: -

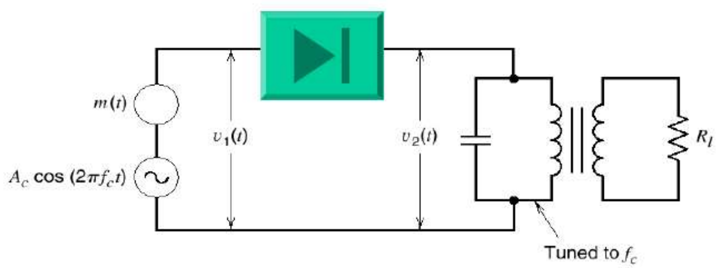
- AM Radio Broadcasting

AM Generation:

1. Square law modulator
2. Switching modulator

These two techniques use non-linear device for their implementation. These are well suited for low power modulation purposes.

Switching modulator

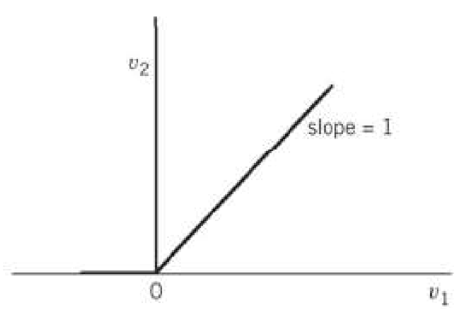


- A diode can be used as switch.
- The carrier wave C(t) applied to the diode is large in amplitude.
- The diode acts as ideal switch.

The input voltage to diode is given by $v_1(t) = m(t) + A_C \cos(2\pi f_c t)$
where $|m(t)| \ll A_C$.

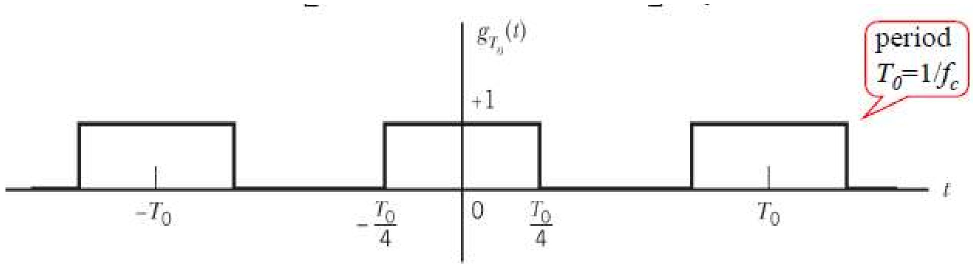
$$v_2(t) = \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

The load voltage, $v_2(t)$ varies periodically between $v_1(t)$ and zero at a rate equal to frequency, f_c. Assuming $|m(t)| \ll A_C$, we may effectively replace the diode behaviour by an approximately equivalent linear time-varying operation.



$$v_2(t) = [m(t) + A_C \cos(2\pi f_c t)]\, g_p(t)$$

Where $g_p(t)$ is the periodic pulse train with duty cycle one-half and period $T_c = 1/f_c$ and which is given by



Representing $g_p(t)$ by its Fourier series, we have

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \text{odd harmonic components}$$

Therefore, $v_2(t) = \frac{A_C}{2}\left[1 + \frac{4}{\pi A_c}m(t)\right]\cos(2\pi f_c t) + \text{unwanted terms}$

Now design the tuned filter /Band pass filter with center frequency f_c and pass band frequency width 2W. We can remove the unwanted terms by passing this output voltage $V_0(t)$ through the band pass filter and finally we will get required AM signal.

Assume the message signal m(t) is band limited to the interval $-W \le f \le W$

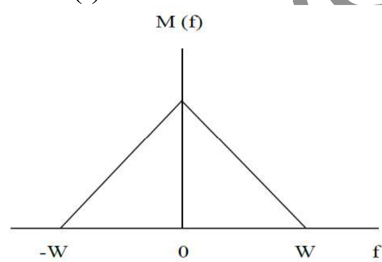
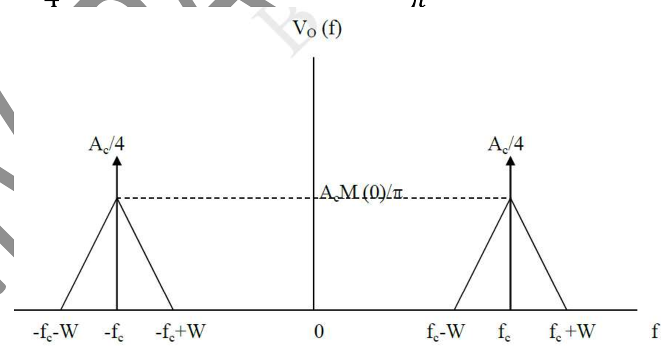


Fig: Spectrum of message signal

The Fourier transform of output voltage $V_O(t)$ is given by

$$V_o(f) = \frac{A_c}{4}[\delta(f-f_c)+\delta(f+f_c)] + \frac{A_c}{\pi}[M(f-f_c)+M(f+f_c)]$$



AM detection (demodulation):

- Detection is the inverse of modulation.
- It is a process of recovering the message signal from the incoming AM wave at the receiver.
- The complexity of the detector is greatly simplified if the transmitter produces an envelope a(t) that has the same shape of message signal, m(t).

For a distortion-less demodulation, we must satisfy two conditions:

1. The percentage modulation is less than 100%, so as to avoid envelope distortion.

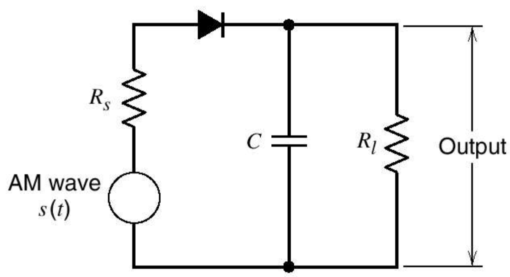
2. The message bandwidth, W is small compared to the carrier frequency f_c, so that envelope a(t) may be visualized satisfactorily. Here it is assumed that the message signal is band-limited to W ie $-W \leq f \leq W$.

Envelope detection

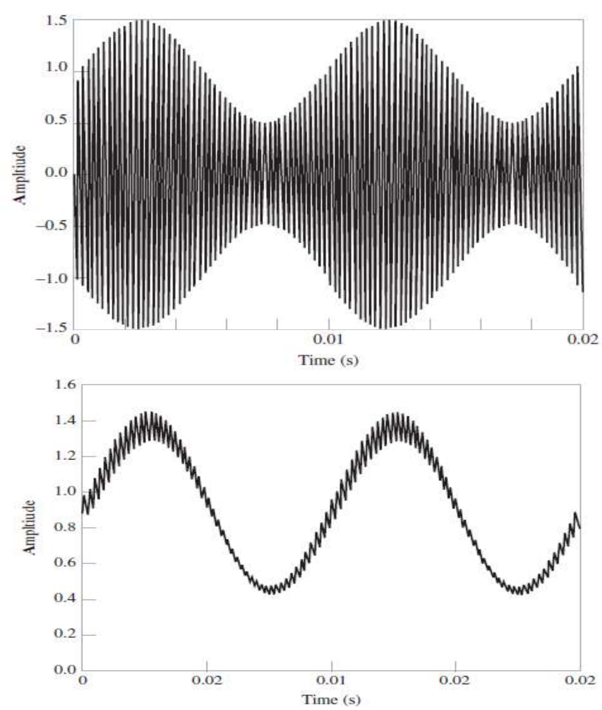
- It is simple AM detector.
- It is well suited for demodulation of narrowband [$f_c \gg W$] AM signal.
- Percentage modulation should be less than 100%.
- It is used in almost all commercial AM radio receivers.

Ideally, an envelope detector produces an output signal that follows the envelope of the input waveform exactly; hence the name.

Circuit diagram:



Waveform:



<u>Working</u>: The envelope detector consists of a diode and an RC filter.

On the positive half-cycle of the input signal, the diode is farward biased and the capacitor charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reverse biased and the capacitor discharges slowly through the load resistor R_L. The discharging process continues until the next positive half-cycle.

When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

Assumptions:

1. Diode is ideal
2. Internal resistance of AM generator is R_s.

Charging time constant , R_s C mus be short compared with the carrier period $\frac{1}{f_c}$, ie

$$\mathbf{R_s\, C} \ll \frac{1}{f_c}$$

Discharging time constant , R_L C should satisfy the condition:

$$\frac{1}{f_c} \ll \mathbf{R_L\, C} \ll \frac{1}{W}$$ where 'W' is the message bandwidth.

The detector voltage (capacitor voltage) is very nearly same as the envelope of the AM wave.
The small ripples in the output at carrier frequency is removed by low pass filtering.
The standard form of AM wave is defined by

$$S(t) = A_c[1 + \mathrm{k_a m(t)}]\cos(2\pi f_c t)$$

The envelope of a signal is given by

$$a(t) = \sqrt{(Inphase\ component)^2 + (quadrature\ component)^2}$$
$$= \sqrt{(A_c[1 + \mathrm{k_a m(t)}])^2 + (0)^2} = A_c[1 + \mathrm{k_a m(t)}]$$

The envelope of the AM wave s(t) is given by $a(t) = A_c + A_c \mathrm{k_a m(t)}]$

Advantages:

- It is very simple to design
- It is inexpensive
- Efficiency is very high when compared to Square Law detector

Disadvantage:

- Due to large time constant, some distortion occurs which is known as diagonal clipping i.e., selection of time constant is somewhat difficult

Application:

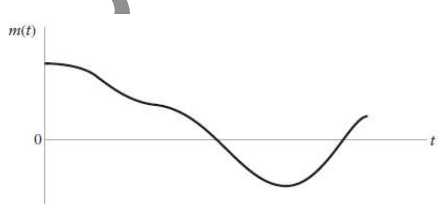
- It is most commonly used in almost all commercial AM Radio receivers.

Double Side Band Suppressed Carrier (DSBSC) Modulation

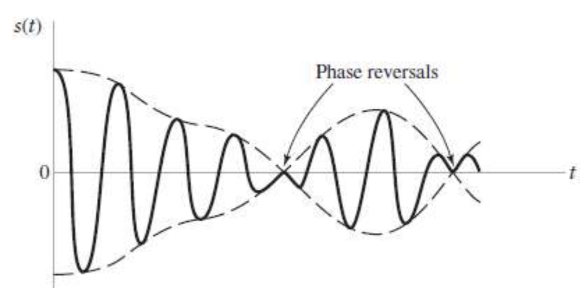
- In standard form of AM wave, transmission of unmodulated carrier represents waste of power. Maximum of 33.33% of total power is utilized to represent the information. This is the main drawback of standard AM.
- The DSBSC modulation results when the carrier component is suppressed in standard AM wave.

<u>Time-domain description:</u> Let m(t) be a message signal, band-limited to 'W' Hz. Then a DSBSC wave is obtained by multiplying the carrier with the message signal.

$S_{DSBSC}\,(t) = \mathrm{m(t)\ c(t)};$ $\boldsymbol{S_{DSBSC}\,(t) = A_c\,\mathrm{m(t)}\cos(2\pi f_c t)}$

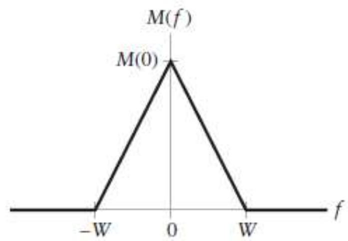


(a)

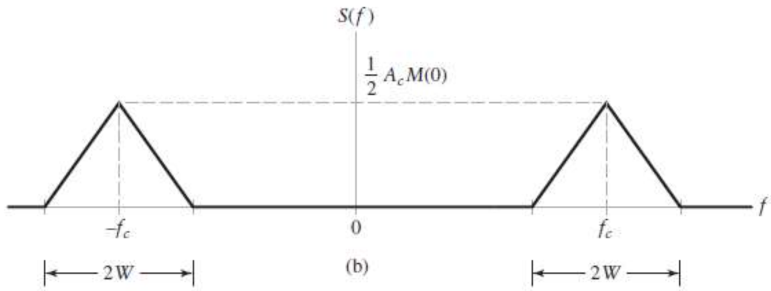


- This modulated wave undergoes a phase reversal whenever the m(t) crosses zero.
- Unlike standard AM, the envelope of DSBSC wave is different from the message signal.

Frequency-domain description: If M(f) is the message spectrum, defined in the range $-W \leq f \leq W$, then the Fourier transform of s(t) gives the spectrum.



$$S(f) = \frac{A_c}{2} \quad [M(f - f_c) + M(f + f_c)]$$



- Modulation process translates baseband (message) spectrum by $\pm f_c$.
- The transmission bandwidth of DSBSC wave is same as that of standard AM wave
  ie Bandwidth = 2 W, Hz
- The DSBSC wave contains two side bands only.

Single-tone modulation of DSBSC wave:-

In single-tone modulation modulating signal consists of only one frequency component where as in multi-tone modulation modulating signal consists of more than one frequency components.

The standard time domain equation for the DSB-SC modulation is given by

$$S(t) = A_c \, m(t)\cos(2\pi f_c t)$$

Assume $m(t) = A_m \cos(2\pi f_m t)$

$$S(t) = A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t)$$

$$S(t) = \frac{A_c A_m}{2}[\cos[2\pi(f_c - f_m)t] + \cos[2\pi(f_c + f_m)t]] \ldots\ldots\ldots\ldots\ldots (3)$$

The Fourier transform of s (t) is

$$S(f) = \frac{A_c A_m}{4}\{\delta[f - (f_c + f_m)] + \delta[f + (f_c + f_m)]\}$$
$$+ \frac{A_c A_m}{4}\{\delta[f - (f_c - f_m)] + \delta[f + (f_c - f_m)]\}$$

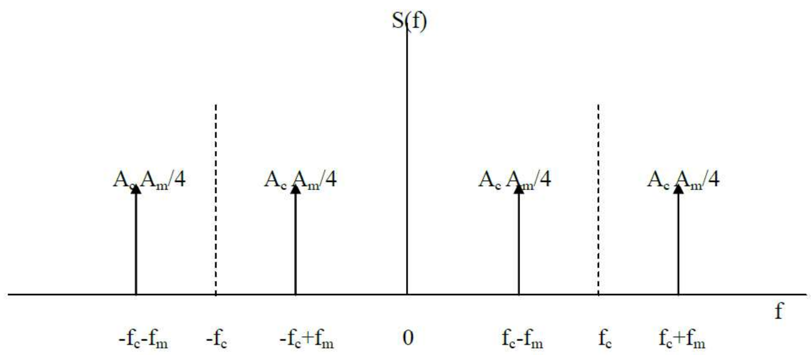


Fig. Spectrum of Single tone DSBSC wave

Power calculations of DSB-SC waves:-

Total power $P_t = P_{LSB}+P_{USB}$

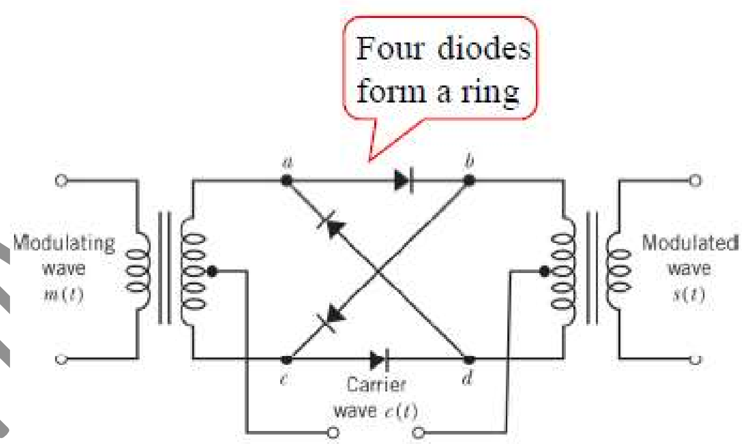
Total power $P_t = A_c^2 A_m^2 /8 + A A_c^2 A_m^2 /8$

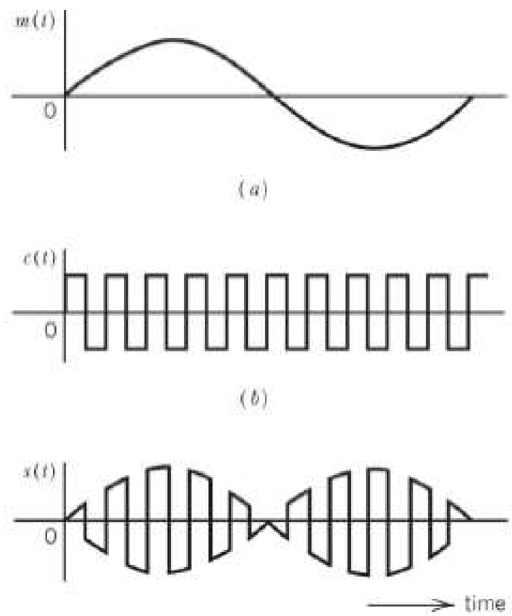
Total power $P_t = A_c^2 A_m^2 /4$

Generation of DSBSC waves

1. Balanced Modulator
2. Ring Modulator

Ring Modulator





- One of the most useful product modulator, well suited for generating a DSBSC wave, is the ring modulator shown in above figure. The four diodes form ring in which they all point in the same way-hence the name.
- The diodes are controlled by a square-wave carrier c(t) of frequency f_c, which applied longitudinally by means of to center-tapped transformers. If the transformers are perfectly balanced and the diodes are identical, there is no leakage of the modulation frequency into the modulator output.

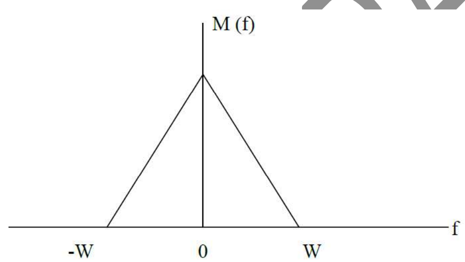


Fig. Spectrum of message signal

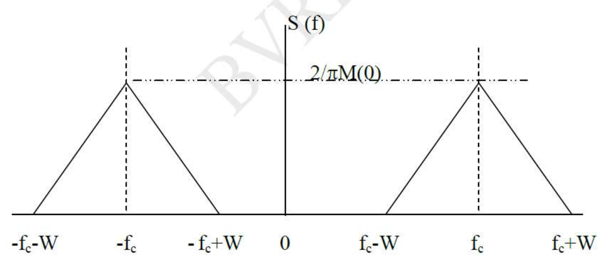


Fig. Spectrum of DSBSC wave

On one half-cycle of the carrier, the outer diodes are switched to their forward resistance r_f and the inner diodes are switched to their backward resistance r_b .

On other half-cycle of the carrier wave, the diodes operate in the opposite condition. The square wave carrier c (t) can be represented by a Fourier series as follows:

$$C(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))$$

$$= \frac{4}{\pi}\cos(2\pi f_c t) + \text{odd harmonic components}$$

When the carrier supply is positive, the outer diodes are switched ON and the inner diodes are switched OFF, so that the modulator multiplies the message signal by +1

When the carrier supply is positive, the outer diodes are switched ON and the inner diodes are switched OFF, so that the modulator multiplies the message signal by +1.when the carrier supply is negative, the outer diodes are switched OFF and the inner diodes are switched ON, so that the modulator multiplies the message signal by -1.

Now, the Ring modulator output is the product of both message signal m (t) and carrier signal c (t).

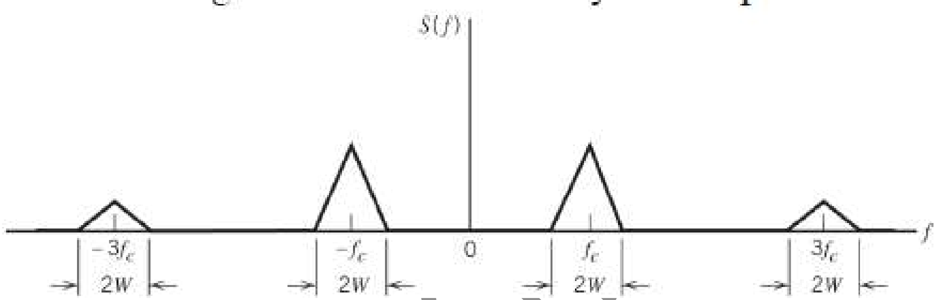
S (t) =c (t) m (t)

$$S(t) = m(t)\left[\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\big(2\pi f_c t(2n-1)\big)\right]$$

$$S(t) = \frac{4}{\pi}\cos(2\pi f_c t)\ m(t) - \frac{4}{3\pi}\cos(6\pi f_c t)\ m(t) + \ldots\ldots\ldots$$

The Fourier transform of s (t) is

$$S(f) = \frac{2}{\pi}\left[M(f-f_c) + M(f+f_c)\right] - \frac{2}{3\pi}\left[M(f-3f_c) + M(f+3f_c)\right] + \ldots\ldots$$



When the output is filtered appropriately with a bandpass filter,

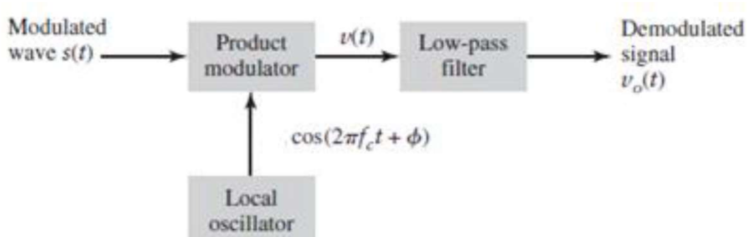
$$S(f) = \frac{2}{\pi}\left[M(f-f_c) + M(f+f_c)\right]$$

$$\boldsymbol{S(t) = \frac{4}{\pi}\cos(2\pi f_c t)\ m(t)}$$

The ring modulator is sometimes referred to as *a double-balanced modulator*, because it is balanced with respect to both the message signal and the square wave carrier signal.

Coherent Detection of DSBSC modulated waves

The message signal *m(t)* can be uniquely recovered from a DSBSC wave *s(t)* by first multiplying *s(t)* with a locally generated sinusoidal wave and then low pass filtering the product as shown.



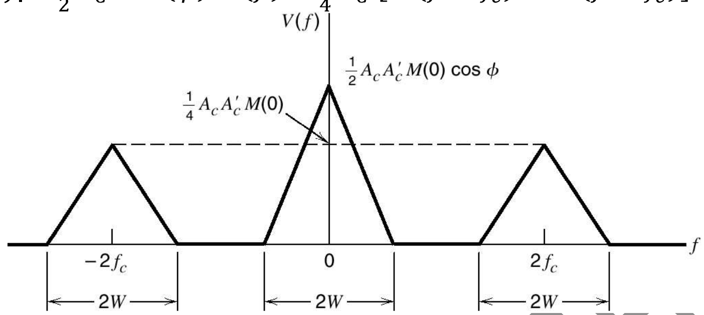
It is assumed that the local oscillator signal is exactly coherent or synchronized, in both frequency and phase, with the carrier wave $c(t)$ used in the product modulator to generate $s(t)$. This method of demodulation is known as coherent detection or synchronous detection.

Consider a DSBSC wave, described by, s(t) = $A_C\, m(t) \cos(2\pi f_c t)$
Assuming local oscillator signal to be $\cos(2\pi f_c t + \varphi)$, product modulator output is given by

$$v(t) = s(t)\cos(2\pi f_c t + \varphi)$$
$$= A_C\, m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \varphi)$$
$$= \frac{1}{2} A_C\, m(t)\cos(\varphi) + \frac{1}{2} A_C\, m(t)\cos(4\pi f_c t + \varphi)$$

The amplitude spectrum of $v(t)$ is given by

$$|V(f)| = \frac{1}{2} A_C \cos(\varphi)\, M(f) + \frac{1}{4} A_C\,[M(f - f_c) + M(f + f_c)]$$



The LPF removes the unwanted term in the product modulator output.
The specifications of LPF are:

Mid-band frequency = f_c
Bandwidth = 2W

Condition for no-distortion = ($2f_c$ −W) > W or $\boldsymbol{f_c > W}$

The overall output $v_o(t)$ is

$$v_o(t) = \frac{1}{2} A_C\, m(t)\cos(\varphi)$$

Therefore, $v_o(t) \propto m(t)$ when phase error φ is constant.

Case (i) : when $\varphi = 0$, $v_o(t) = \frac{1}{2} A_C\, m(t)$

Case (ii) : when $\varphi = \pm\frac{\pi}{2}$, $v_o(t) = 0$

The zero demodulated signal, which occurs for $\varphi = \pm\frac{\pi}{2}$ represents '***Quadrature null effect'*** of the coherent detector. Thus the phase error φ in the local oscillator causes the detector output to be attenuated by a factor equal to $\cos(\varphi)$. As long as phase error φ is constant, the detector output provides an undistorted version of $m(t)$.

In practice, the phase error φ varies randomly with time. Therefore, a circuitry must be provided in the receiver to maintain the local oscillator in perfect synchronism, in both frequency and phase, with the carrier signal used at the transmitter. This results in receiver complexity.

Frequency error: This error occurs when the frequency, f_1 of the local oscillator at the receiver is different from the carrier frequency f_c used at the transmitter. Usually, f_1 differes from f_c by a smaller value ie $f_1 = f_c + \Delta f$

$$v(t) = s(t)\cos(2\pi f_1 t)$$
$$= A_C\, m(t)\cos(2\pi f_c t)\cos(2\pi f_1 t)$$
$$= \frac{1}{2} A_C\, m(t)\cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_C\, m(t)\cos[2\pi(f_c - f_m)t]$$

The amplitude spectrum of $v(t)$ is given by

$$|V(f)| = \frac{1}{2} A_C \cos(\varphi)\, M(f) + \frac{1}{4} A_C\,[M(f - f_c) + M(f + f_c)]$$

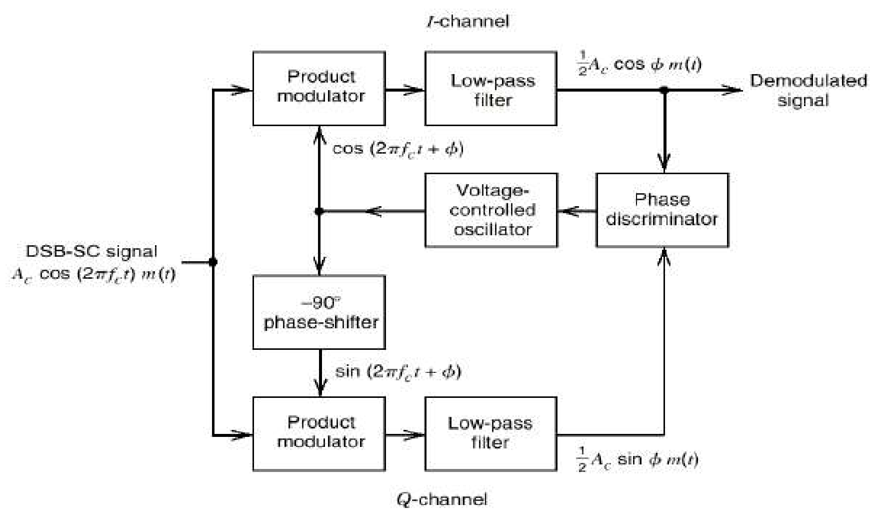
Costas Receiver (Costas loop)

Costas receiver is a synchronous receiver system, suitable for demodulating DSBSC waves. It consists of two coherent detectors supplied with the same input signal, that is the incoming DSBSC wave

$$s(t) = A_C\, m(t) \cos(2\pi f_c t)$$

but with individual local oscillator signals that are in phase quadrature with respect to each other as shown below.

- The frequency of the local oscillator is adjusted to be the same as the carrier frequency f_c.
- The detector in the upper path is referred to as the in-phase coherent detector or I - channel, and that in the lower path is referred to as the quadrature-phase coherent detector or Q-channel. These two detector are coupled together to form a negative feed back system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.
- Suppose the local oscillator signal is of the same phase as the carrier wave, $C(t) = A_C\ \cos(2\pi f_c t)$ used to generate the incoming DSBSC wave. Then we find that the I-channel output contains the desired demodulated signal *m(t)*, where as the Q-channel output is zero due to quadrature null effect of the Q-channel.



- Suppose that the local oscillator phase drifts from its proper value by a small anglef radiations. The I-channel output will remain essentially unchanged, but there will be some signal appearing at the Q-channel output, which is proportional to $\sin(\emptyset) = \emptyset$ for small $\emptyset$. This Q-channel output will have same polarity as the I-channel output for one direction of local oscillator phase drift and opposite polarity for the opposite direction of local oscillator phase drift.
- Thus by combining the I-channel and Q-channel outputs in a phase discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained that automatically corrects for the local phase errors in the voltage-controlled oscillator.

Quadrature-Carrier Multiplexing

The quadrature null effect of the coherent detector may also be put to good use in the construction of the so-called *quadrature-carrier multiplexing* or *quadrature-amplitude modulation* (QAM).

This scheme enables two DSB-SC modulated waves (resulting from the application of two physically *independent* message signals) to occupy the same channel bandwidth. Yet it allows for the separation of the two message signals at the receiver output.

Quadrature-carrier multiplexer is therefore a *bandwidth-conservation system*.

A block diagram of this system is shown below.

Transmitter: The transmitter part of the system involves the use of two separate product modulators that are supplied with two carrier waves of the same frequency but differing in phase by -90 degrees. The transmitted signal consists of the sum of these two product modulator outputs, as shown by

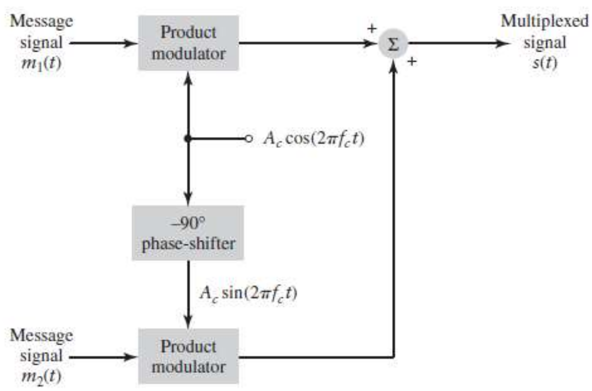
$$S(t) = A_c\, m_1(t) \cos(2\pi f_c t) + A_c\, m_2(t) \cos(2\pi f_c t)$$

where $m_1(t)$ and $m_2(t)$ denote the two different message signals applied to the product modulators.

The multiplexed signal occupies a channel bandwidth of $2W$ centered on f_c where W is the message bandwidth, assumed to be common to both messages.

$A_c\ m_1(t)$ ------- the in-phase component

$-A_c\ m_2(t)$ ------- quadrature component.



The receiver :

Specifically, the multiplexed signal is applied simultaneously to two separate coherent detectors that are supplied with two local carriers of the same frequency, but differing in phase by -90 degrees.

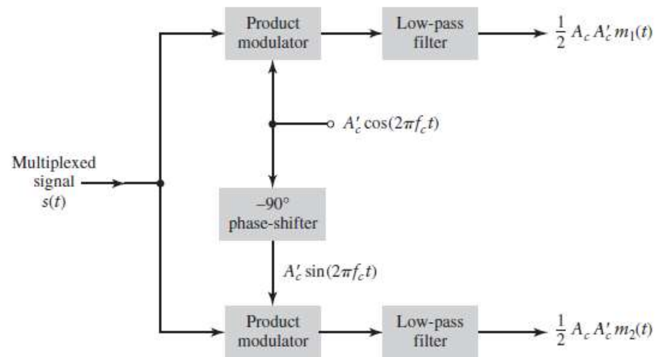
The output of the top detector is $\frac{1}{2} A_c A_c' m_1(t)$

the output of the bottom detector is $\frac{1}{2} A_c A_c' m_2(t)$

For the system to operate satisfactorily, it is important to maintain the correct phase and frequency relationships between the oscillator used to generate the carriers in the transmitter and the corresponding local oscillator used in the receiver.

To maintain this synchronization, we may use a Costas receiver.

Another commonly used method is to send a *pilot signal* outside the passband of the modulated signal.



Hilbert transform

- The Fourier transform is useful for evaluating the frequency content of an energy signal, or in a limiting case that of a power signal. It provides mathematical basis for analyzing and designing the frequency selective filters for the separation of signals on the basis of their frequency content.
- Another method of separating the signals is based on phase selectivity, which uses phase shifts between the appropriate signals (components) to achieve the desired separation.
- In case of a sinusoidal signal, the simplest phase shift of 180^o is obtained by "Ideal transformer" (polarity reversal). When the phase angles of all the components of a given signal are shifted by 90^o, the resulting function of time is called the "Hilbert transform" of the signal.

Consider an LTI system with transfer function defined by

$$H(f)=\begin{cases}-j, f>0\\0, f=0\\j, f<0\end{cases}$$

and the Signum function given by

$$\operatorname{sgn}(f)=\begin{cases}1, f>0\\0, f=0\\-1, f<0\end{cases}$$

The function *H(f)* can be expressed using Signum function as given by

$$H(f)=-j\operatorname{sgn}(f)$$

Therefore,

$$H(f)=\begin{cases}1e^{-j\pi/2}, f>0\\1e^{j\pi/2}, f<0\end{cases}$$

Thus the magnitude, $|H(f)|=1$ for all *f*, and angle

$$\angle H(f)=\begin{cases}-\pi/2, f>0\\+\pi/2, f<0\end{cases}$$

The device which possesses such a property is called *Hilbert transformer*.

Whenever a signal is applied to the Hilbert transformer, the amplitudes of all frequency components of the input signal remain unaffected. It produces a phase shift of -90° for all positive frequencies, while a phase shift of 90° for all negative frequencies of the signal. If *x(t)* is an input signal, then its Hilbert transformer is denoted by $\hat{x}(t)$ and shown in the following diagram.

We have

$$H(f) = -j\,\mathrm{sgn}(f)$$

$$H(f) \leftrightarrow \frac{1}{\pi t}$$

Therefore the impulse response *h(t)* of an Hilbert transformer is given by the

$$\boxed{h(t) = \frac{1}{\pi t}}$$

Now consider any input *x(t)* to the Hilbert transformer, which is an LTI system.
Let the impulse response of the Hilbert transformer is obtained by convolving the input *x(t)* and impulse response *h(t)* of the system.

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)}\,d\tau}$$

The above equation gives the Hilbert transform of *x(t)*.
We have

$$\hat{x}(t) = x(t) * h(t)$$

The Fourier transform $\hat{X}(f)$ of $\hat{x}(t)$ is given by

$$\hat{X}(f) = X(f)H(f)$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f)$$

Applications of Hilbert transform

- It is used to realize phase selectivity in the generation of special kind of modulation called Single Side Band modulation.
- It provides mathematical basis for the representation of band pass signals.

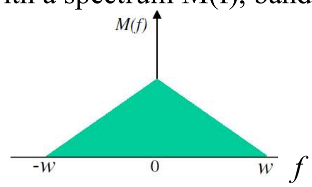
Note: Hilbert transform applies to any signal that is Fourier transformable.

Single Side band Modulation (SSBSC) or (SSB)

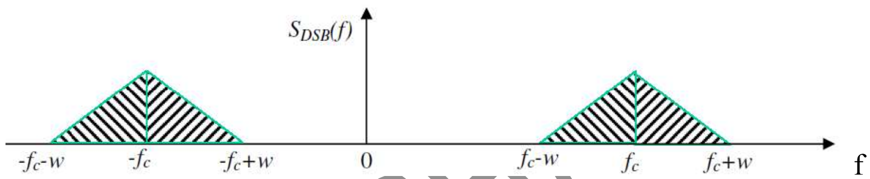
Standard AM and DSBSC require transmission bandwidth equal to twice the message bandwidth. In both the cases, spectrum contains two side bands of width '*W*' Hz, each. But the upper and lower sides are uniquely related to each other by the virtue of their symmetry about the carrier frequency. That is, given the amplitude and phase spectra of either side band, the other can be uniquely determined. Thus if only one side band is transmitted, and if both the carrier and the other side band are suppressed at the transmitter, no information is lost. This kind of modulation is called SSBSC.

Frequency domain description:

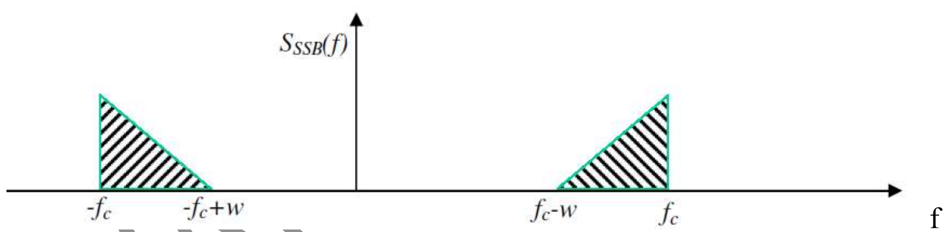
Consider a message signal m(t), with a spectrum M(f), band limited to the interval $-w < f < w$.

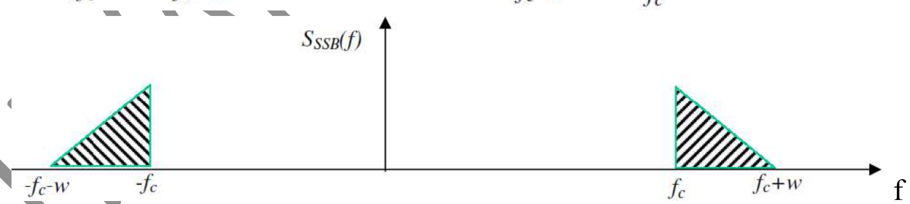


The DSBSC wave obtained by multiplying *m(t)* by the carrier wave $A_C \cos(2\pi f_c t)$.



The SSB wave with only upper sideband or only lower sideband is shown below:





Thus the bandwidth of SSB wave is equal to bandwidth of the message signal. It is equal to half of that of the AM wave (or DSBSC wave).

Advantage of SSB:

- Reduced bandwidth requirement when compared with AM wave (or DSBSC wave).
- Elimination of high power carrier wave

Disadvantage:

- Cost of implementation
- Complexity of implementation

The canonical form of SSB wave:

$$s(t) = \frac{1}{2} A_c\, m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c\, \widehat{m}\,(t) \sin(2\pi f_c t)$$

Single-tone SSB modulation

Consider a single-tone message signal $m(t) = A_m \cos(2\pi f_m t)$ and its Hilbert transform, $\widehat{m}\,(t) = A_m \sin(2\pi f_m t)$. The SSB wave, obtained by transmitting only the upper side-frequency, is defined by

$$s_u(t) = \frac{1}{2} A_c\, m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c\, \widehat{m}\,(t) \sin(2\pi f_c t)$$

$$= \frac{1}{2} A_c A_m \cos(2\pi f_m t) \cos(2\pi f_c t) - \frac{1}{2} A_c A_m \sin(2\pi f_m t) \sin(2\pi f_c t)$$

$$= \frac{1}{2} A_c A_m [\cos(2\pi f_m t) \cos(2\pi f_c t) - \sin(2\pi f_m t) \sin(2\pi f_c t)]$$

$$\boldsymbol{s_u(t) = \frac{1}{2} A_c A_m \cos[2\pi(f_c + f_m)t]}$$

Similarly, SSB wave with only lower sideband is given by

$$\boldsymbol{s_l(t) = \frac{1}{2} A_c A_m \cos[2\pi(f_c - f_m)t]}$$

Vestigial side band Modulation

Single-sideband modulation works satisfactorily for an information-bearing signal (e.g., speech signal) with an energy gap centered around zero frequency. However, for the spectrally efficient transmission of *wideband signals*, we have to look to a new method of modulation for two reasons:

1. Typically, the spectra of wideband signals (exemplified by television video signals and computer data) contain significant low frequencies, which make it impractical to use SSB modulation.
2. The spectral characteristics of wideband data befit the use of DSB-SC. However, DSBSC requires a transmission bandwidth equal to twice the message bandwidth, which violates the bandwidth conservation requirement.

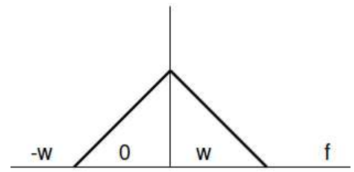
To overcome these two practical limitations, we need a *compromise* method of modulation that lies somewhere between SSB and DSB-SC in its spectral characteristics. *Vestigial sideband*, the remaining modulation scheme to be considered in this section, is that compromise scheme.

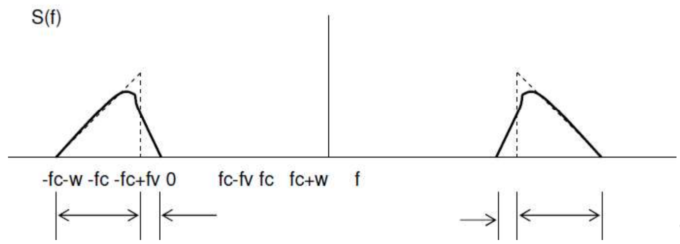
Vestigial sideband (VSB) modulation distinguishes itself from SSB modulation in two practical respects:

1. Instead of completely removing a sideband, a trace or *vestige* of that sideband is transmitted; hence, the name "vestigial sideband."
2. Instead of transmitting the other sideband in full, *almost* the whole of this second band is also transmitted.

Frequency domain Description:

Fig illustrates the spectrum of VSB modulated wave s(t) in relation to the message m(t), assuming that the lower sideband is modified into vestigial sideband.

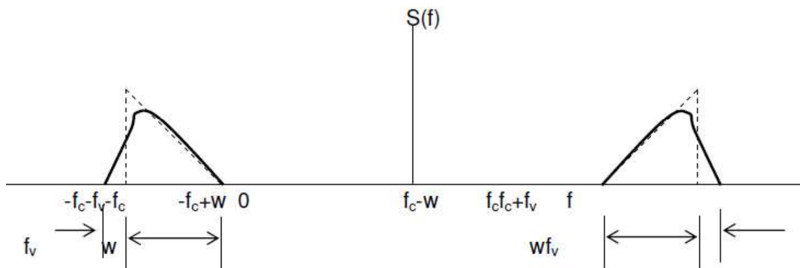




The transmitted vestige of lower sideband compensates for the amount removed from the upper sideband. The bandwidth required to send VSB wave is

B = w+f$_v$ Where f_v is the width of the vestigial side band.

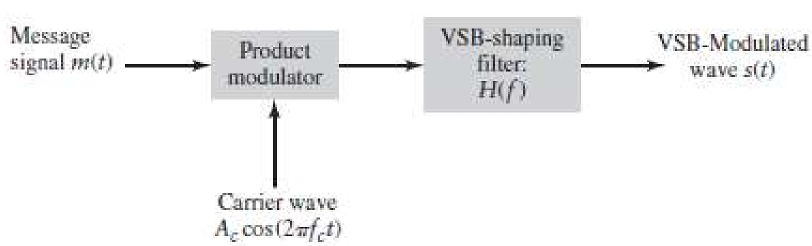
Similarly, if Upper side band is modified into the vestigial side band then,



Therefore, VSB has the virtue of conserving bandwidth almost as efficiently as SSB modulation, while retaining the excellent low-frequency base band characteristics of DSBSC and it is standard for the transmission of TV signals.

Generation of VSB modulated wave:

VSB modulated wave is obtained by passing DSBSC through a sideband shaping filter as shown in fig below.



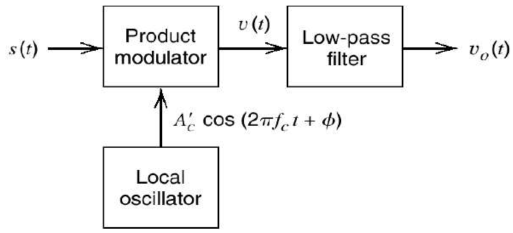
The exact design of this filter depends on the spectrum of the VSB waves. The relation between filter transfer function H(f) and the spectrum of VSB waves is given by

$$S(f) = A_c/2\ [M(f - f_c) + M(f + f_c)]\ H(f)$$

Where M(f) is the spectrum of Message Signal.

Now, we have to determine the Specification for the Filter transfer function H(f)

It can be obtained by passing S(t) to a coherent detector and determining the necessary condition for Undistorted version of the message signal m(t). Thus, S(t) is multiplied by a Locally generated sinusoidal wave $\cos 2\pi f_c t$, which is synchronous with the carrier wave Ac $\cos 2\pi f_c t$ in both frequency and phase, as in fig below,



Then, $v(t) = s(t).\cos 2\pi f_c t$

In frequency domain,

$V(f) = ½ [S(f - f_c) + S(f + f_c)]$

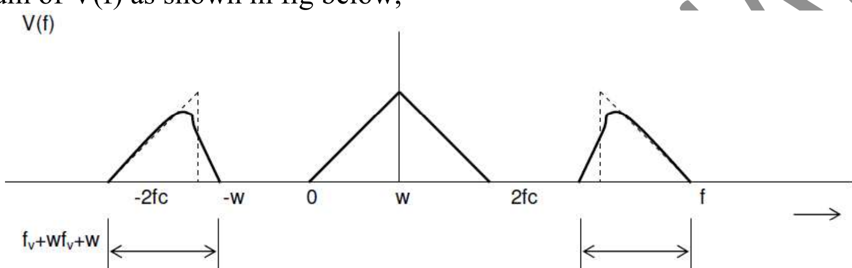
$V(f) = ½[Ac /2 [M (f - fc - fc) + M(f - fc + fc)]H(f – fc)$

$+ ½[Ac /2 [M (f + fc - fc) + M(f + fc + fc)]H(f + fc)$

$V(f) = ½[Ac /2 [M (f -2 fc) + M(f)]H(f – fc) + ½[Ac /2 [M (f) + M(f +2fc)]H(f + fc)$

$V(f) = Ac /4\, M(f)[H (f - fc) + H(f + fc)] + Ac /4 [M(f -2 fc) H (f - fc) + M(f + 2fc) H(f + fc)]$

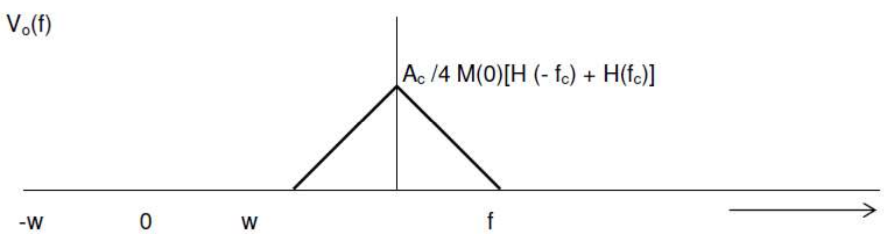
The spectrum of V(f) as shown in fig below,



Pass v(t) to a Low pass filter to eliminate VSB wave corresponding to 2fc.

$V_o(f) = Ac /4\, M(f)[H (f - fc) + H(f + fc)]$

The spectrum of $V_o(f)$ is in fig below,

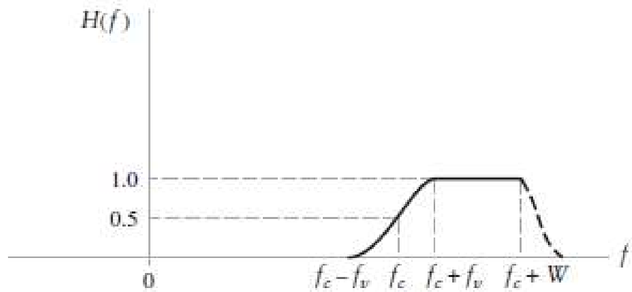


For a distortion less reproduction of the original signal m(t), Vo(f) to be a scaled version of M(f). Therefore, the transfer function H(f) must satisfy the condition

$$H (f - f_c) + H(f + f_c) = 2H(f_c)$$

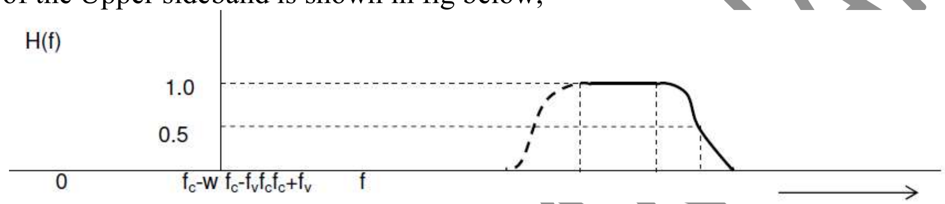
Where $H(f_c)$ is a constant

Since m(t) is a band limited signal, we need to satisfy above equation in the interval $-w \leq f \leq w$. The requirement is satisfied by using a filter whose transfer function is shown below



The Response is normalized so that H(f) at fc is 0.5. Inside this interval (fc-fv) ≤ f ≤ (fc+fv) response exhibits odd symmetry. i.e., Sum of the values of H(f) at any two frequencies equally displaced above and below is Unity.

Similarly, the transfer function H(f) of the filter for sending Lower sideband along with the vestige of the Upper sideband is shown in fig below,



Comparison of Amplitude modulation techniques

1. In standard AM systems, two sidebands are transmitted along with carrier. Demodulation is accomplished by using an envelope detector or square-law detector. The complex of receiver is less when envelope detector is used.
2. The suppressed carrier systems (DSBSC, SSB, VSB) require much less power than AM to transmit same amount of information. This makes them less expensive compared to standard AM. The receiver is more comples because additional circuitry is required for carrier recovery.
3. SSB requires minimum bandwidth and minimum transmission power. SSB is preferred for long distance communication of voice over metallic circuits (copper wire) because it permits longer spacing between *repeaters*.
4. VSB requires bandwidth that is intermediate between that required for SSB or DSBSC modulation. For signals with large bandwidth (TV signal and digital data) the saving can be significant.
5. DSBSC, SSB and VSB are *linear modulation* techniques whereas standard AM is not purely linear.
6. In SSB and VSB schemes, the role of quadrature component is merely to interfere with the in-phase component. However the message signal can be recovered from the modulated signal S(t) with the use of coherent detector.
7. Quadrature Amplitude modulation permits the multiplexing of two message signals with a single carrier and it is called ‘Quadrature carrier multiplexing’.

Frequency Translation

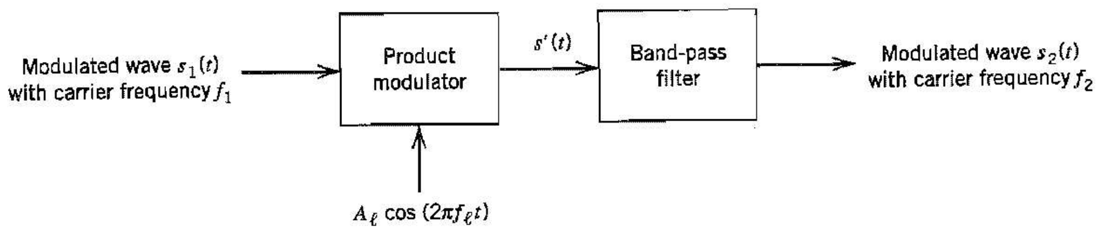
- In communication systems, it is often necessary to translate the modulated wave upward or downward in frequency, so that it occupies a new frequency band.

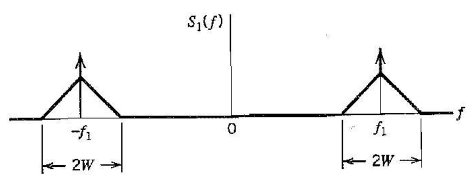
- The frequency translation involves:
  - (i) multiplication of the signal by a locally generated signal and
  - (ii) bandpass filtering.

  Consider a DSBSC wave, $s_1(t) = A_c\, m(t)\cos(2\pi f_1 t)$.

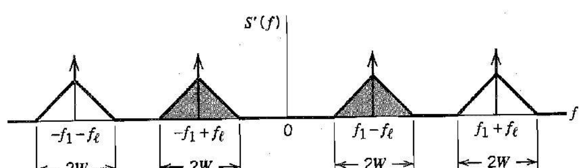
Suppose that it is required to translate the modulated wave downward in frequency, so that its carrier frequency is changed from f_c to a new value f_o, where $f_o < f_1$. Let f_l be local oscillator frequency, defined by

$$\boldsymbol{f_1 - f_l = f_o}$$





(a)



$$s'(t) = s(t)\cos(2\pi f_l t)$$
$$= A_c\, m(t)\cos(2\pi f_1 t)\cos(2\pi f_l t)$$
$$= \frac{1}{2}A_c\, m(t)\cos[2\pi(f_1 - f_l)t] + \frac{1}{2}A_c\, m(t)\cos[2\pi(f_1 + f_l)t]$$

The multiplier output $v_1(t)$consists of two DSBSC waves, one with carrier frequency, $(f_1 - f_l)$ and the other with carrier frequency of $(f_1 + f_l)$.

The modulated wave with desired carrier frequency $f_o = f_1 - f_l$ is extracted using a band-pass filter of mid-band frequency f_o and bandwidth 2W, provided

$$f_l > w.$$

The filter output is $s_2(t) = \frac{1}{2}A_c\, m(t)\cos[2\pi(f_1 - f_l)t] = \frac{1}{2}A_c\, m(t)\cos[2\pi(f_o)t]$

The output is the modulated wave, translated downward.

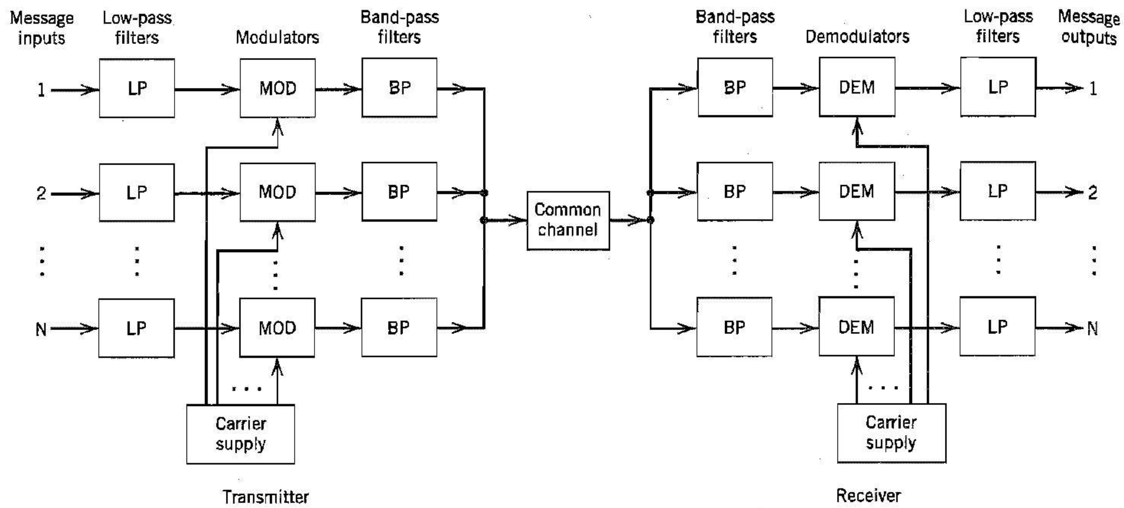
The device that carries out the frequency translation of a modulated wave is called *mixer.* The operation is called ***mixing*** or ***heterodyning*** .

Mixing is a linear operation.

Frequency Division Multiplexing (FDM)

- Multiplexing is a technique whereby a number of independent signals can be combined into a composite signal suitable for transmission over a common channel.

- Multiplexing requires that the signals be kept apart so that they do not interfere with each other and thus they can be separated at the receiving end.
- The technique of separating the signals in frequency is called '***Frequency Division Multiplexing'*** (FDM).



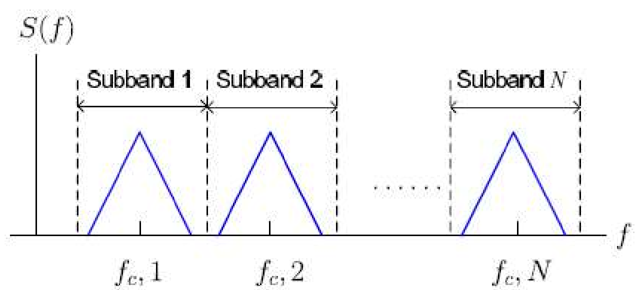


Figure 2: Spectrum of the FDM signal.

- The block diagram of FDM system is shown in fig. 1. The incoming message signals are assumed to be of the low-pass type and they are band-limited using LPFs.
- The filtered signals are applied to modulators that shift the frequency ranges of the signals so as to occupy mutually exclusive frequency intervals.
- The necessary carrier frequencies to perform frequency translations are obtained from carrier supply.
- The most used method of modulation in FDM is single side band modulation, which requires bandwidth that is approximately equal to message bandwidth.
- The band pass filters following the modulators are used to restrict the band of each modulated wave to its prescribed range.
- The BPF outputs are combined in parallel to form the input to common channel.
- At receiving end, received signal is applied to a bank of BPF connected in parallel. These filters are used to separate the modulated signals.
- Demodulators are used to extract the message signal.