Module – 4
Electrical Conductivity in metals

1. Explain the Resistivity (ρ) and Mobility of Electrons.

Resistivity (ρ): For a metal of uniform cross-section, the resistance R is directly proportional to its area of cross section A,

$$\text{i.e., } R \; \alpha \; \frac{L}{A}$$

$$\text{Or } R = \rho \left(\frac{L}{A}\right)$$

Where ρ is the constant of proportionality.

$$\rho = R \left(\frac{A}{L}\right)$$

ρ is called resistivity. It is a property of the material and gives the measure of the opposition offered by the material during a current flow in it.

Mobility of Electrons:

The mobility of electrons is defined as the magnitude of the drift velocity acquired by the electrons in a unit field. Thus, if E is the applied electric field in which the electrons acquire a drift velocity v_d then the mobility of electrons is given by

$$\mu = \frac{v_d}{E}$$

We have the equation for drift velocity, $V_d = \frac{eE\tau}{m}$

$$\mu = \frac{1}{E}\left(\frac{eE\tau}{m}\right)$$

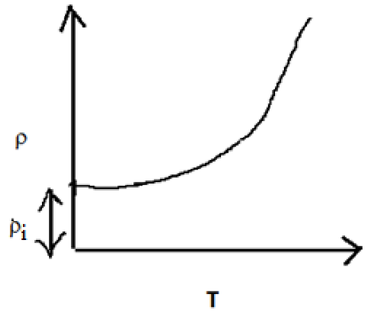
$$\mu = \frac{e\tau}{m}$$

2. Explain the concept of Phonon.

During the flow of current in a metal, the electrons leave the atoms to which they originally being bound to. Because of loss of electrons, the atoms become positive ionic cores. Due to the thermal excitation, the ion will always be oscillating about fixed positions in the frame- work of the metal. These vibrations are called lattice-vibrations (phonons).

3. Explain the Effect of Temperature and impurity on Electrical resistivity of metals.

The variation of electrical resistivity ρ of a metal with temperature T is as shown in the graph. The resistivity of metals is attributed to the scattering of conduction electrons. In metals the scattering takes place basically under two mechanism which give rise to two components of resistivity which are described below.



1. Resistivity ρ_{ph}due to scattering of electrons by lattice vibrations (phonons) which increases with temperature.ρ_{ph} is therefore temperature dependent. It is the resistivity exhibited by a pure specimen that is free of all defects, and hence called the ideal resistivity.
2. Resistivity ρ_i due to scattering of conduction electrons by the presence of impurities, this type of scattering is independent of temperature and contributes to resistivity even at the temperature T= 0 K. ρ_i is therefore called residual resistivity.

Since the two scattering mechanisms mentioned above act independently, the two resistivity's are additive. If ρ is the total resistivity of the metal, then ρ is given by

$$\boldsymbol{\rho = \rho_{ph} + \rho_i}$$

The above equation is called Matthiessen's rule.

Matthiessen's rule states that the total resistivity of a metal is the sum of the resistivity due to phonon scattering, which is temperature dependent and, the resistivity due to scattering by impurities which is temperature independent.

4. Enumerate the failures of classical free electron theory. (MQP2, JAN/FEB 2023)

1. Specific heat:
2. Temperature dependence of electrical conductivity (σ)
3. Dependence of electrical conductivity on electron concentration 'n'

1. <u>Specific heat</u>: The theory compares free electron to the gas molecules. The same specific heat should be applicable for both. The specific heat of a gas at constant volume is given by,

$$\mathbf{C_V} = \frac{3}{2}\boldsymbol{R} \quad \text{............. (1)}$$

But for a metal, $\mathbf{C_V = 10^{-4}RT}$ .... (2)

From (1) & (2), we see that, C_V for a metal is not only far smaller than C_V for gas molecule but also its dependent on temperature.

<u>2. Temperature dependence of electrical conductivity (σ)</u>

Experimentally observed value of σ for metals is inversely proportional to temperature T.

$$\text{i.e., } \boldsymbol{\sigma_{exp}\ \alpha\ \frac{1}{T}} \text{.......... (1)}$$

According to Classical free electron theory, dependence of electrical conductivity on temperature is given by,

$$\sigma\ \alpha\ \frac{1}{\sqrt{T}} \text{....................... (2)}$$

Since both the equations contradict each other, this is considered as a failure.

3. Dependence of electrical conductivity on electron concentration 'n'

σ is given by,

$$\sigma = \frac{n\tau e^2}{m}$$

As per the equation of conductivity, conductivity of a metal is proportional to 'n' (n – free electron concentration). The 'n' values for Zinc and Copper are $13.10X10^{28}$ /m^3 and $8.45X10^{28}$/m^3 respectively. However, the conductivity of copper is greater than Zinc. This is also significant failure of the theory.

5. Enumerate the Assumptions of Quantum free electron theory of Metals. (MQP1)

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy levels.
- The distribution of electrons in the various allowed energy levels occurs as per Pauli Exclusion Principle.
- The distribution of electrons in the various allowed energy levels obey F-D statistics.
- The free electrons travel in a constant potential inside the metal butstay confined within its boundaries.
- The attraction between the free electrons and the lattice ions, and therepulsion between the electrons themselves are ignored.

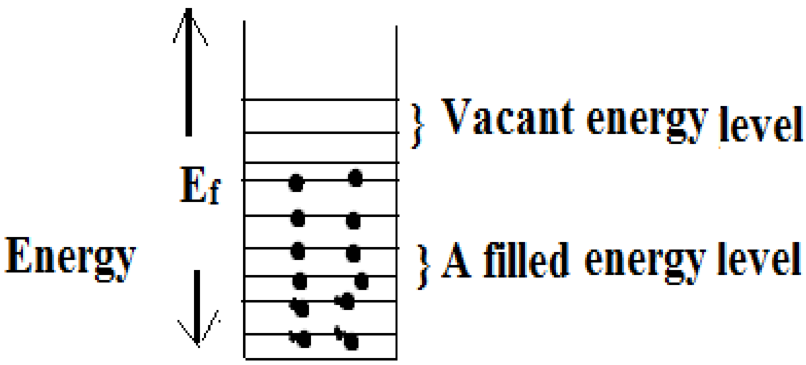
6. Explain Fermi energy. (MQP1)

We know that for a metal containing N atoms, there will be N allowed energy levels in each band. The electrons will occupy energy levels according to Pauli exclusion principle (each energy level can accommodate a maximum of two electrons, one with spin up and one with spin down). This is the picture when no external energy is supplied for the electrons, such as thermal energy or electrical energy.

Here we define Fermi energy as follows. "The energy corresponding to the highest occupied level at zero-degree absolute is called Fermi energy" and the energy level is referred to as Fermi level.

Fermi energy is denoted as E_F.

Thus, at T=0K, all the energy levels lying above the Fermi level are empty and those lying below are completely filled.

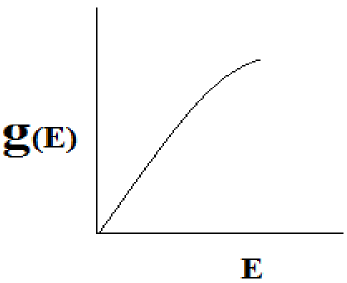


7. Explain Density of States.

The exact dependence of density of energy states on the energy is realized through a function denoted as g(E) known as density of states function. It is defined as "The number of allowed energy states per unit volume per unit energy range. In an energy band if E varies g(E) also varies as shown in the graph.

It is given by the equation

$$g(E)\,dE = \left[\frac{8\sqrt{2}\,\pi m^{3/2}}{h^3}\right] E^{1/2}\,dE$$



Where, m= mass of electron

h is the Planck's constant.

$g(E) \ \alpha \ E^{1/2}$

8. Define Fermi Factor and discuss the variation of Fermi factor with Temperature and energy (MQP1, JAN/FEB 2023).

Fermi-Dirac Statistics permits the evaluation of probability of finding electrons occupying energy levels in a certain energy range. This is done through a function called Fermi Factor f (E). It is given by,

$$\mathbf{f\,(E)} = \frac{1}{e^{\frac{E-E_F}{KT}} + 1}$$

"Fermi factor is the probability of occupation of a given energy state for a material in thermal equilibrium".

Probability of occupation is considered for following cases.

i) Probability of occupation for $E<E_F$ at T=0K

We have, $F(E) = \frac{1}{e^{\frac{E-E_F}{KT}} + 1}$

If $E<E_F$, $E-E_F$ will be negative and T=0,

$$f(E) = \frac{1}{e^{-\infty} + 1}$$

$$f(E) = \frac{1}{0+1} \quad f(E) = 1$$

- f(E)=1 means, the energy level is certainly occupied. ie, there is 100% probability that the electrons occupy the energy level below Fermi energy.
- All the energy levels below Fermi level are occupied.
- $E<E_F$ applies to all the energy levels below E_F.

i) Probability of occupation for $E>E_F$ at T=0K

If $E>E_F$, $E-E_F$ will be positive and for T=0,

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty} \quad f(E) = 0$$

- All the energy levels above Fermi level are unoccupied. I.e, 0% probability for the electrons to occupy the energy level above the Fermi level.

ii) Probability of occupation at ordinary temperature.

At ordinary temperature, though value of probability remains 1 for $E<<E_F$, it starts decreasing from 1 as the value of E become closer to E_F.

For $E=E_F$,

$$f(E) = \frac{1}{e^{0} + 1} = \frac{1}{1+1} = \frac{1}{2} \quad \text{i.e } f(E) = \frac{1}{2}$$

There is 50% probability for the electrons to occupy the Fermi energy level.

Variation of f (E) with E is as shown in Fig.