# Module – 5
# Physics of Animation

**1. Elucidate the importance of size & scale and weight and strength inanimations. Size and Scale. (MQP2)**

The size and scale of characters often play a central role in a story's plot. What would Superman be without hisheight and bulging biceps? Some characters, like the Incredible Hulk, are even named after their body types.

We often equate large characters with weight and strength, and smaller characters with agility and speed. There is a reason for this. In real life, larger people and animals do have a larger capacity for strength, while smaller critters can move and maneuver faster than their large counterparts. When designing characters, you can run into different situationshaving to do with size and scale, such as:

1. Human or animal-based characters that are much larger than we see in our everyday experience. Superheroes, Greek gods, monsters,
2. Human or animal-based characters that are much smaller than we are accustomed to, such as fairies and elves.
3. Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.
4. Characters that are child versions of older characters. An example would be an animation featuring a mother cat and her kittens. If the kittens are created and animated with the same proportions and timing as the mother cat, they won't look like kittens; they'll just look like very small adult cats.

**Proportion and Scale**

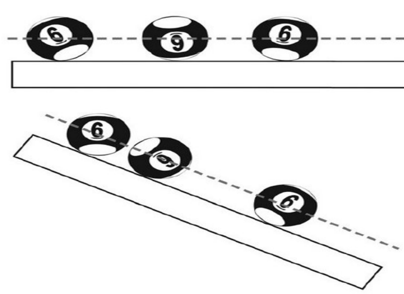
Creating a larger or smaller character is not just a matter of scaling everything about the character uniformly. To understand this, let's look at a simple cube. When you scale a cube, its volume changes much more dramatically thanits surface area. Let us say each edge of the cube is 1 unit length. The area of one side of the cube is 1 square unit, and the volume of the cube is 1 cubed unit. If you double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times, and its volume increases by 8 times. While the area increases by squares as you scale the object, the volume changes by cubes.

**Wight and strength**

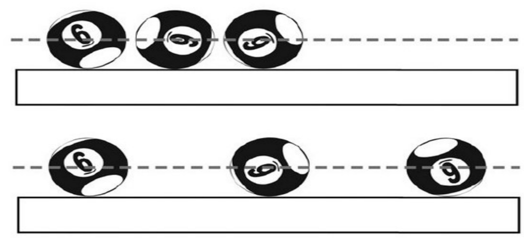
Body weight is proportional to volume. The abilities of your muscles and bones, however, increase by area because their abilities depend more on cross-sectional area than volume. To increase a muscle or bone's strength, you need to increase its cross- sectional area. To double a muscle's strength, for example, you would multiply its width by
$\sqrt{2}$. To triple the strength, multiply the width by $\sqrt{3}$. Since strength increases by squares and weight increases by cubes, the proportion of a character's weight that it can lift does not scale proportionally to its size.
Let us look at an example of a somewhat average human man. At 6 feet tall, he weighs 180 pounds and can lift90 pounds. In other words, he can lift half his body weight. If you scale up the body size by a factor of 2, the weight increases by a factor of 8. Such a character could then lift more weight. But since he weighs more than 8 times more than he did before, he cannot lift his arms and legs as easily as a normal man. Such a giant gains strength, but loses agility.

**2. Discuss timing in linear motion, Uniform motion, slow in and slow out. (MQP1,JAN/FEB2023)**

Linear motion refers to motion in a straight line, always in the same direction. An object moving with linear motion might speed up or slow down as it follows a linear path. A heavy ball rolling on a table or incline is an example of linearmotion. The ball is rotating, but its center of gravity follows a linear path. A heavy ball rolling on a table or incline is anexample of linear motion. The ball is rotating, but its center of gravity follows a linear path.



Linear motion.



Uniform motion.

**Uniform Motion Timing**

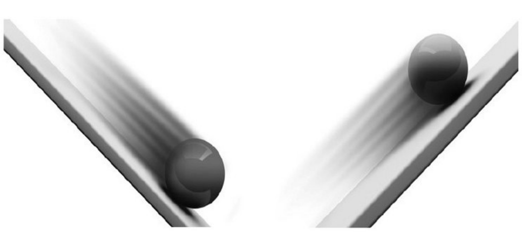
When uniform motion occurs, the net force on the object is zero. Net force is the total of all forces added up. There might be several forces acting on the object, but when both the magnitude and direction of the forces are added up, they add up to zero. Uniform motion is the easiest to animate because the distance the object travels between framesis always the same. Uniform motion is a type of linear motion with constant speed and no acceleration or deceleration. The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.

**Slow in and slow out**

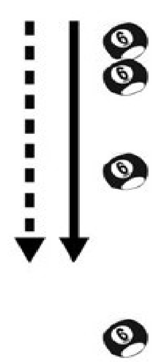
When motion is accelerating or decelerating, we refer to this type of motion as a slow in or slow out. This type of motion is sometimes called ease in or ease out. In this book, we use the hyphenated forms slow-in and slow-out for easier understanding.

1. Slow in, ease in—the object is slowing down, often in preparation for stopping.
2. Slow out, ease out—the object is speeding up, often from a still position.

The term slow out can be confusing, since it essentially means “speed up.” one can think of slow out as the same as easeout, as in easing out of a still position and speeding up to full speed.
For example, a ball rolling down an incline or dropping straight down is slowing out, as it goes from a still position orslow speed to a fast speed. A ball rolling up an incline is slowing in.



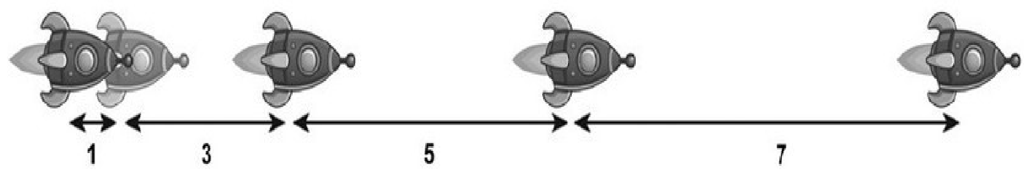
Slowing out: speeding up. Slowing in: slowing down.



**3. Illustrate the odd rule and odd rule multipliers with a suitable example. (MQP1,JAN/FEB2023)**

When acceleration is constant, one can use the Odd Rule to time the frames. With this method, one calculate the distance the object moves between frames using a simple pattern of odd numbers. Between consecutive frames, the distance the object moves is a multiple of an odd number. For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.

For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.




Rocket speeding up using the Odd Rule.


Sled coming to a stop using the Odd Rule.

The Odd Rule is a multiplying system based on the smallest distance traveled between two frames in the sequence. Fora slow-out, this is the distance between the first two frames; for a slow-in, it's the distance between the last two frames. This distance, the base distance, is used in all Odd Rule calculations.

**Odd Rule Multipliers**

The Odd Rule in its simplest form, as described above, is just one way to use it. For example, one can instead calculate the distance from the first frame to the current frame and use these distances to place the object on specific frames.

| | Multiply by base distance to get distance between: | |
|---|---|---|
| Frame # | Consecutive frames | First frame and this frame |
| 1 | n/a | 0 |
| 2 | 1 | 1 |
| 3 | 3 | 4 |
| 4 | 5 | 9 |
| 5 | 7 | 16 |
| 6 | 9 | 25 |
| 7 | 11 | 36 |

Calculating the distance for a large number of frames and a chart like this isn't practical, one can figure out the odd number multiplier for consecutive frames with this formula:

Odd number multiplier for consecutive frames = ((frame # – 1) * 2) – 1

In the charts above, note that the distances in the last column are squared numbers: 4 = 22, 9 = 32, 16 = 42,and so on. One of the benefits of the Odd Rule is one can calculate the total distance traveled from the start point to the current frame with the following formula:

Multiplier for distance from first frame to current frame = (current frame # – 1)^ 2

When setting the keys, one can use either the consecutive key multipliers or total distance multipliers but need to choose the one that's easiest to use for the animated sequence.

**4. Describe Jumping and parts of jump.(MQP1,JAN/FEB2023)**

A jump can be divided into several distinct parts:

- **Crouch**—A squatting pose taken as preparation for jumping.

- **Takeoff**—Character pushes up fast and straightens legs with feet still on the ground. The distance from the character's center of gravity (CG) in the crouch to the CG when the character's feet are just about to leave the ground is called the push height. The amount of time (or number of frames) needed for the push is called the push time.

- **In the air**—Both the character's feet are off the ground, and the character's center of gravity (CG) moves in a parabolic arc as any Para free-falling body would. First it reaches an apex, and then falls back to the ground at the same rate at which it rose. The height to which the character jumps, called the jump height, is measured from the CG at takeoff to the CG at the apex of the jump. The amount of time the character is in the air from takeoff to apex is called the jump time. If the takeoff pose and the landing pose are similar, then the jump height and jump time are about the same going up as they are going down.

- **Landing**—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.

**5. Describe the calculation of Push time and stop time with examples.(MQP2,**

Push:

Jump Height/Jump Time = Push Height/Push Time

$$\frac{Jump\ Height}{Push\ Time} = \frac{Push\ Height}{Push\ Time}$$

**Push Time**

The JM also gives you the ratio of the jump time to the push time.

JM = Jump Time/Push Time

Working a little algebra, we can express the equation in a way that directly calculates the push time:
Push Time = Jump Time/JM

Example:

JM = 3

Jump Time: 15 frames

Push Time = 15/3 = 5 frames

## Stop Time

The stop height is often a bit larger than the push height, but the timing ofthe push and stop are the same in the sense that the CG moves the same distance per frame in the push and stop. If the stop height is larger than the push height, you'll just need more frames for the stop than the push.

Push Height/Push Frames = Stop Height/Stop FramesThis can also be expressed as:
Push Height/Push Time = Stop Distance/Stop TimeYou can also flip everything over and express it as:
Push Time/Push Height = Stop Time/Stop Distance

Using algebra, we can get the following equation for stop time:

Stop Time = (Push Time * Stop Distance)/Push Height

## Example:

Push Time: 5 frames
Push Height: 0.4m
Stop Height: 0.5m
Stop Time = (5 * 0.5) / 0.4 = 6 frames

**6. Sketch and explain the motion graphs for linear, easy ease, easy ease in andeasy ease out cases of animation. (MQP2)**

**Motion Graphs**

A motion graph plots an object’s position against time. If one is using animation software, understanding and using motion graphs is a key skill in animating anything beyond the simplest of motions. If one is drawing the animation, drawing motion graphs before animating can help one to visualize the motion. On a motion graph, the time goes from left to right across the bottom of the graph, while the object’s position is plotted vertically against the time. Each axis in3D space (X, Y, Z) has its own line showing the object’s position along that axis. At the very least, one will need to understand the types of lines in a motion graph and what they represent in terms of visible motion. one can also look atmotion graphs to get a better understanding of any difficulties one is having with the timing or action.

- A motion graph plots objects position against time.
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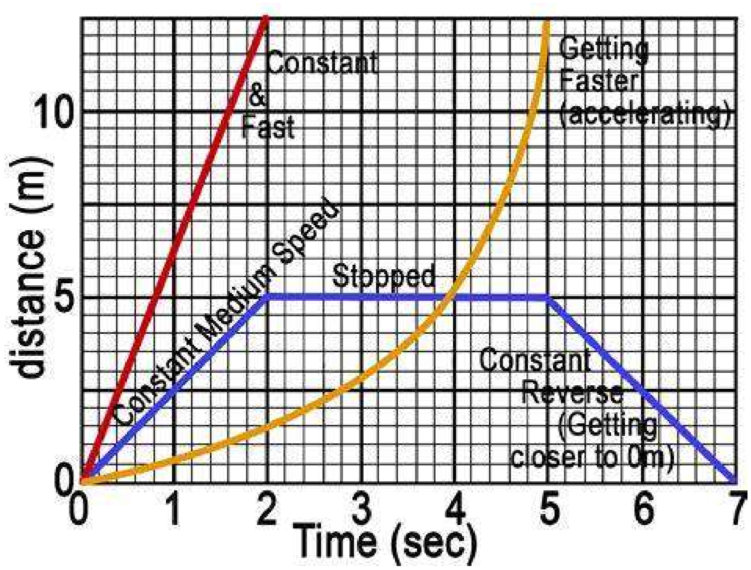

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When motion is accelerating or decelerating – slow in or slow out (ease in or ease out)
Slow in/ ease in – Object is slowing down (preparation for stopping)
Slow out/ease out – The object is speeding up (often from a still position)

**7. Distinguish between descriptive and inferential statistics. (MQP1)**

| Sl. No. | Descriptive Statistics | Inferential Statistics |
|---|---|---|
| 1 | It gives information about raw data which describesthe data in some manner. | It makes inferences about the population using datadrawn from the population. |
| 2 | It helps in organizing, analyzing, and to present data in a meaningful manner. | It allows us to compare data, and make hypothesesand predictions. |
| 3 | It is used to describe a situation. | It is used to explain the chance of occurrence of anevent. |
| 4 | It explains already known data and is limited to a sample or population having a small size. | It attempt s to reach the conclusion about thepopulation. |
| 5 | It can be achieved with the help of char t s, graphs, tables, etc. | It can be achieved by probability. |

**8. Discuss modeling the probability for proton decay. (MQP2, JAN/FEB2023)**

The experimental search for Proton Decay was undertaken because of the implications of the Grand unification Theories. The lower bound for the lifetime is now projected to be on the order of $\tau = 10^{33}$ Years. The probability for observing a proton decay can be estimated from the nature of particle decay and the application of Poisson Statistics. The number of protons N can be modeled by the decay equation

$$N=N_0e^{-\lambda t}$$

Here $\lambda = 1/t= 10^{-33}/$ year is the probability that any given proton will decay in a year. Since the decay constant $\lambda$ is so small, the exponential can be represented by the first two terms of the Exponential Series.

$$e^{-\lambda t}=1-\lambda t, \text{ thus } N\approx N_0(1-\lambda t)$$

For a small sample, the observation of a proton decay is infinitesimal, but suppose we consider the volume of protons represented by the Super Kameokande neutrino detector in Japan. The number of protons in the detector volume is reported by Ed Kearns of Boston University to be 7.5

x $10^{33}$protons. For one year of observation, the number of expected proton decays is then

$$N-N_0 = N_0\lambda t = (7.5X10^{33}\text{protons})\ (10^{-3}\ /\text{year})(1\ \text{year})=7.5$$

About 40% of the area around the detector tank is covered by photo-detector tubes, and if we take that to be the nominalefficiency of detection, we expect about three observations of proton decay events per year based on a $10^{33}$year lifetime.

So far, no convincing proton decay events have been seen. Poisson statistics provides a convenient means for assessing the implications of the absence of these observations. If we presume that λ = 3 observed decays per year is the mean, then the Poisson distribution function tells us that the probability for zero observations of a decay is

$$P(K) = \frac{\lambda^k e^{-\lambda}}{K!} \quad P(K) = \frac{3^0 e^{-3}}{0!} = 0.05$$

This low probability for a null result suggests that the proposed lifetime of $10^{33}$ years is too short. While this is not a realistic assessment of the probability of observations because there are a number of possible pathways for decay, it serves to illustrate in principle how even a non-observation can be used to refine a proposed lifetime.

**9. Mention the general pattern of Monte Carlo method and hence determinethe value of π. (MQP2)**

Monte Carlo method also referred as Multiple Probability Simulation is a statisticaltechnique that uses randomness to solve probabilistic problems. It is used in predicting and forecasting business models, supply chain, science, engineering,and particle physics and so on.

**Estimation of Pi ($\pi$) using Monte Carlo method**

For example, consider a quadrant (circular sector) inscribed in a unit square. Giventhat the ratio of their areas is $\pi/4$ the value of $\pi$ can be approximated using MonteCarlo method.

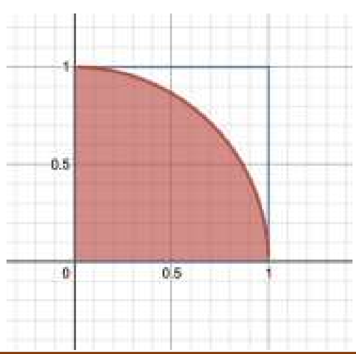
1. Draw a square, then inscribe a quadrant within it.
2. Uniformly scatter a given number of points over the square

Count the number of points inside the quadrant, i.e. having a distance fromthe origin of less Than 1.

$$\text{Then, } \frac{Area\ of\ circle}{Area\ of\ square} = \frac{\pi r^2/4}{r^2}$$

$$\text{Further, } \frac{\pi}{4} = \frac{No.of\ points\ generated\ inside\ the\ circle}{Total\ number\ of\ points\ generated\ in\ the\ square}$$

$$\pi = 4 \times \frac{No.of\ points\ generated\ inside\ the\ circle}{Total\ number\ of\ points\ generated\ in\ the\ square}$$



Using the above equation, the value of $\pi$ can be approximated. There are twoimportant considerations:

1. If the points are not uniformly distributed, then the approximation will be poor.
2. There are many points. The approximation is generally poor if only a few points are randomly placed in the whole square. On average, the approximation improves as more points are placed.

**10. Discuss the salient features of Normal distribution using bell curves. (MQP1)**

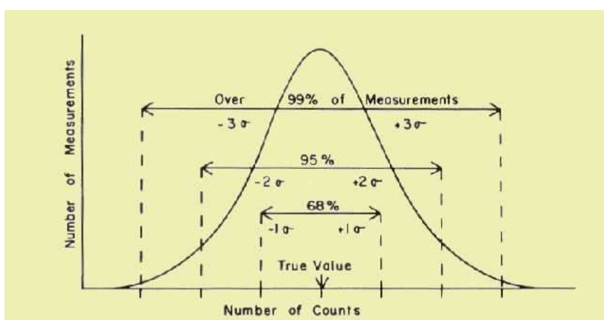
A bell curve is a common type of distribution for a variable, also known as the normal distribution. The term "bell curve" originates from the fact that the graph used to depict a Normal Distribution consists of a symmetrical bell-shapedcurve.

The highest point on the curve, or the top of the bell, represents the most probable event in a series of data (its Mean, Mode and Median in this case), while all other possible occurrences are symmetrically distributed around the mean, creating a downward-sloping curve on each side of the peak. The width of the bell curve is described by its Standard Deviation

The term "bell curve" is used to describe a graphical depiction of a normal probability distribution, whose underlying standard deviations from the mean create the curved bell shape. A standard deviation is a measurement used to quantify the variability of data dispersion, in a set of given values around the mean. The mean, in turn, refers to the average of alldata points in the data set or sequence and will be found at the highest point on the bell curve.

**Standard Deviation**

The Standard Deviation is a measure of how spread out numbers are. 68% of values are within 1 standard deviation of the mean. 95% of values are within 2 standard deviations of the mean. 99.7%of values are within three deviations of the mean




**11. Explain Poisson distribution**

If the probability p is so small that the function has significant value only for very small k, then the distribution ofevents can be approximated by the Poisson distribution.

Probability mass function

A discrete Radom variable $X$ is said to have a Poisson distribution, with parameter, if it has a probability MassFunction given by

$$\mathbf{f\ (k;\ \lambda)=P\ (X=k)} = \frac{\lambda^k e^{-\lambda}}{K!}$$

Here $k$ is the number of occurrences, $e$ is Euler's Number,! Is the factorial function. The positive real number $\lambda$ is equalto the expected value of $X$ and also to its Variance. The Poisson distribution may be used in the design of experiments such as scattering experiments where a small number of events are seen.

Example of probability for Poisson distribution

On a particular river, overflow occur once every 100 years on average. Calculate the probability of k= 0,1,2,3,4,5,or 6 overflow floods in a 100 year interval, assuming the poission model is appropriate.

Because the average event rate is one overflow flood per 100 years, $\lambda = 1$

$$\mathbf{f\ (k;\ \lambda)=P\ (X=k)} = \frac{\boldsymbol{\lambda^k e^{-\lambda}}}{\boldsymbol{K}!}$$

$$\text{P(k overflow floods in 100 years)} = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{1^k e^{-1}}{k!}$$

$$\text{P(k=0 overflow floods in 100 years)} = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} = 0.368$$

$$\text{P(k=1 overflow floods in 100 years)} = \frac{\lambda^k e^{\ \lambda}}{k!} = \frac{1^1 e^{\ 1}}{1!} = \frac{e^{\ 1}}{1} = 0.368$$

$$\text{P(k=2 overflow floods in 100 years)} = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} = 0.184$$

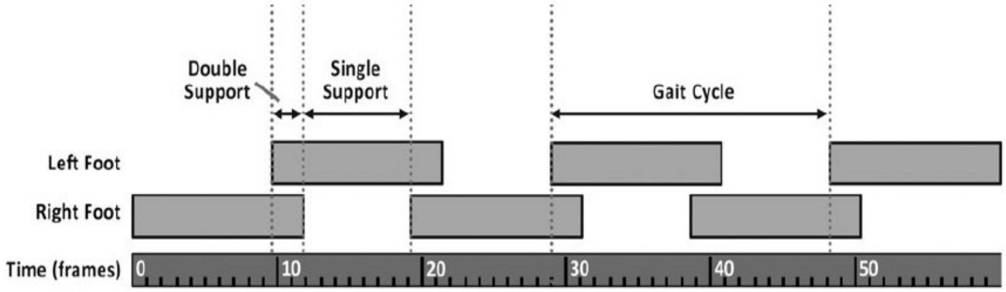
**12. Explain Walking, Strides and Steps.**

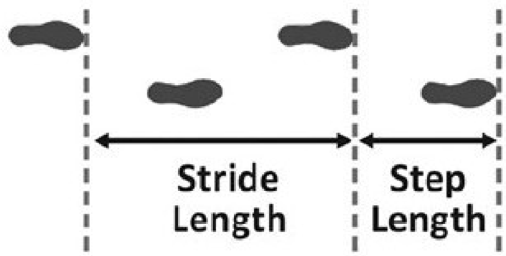
Walks feature all the basics of mechanics while including personality. The ability to animate walk cycles is one of themost important skills a character animator needs to master.

**Strides and Steps**

A step is one step with one foot. A stride is two steps, one with each foot. Stride length is the distance the character travels in a stride, measured from the same part of the foot. Step and stride length indicate lengthwise spacing for the feet during a walk.

Gait is the timing of the motion for each foot, including how long each foot is on the ground or in the air. During a walk, the number of feet the character has on the ground changes from one foot (single support) to two feet (double support) and then back to one foot. You can plot the time each foot is on the ground to see the single and doublesupport times over time. A normal walking gait ranges from 1/ 3 to 2/3 of a second per step, with 1/2 second being average.




**Walk Timing**

Walking is sometimes called "controlled falling." Right after you move past the passing position, your body's center of gravity is no longer over your base of support, and you begin to tip. Your passing leg moves forward to stop the fall, creating your next step. Then the cycle begins again. The horizontal timing for between the four walk poses is not uniform. The CG slows in going from the contact to passing position, then slows out from passing to contact. The CG also rises and falls, rising to the highest position during passing and the lowest during contact. The head is in the highest position during passing.

## Solved Problems

**1. slowing-in object in an animation has a first frame distance 0.5m and the first slow in frame 0.35m. Calculate the base distance and the number of frames in sequence.**

**Solution:**

**Base Distance** = $\frac{\textbf{Total distance} - \textbf{first frame distance}}{2}$

$= \frac{0.5 - 0.35}{2} = 0.07$ **m**

**No of frames** = $\frac{\textbf{Total distance}}{\textbf{base distance}}$

$= \frac{0.5}{0.07} = 7$

**2. The number of particles emitted per second by a random radioactive source has a Poisson's distribution with λ = 4. Calculate the probabilityof P(X = 0) and P(X = 1).**

**Solution:**

Let x is said to be Poisson's Distribution

f (k)=P(x=k)= $\frac{e^{-\lambda}\lambda^{k}}{k!}$

1) P(X = 0)= k=0 , λ=4

   P(x=0) = $\frac{e^{-4}4^{0}}{0!} = \frac{0.018}{1} = 0.018$

2) P(X = 1)= k=1 , λ=4

   P(x=0) = $\frac{e^{-4}4^{1}}{1!} = \frac{0.018 \quad 4}{1} = 0.07$