Model Question Paper- I Solutions 2022-23 (CBCS Scheme)
Engineering Physics – BPHYS102 / 202

1. a Define LASER and Discuss the interaction of radiation with matter.

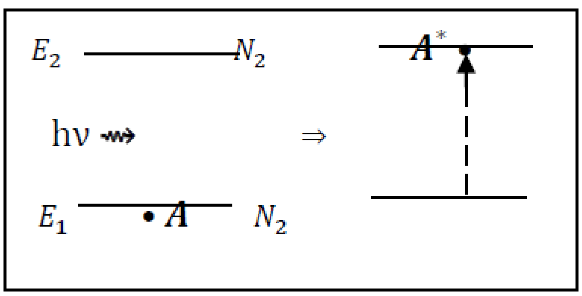
The term **LASER** is an acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. Laser is a device that produces a highly coherent, monochromatic, intense beam of light with very small divergence.

Einstein's explanation of interaction of radiation with matter (or) Explaination of Induced absorption, Spontaneous emission & Stimulated emission.

Consider a system of energy density $E\nu$ andLet N_1& N_2 be the population of the energy states E_1& E_2 respectively so that $(E_2 - E_1) = h\nu$ & $E_2 > E_1$

According to Einstein radiation interacts with matter in 3 ways namely:

1) Induced absorption:



Induced absorption is the phenomenon in which an atom(A) in the lower energy state E_1 absorb the incident photon of energy 'hν' & excite to the higher energy state E_2

If $(E_2 - E_1) = h\nu$.

Mathematically it (induced absorption) is represented as

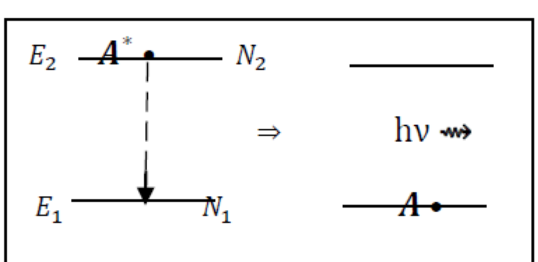
hν +A $\rightarrow A^*$**or** photon + atom$\rightarrow atom^*$

Also, Rate of induced absorption = $B_{12}N1E\nu$,

where $E\nu$ = *Energydensity* of radiations &

B_{12} =*Einstein's* coefficient for *inducedabsorption*.

2. Spontaneous emission:

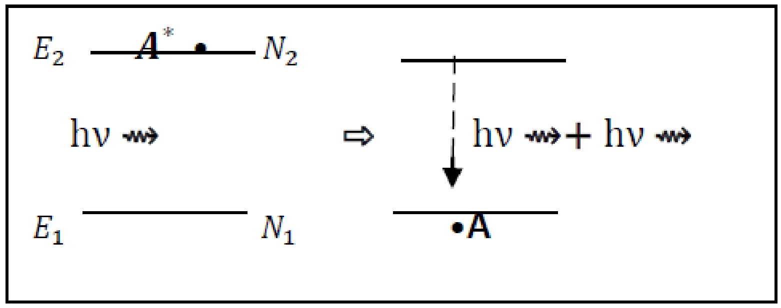


Spontaneous emission is the phenomenon in which an atom (A) in the excited

state of energy E_2 de-excite to the lower energy state E_1 without any external influence by emitting a photon of energy $h\nu = (E_2 - E_1)$.
Mathematically, it is represented as $\boldsymbol{A^* \rightarrow A + h\nu}$

Also, Rate of Spontaneous emission = $A_{21}N_2$
$where A_{21} = Einstein's coefficient$ for $spontaneous emission$.

3. Stimulated emission:



Stimulated emission is the phenomenon in which an atom ($\boldsymbol{A^*}$) in the excited state of energy E_2 de-excite to the lower energy state E1 under the influence of an external photon ($h\nu$) by emitting an identical photon of energy $h\nu = (E_2 - E_1)$.
Mathematically,$s represented as$ $\boldsymbol{h\nu + A^* \rightarrow A + h\nu + h\nu}$

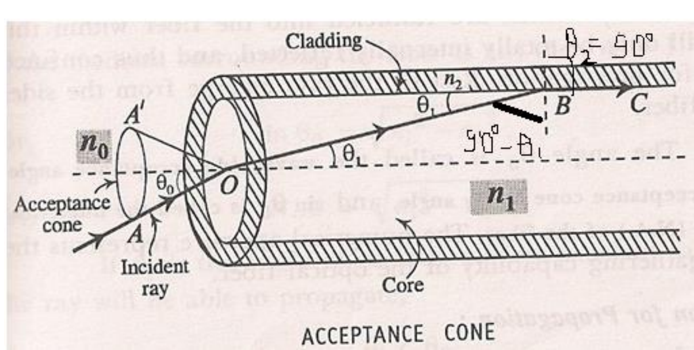
Also, Rate of Stimulated emission= $B_{21}N_2\,E\nu$,

where $U\nu = energy density$&

$B_{21} = Einstein's coefficient$ for $Spontaneous emission$.

1 b **Define Acceptance angle and Numerical Aperture and hence derive an expression for NA in terms of RIs core, cladding and surrounding.**

Acceptance angle (θ_0) is the maximum angle of incidence on the core for which TIR takes place inside the core.
Numerical aperture (NA) is the most important parameter of an optical fiber. It measures how much light signals can be collected by an optical fiber. Based on the refractive indices of core and cladding, we can measure the value of NA.
Derive an expression for acceptance angle or Numerical aperture of an optical fiber.



Consider a light ray AO incident at an angle 'θ_0' enters into the fiber. Let

'θ_1' be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle (90^0- θ_1) at B grazes the interface between core and cladding along BC.

If the angle of incidence is greater than critical angle, it undergoes total internal reflection. Thus θ_0 is called the waveguide acceptance angle and $\sin\theta_0$ is called the numerical aperture.

Acceptance angle is the maximum angle submitted by the ray with the axis of the fiber so that light can be accepted and guided along the fiber.

Let n_1, n_2&nobe the RI of core, cladding and launch medium respectively. Also OA incident ray, AB refracted ray, BC totally reflected ray, θ_i&θrbe the angles of incidence, refraction at A &θ&θrbe the angle of incidence and angle of refraction at B respectively.

By Snell's law at position "O",

Let n0, n1 and n2 be the refractive indices of the medium, core and cladding respectively.

From Snell's law,

$n_o\sin\theta_0 = n_1\sin\theta_1 \rightarrow (1)$

From Snell's law,

At 'B' the angle of incidence is ($90^0 - \theta_1$)

$$n_1\sin(90^0 - \theta_1) = n_2 \sin 90^0$$

$$n_1\cos\theta_1 = n_2$$

$$\cos\theta_1 = \frac{n_2}{n_1} \quad \rightarrow (2)$$

From equation (1), we have

Rearrange the equation (1)

$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{(1 - cos^2\theta_1)} \quad .... (3) \quad (\sin\theta 1 = \sqrt{(1 - cos^2\theta_1)})$$

Using eqn. (2) in (3)

Put the value of cos θ_1 in equn(3) from equn(2)

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{(1 - \frac{n_2^2}{n_1^2})}$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0} \text{ x } \frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad .... (4)$$

If the surrounding medium is air $n_0 = 1$, then

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

Where Sinθo is called numerical aperture (N.A).

$$\text{N.A} = \sqrt{n_1^2 - n_2^2}$$

Therefore, for any angle of incidence equal to θi equal to or less than θ_0, the incident ray is able to propagate.

$$\theta_i < \theta_0$$
$$\sin\theta_i < \sin\theta_0$$
$$\sin\theta_i < \sqrt{n_1^2 - n_2^2}$$

$\sin\theta_i < N.A$ is the condition for propagation.

Light will be transmitted through the fiber with multiple TIR, when the above condition is satisfied.

1 c **A LASER source has a power output of 10^{-3} W. Calculate the number of photons emitted per second given the wavelength of LASER 692.8 nm.**

P=$\mathbf{10^{-3}}$ W , λ =694.8 nm=694.8x10^{-9} m ,t= 1 s, C=3x108m/s ;
h=6.63x10^{-34} J-s, n =?

Using E=P.t and $E = \frac{n.hC}{\lambda}$,

$$we\ get\ n = \frac{Pt\lambda}{hC} = \frac{10^{-3} * 1 * 694.8\ x\ 10^{-9}}{6.63\ x\ 10^{-34} x\ 3x10^{-8}}$$

n =3.489x1011 photons/pulse.

2 a **Illustrate the construction and working of Semiconductor LASER with a neat sketch and energy level diagram also mention its applications.**

<u>Semiconductor Diode laser:</u>

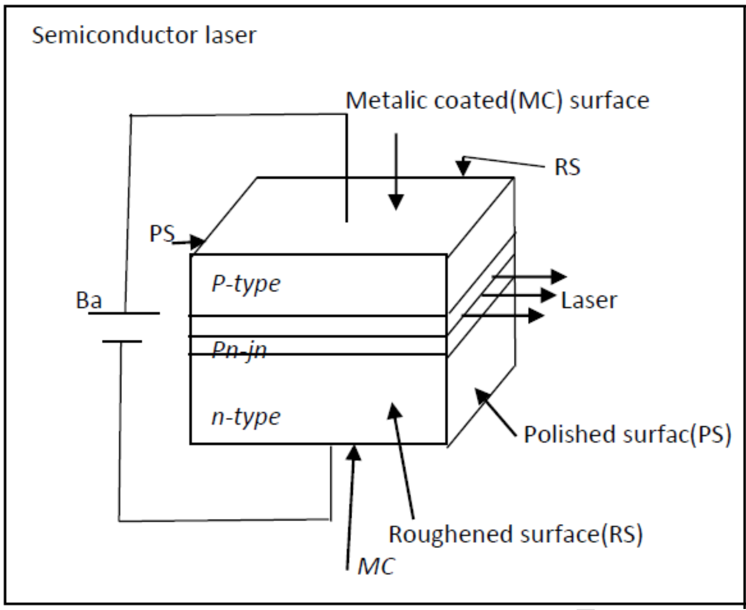
Principle, construction and working of Semiconductor Diode laser.

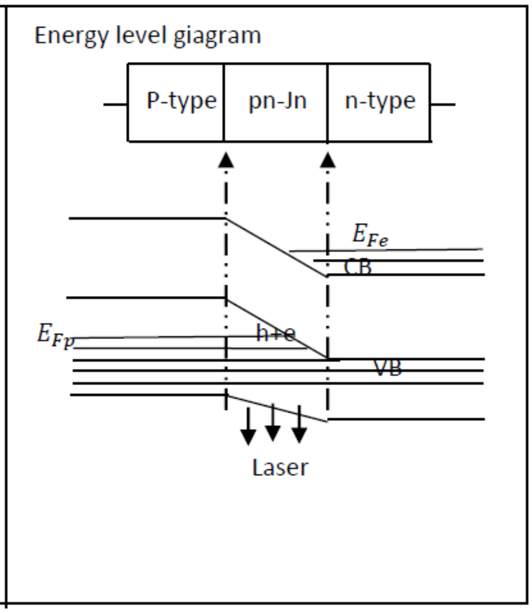
Construction: Metallic contacts are provided to the *P* and *N* types in a heavily doped *P-N* junction diode. Two opposite faces which are perpendicular to the plane of the junction are polished and made parallel to each other. These parallel faces constitute the resonant cavity and laser is obtained through these faces as shown in Figure. The remaining two faces are roughened to prevent lasing action in that direction.

1. The schematic diagram of Ga-As semiconductor device is as shown in the diagram.
2. It consists of heavily doped n-region of Ga-As doped with tellurium and p-region of Ga-As doped with zinc.
3. The dopants are added in the concentration of the order 10^{17} to 10^{19} number of dopants per cm^3.
4. The upper and lower surfaces are metalized so that pn-junction is forward

biased.

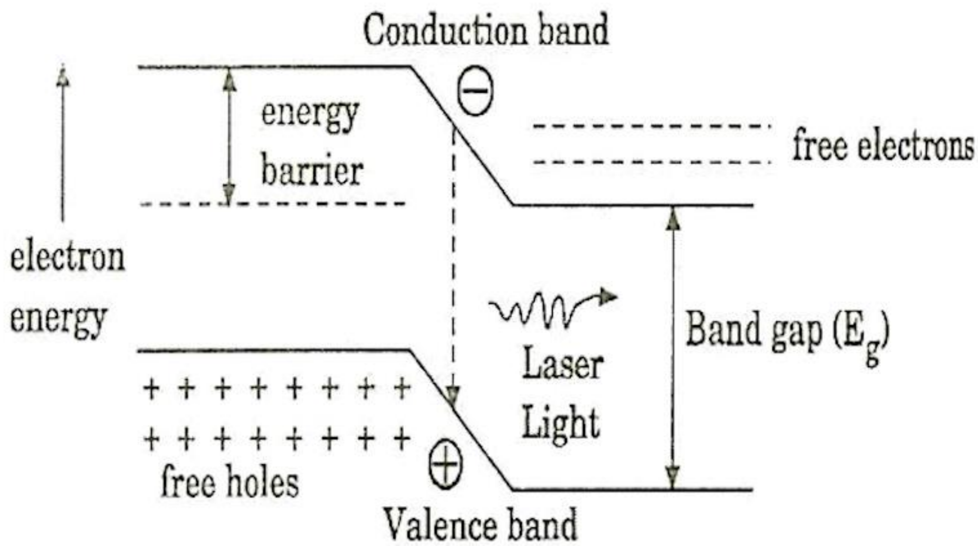
5. Two surfaces perpendicular to the junction are polished so that they act as optical resonators and the other two surfaces roughened to prevent lasing in that direction.





Principle: Semiconductor diode laser works on the principle of stimulated emission.

Energy Level Diagram :



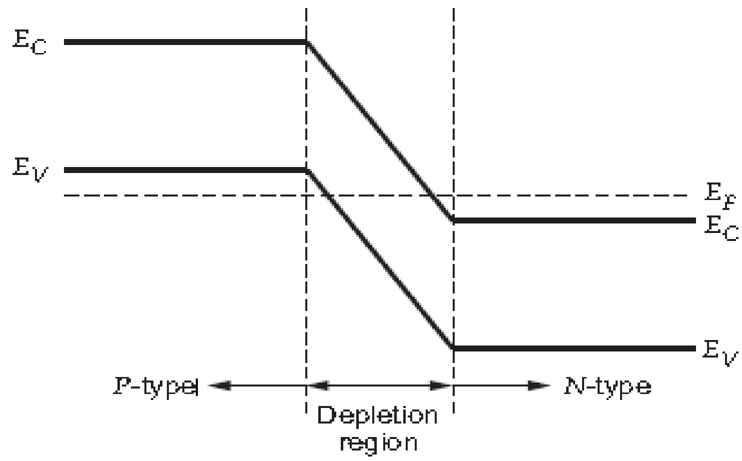
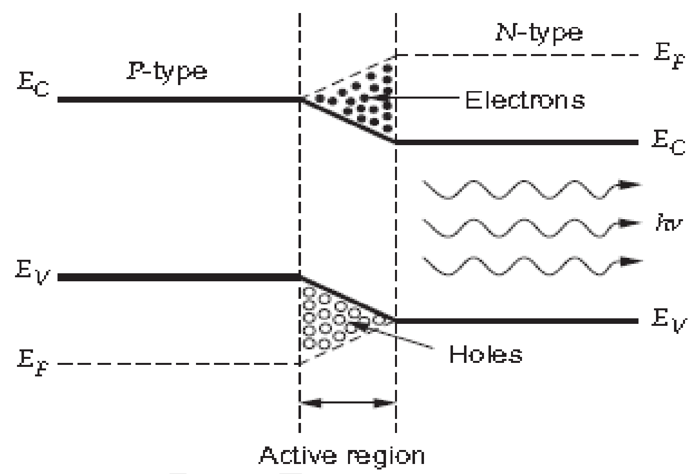


Fig. 5.2 (a)



Working:

1. Semi-conductor laser are made up of highly de-generate semi-conductors having direct band gap like Gallium Arsenide (GaAs).

2. When GaAS diode is forward biased with voltage nearly equal to the energy gap voltage, electrons from n-region & holes from p-region flow across the junction creating population inversion in the active jn region.

3. As the voltage is gradually increased due to forward biasing population inversion is achieved between the valence band and conduction band which in turn result in stimulated emission.

4. Photons produced are amplified between polished optical resonator surfaces producing laser beam.

5. GaAs laser produce laser beam of wavelength 8870Å in IR region , GaAsP produce laser beam of 6500Å in visible region etc.

Spontaneously emitted photon may trigger stimulated emissions over a large number of recombinations leading to build of laser radiation of high power. The energy gap of GaAs is 1.4 eV, the wavelength of emitted light

$$\lambda = \frac{hc}{E_g} = 8400A^0$$

Since energy gap is a function of temperature, the wavelength of laser can be tuned anywhere between 8400A^0 to 9000A^0 .

2 b

Discuss the types of optical fibers based on Modes of Propagation and RI profile.

Modes of propagation:

The paths along which the light is guided in the fiber are called modes of propagation and the number of modes of the fiber is given by N= $\frac{V^2}{2}$.

Types of optical fibers:

In an optical fiber the refractive index of cladding is uniform and the refractive index of core may be uniform or may vary in a particular way such that the refractive index decreases from the axis radialy.

Refractive index profile:

The curve which represents tha variation of R.I. w.r.t. the radial distance from the axis of the fiber is called *refractive index profile*.

Following are the different types of fibers:

1. Single mode fiber
2. Step index multimode fiber
3. Graded index multimode fiber

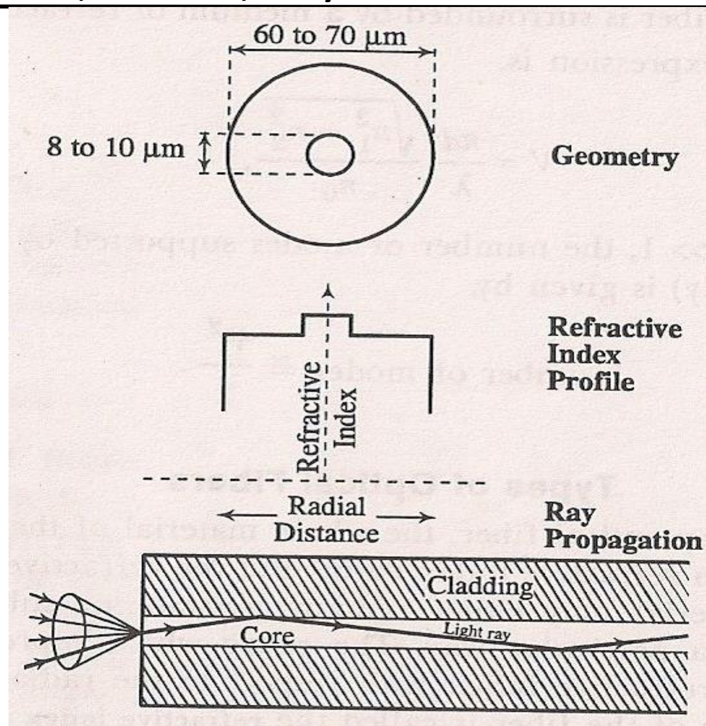
This classification is done depending on the refractive index profile and the number of modes that the fiber can guide.

***1. Single mode fiber*:**

A single mode fiber has a core material of uniform R.I. value. The cladding is also of uniform R.I. But the R.I. of cladding is less than that of the core. This results in a sudden increase in the value of R.I. from cladding to core.

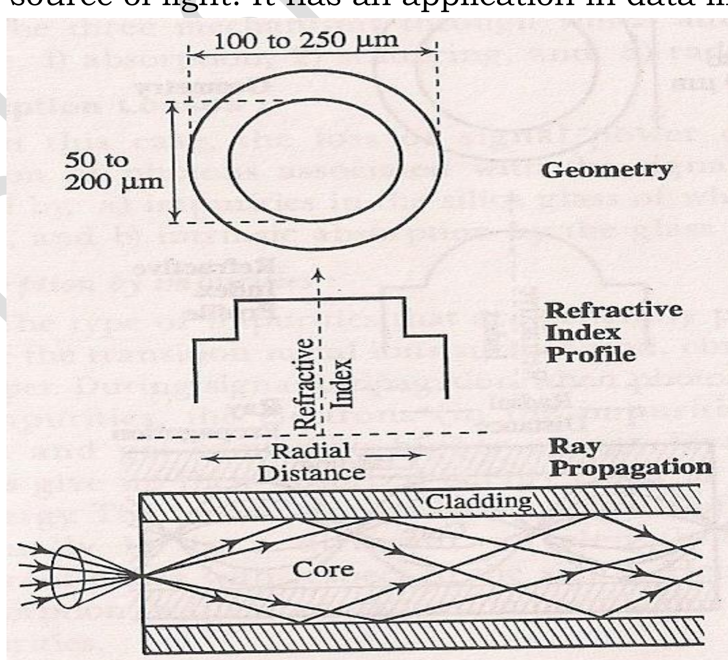
Thus its R.I. profile takes the shape of a step. The diameter value of the core is about 8 to 10μm and external diameter of cladding is 60 to 70μm. Because of its narrow core, it can guide just a single mode as shown in figure. Refractive index of core and cladding has uniform value; there is an increase in refractive index from cladding to core.

Applications: They are used in submarine. It is used in long hand communication due to higher bandwidth. They transmit transformation to longer distance due to negligible dispersion.



SINGLE MODE FIBER

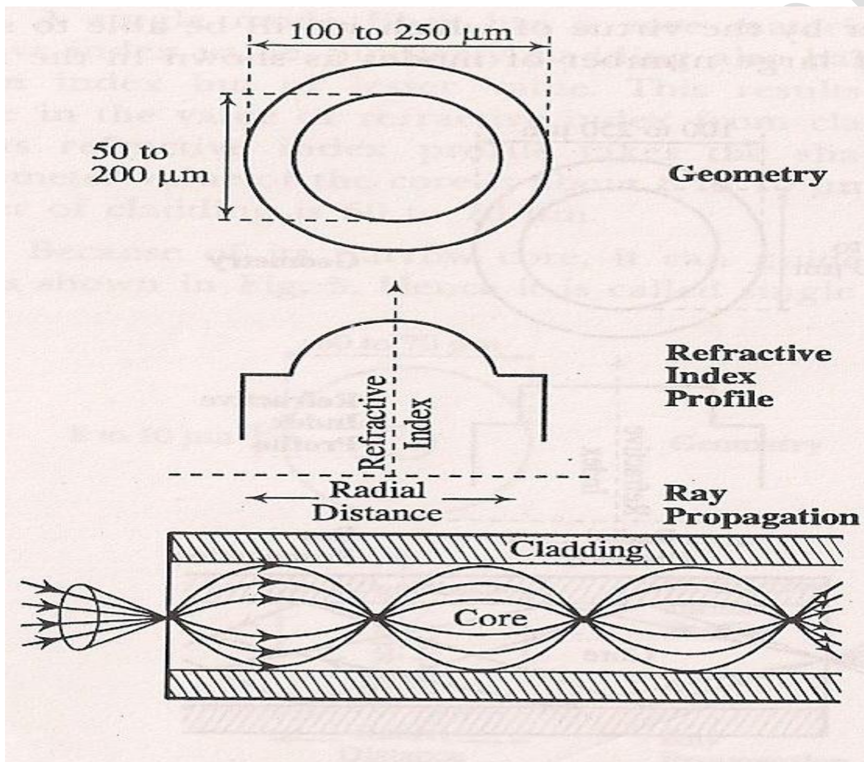
2. *Step index multimode fiber:* It is similar to single mode fiber but core has large diameter. The diameter value of the core is about 50 to 200μm and external diameter of cladding is 100 to 250μm. But the core is comparatively larger in diameter. It can propagate large number of modes as shown in figure. Laser or LED is used as a source of light. It has an application in data links.



STEP - INDEX MULTIMODE FIBER

Applications: They are used in submarine. It is used in short distance communication due to lower bandwidth. They transmit transformation to shorter distance due to negligible dispersion.

3.*Graded index multimode fiber:* It is also called GRIN. The geometry of the GRIN multimode fiber is similar to that of step index multimode fiber. Its core material has a special feature that its R.I. value decreases in the radially outwards direction from the axis and becomes equal to that of the cladding at the interface. But the R.I. of the cladding remains uniform. The refractive index profile is shown in figure. The incident rays bends and takes a periodic path along the axis. The rays have different paths with same period. Laser or LED is used as a *source of light*. It is the expensive of all. It is used in telephone trunk between central offices.



GRADED - INDEX MULTIMODE FIBER

Applications: They are used in the telephone trunk between central offices. It is used in short distance communication due to lower bandwidth. They transmit transformation to shorter distance.

2 c **Obtain the attenuation co-efficient of the given fiber of length 1500 m given the input and output power 100 mW and 70 mW.**

Given data : $P_{out} = 70 \times 10^{-3}$ W $P_{in} = 100 \times 10^{-3}$ W ; L = 1500m = 1.5 km; α=? ,

Using $\alpha = -\frac{10}{L}\log_{10}\left[\frac{P_{out}}{P_{in}}\right] = -\frac{10}{1.5}\log_{10}\left[\frac{70}{100}\right]$

$$= \mathbf{1.412\ dB/km}$$

3 a **Set-up Schrodinger's time independent wave equation in one dimension.**

Expression for Time independent Schrodinger wave equation:

Consider a particle of mass 'm' moving with velocity 'v'.
The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{mV} = \frac{h}{P} \text{.................} \quad (1)$$

Where 'mv' is the momentum of the particle.
The wave eqn is one dimensional wave function **Ψ** for the de-Broglie wave of a particle moving along the positive direction of x-axis is given by

$$\mathbf{\Psi} = Ae^{i(kx-\omega t)} \quad \text{.........} \ (2)$$

Where 'A' is a constant and 'ω' is the angular frequency of the wave.
Where Ae^{ikx} *represent the time independent part of the wave function and is represented* by $\psi = Ae^{ikx}$

Differentiating eqn (2) w.r.t 't' twice we get

$\frac{d\psi}{dt} = (-i\omega)^{(kx-\omega t)}$ and

$$\frac{d^2\psi}{dt^2} = A(-i\omega)^2 e^{i(kx-\omega t)}$$

$$= (-i)^2 \omega^2 A\, e^{i(kx-\omega t)}$$

$$= (-1)\, \omega^2 A\, e^{i(kx-\omega t)}$$

$$\frac{d^2\psi}{dt^2} = -\omega^2\psi \text{ ....} \ (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Where 'y' is the displacement and 'v' is the velocity.
Similarly for the de- Broglie wave associated with the particle
By analogy, we can write the wave equation for the de-Broglie wave for the motion of a free particle as

Here, y = ψ

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \quad \text{.......} \ (4)$$

The above equation represents waves propagating along the x-axis with a velocity "v" and 'ψ' is the displacement at time 't'.

From eqns 3 & 4, we get

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi \quad \ldots\ldots. (5)$$

If λ and ν are the wavelength and frequency of the wave, then

But ω = 2πf = 2πν and w.k.t. c = ν λ or v = ν/ λ $\nu = \frac{v}{\lambda}$

ν be the frequency of radiations and λ be the wavelength of the radiations

The angular frequency, $\omega = 2\pi\nu = 2\pi\frac{v}{\lambda}$

Squaring on both sides, we get

$$\omega^2 = \frac{4\pi^2 v^2}{\lambda^2} \quad \text{or} \quad v^2 = \frac{\omega^2\lambda^2}{4\pi^2}$$

Put the value of v^2 in the equation (5), we get

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

$$\frac{d^2\psi}{dx^2} = \frac{-4\pi^2}{\lambda^2}\psi \ldots\ldots.. (6)$$

$$\frac{1}{\lambda^2} = -\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2} \ldots\ldots..(7)$$

For a particle of mass "m"moving with a velocity "v"

Kinetic energy = K.E. = $\frac{1}{2}mv^2$

K.E. = $\frac{m^2v^2}{2m}$ (Dividing & multiplying by "m")

$$\text{K.E.} = \frac{p^2}{2m} \quad \ldots\ldots\ldots (8) \quad (p = mv)$$

But, from equation(1), we have

But, de-Broglie wave length $\lambda = \frac{h}{p}$

$$p = \frac{h}{\lambda}$$

By substituting for "p" in equation (8)

$$\text{K.E.} = \frac{(h/\lambda)^2}{2m} \text{K.E.} = \frac{h^2}{2m\lambda^2}$$

$$\text{K.E.} = \frac{h^2}{2m}\cdot\frac{1}{\lambda^2} \quad \ldots\ldots(9)$$

By substituting $\frac{1}{\lambda^2}$ from equation (7)

Equation (9) becomes.

$$\text{K.E.} = \frac{h^2}{2m}\left[-\frac{1}{4\pi^2\psi}\frac{d^2\psi}{dx^2}\right]$$

$$\text{K.E.} = -\frac{h^2}{8\pi^2 m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} \quad \ldots\ldots(10)$$

Let there be a field where the particle is present. Depending on its position in the field, the particle will access certain potential energy V. Then, the total energy E of the particle is the sum of kinetic energy and & the potential energy (V) is given by

Total energy(E) = kinetic energy + potential energy(V)

$$E = -\frac{h^2}{8\pi^2 m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V$$

$$-\frac{h^2}{8\pi^2 m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} = E - V$$

$$\frac{d^2\psi}{dx^2} = \frac{-8\pi^2 m}{h^2}(E - V)\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \quad \ldots\ldots\ldots(11)$$

This is Schrodinger time independent equation for a particle.

For a free particle, V=0

$$\therefore \frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE\psi}{h^2} = 0$$

This is Schrodinger time independent equation for free particle.

3 b

State and Explain Heisenberg's Uncertainty principle and Principle of Complementarity.

Statement: The Heisenberg's uncertainty principle states that "It is impossible to determine both the exact position and the exact momentum of a particle at the same time. The product of uncertainties in these quantities is always greater than or equal to $\frac{h}{4\pi}$ ".

$$\Delta x . \Delta P_x \geq \frac{h}{4\pi}$$

Heisenberg's Uncertainty Principle states that "the product of the uncertainties in the simultaneous measurement of the position (Δx) in the position and momentum (ΔP_x) of a particle is equal to or greater than (h/4**π**)"

$$\therefore \Delta x.\Delta P x \geq \frac{h}{4\pi}, \quad \text{where h is Planck's constant.}$$

The significance of HUP is that, it is impossible to determine simultaneously both the position and momentum of the particle accurately at the same instant.

$\Delta \boldsymbol{E}.\Delta \boldsymbol{t} \geq \frac{h}{4\pi}$, ***where*** $\Delta \boldsymbol{E}$=***energy***, $\Delta \boldsymbol{t}$= **time,**

and $\Delta \boldsymbol{L}.\Delta \boldsymbol{\theta} \geq \frac{h}{4\pi}$,**where,** $\Delta \boldsymbol{L}$=***angular momentum*** & $\Delta \boldsymbol{\theta}$ =***displacement***.

Principle of Complementarity :

Statement: Bohr stated as "In a situation where the wave aspect of a system is revealed, its particle aspect is concealed; and, in a situation where the particle aspect is revealed, its wave aspect is concealed. Revealing both simultaneously is impossible; the wave and particle aspects are complementary."

Explanation:

We know that the consequence of the uncertainty principle is both the Wave and particle nature of the matter cannot be measured simultaneously. In other words, we cannot precisely describe the dual nature of Light.

3 c **An electron is kinetic energy 500 keV is in vacuum. Calculate the group velocity and de Broglie wavelength assuming the mass of the moving electron is equal to the rest mass of electron.**

4 a **Discuss the motion of a quantum particle in a one-dimensional infinite potential well of width 'a' and also obtain the eigen functions and energy eigen states.**

Eigen functions:

Eigen functions are those wave functions in Quantum mechanics which possesses the properties:

1. They are single valued.
2. Finite everywhere and
3. The wave functions and their first derivatives with respect to their variables are continuous.

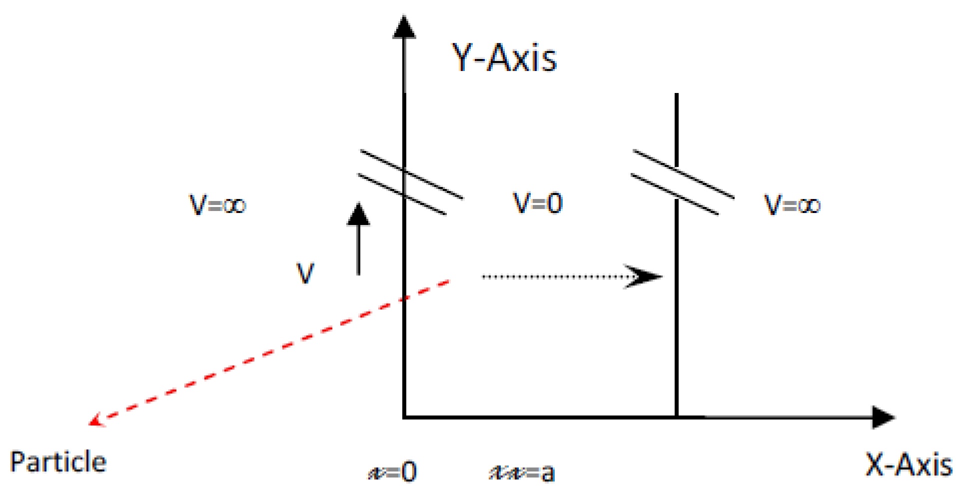
Eigen values:

According to the Schrodinger equation there is more number of solutions. The wave functions are related to energy E. The values of energy En for which Schrodinger equation solved are called Eigen values.

Application of Schrodinger wave equation:

Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box.

Wave function for a free particle in a infinite walled potential Box/Well using Schrodinger 1D time independent equation.



Consider a particle of a mass 'm' free to move in one dimension along positive x -direction between x =0 to x =a. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2 2m(E-V)\psi}{h^2} = 0$$

This is Schrodinger time independent equation for a particle.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E-\infty)\psi}{h^2} = 0 \;........ (1) \qquad (\because V = \infty)$$

For outside, the equation holds good if ψ = 0 & $|\psi|^2 = 0$. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E)\psi}{h^2} = 0 \;..... (2) \qquad (\because V = 0)$$

This is an Eigen-value equation.

Putting $\frac{8\pi^2 m(E)}{h^2} = K^2$ .....(3) in equation (2) , we get

$\frac{d^2\psi}{dx^2} + K^2 = 0$ ......(4)

The general solution of the quadratic equation (4) is of the form

The solution of this equation is:

$\psi = C \cos kx + D \sin kx \rightarrow (5)$

Where C & D are constants determined from boundary conditions as follows :

$\psi(x) = 0$ at $x = 0$ from eqn (5),

at $x = 0 \rightarrow \psi = 0$

$0 = C \cos 0 + D \sin 0$

$\therefore C = 0$

Also $x = a \rightarrow \psi = 0$

$0 = C \cos ka + D \sin ka$

But C = 0

$\therefore D \sin ka = 0$ (5)

$D \neq 0$ (because the wave concept vanishes)

$Ka = \sin^{-1}(0)$

i.e. $ka = n\pi$ where $n = 0, 1, 2, 3, 4...$ (Quantum number)

$k = \frac{n\pi}{a}$ .... (6)

Using this equation (5), we have

$$\psi_n = D \sin \frac{n\pi}{a} x \rightarrow (7)$$

Which gives permitted wave functions.

To find out the value of D, normalization of the wave function is to be done.

$$\text{i.e.} \int_0^a |\psi_n^2| dx = 1 \rightarrow (8)$$

using the values of ψ_n from eqn (7)

4 b **Explain the physical significance of the Wave Function.**

Physical significance of wave function:

Probability density: If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If 'τ' is the volume in which the particle is present but where it is exactly present is not known. Then the probability of finding a particle in certain elemental volume dτ is given by $|\psi|^2 d\tau$. Thus $|\psi|^2$ is called probability density. The probability of finding an event is real and positive quantity.

In the case of complex wave functions, the probability density is

$|\psi|^2 = \psi^* \psi$ where ψ^* is Complex conjugate of ψ.

Normalization:

The probability of finding a particle having wave function 'ψ' in a volume 'dτ' is '|ψ|²dτ'. If it is certain that the particle is present in finite volume 'τ', then

$$\int_0^\tau |\psi|^2 d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^{\infty} |\psi|^2 d\tau = 1$$

4 c **The speed of electron is measured to within an uncertainty of 2×10^4 ms^{-1} in one dimension. What is the minimum width required by the electron to be confined in an atom?**

5 a

Define a bit and qbit and explain the properties of qubit.

Concept of Qubit and its properties

Concept of Qubit

The counterpart of a classical bit in quantum computing is Qubit. It's the basic unit in which of information in a quantum computer. Superposition, Entanglement, and Tunneling are all special properties that define a qubit.

Properties of Qubits

1. A qubit can be in a superposed state of the two states 0 and 1.
2. If measurements are carried out with a qubit in superposed state then the results that we get will be probabilistic unlike how it's deterministic in a classical computer.
3. Owing to the quantum nature, the qubit changes its state at once when

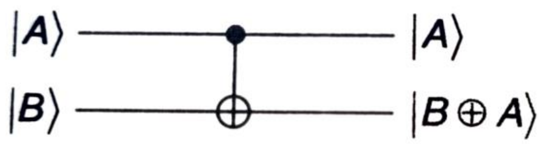
subjected to measurement. This means, one cannot copy information from qubits the way we do in the present computers, as there will be no similarity between the copy and the original. This is known as "no cloning principle".

A Qubit can be physically implemented by the two states of an electron or horizontal and vertical polarizations of photons as $|\downarrow\rangle$ and $|\uparrow\rangle$.

5 b **Discuss the CNOT gate and its operation on four different input states.**

Controlled Not Gate or CNOT Gate

The CNOT gate is a typical multi-qubit logic gate and the circuit is as follows.
The Matrix representation of CNOT



CNOT gate

Gate is given by

$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Transformation could be expressed as

$$|A, B\rangle \rightarrow |A, B \oplus A\rangle$$

Consider the operations of CNOT gate on the four inputs

$|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$.

Operation of CNOT Gate for input $|00\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$.
Hence no change in the state of Target qubit $|0\rangle$

$$|00\rangle \rightarrow |00\rangle$$

Operation of CNOT Gate for input $|01\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$.
Hence no change in the state of Target qubit $|1\rangle$

$$|01\rangle \rightarrow |01\rangle$$

Operation of CNOT Gate for input $|10\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|1\rangle$.
Hence the state of Target qubit flips from $|0\rangle$ to $|1\rangle$.

$$|10\rangle \rightarrow |11\rangle \quad (6.26)$$

Operation of CNOT Gate for input $|11\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|1\rangle$.
Hence the state of Target qubit flips from $|1\rangle$ to $|0\rangle$.

$$|11\rangle \rightarrow |10\rangle$$

The Truth Table of operation of CNOT gate is as follows

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

5 c **A Linear Operator 'X' operates such that X $|0\rangle = |1\rangle$ and X $|1\rangle = |0\rangle$. Find the matrix representation of 'X'.**

Matrix Representation of 0 and 1 States

The wave function could be expressed in ket notation as $|\psi\rangle$ (ket Vector), ψ is the wave function.

The $|\psi\rangle$ =

The $|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$

The matrix for of the states $|0\rangle$ and $|1\rangle$. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

6 a **State the Pauli matrices and apply Pauli matrices on the states $|0\rangle$ and $|1\rangle$.**

Pauli Matrices

Pauli Matrices and Their operation on $|0\rangle$ and $|1\rangle$ States
There are four extremely useful matrices called Pauli Matrices.
The Pauli matrices of the following form

Pauli Matrices operating on $|0\rangle$ and $|1\rangle$ States

1. $\sigma_0 |0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle.$

$\sigma_0 |1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle.$

2. $\sigma_x |0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$

$\sigma_x |1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle.$

3. $\sigma_y |0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i\,|1\rangle$

$\sigma_y |1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i\,|0\rangle.$

4. $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -\,|1\rangle.$

6 b

Elucidate the differences between classical and quantum computing.

Differences Between Classical and Quantum Computing:

Classical Computing:
1. Used by large scale, multipurpose and devices.
2. Information is stored in bits.
3. There is a discrete number of possible states. Either 0 or 1.
4. Calculations are deterministic. This means repeating the same inputs results in the same output.
5. Data processing is carried out by logic and in sequential order.
6. Operations are governed by Boolean Algebra.
7. Circuit behavior is defined by Classical Physics.

Quantum Computing:
1. Used by high speed, quantum mechanics-based computers.
2. Information is based on Quantum Bits.
3. The is an infinite, continuous number of possible states. They are the result of quantum superposition.
4. The calculations are probabilistic, meaning there are multiple possible outputs to the same inputs.

5. Data processing is carried out by quantum logic at parallel instances.
6. Operations are defined by linear algebra by Hilbert Space.
7. Circuit behaviour is defined by Quantum Mechanics.

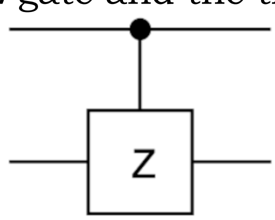
6 c **Describe the working of controlled-Z gate mentioning its matrix representation and truth-table.**

Controlled Z Gate

In Controlled Z Gate, The operation of Z Gate is controlled by a Control Qubit. If the control Qubit is $|A\rangle = |1\rangle$ then only the Z gate transforms the Target Qubit $|B\rangle$ as per the Pauli-Z operation. The action of Controlled
Z-Gate could is specified by a matrix as follows.

$$U_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The controlled Z gate and the truth table are as follows.



Define Fermi Factor and Discuss the variation of Fermi factor with temperature and energy.

7 a **Define Fermi Factor and Discuss the variation of Fermi factor with temperature and energy.**

Fermi factor:
Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

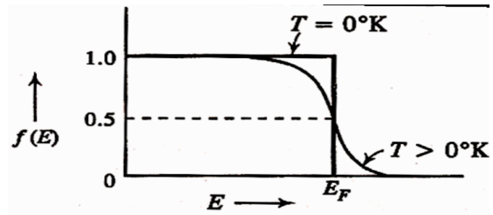
The probability f(E) that a given energy state with energy E is occupied by the electrons at a steady temperature T is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

f(E) is called the Fermi factor, E_F is the Fermi energy, T be the absolute temperature.

Dependence of Fermi factor with temperature and energy:

The dependence of Fermi factor on temperature and energy is as shown in the figure.



VARIATION OF $f(E)$ WITH E

i) Probability of occupation for E < E_F at T = 0^0 K**:**

When T = 0^0 K and E < E_F

$f(E) = \frac{1}{e^{(E-EF)/kT}+1}$ f(E) = $\frac{1}{e^{-\infty}+1}$ $= \frac{1}{0+1}$ = 1

The probability of occupation of energy state is 100%

f(E) = 1 for E < E_F.

f(E) = 1 means the energy level is certainly occupied and E < E_F applies to all energy levels below E_F. Therefore at T = 0^0 K all the energy levels below the Fermi level are occupied.

ii) Probability of occupation for E > E_F at T = 0^0 K**:**

When T = 0^0 K and E > E_F

$f(E) = \frac{1}{e^{(E-EF)/kT}+1}$ f(E)= $\frac{1}{e^{\infty}+1}$ $= \frac{1}{\infty}$ = 0

The probability of occupation of energy state is 0%

f(E) = 0 for E > E_F

… At T = 0K, all the energy levels above Fermi levels are unoccupied. Hence at T = 0^0 K the variation of f(E) for different energy values, becomes a step function as shown in the above figure.

ii) The probability of occupation at ordinary temperature(for E ≈ E_F at T > 0^0 K)

At ordinary temperatures though the value of probability remains 1, for E<EF it starts reducing from 1 for values of E close to but lesser than EF as in the figure.

The values of f(E)becomes ½ at E=E_F

This is because for E=E_F

$f(E) = \frac{1}{e^{(E-EF)/kT}+1}$ $f(E) = \frac{1}{e^{0}+1}$ $= \frac{1}{1+1} = \frac{1}{2}$

The probability of occupation of energy state is 50%

Further for E > E_F the probability value falls off to zero rapidly.

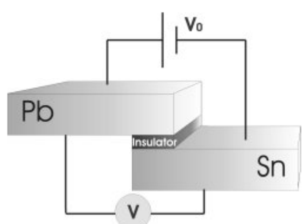
Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute. This may be considered as the physical basis for the concept of Fermi energy.

7 b **Explain DC and AC Josephson effects and mention the applications of superconductivity in quantum computing.**

JOSEPHSON JUNCTION

In 1962, Bian Josephson an English graduate student, made a startling prediction that cooper pairs can tunnel through a type of junction which came to be known later as Josephson junction.

A Josephson juntion is simply an arrangement of two superconductors separated by an insulating barrier as shown in Fig. A. When the barrier is thin enough, cooper pairs from one superconductor can tunnel through the barrier and reach the other superconductor. Based on his theory. Josephson proposed that this kind of tunneling leads to three kinds of effects. namely (1) dc Josephson effect, (2) ac Josephson effect and (3) quantum interference.



In quantum mechanics we know that an electron can tunnel through a barrier, even if its kinetic energy is less than the barrier potential, thereby producing a tunneling current. In this report, I shall describe a different kind of tunneling in the superconductors, in which superconducting pairing between electrons is important. This kind of tunneling, predicted by B. D. Josephson in 1962 [1], was first confirmed experimentally in 1963 by P. W. Anderson and J. M. Rowell [2]. Their experimental sample, a superconductor-insulator-superconductor (SIS) junction, is illustrated in Figure 1.

There are two kinds of Josephson tunneling, known as the DC and AC Josephson effects. We focus here only on the DC Josephson effect, which was the subject of the Anderson and Rowell paper. To begin with, I shall show how Anderson and Rowell measured this tunneling current by giving the (sketchy) plots of the apparatus and the data they had. After a brief review of Josephson's thoery, we will be able to understand how they can make sure at that time that what they measured was indeed the Josephson current based on a few experiment point of views, and

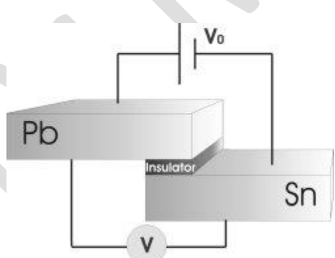


Fig. 1: The SIS Josephson junction used to detect the Josephson tunneling currect. The junc dimensions 0.025x0.065 cm^2.

hence ruled out the possibilities of ordinary electron tunneling.

Measurement of the Josephson Effect

The junction illustrated in Fig. 1 consists of an insulating oxide barrier between two thin lead and tin films, both of which are approximately 2000 angstrom thick. One possible way to fabricate the insulating barrier is by exposing the connection part between the two metals to the air. Anderson and Rowell applied a known voltage difference, say V_0, across the two metals (in the super-conducting state), and the voltage drop across the two metals can be determined by attaching a potentiometer across them. The way they did in 1963 was by measuring the corresponding voltage drop across a 10 ohm resistor on the loop connecting the two metals, and then the resultant Josephson tunneling current through the barrier can be known from the Ohm' law.

Their measured (tunneling) current versus (applied) voltage characteristic plot is given in Figure 2. It should be pointed out that the tunneling current is quite sensitive about the magnetic field as is shown in the figure. Even a few gauss difference can cause a lot of difference in the tunneling current (see the discussion in the Josephson's theory part). The linear part (in Figure 2) involves quasi-particle tunneling for the driven voltage is now large enough, while the current with voltage smaller than the threshold value is recognized as the Josephson tunneling current (the dashed line).

DC and AC Josephson effects :

Insulating barrier Superconductor-2 Quantum Interference :

As per dc Josephson effect, the tunneling of cooper pairs through the junction occurs without any resistance, which results in a steady dc current without any application of voltage between the two superconductors (Fig. A). But if a dc voltage is applied across the junction, the tunneling occurs in such a way that an ac current would develop in the junction & this effect is called ac Josephson effect. Now let us understand the effect known as quantum interference based upon which a SQUID is devised.

Applications in Quantum Computing: Charge, Phase and Flux qubits:

Charge Qubit :

In quantum computing, a **charge qubit** (also known as **Cooper-pair box**) is a qubit whose basis states are charge states (i.e. states which represent the presence or absence of excess Cooper pairs in the island). In superconducting quantum computing, a charge qubit[4] is formed by a tiny superconducting island coupled by a Josephson junction (or practically, superconducting tunnel junction) to a superconducting reservoir (see figure). The state of the qubit is determined by the number of Cooper pairs that have tunneled across the junction. In contrast with the charge state of an atomic or molecular ion, the charge states of such an "island" involve a macroscopic number of conduction electrons of the island.

Phase qubits:

In quantum computing, and more specifically in superconducting quantum computing, the **phase qubit** is a superconducting device based on the superconductor–insulator–superconductor (SIS) Josephson junction, designed to operate as a quantum bit, or qubit.

The phase qubit is closely related, yet distinct from, the flux qubit and the charge qubit, which are also quantum bits implemented by superconducting devices. The major distinction among the three is the ratio of Josephson energy vs charging energy (the necessary energy for one Cooper pair to *charge* the total capacitance in the circuit):

- For phase qubit, this ratio is on the order of 10^6, which allows for macroscopic bias current through the junction;
- For flux qubit it's on the order of 10, which allows for mesoscopic supercurrents (typically ~300 nA);
- For charge qubit it's less than 1, and therefore only a few Cooper pairs can tunnel through and charge the Cooper-pair box. However, transmon can have a very low charging energy due to the huge shunt capacitance, and therefore have this ratio on the order of 10~100

Flux qubits:

In quantum computing, more specifically in superconducting quantum computing, **flux qubits** (also known as **persistent current qubits**) are micrometer sized loops of superconducting metal that is interrupted by a number of Josephson junctions. These devices function as quantum bits. The flux qubit was first proposed by Terry P. Orlando et al. at MIT in 1999 and fabricated shortly thereafter.[1] During fabrication, the Josephson junction parameters are engineered so that a persistent current will flow continuously when an external magnetic flux is applied. Only an integer number of flux quanta are allowed to penetrate the superconducting ring, resulting in clockwise or counter-clockwise mesoscopic supercurrents (typically 300 nA) in the loop to compensate (screen or enhance) a non-integer external flux bias. When the applied flux through the loop area is close to a half integer number of flux quanta, the two lowest energy eigenstates of the loop will be a quantum superposition of the clockwise and counter-clockwise currents. The two lowest energy eigenstates differ only by the relative quantum phase between the composing current-direction states. Higher energy eigenstates correspond to much larger (macroscopic) persistent currents, that induce an additional flux quantum to the qubit loop, thus are well separated energetically from the lowest two eigenstates.

Flux qubits are fabricated using techniques similar to those used for microelectronics. The devices are usually made on silicon or sapphire wafers using electron beam lithography and metallic thin film evaporation processes. To create Josephson junctions, a technique known as shadow evaporation is normally used; this involves evaporating the source metal alternately at two angles through the lithography defined mask in the electron beam resist. This results in two overlapping layers of the superconducting metal, in between which a thin layer of insulator (normally aluminum oxide) is deposited.

7 c **Calculate the probability of occupation of an energy level 0.02 eV above fermi level at temperature 27°C.**

Given: $(E - E_F) = 0.02\ eV = 0.02x1.6x10^{-19}$ J , k=1.38x10^{-23} J/K and T=300K

Using $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

$$= \frac{1}{1+e^{(0.02x1.6x10^{-19})/1.38x10^{-23}\ x\ 300}}$$

$$= \frac{1}{1+e^{0.773}}$$

$$= \frac{1}{1+2.166}$$

$$= \frac{1}{3.166}$$

=0.32 or 32 %

8 a **Describe Meissner's Effect and hence classify superconductors into Soft and Hard superconductors using M-H graphs.**

Meissner effect:

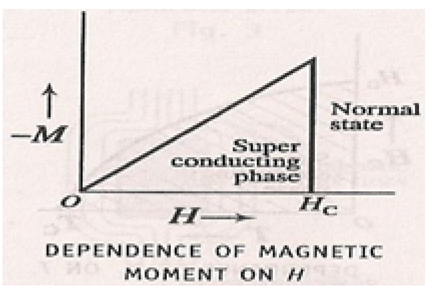
A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnetic. This effect is called Meissner effect.

Type of superconductors

1.Type I superconductors:

These materials exhibit complete Meissner effect and have well defined critical field H_c. These are perfect diamagnetic in the superconducting state and possesses negative magnetic moment.

The material remains in the superconducting when the field is less than the critical field. It expels the magnetic lines of force from the body of the mater immediately after H_c the material transits to normal state and the flux penetrates the material i.e. Meissner effect is absent. H_c is of the order of 0.1 T or less. Since H_c very low, even weak magnetic field can destroy the phenomenon. As weak magnetic field can penetrate the material more easily and they are also called *soft superconductor*.



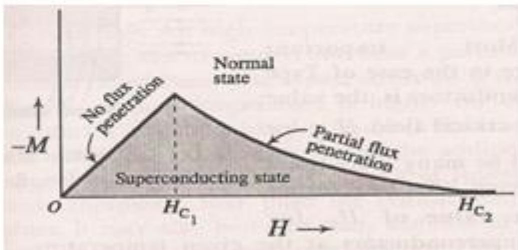
Element	T_c (K)
Aluminium	1.196
Cadmium	0.52
Gallium	1.09
Indium	3.40
Tin	3.72
Mercury	4.12
Lead	7.175

2.Type II Superconductors:

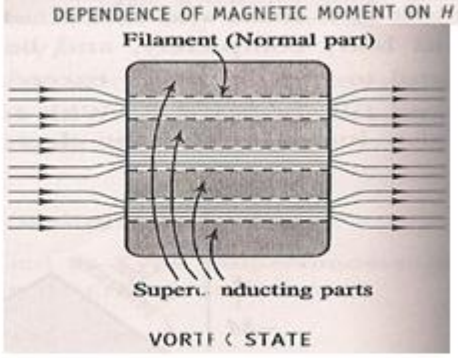
These materials are having two critical fields H_{c1} and H_{c2}. For the field less thenH_{c1} (lower critical field), it expels the magnetic field completely and there is no flux penetration. It becomes a perfect diamagnetic and the material is in the super conducting state. After Hc_1 the flux penetrates and partially fills the body of the material through channels called filaments. As the field is increased these filaments broaden and by Hc_2 (the material possesses both normal and superconducting state, hence the state is called *'Mixed State'*. This is also referred to as *'vortex state'* where the material is in a magnetically mixed state but electrically it is a superconductor. After Hc_2 the material transits to normal state and the resistance is finite. Type II superconductors can carry larger currents when the magnetic field is between Hc_1 and Hc_2.

Hc_2 the upper critical field is many a folds greater than Hc_1 the lower critical field. Only strong magnetic field of the order of 10T can penetrate the material hence these are called hard super conductor. Type-II superconductors are used in the

	manufacturing of the superconducting magnets of high magnetic fields above 10 Tesla.



DEPENDENCE OF MAGNETIC MOMENT ON H



Element	T_c (K)
Tantalum	4.5
Thallium	2.4
Niobium	9.3

8 b **Enumerate the assumptions of Quantum free Electron Theory of Metals**

Assumptions of quantum free electron theory:

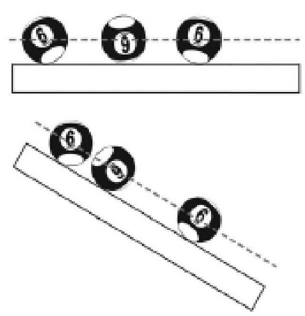
1. The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
2. The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
3. The electrons travel with a constant potential inside the metal but confined within its boundaries.
4. The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

8 c **Lead has superconducting transition temperature of 7.26 K. If the initial field at 0K is 50 × 10^3 Am^{-1} . Calculate the critical field at 6k.**

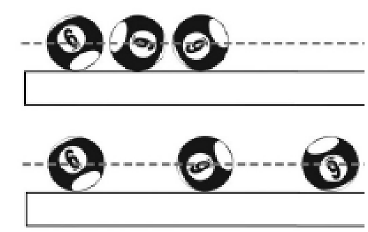
9 a **Discuss timing in Linear motion, Uniform motion, slow in and slow out.**

Linear Motion Timing

Linear motion refers to motion in a straight line, always in the same direction. An object moving with linear motion might speed up or slow down as it follows a linear path. A heavy ball rolling on a table or incline is an example of linear motion. The all is rotating, but its center of gravity follows a linear path. A heavy ball rolling on a table or incline is an example of linear motion. The ball is rotating, but its center of gravity follows a linear path.



Linear motion.



Uniform motion.

Uniform Motion Timing

When uniform motion occurs, the net force on the object is zero. Net force is the total of all forces added up. There might be several forces acting on the object, but when both the magnitude and direction of the forces are added up, they add up to zero. Uniform motion is the easiest to animate because the distance the object travels between frames is always the same. Uniform motion is a type of linear motion with constant speed and no acceleration or deceleration.

The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.

Slow in and Slow out

When motion is accelerating or decelerating, we refer to this type of motion as a slow in or slow out. These types of motion are sometimes called ease in or ease out. In this book, we use the hyphenated forms slow-in and slow-out for easier understanding.

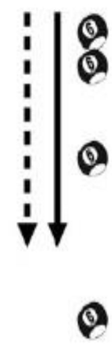
1. Slow in, ease in— The object is slowing down, often in preparation for stopping.
2. Slow out, ease out—The object is speeding up, often from a

still position.

The term slow out can be confusing, since it essentially means "speed up." one can think of slow out as the same as ease out, as in easing out of a still position and speeding up to full speed.

For example, a ball rolling down an incline or dropping straight down is slowing out, as it goes from a still position or slow speed to a fast speed. A ball rolling up an incline is slowing in.

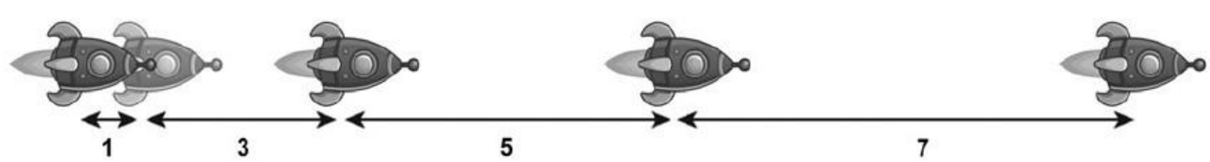
Slowing out: speeding up. Slowing in: slowing down.



9 c **Illustrate the odd rule and odd rule multipliers with a suitable example.**

The Odd Rule

When acceleration is constant, one can use the Odd Rule to time the frames. With this method, one calculate the distance the object moves between frames using a simple pattern of odd numbers. Between consecutive frames, the distance the object moves is a multiple of an odd number. For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.



Rocket speeding up using the Odd Rule.

For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.

Sled coming to a stop using the Odd Rule.

The Odd Rule is a multiplying system based on the smallest distance travelled between two frames in the sequence. For a slow-out, this is the distance between the first two frames; for a slow-in, it's the distance between the last two frames.

This distance, the base distance, is used in all Odd Rule calculations.

Odd Rule Multipliers:

The Odd Rule in its simplest form, as described above, is just one way to use it. For example, one can instead calculate the distance from the first frame to the current frame and use these distances to place the object on specific frames.

	Multiply by base distance to get distance between:	
Frame #	Consecutive frames	First frame and this frame
1	n/a	0
2	1	1
3	3	4
4	5	9
5	7	16
6	9	25
7	11	36

Calculating the distance for a large number of frames and a chart like this isn't practical, one can figure out the odd number multiplier for consecutive frames with this formula:

Odd number multiplier for consecutive frames = ((frame # – 1) * 2) – 1

In the charts above, note that the distances in the last column are squared numbers: $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, and so on.

One of the benefits of the Odd Rule is one can calculate the total distance travelled from the start point to the current frame with the following formula:

Multiplier for distance from first frame to current frame = (current frame # – 1) 2

When setting the keys, one can use either the consecutive key multipliers or total distance multipliers but need to Choose the one that's easiest to use for the animated sequence.

10 a **Describe Jumping and parts of jump.**

Jumping

A jump is an action where the character's entire body is in the air, and both the character's feet leave the ground at roughly the same time. A jump action includes a takeoff, free movement through the air, and a landing.

Parts of Jump

A jump can be divided into several distinct parts:

- **Crouch**—A squatting pose taken as preparation for jumping.
- **Takeoff**—Character pushes up fast and straightens legs with feet still on the ground. The distance from the character's center of gravity (CG) in the crouch to the CG when the character's feet are just about to leave the ground is called the push height. The amount of time (or number of frames) needed for the push is called the push time.
- **In the air**—Both the character's feet are off the ground, and the character's center of gravity (CG) moves in a parabolic arc as any free-falling body would. First it reaches an apex, and then falls back to the ground at the same rate at which it rose. The height to which the character jumps, called the jump height, is measured from the CG at takeoff to the CG at the apex of the jump. The amount of time the character is in the air from take-off to apex is called the jump time. If the takeoff pose and the landing pose are similar, then the jump height and jump time are about the same going up as they are going down.
- **Landing**—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.

Calculating Jump Actions

When working out the timing for a jump, one will need to first decide on:

1. Jump height or jump time
2. Push height
3. Stop height
4. Horizontal distance the character will travel during the jump

From these factors, one can calculate the timing for the jump sequence.

Calculating Jump Timing

When planning the jump animation, the most likely scenario is that you know the jump height, expressed in the units you are using for the animation (e.g., inch or cm).

Placement and timing for frames while the character is in the air follow the same rules as any object thrown into the air against gravity. Using the tables in the Gravity chapter (or an online calculator), one can figure out the jump time for each frame. Look up the amount of time it takes an object to fall that distance due to gravity, and express the jump time in frames based on the fps one is using.

Example:

Jump height = 1.2m
Jump time for 1.2m = 0. 5 seconds
Jump time at 30fps = 0. 5 * 30 = 15 frames

Jump Magnification

When calculating the remainder of the timing for the entire jump action, you can use a factor called jump magnification (JM). The JM can be used to calculate the push timing and stop timing.
The JM is the ratio of the jump height to the push height.

$$JM = \frac{Jump\ Height}{Push\ Height}$$

Since you already know the jump height and push height, you can calculate the JM. Then you can use the JM to calculate other aspects of the jump.

Example:

Jump Height = 1m
Push Height = 0.33m

$$\text{JM} = \frac{Jump\ Height}{Push\ Height} = 3$$

10 b **Discuss the salient features of Normal distribution using bell curves.**

Normal Distribution and Bell Curves

A bell curve is a common type of distribution for a variable, also known as the normal distribution. The term "bell curve" originates from the fact that the graph used to depict a Normal Distribution consists of a symmetrical bell-shaped curve. The highest point on the curve, or the top of the bell, represents the most probable event in a series of data (its Mean, Mode and Median in this case), while all other possible occurrences are symmetrically distributed around the mean, creating a downward-sloping curve on each side of the peak. The width of the bell curve is described by its Standard Deviation. The term "bell curve" is used to describe a graphical depiction of a normal probability distribution, whose underlying standard deviations from the mean create the curved bell shape. A standard deviation is a measurement used to quantify the variability of data dispersion, in a set of given values around the mean. The mean, in turn, refers to the average of all data points in the data set or sequence and will be found at the highest point on the bell curve.