CAMBRIDGE INSTITUTE OF TECHNOLOGY

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QUESTION BANK

MODULE 1- CALCULUS

1) Find the angle of intersection between the curves $r^2 sin2\theta = 4$ and $r^2 = 16Sin2\theta$.
2) Determine the angle of intersection between the curves $r^n = a^n(\sin n\theta + \cos n\theta)$ and $r^n = a^n \sin n\theta$.
3) Determine the angle of intersection for the curves $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$.
4) Find the angle between two curves $r = alog\theta$ and $r = \frac{a}{log\theta}$.
5) Find the angle between curves the $r = \frac{a}{1+cos\theta}$ and $r = \frac{b}{1-cos\theta}$
6) Find the angle between the curves $r = a(1 + cos\,\theta)\; and\; r = b(1 - cos\,\theta)$.
7) Find the angle of intersection for the curves $r = sin\,\theta + cos\,\theta\; andr = 2\,sin\theta$.
8) Find the angle between polar curves $r = a(1 + cos\,\theta)\; and\; r^2 = a^2 cos\,2\theta$.
9) Find the angle between the curves $r = a(1 - cos\,\theta)\; and\; r = 2a\,cos\,\theta$.
10) Show that the curves $r^n = a^n \cos n\theta\; and\; r^n = b^n \sin n\theta$ cut each other orthogonally.
11) Find the angle of intersection of the curves $r = sin\theta + cos\theta\; and\; r = 2\,sin\theta$.
12) Find the angle of intersection for the curves $r = 6\cos\theta\; and\; r = 2(1+\cos\theta)$.
13) find the angle between the radius vector and the tangent and also find the slop of the tangent $r = a(1 + cos\theta) at\; \theta = \pi/3$
14) Determine the angle of intersection for $r^2 \sin 2\theta = a^2 \quad and \quad r^2 \cos 2\theta = b^2$.
15) Determine the Pedal equation of the curve $r^2 = a^2 sec2\theta$.
16) Find the Pedal equation of the curve $r = a\,\text{cosec}^2 \frac{\theta}{2}$.
17) Find the Pedal equation of the curve $r^n = a^n(\sin n\theta)$.
18) Find the Pedal equation of the curve $r = 2(1 + \cos\theta)$.
19) Find the Pedal equation of the curve $r = a(1 - cos\theta)$.
20) Find the Pedal equation of the curve $r(1 - cos\theta) = 2a$.
21) Find the Pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$.
22) Determine the Pedal equation of the curve $r^m\, cosm\theta = a^m$.
23) Determine the Pedal equation of the curve $\frac{l}{r} = 1 + e\cos\theta$.
24) Determine the Pedal equation of $r^m = a^m(\cos m\theta + \sin m\theta)$.
25) Find the pedal equation of the curve $r = a\; e^{\theta\, cot\alpha}$.
26) Find the radius of curvature of the curve $x^3 + y^3 = 3xy\; at \left(\frac{3}{2}, \frac{3}{2}\right)$
27) Determine the radius of curvature of the curve $x^3 + y^3 = 3axy\; at\; \left(\frac{3a}{2}, \frac{3a}{2}\right)$ and show that $\rho = \frac{3a}{8}$.
28) Show that the radius of curvature at (a,0) on the curve $y^2 = \frac{a^2(a-x)}{x}$ is $\frac{a}{2}$
29) Determine the radius of curvature of the curve $y = 4sinx - sin2x$, at $x = \frac{\pi}{2}$ is $\frac{5\sqrt{5}}{4}$
30) Determine the radius of curvature of $x = a(cost + t\sin t), y = a(\sin t - t\, cost)$.
31) Find the radius of curvature for the cardioids $r = a(1 + \cos\theta)$.

32) Find the radius of curvature of the cure $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point where it cutes the line $y=x$.
33) Find the radius of curvature of the cure y=ax^2+bx+c $at\ x=1/2a(\sqrt{a^2-1-b}$
34) Determine the radius of curvature of the curve $y^2=\frac{a^2(a-x)}{x}at(a,0)$.
35) Show that the radius of curvature for the curve $r^n=a^n\cos n\,\theta$ varies inversely as r^{n-1}
36) Determine the radius of curvature of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ as $3(axy)^{\frac{1}{3}}$.
37) Determine the radius of curvature of the curve $r=a(1+\cos\theta)$, S.T $\frac{\rho^2}{r}=constant.$
38) Determine the radius of curvature of $r(1-cos\theta)=2a$ hence show that ρ^2 varies as r^3.
39) Show that the radius of curvature at any point of cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)\ is\ 4a\ cos\frac{\theta}{2}$
40) Determine the radius of curvature of $r^n=a^n cosn\theta$ and hence show that it varies inversely as r^{n-1}.
41) Find the radius of curvature of the curve $y=x^3(x-a)$ at point (a,0)

42) Find the radius of curvature of the curve $x^4+y^4=2$ at point (1,1)
43) Find the radius of curvature of the curve $a^2y=x^3+a^3$ at the point where the curves cut x-axis.
44) Determine the radius of curvature of the curve $r=a(1-\sin\theta)$.
45) Determine the center of curvature of the parabola $y^2=4ax$ at (x, y). Also find the equation of the evolute of the given parabola.
46) Find the center of curvature of the curve $y=x^3-6x^2+3x+1\ at\ (1,-1)$.
47) Determine the Evolute of $x^{2/3}+y^{2/3}=a^{2/3}$.
48) Determine the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

MODULE 2- SEREIS EXPANSION AND MULTIVARIABLE CALCULUS QUESTIONS BANK

1) Determine the Maclaurin's series expansion of $\log(\sec x)$ up to the terms containing fourth degree.
2) Determine the Maclaurin's series expansion of $log(\cos x)$ up to the terms containing fourth degree.
3) Expand $log(1+e^x)$ by Maclaurin's series the terms containing fourth degree.
4) Expand e^{sinx} by Maclaurin's series the terms containing fourth degree.
5) Express $\log(1+x)$ as Maclaurin's series up to fifth degree terms and hence prove that $$\log\sqrt{\frac{1+x}{1-x}}=x+\frac{x^3}{3}+\frac{x^5}{5}+\dots.$$
6) Express $log(1+sin\,x)$ in Maclaurin's series up to fourth degree term.
7) Express $log(1+cos\,x)$ in Maclaurin's series up to fourth degree term.
8) Express $\log(1+\sin x)$ in Maclaurin's series up to fourth degree term.
9) Express $\sqrt{1+\sin 2x}$ in Maclaurin's series up to the terms containing x^4.
10) Express $e^{\cos x}$in Maclaurin's series up to the term containing x^4.

11) Express $e^{\tan^{-1}x}$ in Maclaurin's series up to the term containing x^4.

12) Express $\tan^{-1}x$ in Maclaurin's series up to the terms containing fifth degree.

13) Find the value of the given indeterminate form $lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$.

14) Find the value of the given indeterminate form $lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{4}\right)^{\frac{1}{x}}$.

15) Find the value of the given indeterminate form $\lim_{x\to 0}\left(\frac{2^x+3^x+4^x}{3}\right)^{\frac{1}{x}}$.

16) Find the value of the indeterminate form $lim_{x\to\frac{\pi}{2}}(tan\,x)^{cos\,x}$.

17) Find the value of the indeterminate form $\lim_{x\to 0}(1+\sin x)^{\cot x}$.

18) Find the value of the indeterminate form $lim_{x\to 0}\left(\frac{sin\,x}{x}\right)^{\frac{1}{x^2}}$.

19) Find the value of the indeterminate form $lim_{x\to 0}(a^x+x)^{\frac{1}{x}}$.

20) Find the value of the given indeterminate form $lim_{x\to 0}(cosx)^{\frac{1}{x^2}}$.

21) Find the value of the given indeterminate form $lim_{x\to 0}(tan\,x)^{tan\,x}$.

22) Find the value of the given indeterminate form $lim_{x\to 0}(cot\,x)^{tan\,x}$.

23) Find the value of the indeterminate form $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$.

24) Find the value of the given indeterminate form $lim_{x\to\frac{\pi}{2}}(tan\,x)^{tan\,2x}$.

25) Find the value of the given indeterminate form $lim_{x\to\frac{\pi}{2}}(sin\,x)^{tan\,x}$.

26) Find the value of the given indeterminate form $\lim_{x\to 0}(\cot x)^{\tan x}$.

27) Find the value of the indeterminate form $\lim_{x\to a}\left(2-\frac{x}{a}\right)^{\tan\left(\pi x/2a\right)}$

28) Determine the total derivative of $u=x^3y^2+x^2y^3$ where $x=at^2$, $y=2at$.

29) Determine the total derivative of $u=xy^2+x^2y$ where $x=at^2$, $y=2at$.

30) Determine the total derivative of $u=\tan^{-1}\left(\frac{x}{y}\right)$ where $x=2t, y=1-t^2$.

31) Determine the total derivative of $u=tan^{-1}\left(\frac{y}{x}\right)$where $x=e^t-e^{-t}, y=e^t+e^{-t}$, find the total derivative $\frac{\partial u}{\partial t}$ using partial differentiation.

32) Determine the total derivative of $u=x^2-y^2$ where $x=e^t\cos t, y=e^t\sin t$.

33) If $u=f\left(\frac{x}{y},\frac{y}{z}\right)$ determine the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$.

34) If $z=\frac{x^2+y^2}{x+y}$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$.

35) If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ determine the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$.

36) If $u=f(2x-3y,3y-4z,4z-2x)$ prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$.

37) If $u=f(2x-3y,3y-4z,4z-2x)$ prove that $6\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial y}+3\frac{\partial u}{\partial z}=0$.

38) If $z=f(x+ay)+g(x-ay)$ show that $\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}$