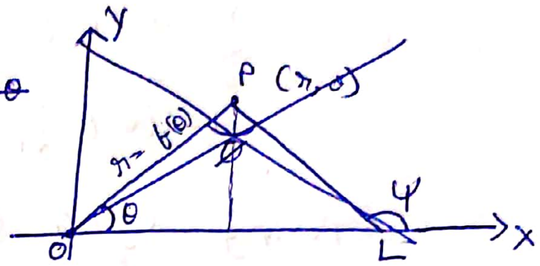
1) Find the expression for angle b/w radius vector & tangent ($\tan \phi = r \frac{d\theta}{dr}$)

Sol) Statement: Question:

let $P(r, \theta)$ be point on the curve $r = f(\theta)$, $\angle OPX = \theta$ & $OP = r$. Let PL be the tangent on the curve at point P. Subtending an angle $\psi$ with the ~~pos~~ positive direction of the inital line (x-axis) & $\phi$ is an ~~an~~ angle b/w radius vector & tangent.

From the figure ~~wh~~ we have ~~$\psi = \phi + \theta$~~

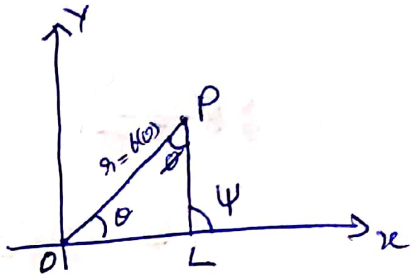



$$\psi = \theta + \phi$$

Tan on both sides

$$\tan \psi = \tan(\phi + \theta)$$

$$\tan \psi = \frac{\tan \phi + \tan \theta}{1 - \tan\theta \tan\phi} \longrightarrow (1)$$




let $x, y$ be the Cartisian coordinate of P ~~that we~~

$x = r\cos\theta$

$y = r\sin\theta$

Since r is an function of '$\theta$'

$$\therefore \tan\psi = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \tan\psi = \frac{\frac{d(r\sin\theta)}{d\theta}}{\frac{d(r\cos\theta)}{d\theta}} \Rightarrow \tan\psi = \frac{r\cos\theta + \sin\theta \frac{dr}{d\theta}}{-r\sin\theta + \cos\theta \frac{dr}{d\theta}}$$

$$\Rightarrow \tan\psi = \frac{r\cos\theta + \sin\theta\, r'}{\cos\theta\, r' - r\sin\theta}$$

$$\Rightarrow \tan\psi = \frac{r\cos\theta + \sin\theta\, r'}{r'\cos\theta - \sin\theta\, r'}$$

divide $r'\cos\theta$ in both numerator & denominator

$$\tan\psi = \frac{\frac{r\cancel{\cos\theta}}{r'\cancel{\cos\theta}} + \frac{\sin\theta\,\cancel{r'}}{\cos\theta\,\cancel{r'}}}{\frac{\cancel{r'}\cancel{\cos\theta}}{\cancel{r'}\cancel{\cos\theta}} - \frac{\sin\theta\, r}{\cos\theta\, r'}}$$

$$\tan\psi = \frac{\frac{r}{r'} + \tan\theta}{1 - \tan\theta\,\frac{r}{r'}} \rightarrow (2)$$

Comparing Eq (1) & (2) $\qquad \tan\psi = \dfrac{\tan\phi + \tan\theta}{1 - \tan\phi\,\tan\theta}$

~~$\tan\phi = \frac{r}{r'}$~~

$$\therefore \tan\psi = \frac{\frac{r}{r'} + \tan\theta}{1 - \tan\theta\,\frac{r}{r'}} = \frac{\tan\phi + \tan\theta}{1 - \tan\theta\,\frac{r}{r'}}$$

$$\therefore \tan\phi = \frac{r}{r'}$$

$$= \tan\phi = \frac{r}{\frac{dr}{d\theta}}$$

$$= \tan\phi = r\,\frac{d\theta}{dr}$$

$$\Rightarrow \cot\phi = \frac{1}{r}\,\frac{dr}{d\theta}$$

2) Determine the ~~curve~~ angle

$r = a(1+\cos\theta) \to ①$ $\qquad r = b(1-\cos\theta) \to ②$

Take log on both sides ①

$\log r = \log a + \log(1+\cos\theta)$

diff w.r.t $\theta$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cot\phi_1 = \frac{-\cancel{2}\sin\theta/2 \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$

$\cot\phi_1 = -\tan\theta/2$

$\cot\phi_1 = \cot\left(\frac{\pi}{2}+\theta/2\right)$

$\phi_1 = \frac{\pi}{2} + \theta/2$

② Take log on both sides ②

$\log r = \log b + \log(1-\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1-\cos\theta}(+\sin\theta)$

$\cot\phi_2 = \frac{\cancel{2\sin\theta/2}\ \cancel{2\sin\theta/2}\cos\theta/2}{\cancel{2}\sin^{\cancel{2}}\theta/2}$

$\cot\phi_2 = \cot\theta/2$

$\phi_2 = \frac{\theta}{2}$

$|\phi_2 - \phi_1| = \left|\frac{\theta}{2} - \frac{\pi}{2} - \theta/2\right| = \frac{\pi}{2}$

(3) $\frac{2a}{r} = 1 + \cos\theta$

log on both sides

$\log 2a - \log r = \log(1+\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cancel{-}\frac{1}{r}\frac{dr}{d\theta} = \cancel{-}\frac{\cancel{2}\sin\theta/2 \ \cancel{\cos\frac{\theta}{2}}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$

$\cot\phi = \tan\theta/2$

~~$\cot\phi = \frac{\pi}{2}$~~

$\cot\phi = \cot\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$\phi = \frac{\pi}{2} - \theta/2$

(4) ~~$r = a\log\theta$ & $r = \frac{a}{\log\theta}$~~

$p = r\sin\phi$

5) $r^n = a^n\cos m\theta + b^n\sin m\theta.$

6) $x = a\cos^3\theta \qquad y = b\sin^3\theta \qquad \theta = \frac{\pi}{4}$

diff w.r.t $\theta$

$$\rho = \frac{\left(1+(y')^2\right)^{3/2}}{y''}$$

$$\frac{dx}{d\theta} = 3\cancel{\cos^3} - 3\sin\theta\cos^2\theta\, a$$

$$\frac{dy}{d\theta} = 3\cos\theta\sin^2\theta\, b$$

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\cos\theta\sin^2\theta\, b}{-3\sin\theta\cos^2\theta\, a} = y' = -\frac{\sin\theta\, b}{\cos\theta\, a}$$

$$\cancel{\frac{dy}{dx} = \cot^3\theta} \Rightarrow y' = \cancel{\cot^3\theta} - \tan\theta\,\frac{b}{a}$$

$$y'' = \cancel{3\operatorname{cosec}^6\theta} - \sec^2\theta\,\frac{b}{a}$$

$$\cancel{\rho = \frac{(1+\cot^6\theta)^{3/2}}{-3\operatorname{cosec}^6\theta}}$$

$$\rho = \frac{\left(1+\tan^2\theta\,\frac{b^2}{a^2}\right)^{3/2}}{-\sec^2\theta\,\frac{b}{a}} \Rightarrow \rho = \frac{\left(\sec^2\theta\,\frac{b^2}{a^2}\right)^{3/2}}{-\sec^2\,\frac{b}{a}}$$

$$\rho = \frac{\sec^3\theta\,\frac{b^3}{a^3}}{-\cancel{\sec^2\theta}\,\frac{b}{a}} \Rightarrow \rho = \frac{\frac{\sec\theta\, b^3}{a^3}}{\frac{b}{a}}$$

$$\rho = \sec\theta\,\frac{b^2}{a^2}$$

7) $r = a e^{\theta \cot a}$

~~ta~~ Apply log

$\log r = \log a + \theta \cot a \log e$

diff w.r.t $\theta$

$\frac{1}{r} r' = 0 + 1 \cot a$

$\frac{1}{r} r' = \cot a$

$r' = r \cot a$

$\cancel{r'' = r' \cot a + r' \operatorname{cosec}^2 a}$

$r'' = r' \cot a$

$r'' = r \cot a \cot a$

$r'' = r \cot^2 a$

$\cancel{\rho = \frac{(1+(r')^2)^{3/2}}{r''}}$

$$\rho = \frac{(r^2+(r')^2)^{3/2}}{r^2+2r'^2-rr''}$$

$\cancel{\rho = \frac{(1+r^2\cot^2 a)^{3/2}}{r\cot^2 a}}$

$$\rho = \frac{((a e^{\theta \cot a})^2 + r^2\cot^2 a)^{3/2}}{r^2+2r^2\cot^2 a - r(r\cot^2 a)}$$

$$\rho = \frac{r^3 + r^3\cot^3 a}{r^2+2r^2\cot^2 a - r^2\cot^2 a}$$

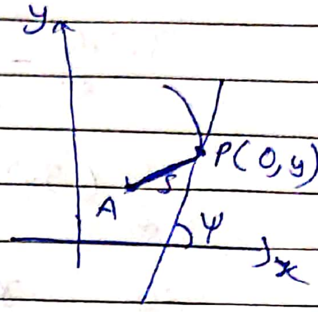
$$\rho = \frac{r^3(1+\cot^3 a)}{r^2(1+2\cot^2 a - \cot^2 a)} \Rightarrow \frac{r(1+\cot^3 a)}{1+\cot^2 a}$$

$$\frac{f}{\pi} = \frac{1+\cot^3 a}{1+\cot^2 a}$$

DATE / /
PAGE

8) Derive the formula for radius of curvature of the curve $\rho = \frac{(1+y_1^2)^{3/2}}{y''}$

Sol) Let $y=f(x)$ be the eqn of the cartesian curve & A be a fixed point on the curve such that $AP=s$




Let $\psi$ be the angle made by tangent at P with x-axis

We know that

$$\tan\psi = \frac{dy}{dx}$$

diff w.r.t s

$$\sec^2\psi \cdot \frac{d\psi}{ds} = \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds} \quad \because \frac{dx}{ds} = \cos\psi$$

$$\sec^2\psi \cdot \frac{1}{\rho} = \frac{d^2y}{dx^2}\cos\psi$$

$$\sec^2\psi = \rho\frac{d^2y}{dx^2}\cancel{\cos\psi}\frac{1}{\sec\psi} \Rightarrow \sec^3\psi = \rho\frac{d^2y}{dx^2}$$

$$\rho = \frac{\sec^3\psi}{\frac{d^2y}{dx^2}} \Rightarrow \rho = \frac{\sec^3\psi}{y''}$$

(divide & multiple 2 in the power)

$$\Rightarrow \rho = \frac{\sec^{\frac{3}{2}\times 2}\psi}{y''}$$

$$\Rightarrow \rho = \frac{(\sec^2\psi)^{3/2}}{y''} \Rightarrow \rho = \frac{(1+\tan^2\psi)^{3/2}}{y''}$$

$$\Rightarrow \rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{y''} \Rightarrow \rho = \frac{(1+(y')^2)^{3/2}}{y''}$$

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9) $x^3+y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

diff w.r.t $x$

$$3x^2 + 3y^2\frac{dy}{dx} = 3ay + 3ax\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 3ax\frac{dy}{dx} = 3ay - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 3ax) = 3ay - 3x^2$$

$$\frac{dy}{dx} = \frac{\cancel{3}(ay - x^2)}{\cancel{3}(y^2 - ax)}$$

$$\frac{dy}{dx} = \frac{a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2}{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)} \Rightarrow \frac{dy}{dx} = \frac{-\left(\cancel{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)}\right)}{\cancel{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)}}$$

$$\Rightarrow \frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

$$\frac{d^2y}{dx^2} = \frac{(ay' - 2x)(y^2 - ax) - \left(2y\frac{dy}{dx} - a\right)(ay - x^2)}{(y^2 - ax)^2}$$

$$y'' = \frac{(ay' - 2x)(y^2 - ax) - (2yy' - a)(ay - x^2)}{(y^2 - ax)^2}$$

$$y'' = \frac{\left(-a - \cancel{x}\,\frac{3a}{\cancel{x}}\right)\left(\left(\frac{3a}{2}\right)^2 - \boxed{\left(\frac{3a}{2}\right)}a\right) - (-3a - a)\left(\left(\frac{3a}{2}\right)a - \left(\frac{3a}{2}\right)^2\right)}{\left(\left(\frac{3a}{2}\right)^2 - \left(\frac{3a}{2}\right)a\right)^2}$$

$$y'' = \frac{(-4a)\left(\frac{9a^2}{4} - \frac{3a^2}{2}\right) - (-4a)\left(\frac{3a^2}{2} - \frac{9a^2}{4}\right)}{\left[\frac{9a^2}{4} - \frac{3a^2}{2}\right]^2}$$

DATE / /
PAGE

$$y'' = \frac{(-\cancel{4}a)\left(\frac{3a^2}{\cancel{4}}\right) - (-\cancel{4}a)\left(-\frac{3a^2}{\cancel{4}}\right)}{\left(\frac{3a^2}{4}\right)^2}$$

$$y'' = \frac{-3a^3 - 3a^3}{\frac{9a^4}{16}} \Rightarrow y'' = \frac{-\cancel{6}^{2}a^3}{\frac{3\cancel{9}a^4}{16}}$$

$$y'' = \frac{-32}{3a}$$

$$\rho = \frac{(1+(y')^2)^{3/2}}{y''} \Rightarrow \rho = \frac{(1+1)^{3/2}}{\frac{-32}{3a}}$$

$$\rho = \frac{(2)^{3/2}(3a)}{-32}$$

10) $x = a(\cos t + t\sin t)$ . $y = a(\sin t - t\cos t)$

diff. w.r.t      diff. w.r.t $t$

$\frac{dx}{dt} = a(-\sin t + \sin t + t\cos t)$      $\frac{dy}{dt} = a(\cos t - \cos t + t\sin t)$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{\cancel{a}(\cancel{t}\sin t)}{\cancel{a}\cancel{t}\cos t}$$

$\frac{dy}{dx} = +\tan t$      $y'' = +\sec^2 t \frac{dt}{dx}$

$$\therefore y'' = \frac{\sec^2 t}{a t\cos t} . \qquad \frac{1}{\cos t} = \sec t$$

$$y'' = \frac{\sec^2 t}{at}\sec t \Rightarrow y'' = \frac{\sec^3 t}{at}$$

$$f = \frac{(1+(y')^2)^{3/2}}{y''}$$

$$f = \frac{(1+(\tan t)^2)^{3/2}}{\frac{\sec^3 t}{at}} \Rightarrow f = \frac{\sec^3 t}{\frac{\sec^3 t}{at}}$$

$$\Rightarrow f = at$$

11) $r^n = a^n \cos n\theta$ $\qquad r^{n-1}$ $\qquad f = \frac{(r^2+(r_1)^2)^{3/2}}{2r_1^2 + r^2 - r r''}$

$n \log r = n \log a + \log \cos n\theta$

diff. w.r.t $\theta$

$$\frac{n}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-n \sin n\theta)$$

$$\frac{\cancel{n}}{r} r' = -\cancel{n} \tan n\theta$$

$$r' = -r \tan n\theta$$

$$r'' = -(r' \tan n\theta + n\sec^2 n\theta\, r) \Rightarrow r'' = -r'\tan n\theta - n\sec^2 n\theta\, r$$

$$f = \frac{(r^2 + (r_1)^2)^{3/2}}{2r_1^2 + r^2 - r r''}$$

$$f = \frac{(r^2 + r^2 \tan^2 n\theta)^{3/2}}{2r^2 \tan^2 n\theta + r^2 - r(-n\sec^2 n\theta\, r - r'\tan n\theta)}$$

$$f = \frac{r^3(1+\tan^2 n\theta)^{3/2}}{2r^2 \tan^2 n\theta + r^2 + n\sec^2 n\theta\, r^2 + r((-r\tan n\theta)\tan n\theta)}$$

$$f = \frac{\cancel{r^3}\{\sec^3 n\theta}{\cancel{r^2}(2\tan^2 n\theta + 1 + n\sec^2 n\theta - \tan^2 n\theta)}$$

$$f = \frac{r \sec^3 n\theta}{n\sec^2 n\theta + (\tan^2 n\theta + 1)} \Rightarrow \cancel{\frac{\sec r\sec^3 n\theta}{n\sec^2 n\theta}}$$

$$f = \frac{r\sec^{\cancel{3}} n\theta}{\cancel{\sec^2 n\theta}(n+1)} \Rightarrow f = \frac{r\sec n\theta}{n+1} \to (1)$$

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DATE / /
PAGE

From given

$$r^n = a^n \cos n\theta$$

$$\frac{1}{\cos n\theta} = \frac{a^n}{r^n}$$

$$\sec n\theta = \frac{a^n}{r^n} \rightarrow (2)$$

(2) in (1)

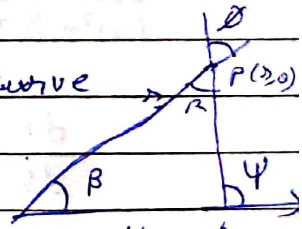
$$\rho = \frac{r\left(\frac{a^n}{r^n}\right)}{1+n} \Rightarrow \frac{\rho}{\cancel{r}} = \frac{r}{1+n}\ \frac{a^n}{r^n}$$

$$\rho = \frac{a^n}{r^{(n-1)}}\ \frac{1}{1+n}$$

12) Derive the formula for radius of curvature of the curve.

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{2r_1^2+r^2-r r_1''}$$

Sol) We have $r=f(\theta)$ be an eq.n of a ~~plo~~polar curve let $OP=r$ be the radius vector & $\phi$ be the angle made by the radius vector with the tangent at $P(r,\theta)$. $\psi$ be angle made by the tangent at $p$ with the & initial line. let $A$ be the fixed point on the curve & angle $AP=S$.




$$\psi = \theta + \phi$$

$$\frac{d\psi}{ds} = \frac{d\theta}{ds} + \frac{d\phi}{ds}$$

$$\Rightarrow \frac{d\psi}{ds} = \frac{d\theta}{ds} + \frac{d\phi}{d\theta} \times \frac{d\theta}{ds}$$

$$\Rightarrow \frac{d\psi}{ds} = \frac{d\theta}{ds}\left[1+\frac{d\phi}{d\theta}\right] \rightarrow (1)$$

We know that $\frac{d\psi}{ds} = \frac{1}{\rho}$ & $\tan\phi = r\frac{d\theta}{dr}$.

(2)

$\therefore \tan\phi = \frac{r}{\frac{dr}{d\theta}} \Rightarrow \tan\phi = \frac{r}{r'}$

& diff w.r.t $\theta$

$\sec^2\phi \frac{d\phi}{d\theta} = \frac{r'r' - r''.r}{(r')^2}$

$\frac{d\phi}{d\theta} = \frac{r^2 - r''r}{\sec^2\phi \, (r')^2}$ //

$\sec^2\phi = 1 + \tan^2\phi \Rightarrow 1 + \left(\frac{r}{r'}\right)^2$

$\therefore \frac{d\phi}{d\theta} = \frac{r^2 - r''r}{\left(1 + \frac{r^2}{r'^2}\right)(r')^2}$

$= \frac{d\phi}{d\theta} = \frac{r^2 - r''r}{\frac{r'^2 + r^2}{\cancel{r'^2}}} = \frac{d\phi}{d\theta} = \frac{r^2 - r''r}{r_1^2 + r^2} \rightarrow (3)$

The arc length of $\theta$ w.r.t $s$

$\sqrt{r^2 + r_1^2} = \frac{ds}{d\theta}$

$\therefore \frac{d\theta}{ds} = \frac{1}{(r^2 + r_1^2)^{1/2}} \rightarrow (4)$

$\frac{d\psi}{ds} = \frac{1}{\rho}$

$\frac{d\phi}{d\theta} = \frac{(r_1)^2 - r''r}{r_1^2 + r^2}$

$\frac{d\theta}{ds} = \frac{1}{(r^2 + r_1^2)^{1/2}}$

Sub (2), (3) & (4) in (1)

$\frac{1}{\rho} = \frac{1}{(r^2 + r_1^2)^{1/2}} \left[1 + \frac{(r_1)^2 - r''r}{r_1^2 + r^2}\right]$

$\Rightarrow \frac{1}{\rho} = \frac{1}{(r^2 + r_1^2)^{1/2}} \left[\frac{r_1^2 + r^2 + r_1^2 - r''r}{r_1^2 + r^2}\right]$

$\frac{1}{\rho} = \frac{2r_1^2 + r^2 - r''r}{(r^2 + r_1^2)^{3/2}} \Rightarrow \rho = \frac{(r^2 + r_1^2)^{3/2}}{2r_1^2 + r^2 - r''r}$

DATE / /
PAGE

13) $y = \log(1+e^x)$ $\qquad y(0) = \log 2$

$$y' = \frac{e^x}{1+e^x} \qquad y'(0) = \frac{1}{2}$$

$$y'(1+e^x) = e^x$$

$$y''(1+e^x) + y'e^x = e^x \qquad \frac{1-\frac{1}{2}(1)}{2}$$

$$y'' = \frac{e^x - y'e^x}{1+e^x} \qquad y''(0) = \frac{1-\frac{1}{2}}{2} = \frac{\frac{1}{2}}{2}$$

$$y''(0) = \frac{1}{4}$$

$$y''(1+e^x) = e^x - y'e^x$$

$$y'''(1+e^x) + y''(e^x) = e^x - (y''e^x + y'e^x)$$

$$y''' = \frac{e^x - y''e^x - y'e^x - y''(e^x)}{1+e^x}$$

$$y''' = \frac{e^x - 2y''e^x - y'e^x}{1+e^x} \qquad y'''(0) = \frac{1 - \frac{1}{2} - \frac{1}{2}}{2}$$

$$y'''(0) = 0$$

$$y(x) = y(0) + x\,y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0)$$

$$y(x) = \log 2 + \frac{x}{2} + \frac{x^2}{4} + 0$$

$$y(x) = \log 2 + \frac{x}{2} + \frac{x^2}{8}$$

14) $\log(\sec x + \tan x)$

$y = \log(\sec x + \tan x)$ $\qquad\qquad y(0) = 0$

$y' = \dfrac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$y' = \dfrac{\sec x \, \cancel{(\tan x + \sec x)}}{\cancel{\tan x + \sec x}}$

$y' = \sec x$ $\qquad\qquad y'(0) = 1$

$y'' = \sec x \tan x$ $\qquad\qquad y''(0) = 0$

$y''' = \sec x \tan x \tan x + \sec^3 x$

$y''' = \sec x \tan^2 x + \sec^3 x$ $\qquad\qquad \cancel{y'''(0) = 1}$

$\cancel{y^{IV} = \sec x \tan^3 x + \sec x \, 2 \tan x \sec^2 x + \sec^3 x}$

$\cancel{y^{IV} = \sec x \tan^3 x + 2 \tan x \sec^3 x + \sec^3 x}$

$\cancel{y^{IV}(0) = 1}$

$y''' = \sec x(\sec^2 x - 1) + \sec^3 x$

$y''' = \sec^3 x - \sec x + \sec^3 x$

$y''' = 2\sec^3 x - \sec x$

$y''' = \sec x \left( \dfrac{2}{\cos^2 x} - 1 \right)$ $\qquad\qquad y'''(0) = 1$

$y''' = \dfrac{1}{\cos x} \left( \dfrac{2}{\cos^2 x} - 1 \right) \Rightarrow y''' = \dfrac{2}{\cos^3 x} - \dfrac{1}{\cos x}$

$\cos^2 x = 1 - \sin^2$

$y''' = \cancel{\dfrac{2}{\cos^3 x}} - \cancel{\dfrac{1}{\cos x}}$

$y''' = \sec x \left( \dfrac{2}{\cos^2 x} - 1 \right)$

$y^{IV} = \sec x \tan x \left( \dfrac{2}{\cos^2 x} - 1 \right) + \sec x \left( \dfrac{+ 2 \cdot 2 \cos x \sin x}{\cos^3 x} \right)$

$y^{IV} = \sec x \tan x \left( \dfrac{2}{\cos^2 x} - 1 \right) + \dfrac{4 \sec x \sin x}{\cos^3 x}$

$y^{IV}(0) = 1(0) \left( \dfrac{2}{1} - 1 \right) + \dfrac{0}{1}$

$y^{IV}(0) = 0$

$y(x) = y(0) + x y'(0) + \dfrac{x^2}{2!} y''(0) + \dfrac{x^3}{3!} y'''(0) + \dfrac{x^4}{4!} y^{IV}(0)$

$y(x) = 0 + x(1) + \dfrac{x^2}{2}(0) + \dfrac{x^3}{6}(1) + \dfrac{x^4}{4!}(0)$

$y(x) = x + \dfrac{x^3}{6}$

P.T.O

DATE / /

PAGE

15) $y = \log(1+x)$ $\qquad y(0) = 0$

$y' = \frac{1}{1+x}$ $\qquad y'(0) = 1$

$y'(1+x) = 1$

$y''(1+x) + y' = 0$

$y'' = \frac{-y'}{1+x}$ $\qquad y''(0) = -1$

$y''(1+x) = -y'$

$y'''(1+x) + y'' = -y''$

$y''' = \frac{-y'' - y''}{1+x}$ $\qquad y''' = \frac{-2y''}{1+x}$ $\qquad y'''(0) = +2$

$y'''(1+x) = -2y''$

$y^{iv}(1+x) + y''' = -2y'''$

$y^{iv} = \frac{-3y'''}{1+x}$ $\qquad y^{iv}(0) = -6$

$y^{iv}(1+x) = -3y'''$

$y^{v}(1+x) + y^{iv} = -3y^{iv}$

$y^{v} = \frac{-4y^{iv}}{1+x}$ $\qquad y^{v} = +24$

$y(x) = y(0) + x y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y^{iv}(0) + \frac{x^5}{5!} y^{v}(0)$

$y(x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$

$x = -x$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}$

$\log\sqrt{\frac{1+x}{1-x}} = \log\left(\frac{1+x}{1-x}\right)^{1/2}$

$\Rightarrow \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$\Rightarrow \frac{1}{2}\left(\log(1+x) - \log(1-x)\right)$

$\frac{1}{2}\log\sqrt{\frac{1+x}{1-x}} = \frac{1}{2}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}\right) - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}\right)$

$\Rightarrow \frac{1}{2}\left[x - \cancel{\frac{x^2}{2}} + \frac{x^3}{3} - \cancel{\frac{x^4}{4}} + \frac{x^5}{5}\right] + \left[x + \cancel{\frac{x^2}{2}} + \frac{x^3}{3} + \cancel{\frac{x^4}{4}} + \frac{x^5}{5}\right]$

$\Rightarrow \cancel{\frac{1}{2}\left[2x - 2\frac{x^3}{3} - 2\frac{x^4}{4} + 2\frac{x^5}{5}\right]}$

$\Rightarrow \frac{1}{2}\left[2x + 2\frac{x^3}{3} + 2\frac{x^5}{5}\right]$

$\Rightarrow \frac{\cancel{2}}{\cancel{2}}\left[x + \frac{x^3}{3} + \frac{x^5}{5}\right]$

$\therefore \log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5}$

DATE / /
PAGE

16) $K = \lim_{x \to a} \left[2 - \frac{x}{a}\right]^{\tan\left[\frac{\pi x}{2a}\right]}$

$\log K = \lim_{x \to a} \tan\left[\frac{\pi x}{2a}\right] \log\left[2 - \frac{x}{a}\right]$

$\log K = \lim_{x \to a} \frac{\log\left[2 - \frac{x}{a}\right]}{\cot\left(\frac{\pi x}{2a}\right)}$ $\qquad \tan\left[\frac{\pi x}{2a}\right] = \frac{1}{\cot\left[\frac{\pi x}{2a}\right]}$

$\log K = \lim_{x \to a} \frac{\frac{1}{2-\frac{x}{a}}\left(-\frac{1}{a}\right)}{-\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right) \times \frac{\pi}{2a}}$

$\log K = \lim_{x \to a} \frac{\cancel{-} \frac{1}{2-\frac{x}{a}} \frac{1}{a}}{\cancel{-} \operatorname{cosec}^2\left(\frac{\pi x}{2a}\right) \times \frac{\pi}{2a}} \Rightarrow \lim_{x \to a} \frac{\frac{1}{2-\frac{x}{a}}\left(\frac{1}{a}\right)}{\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right) \times \frac{\pi}{2a}}$

$\log K = \frac{\frac{1}{2-1} \frac{1}{a}}{1 \times \frac{\pi}{2a}} \Rightarrow \log K = \frac{\frac{1}{\cancel{a}}}{\frac{\pi}{2\cancel{a}}}$

$\Rightarrow \log K = \frac{2}{\pi} \Rightarrow K = e^{2/\pi}$

P.T.O.

17) $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$ ....... $1^{\infty}$

$\log k = \lim_{x \to 0} \frac{1}{x^2} \log(\cos x)$ ....... $\frac{0}{0}$

$\log k = \lim_{x \to 0} \frac{-\frac{\sin x}{\cos x}}{2x}$

$\log k = \lim_{x \to 0} \frac{-\tan x}{2x}$ ....... $\frac{\infty}{\infty}$ $\frac{\infty}{0}$

$\log k = \lim_{x \to 0} \frac{-\sec^2 x}{2}$

$\log k = -\frac{1}{2}$

$k = e^{-1/2}$

18) $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$

$\log k = \lim_{x \to 0} \frac{1}{x} \log\left(\frac{a^x + b^x + c^x}{3}\right)$

$\log k = \lim_{x \to 0} \frac{\frac{1}{\frac{a^x + b^x + c^x}{\cancel{3}}} \left(\frac{a^x \log a + b^x \log b + c^x \log c}{\cancel{3}}\right)}{1}$

$\log k = \lim_{x \to 0} \frac{a^x \log a + b^x \log b + c^x \log c}{a^x + b^x + c^x}$

$\log k = \frac{\log a + \log b + \log c}{3}$

$\log k = \frac{1}{3} \log(abc)$

$\log k = \log \sqrt[3]{abc}$

$k = \sqrt[3]{abc}$

DATE / /
PAGE

19) $\lim_{x\to 0} (a^x + x)^{1/x}$ . . . . . . $1^\infty$

$\log k = \lim_{x\to 0} \frac{1}{x} \log (a^x + x)$

$\log k = \lim_{x\to 0} \frac{\frac{1}{a^x + x} a^x \log a}{1}$

$\log k = \lim_{x\to 0} \frac{a^x \log a}{a^x + x}$

$\log k = \frac{\log a}{1}$

$\log k = a$

20) $r(1-\cos\theta) = 2a$

$r = \frac{2a}{1-\cos\theta}$

Log on both sides

$\log r = \log 2a - \log(1-\cos\theta)$

diff wrt $\theta$

$\log r = 0 - \frac{1}{1-\cos\theta} (\sin\theta)$

$\log r = \frac{-\sin\theta}{1-\cos\theta}$

$\log r = \frac{-2 \sin\theta/2 \cos\theta/2}{2\sin^2\theta/2}$ ~~log r = cot~~

$\frac{1}{r}\frac{dr}{d\theta} = -\cot\theta/2 \implies r' = -r\cot\theta/2$

~~log r = cot θ/2~~

$r' = -r \cot \theta/2$

diff w.r.t

$r'' = -\left(r' \cot \theta/2 + \left(-\left(r \operatorname{cosec}^2 \theta/2 \cdot \frac{1}{2}\right)\right)\right)$

$r'' = -r' \cot \theta/2 + \frac{r \operatorname{cosec}^2 \theta/2}{2}$

$r'' = -r' \cot \theta/2 + \frac{r \operatorname{cosec}^2 \theta/2}{2}$

2) ~~$r = a(1+\cos\theta)$~~

~~diff w.r.t $\theta$~~

~~$r' = a - \sin\theta$~~

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - r r_{11}}$$

$$\rho = \frac{(r^2 + r^2 \cot^2 \theta/2)^{3/2}}{r^2 + 2r^2 \cot^2 \theta/2 - r\left(-r' \cot \theta/2 + \frac{r \operatorname{cosec}^2 \theta/2}{2}\right)}$$

$$\rho = \frac{r^3 (1 + \cot^2 \theta/2)^{3/2}}{r^2 + 2r^2 \cot^2 \frac{\theta}{2} - \left(r^2 \cdot \cot^2 \theta/2 + \frac{r^2 \operatorname{cosec}^2 \theta/2}{2}\right)}$$

$$\rho = \frac{r^{\cancel{3}} \operatorname{cosec}^3 \theta/2}{\cancel{r^2}\left(1 + 2\cot^2 \frac{\theta}{2} - \cot^2 \theta/2 - \frac{\operatorname{cosec}^2 \theta/2}{2}\right)}$$

$$\rho = \frac{r \operatorname{cosec}^3 \theta/2}{1 + \cot^2 \frac{\theta}{2} - \frac{\operatorname{cosec}^2 \theta/2}{2}} \Rightarrow \rho = \frac{r \operatorname{cosec}^3 \frac{\theta}{2}}{\operatorname{cosec}^2 \theta/2 - \frac{\operatorname{cosec}^2 \theta/2}{2}}$$

$$\rho = \frac{r \operatorname{cosec}^{\cancel{3}} \theta/2}{\cancel{\operatorname{cosec}^2 \theta/2}}$$

$$\rho = r \operatorname{cosec} \theta/2$$

DATE / /
PAGE

7) $r = a(1+\cos\theta)$

diff. w.r.t $\theta$

2a (

$r' = -a\sin\theta$

$r'' = -a\cos\theta$

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{2r_1^2 + r^2 - r r_{11}}$$

$(1+\cos\theta)^2 = 1 + \cos^2\theta + 2\cos\theta$

$$\cancel{\rho = (r^2 + a^2\sin^2)^{3/2}}$$

$$\rho = \frac{(a^2(1+\cos\theta)^2 + a^2\sin^2\theta)^{3/2}}{2a^2\sin^2\theta + a^2(1+\cos\theta)^2 - \{a(1+\cos\theta)\}(-a\cos\theta)}$$

$$\rho = \frac{a^3(1+\cos^2\theta + 2\cos\theta + \sin^2\theta)^{3/2}}{2a^2\sin^2\theta + a^2(1+\cos^2\theta + 2\cos\theta) - (a(1+\cos\theta))(-a\cos\theta)}$$

$$\rho = \frac{a^3(2+2\cos\theta)^{3/2}}{a^2(2\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta + \cos\theta + \cos^2\theta)}$$

$$\rho = \frac{a(2(1+\cos\theta))^{3/2}}{(2\sin^2\theta + 1 + 2\cos^2\theta + 3\cos\theta)}$$

$$\rho = \frac{a(2(1+\cos\theta))^{3/2}}{(3+3\cos\theta)} \Rightarrow \rho = \frac{a(2(1+\cos\theta))^{3/2}}{3(1+\cos\theta)}$$

$$\rho = \frac{a\,2\sqrt{2}\,(1+\cos\theta)^{3/2}}{3\cancel{(1+\cos\theta)}} \Rightarrow \rho = \frac{a\,2\sqrt{2}\,(1+\cos\theta)^{1/2}}{3}$$

$$\rho = \frac{2\sqrt{2}\,a\,(2\cos^2\theta/2)^{1/2}}{3}$$

$$\rho = \frac{2\sqrt{2}\, a \sqrt{2} \cos \theta/2}{3}$$

$$\rho = \frac{4 a \cos \theta/2}{3}$$

SOBS

$$\rho^2 = \frac{16}{9} a^2 \cos^2 \theta/2$$

$$\rho^2 = \frac{8}{9} a^2\, 2 \cos^2 \theta/2$$

$$\rho^2 = \frac{8}{9} a^2 (1 + \cos\theta)$$

$$\rho^2 = \frac{8}{9} a^{\cancel{2}} \frac{r}{\cancel{a}}$$

from ques

$r = (1 + \cos\theta) a$

$\frac{r}{a} = 1 + \cos\theta$

$$\frac{\rho^2}{r} = \frac{8}{9} a \quad \therefore \text{its a Const.}$$