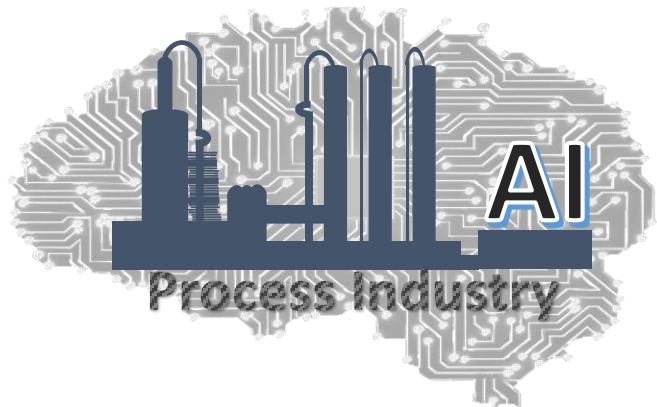


# Statistical Techniques for Monitoring Industrial Processes



*Lecture : CUSUM Control Charts*

*Module : Univariate SPM*

# Course TOC

## ❑ Introduction to Statistical Process Monitoring (SPM)

## ❑ Python Installation and basics (optional)

## ❑ Univariate SPM & Control Charts

- Shewhart Charts
- CUSUM Charts
- EWMA Charts

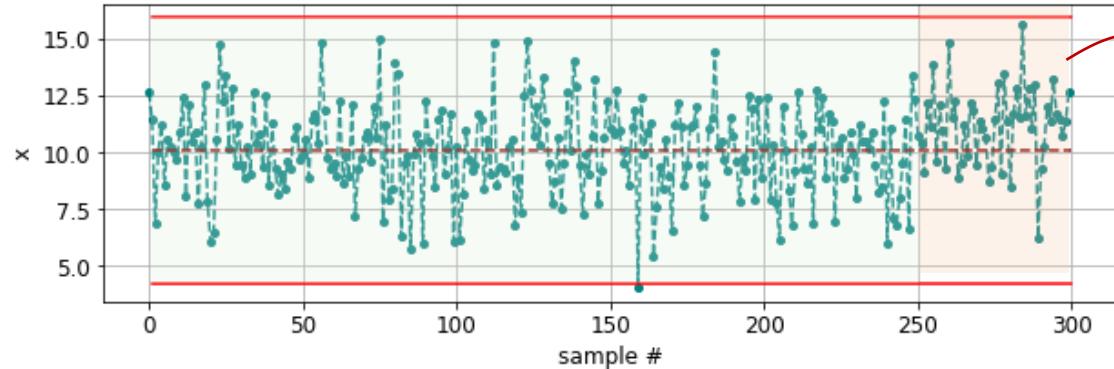


## ❑ Multivariate SPM

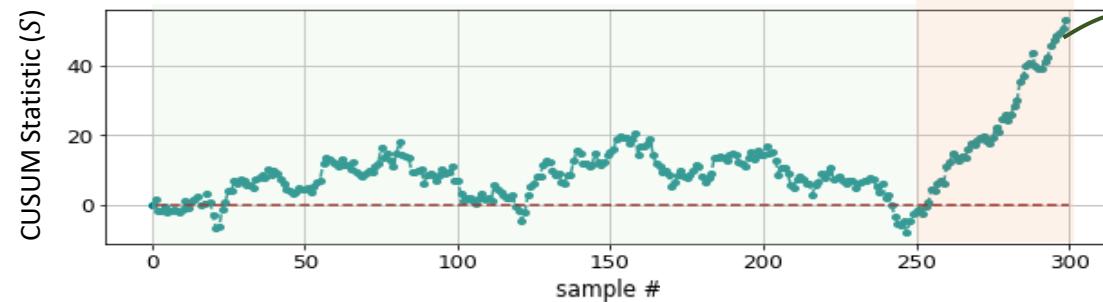
- Fault detection using Principal Component Analysis (PCA)
- Fault detection using Partial Least Squares (PLS) regression
- Fault diagnosis using PCA/PLS contribution charts
- Strategies for handling nonlinear, dynamic, multimode systems

## ❑ Deployment of SPM Solutions

# CUSUM Control Charts: Introduction

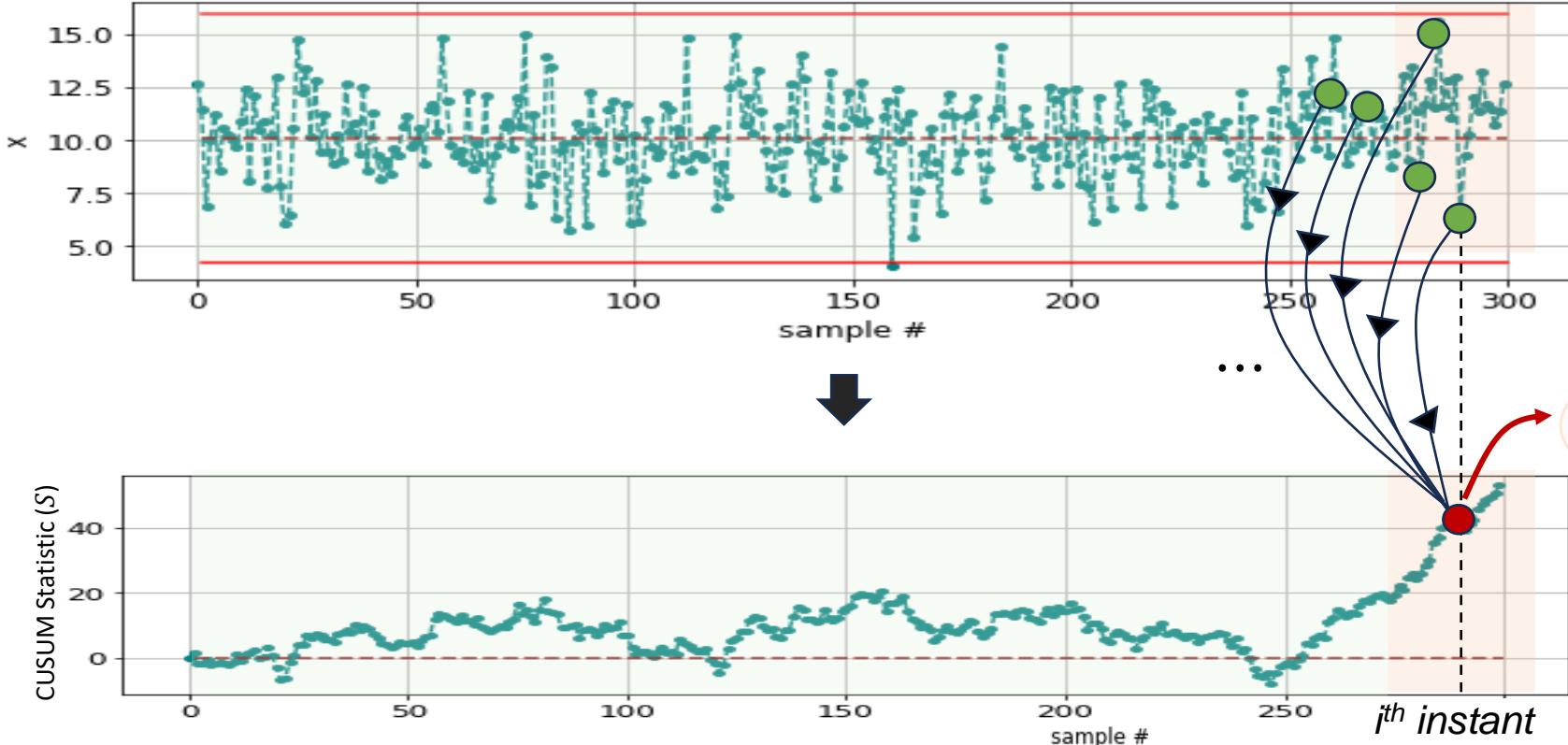


Last 50 samples generated with  $0.5\sigma$  shift in mean



How did the drift become so obvious?

# CUSUM Control Charts: Introduction



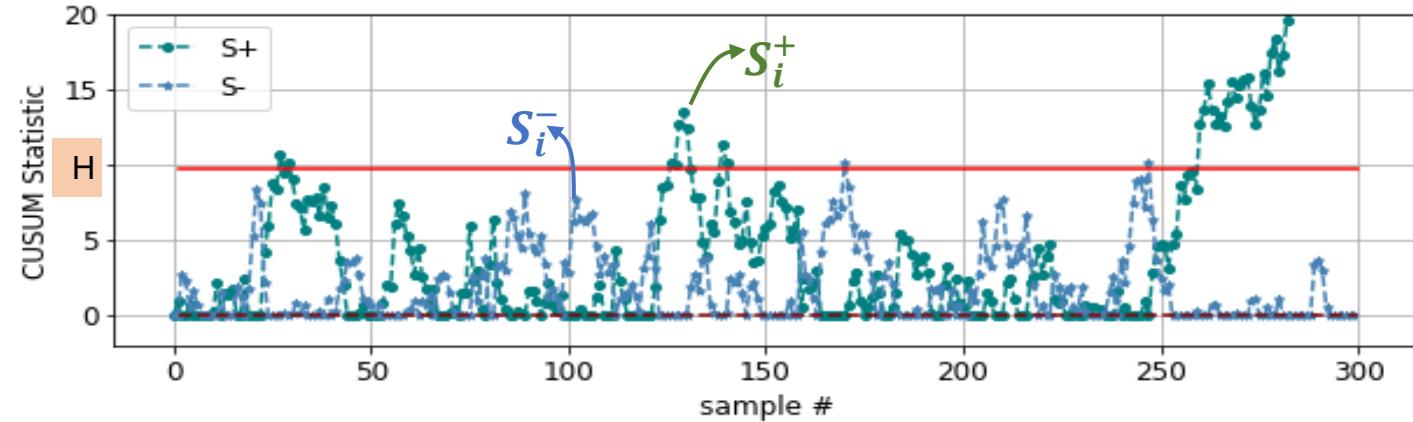
$$S_i = \sum_{j=0}^i (x_j - \mu_0) \quad ; i \geq 0$$

$$S_i = (x_i - \mu_0) + S_{i-1}$$

$$S_0 = 0$$

# Two-Sided CUSUM Control Charts

Violation of control limit by either  $S^+$  or  $S^-$  indicates process not being in statistical control



$S_i^-$  tracks negative shift in mean

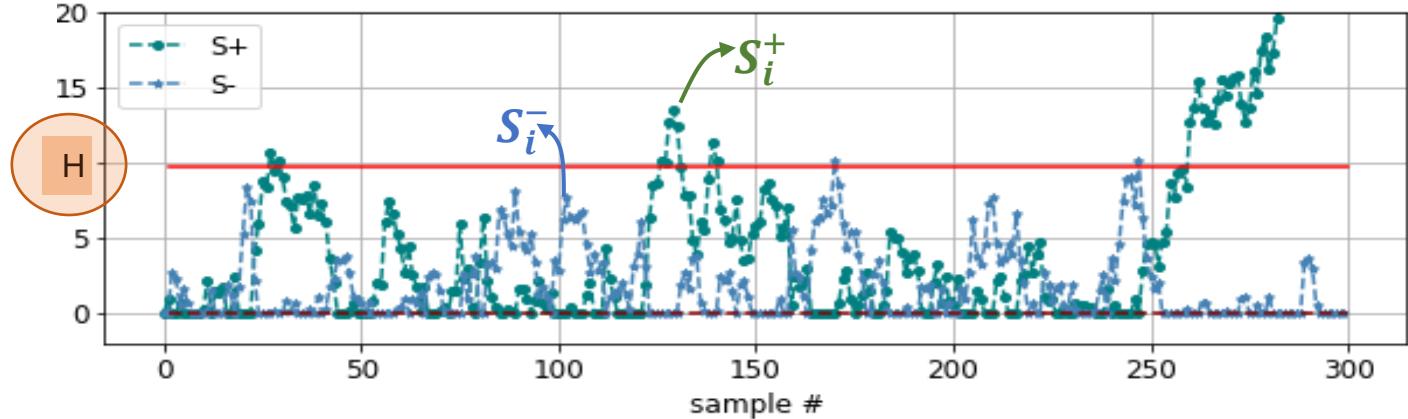
$S_i^+$  tracks positive shift in mean

$$S_i^+ = \max[0, x_i - (\mu_0 + k) + S_{i-1}^+]$$

$$S_i^- = \max[0, (\mu_0 - k) - x_i + S_{i-1}^-]$$

- $k \geq 0$
- Deviation from mean greater than  $k$  increases  $S^+$  or  $S^-$
- $k$  is usually set to be  $\frac{1}{2}$  the size of mean shift that we want to detect quickly

# Construction of CUSUM Control Chart



$S_i^-$  tracks negative shift in mean

$S_i^+$  tracks positive shift in mean

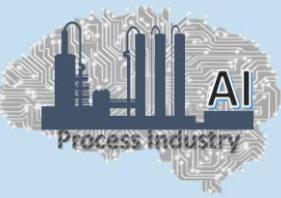
$$S_i^+ = \max[0, x_i - (\mu_0 + k) + S_{i-1}^+]$$

$$S_i^- = \max[0, (\mu_0 - k) - x_i + S_{i-1}^-]$$



## Historical in-control data

- $\mu_0 \approx$  sample mean
- $\hat{\sigma}$  = sample standard deviation
- $H = 4\hat{\sigma}$  or  $5\hat{\sigma}$



# CUSUM Control Charts: Implementation Demo

## 2 Case studies

1

Product purity mean value changes by  $0.5\sigma$  due to process upset

2

Product purity mean value changes by  $2\sigma$  due to process upset

# CUSUM Charts: Pros & Cons

## Pros

- Easy to interpret
- Easy to implement
- Can detect small mean deviations

## Cons

- Shows delay in drift detection

### Shewhart Charts

- Quick detection of large drifts
- Fails for small drifts

### CUSUM Charts

- Detection of small drifts
- Delay in detecting drifts

EWMA charts provide a good compromise between detection of small mean shifts and quick detection of large shifts

# Statistical Techniques for Monitoring Industrial Processes



***Next Lecture :*** Monitoring Airflow in an Aeration tank

***Module :*** Course Introduction

