Problema 2:

a) $\frac{dy}{dx} + 2xy = 4x$, $y(0) = 3$

$\frac{dy}{dx} = -2xy + 4x = x(4 - 2y)$ Fator integrante:

$\frac{dy}{dx} + 2xy = 4x \longrightarrow \mu(x) = e^{\int 2x\,dx} = e^{x^2}$

$e^{x^2}\frac{dy}{dx} + 2xe^{x^2}y = 4xe^{x^2} \rightarrow \frac{d}{dx}[e^{x^2}\cdot y] = 4\cdot x e^{x^2}$

$e^{x^2}y = \int 4xe^{x^2}dx$, $u = x^2$, $du = 2x\,dx$

$\int 4xe^{x^2}dx = 2\int e^u du = 2e^{x^2} + C$

$\hookrightarrow e^{x^2}\cdot y = 2e^{x^2} + C$

$y(x) = 2 + Ce^{-x^2}$ (Condição $y(0) = 3$),

$3 = 2 + C \rightarrow \boxed{C = 1} \rightsquigarrow \boxed{y(x) = 2 + e^{-x^2}}$

b) Solução Numérica: (Com Euler Modificado)

$x_a = 0$ a $x_b = 2$, $h = 0,5$ $\quad \boxed{y_{i+1}^{*} = y_i + hf(x_i, y_i)}$

$\boxed{y_{i+1} = y_i + \frac{h}{2}\left(f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{*})\right)}$

Passo 1: $x_1 = 0,5$

$f(0,3) = 4\cdot 0 - 2\cdot 0\cdot 3 = 0$

$y_1^{*} = 3 + 0,5\cdot 0 = 3 \longrightarrow f(0,5;3) = 4\cdot 0,5 - 2\cdot 0,5\cdot 3 = -1$

$y_1 = 3 + \frac{0,5}{2}[0 + (-1)] = \boxed{2,75}$

$\boxed{y_1 = 2,75}$

$y_{exato} = y(0,5) = 2 + e^{-0,25} \cong 2,778801$

$$\varepsilon_R = \left| \frac{2,75 - 2,778801}{2,778801} \right| \cdot 100\% \cong 1,03700\%$$

$\boxed{\varepsilon_{R1} = 1,03700\%}$

★ Passo 2: $x_2 = 1,0$ $f(0.5, 2.75) = 2 - 2 \cdot 0,5 \cdot 2,75 = -0,75$

$$y_2^* = 2,75 + \frac{0,5}{2}[(-0,75) + (-0,75)] = 2,375$$

$$f(1,0, 2,375) = 4 \cdot 1 - 2 \cdot 1 \cdot 2,375 = -0,75$$

$$y_2 = 2,75 + \frac{0,5}{2}[(-0,75) + (-0,75)] = 2,375$$

$y_{exato} = y(1) = 2 + e^{-1} = 2,367879$

$$\varepsilon_R = \left| \frac{2,375 - 2,367879}{2,367879} \right| \cdot 100\% \cong 0,30080\%$$

$\boxed{\varepsilon_R = 0,30080\%}$

★ Passo 3: $x_3 = 1,5$ $f(1,0; 2,375) = 4 - 4,75 = -0,75$

$y_3^* = 2,375 + 0,5 \cdot (-0,75) = 2,00000$

$f(1,5; 2,0) = 4 \cdot 1,5 - 2 \cdot 1,5 \cdot 2 = 0$

$\boxed{\varepsilon_R \cong 3,89999\%}$

$$y_3 = 2,375 + \frac{0,5}{2}[(-0,75) + 0] = 2,18750$$

$y_{exato}(1,5) = 2 + e^{-2,25} \cong 2,1053999$

$$\varepsilon_R \cong \left| \frac{2,1875 - 2,1053999}{2,1053999} \right| \cdot 100\% \cong 3,899\%$$

Passo 4 → $x_4 = 2{,}00000$

$f(1{,}5;\ 2{,}1875) = 6 - 2 \cdot 1{,}5 \cdot 2{,}1875 = -0{,}5625$

$y_4^* = 2{,}1875 + 0{,}5 \cdot (-0{,}5625) = 2{,}1875 - 0{,}28125 = 1{,}90625$

$f(2{,}0;\ 1{,}90625) = 8 - 4 \cdot 1{,}90625 = 0{,}375$

$y_4 = 2{,}1875 + \frac{0{,}5}{2}[(-0{,}5625) + 0{,}375] = 2{,}140625$

$y_{exato}(2) = 2 + e^{-4} \cong 2 + 0{,}0183156 = 2{,}0183156$

$\varepsilon_R \cong \left| \frac{2{,}140625 - 2{,}0183156}{2{,}0183156} \right| \cdot 100\% \cong 6{,}057\%$

$\varepsilon_R \cong 6{,}057\%$

Tabela:

X	y_{num}	y_{exato}	ε_R %
0,0	3,00000	3,00000	0,00000
0,5	2,75000	2,778801	1,037
1,0	2,37500	2,367879	0,3008
1,5	2,18750	2,105399	3,899
2,0	2,140625	2,0183156	6,057

2.2: Segunda EDO de 2ª Ordem

a) Solução Analítica: $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, $y(0) = 4$, $y'(0) = -5$

Eq. Característica:

$R^2 + R - 2 = 0 \rightsquigarrow R = \frac{-1 \pm 3}{2} \hookrightarrow R_1 = 1,\ R_2 = -2$

Sol. Geral: $y(x) = Ae^x + Be^{-2x}$

$y(0) = A + B = 4 \rightarrow y'(0) = A - 2B \cdot 1 = -5$ - Substitui

$A - 2(4 - A) = -5 \rightsquigarrow A = 1$, então: $B = 3$

$y(x) = e^x + 3e^{-2x}$

$y'(x) = e^x - 6e^{-2x}$

b) Solução Numérica: $\frac{dy_1}{dx} = y_2$ $\frac{dy_2}{dx} = -y_2 + 2y_1$

Com $y_1(0) = 4, y_2(0) = -5$

Euler Modificado: $F(x, y) = [y_2, -y_2 + 2y_1]^T$

$y^* = y_i + h \cdot F(x_i, y_i) \rightsquigarrow y_{i+1} = y_i + \frac{h}{2}[F(x_i, y_i) + F(x_{i+1}, y^*)]$

Passo 1: $x_1 = 0,5$

$F(0, [4, -5]) = [-5, -(-5) + 2 \cdot 4] = [-5, 13]$

$y^* = [4, -5] + 0,5 \cdot [-5, 13] = [1.5, 1.5]$

$y_1 = [4, -5] + \frac{0,5}{2}([-5, 13] + [1,5, 1.5])$

$= [3,125000, -1,375000]$

$y_{exato}(0,5) = e^{0,5} + 3e^{-1} \cong 2,752360$

$y'_{exato}(0,5) = e^{0,5} + 6 \cdot e^{-1} \cong -0,558555$

$Er = \left| \frac{3,125000 - 2,752360}{2,752360} \right| * 100\% \cong 13,53\%$

Passo 2: $x_2 = 1$ $F_1 = [-1,375; 7,625]$

$y^* = [3,125; -1,375] + 0,5 \cdot [-1,375; 7,625]$

$y^* = [2,437500; 2,437500]$

$F^* = [2,4375; 2,4375]$

$y_2 = [3,125; -1,375] + 0,25 \cdot [1,0625; 10,0625]$

$y_2 = [3,390625; 1,140625]$

$$\varepsilon_R = \left| \frac{3,390625 - 3,124288}{-3,124288} \right| * 100\% \cong 8,526\%.$$

$$\boxed{\varepsilon_R = 8,526\%.}$$

★ Passo 3: $X_3 = 1,5$; $Y_2 = [3,390625; 1,140625]$

$F_2 = [1,140625; 5,640625]$

$y_3^* = [3,390625, 1,140625] + 0,5 \cdot [1,140625;$

$5,640625] = [3,9609375; 3,9609375]$

$y_3 = [3,390625; 1,140625] + 0,25[5,1015625;$

$9,6015625] = [4,666016; 3,541016]$

▷ $y_{esatto}(1,5) \cong e^{1,5} + 3e^{-3} \cong 4,631050$

$y'_{esatto}(1,5) \cong e^{1,5} - 6e^{-3} \cong 4,182967$

$$\varepsilon_R = \left| \frac{4,666016 - 4,631050}{4,631050} \right| \cong 0,755\%$$

$\boxed{\varepsilon_R = 0,755\%}$

★ Passo 4: $X_4 = 2,0$; $Y_3 = [4,666016; 3,541050]$

$y_4^* = [4,666016; 3,541050] + 0,5 \cdot [3,541050; 5,791026]$

$y_4^* = [6,436524; 6,436524]$

$F^* = [6,436524; 6,436524]$

$y_4 = [4,666016; 3,541016] + 0,25 \cdot [9,977540;$

$12,227540] = \boxed{y_4 = [7,160401; 6,597901]}$

$y_{exato}(2) = e^2 + 3e^{-4} \cong 7,389056 + 0,054948 \cong 7,444004$

$y'_{exato}(2) = e^2 - 6e^{-4} \cong 7,279160$

$Er = \left| \frac{7,160401 - 7,444004}{7,444044} \right| \times 100\% \cong 3,81\%$

$Er \cong 3,81\%$

Tabela Ex. 2.2

x	y_{num}	y_{exato}	Er %
0,0	4,00000	4,00000	0,0 %
0,5	3,12500	2,75236	13,53%
1,0	3,39063	3,12429	8,526%
1,5	4,16602	4,63105	0,755%
2,0	7,16040	7,44400	3,810%