Supersonic Aerodynamics: Lift and Drag

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Introduction

We briefly review here the fundamentals of generating lift, and what this costs us in inviscid drag at supersonic speeds in the context of the optimum aerodynamic design. The supersonic area rule tells us how to determine the wave drag and this leads to the minimum possible inviscid drag for a supersonic aircraft. We understand from this, then, the trade-off between induced drag and wave drag due to lift. Finally, viscous effects are considered briefly. These determine the altitude at which the aircraft will fly and this sets its C_L and thereby its aerodynamic performance.

Lift and Drag

If we consider a flat plate airfoil, at angle of attack, α, in a flow of subsonic Mach number M, with $\beta^2 = |M^2 - 1|$, then its lift is

$$c_l = 2\pi\alpha/\beta.$$

Since the pressure must be normal to this flat plate one would expect a component of drag with $c_d = \alpha c_l$. And yet we know that in two-dimensional subsonic flow the inviscid drag is zero. How can this be? If we consider the leading edge of the flat plate to be a small circle of radius ε, then we may using conformal mapping to compute the force on this circle and the flat plate. If we then let $\varepsilon \to 0$, the result is, as we know, that the leading edge thrust precisely offsets the drag due to the plate's inclination.

So we escape drag due to lift in two-dimensional subsonic flows. But wings must be finite in span, so we next consider this effect. The wing's lift must be distributed elliptically along its span to minimize the drag that derives from the axial momentum lost to the swirling motion of the wing tip vortices. This itself derives from the pressure difference between the lower and upper surface. This we call the induced drag:

$$D_{induced} = \frac{L^2}{\pi q s^2}.$$

Here L is the lift, distributed elliptically, s is the wing span, and q the dynamic pressure.

At supersonic speeds the same flat plate airfoil gives a lift coefficient of

$$c_l = 4\alpha/\beta.$$

In this case there is no possibility of leading edge suction. The pressure coefficient is uniform and of equal magnitude, but opposite sign, on each side of the plate. So we must accept the resulting drag of αc_l that drives from the waves this airfoil sends to infinity:

$$D_{wave} = \beta L^2/4,$$

but we now have no induced drag.

Busemann,[1,2] and later Jones,[3,4] were the first to point out that, for supersonic flow over an infinite swept wing, it is the normal velocity component that matters. Thus infinite yawed wings with their leading edges swept sufficiently behind the Mach cone (subsonic leading edges) can avoid this wave drag. Again, the tilt of the lift vector is overcome by leading edge suction. For a finite wing not all of the theoretically available suction can be realized. How much can be realized depends on, among other variables such as sweep and leading edge radius, the Reynolds number. See Carlson[5] for a recent review of this subject.

Of course we cannot have infinite wings in supersonic flow either and, consequently, with subsonic leading edges we will have induced drag and wave drag. Sweeping a single wing is more advantageous than considering the wing to have two halves, both of which are swept back or swept forward. As R. T.

Paper presented at the RTO AVT Course on "Fluid Dynamics Research on Supersonic Aircraft", held in Rhode-Saint-Genèse, Belgium, 25-29 May 1998, and published in RTO EN-4.

Jones observed long ago, an oblique wing is the best compromise between forward sweep and backward sweep.

The wave drag of a body of revolution is easily computed. If we let its cross-sectional area be $S(x)$, and its length l, then $2\pi D/q$

$$= \int_0^l S''(x)dx \int_0^l S''(\xi)\log|2(x-\xi)|d\xi$$

plus terms that are multiplied by $S'(l)$.

Wave Drag

The supersonic area rule tells us that the wave drag of an aircraft in a steady supersonic flow is identical to the average wave drag of a series of equivalent bodies of revolution. These bodies of revolution are defined by the cuts through the aircraft made by the tangents to the fore Mach cone from a distant point aft of the aircraft at an azimuthal angle θ, as shown in Fig.1. This average is over all azimuthal angles.

For each azimuthal angle the cross-sectional area of the equivalent body of revolution is given by the sum of two quantities: the cross-sectional area created by the oblique section from the tangent to the fore Mach cone's intersection with the aircraft, projected onto a plane normal to the free stream; and a term proportional to the component of force on the contour of this oblique cut, lying in the θ = constant plane, and normal to the free stream.[6,7] The minimum wave drag associated with a given lift requires that all oblique loadings projected by the Mach planes be elliptical. This is the same as saying that each equivalent body of revolution should be a Kármán ogive, which is the shape that minimizes the fore-body drag of a body of revolution of given base area.

The drag of a supersonic wing with subsonic leading edges and an elliptic spanwise loading can be expressed using our theoretical understanding of drag at supersonic speeds.

This drag is given by:

$$D = qS_fC_f + \frac{L^2}{\pi q s^2} + \frac{\beta^2 L^2}{\pi q l_l^2} + \frac{128 q V^2}{\pi l_V^4}. \qquad (1)$$

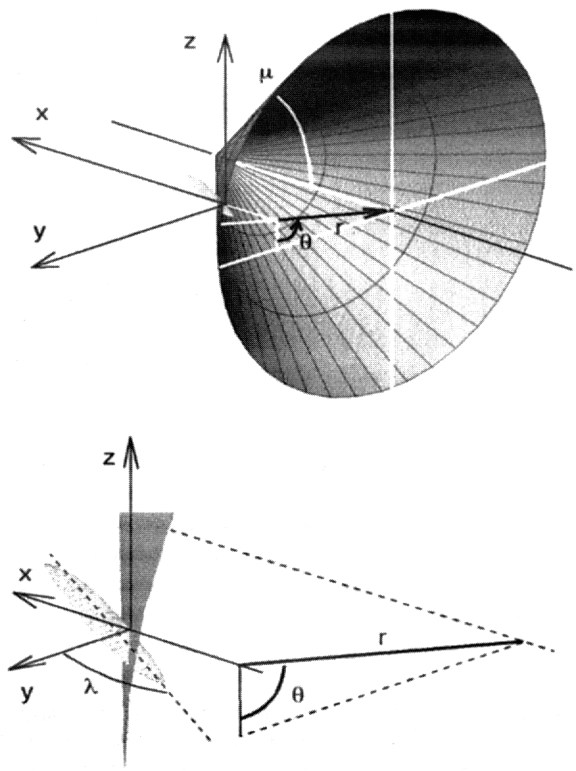

Fig. 1. Fore Mach cone (above) and its intersection with an oblique wing aircraft (below). Courtesy H. Sobieczky.

Here S_f is the wetted area, and C_f is the average skin friction coefficient. Thus the first term represents the skin friction drag which we may accurately approximate by the turbulent drag on a flat plate of the same area and average streamwise chord.

The second term is the induced drag for an elliptically loaded wing. We recognize this expression as the induced drag of the wing in the flow normal to it, that is as $L^2/(\pi q_n b^2)$, where q_n is the dynamic pressure of the normal flow and b is the unswept wing's span. The third and fourth terms are the wave drag due to lift and the wave drag due to volume, where V is the wing's volume. The two lengths, l_l and l_v are the averages over all azimuthal angles of the individual lengths of the equivalent bodies of revolution, appropriately adjusted for the variation of the component of the force lying in the θ = constant plane.

Oblique Wing Aerodynamics

As noted earlier, for the minimum wave drag due to lift, the loading in each oblique plane must be elliptical; that is, the equivalent body of revolution in each azimuthal plane is a Kármán ogive.[8] This is readily realized in an oblique wing with an elliptic planform. With an elliptical spanwise load, every loading projected by the oblique Mach planes is also elliptical.

For the minimum wave drag due to volume each equivalent body must be a Sears-Haack body.[9,10] This minimizes the wave drag for a given volume. If we minimize the wing's thickness, that is, the caliber of the equivalent body, then for the same caliber body as the Sears-Haack body, the volume in Eq. (1) is reduced by $\sqrt{(8/9)}$.

Smith[11] noted that the Sears-Haack area distribution is the product of an elliptic distribution and a parabolic distribution. The lengths of the chords cut by parallel planes on an elliptic planform are distributed elliptically. Thus, if all sections of wing were parabolic, each area distribution of the equivalent body of revolution due to volume would be the product of an elliptic and a parabolic distribution and the wave drag of the wing would be a minimum for given volume.

The lengths in the last two terms are the average over all azimuthal angles of the effective length for lift, and volume, for each azimuthal angle, as determined by the supersonic area rule. To calculate these lengths we must determine the angle at which the tangent to the Mach cone cuts the plane of the wing.

For simplicity we assume that the wing lies in a horizontal plane. We recognize, but ignore, the fact that the wing must incline its lift vector slightly to offset the leading edge suction which occurs on only one side of the wing. In practical cases this results in wing plane inclination of less than two degrees.

If we write down the expression for the fore Mach cone depicted in Fig. 1, and consider its apex to be at a large radial (and thereby axial) location, this equation becomes that for its tangent plane. This plane intersects the horizontal plane and thereby the wing, in a line that makes an angle, φ, given by

$$\tan\varphi = \pm\beta\sin\theta,$$

with the y-axis. Now that we know the angle cut by the tangent to the Mach cone, we also know the length of the equivalent body of revolution for that plane is

$$l(\theta) = b\sin\lambda - \beta\cos\lambda\sin\theta.$$

We may then determine the two lengths l_l and l_v to find the now classical results of an oblique wing:

$$\frac{1}{l_l^2} = \frac{1}{2\pi}\int_0^{2\pi}\frac{(\cos\theta)^2}{l^2(\theta)}d\theta \tag{2}$$
$$= \frac{1}{m^2b^2(\sin\lambda)^2}\left(\frac{1}{\sqrt{1-m^2}}-1\right),$$

$$\frac{1}{l_v^4} = \frac{1}{2\pi}\int_0^{2\pi}\frac{d\theta}{l^4(\theta)} \tag{3}$$
$$= \frac{2+3m^2}{2b^4(\sin\lambda)^4(1-m^2)^{7/2}},$$

where $m = \beta\cot\lambda$.[12,13] For a wing with a subsonic leading edge m is less than 1. The drag arising from the lift of an elliptic wing, i.e., the sum of the induced drag and wave drag due to lift, can also be determined by applying Kogan's theory.[14] And this theory can be used to show that an oblique, elliptically loaded, wing has the minimum inviscid drag for a given lift.[15]

We may use Eqs. (2) and (3) to determine the inviscid drag for an oblique lifting line, that is for m << 1. This gives, for the inviscid flow past a lifting line,

$$D = \frac{L^2}{\pi q s^2\sqrt{1-m^2}}. \tag{4}$$

We should note here that the large sweep approximation, then the drag due to lift is predominately induced drag. Then we may rewrite Eq.(1) in the classical form for minimum drag:[16]

$$D_{min} = qS_fC_f + \frac{L^2}{\pi q s^2} + \frac{\beta^2L^2}{2\pi q\ell^2} + \frac{128qV^2}{\pi\ell^4}. \tag{5}$$

Here ℓ is the aircraft's length, which in the case of an oblique wing is its streamwise length. The linear result for an arbitrary elliptic wing is more complex.[12,13,17]

An oblique elliptic wing simultaneously provides large span and large lifting length. The reduction in the drag for an oblique wing of finite span comes from being able to provide the optimum distribution of lift and volume in all oblique planes. To achieve an elliptic load distribution, twist variation along the wing span, or bending the wing up at the tips, is needed. The proper wing cross-section area distribution then gives the minimum wave drag due to volume or thickness. Fig. 2 provides the inviscid *L/D* for an elliptic planform oblique wing with a 10:1 axis ratio as a function of C_L and sweep as determined by Euler computations.[20] From this we conclude that for inviscid flow the optimum occurs with a sweep of about 68 degrees at $C_L = 0.068$.

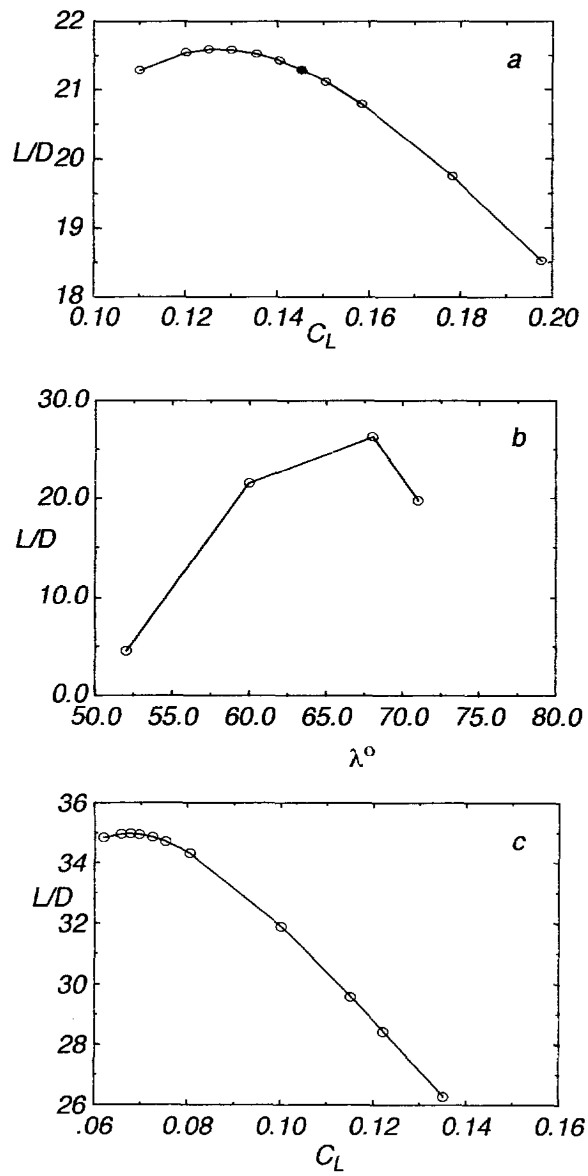

Fig. 2. Inviscid *L/D* as a function of: a) C_L for $\lambda = 60^o$, c) C_L for $\lambda = 68^o$, b) λ for $C_L = .068$.

Viscous Effects

We may approximate the viscous drag by that on a flat plate of equivalent wetted area. Here we follow Peterson[18] in applying the *T'* method of Sommer and Short[19] to determine the skin friction on a flat plate at supersonic Mach numbers. We assume here that the wing is at the recovery temperature. For an oblique wing we may use twice the planform area as the wetted area with good accuracy. Aircraft fly whenever possible at the altitude that, considering viscous effects, maximizes *L/D*. We may improve *L/D* by flying higher and thereby reducing skin friction drag because q is lower, although the increased C_L means a lower inviscid *L/D*.

To determine what altitude, and thereby what C_L, gives maximum viscous *L/D*, we must first determine the appropriate Reynolds numbers and develop a table of C_f, and determine the skin friction drag. The streamwise chord used was the mean chord divided by $\cos\lambda$. Because viscous drag is $2qSC_f$, where C_f is the average skin friction coefficient, and $L = qSC_L$ we can write for the drag:

$$D/L = (D/L)_{inviscid} + 2C_f/C_D. \qquad (6)$$

To be specific, previous design studies[17,20,21] suggest an OFW with a 10:1 axis ratio, a 550 foot span, and a maximum chord of 55 feet. This gives a planform area, S, of 23,758 square feet, and with a maximum thickness of 19% gives a volume of 124,140 cubic feet. Such an aircraft might accommodate 800 passengers. The studies of Rawdon et al.[21] suggest an estimated takeoff weight of 1.575 million pounds, a weight upon entering cruise of 1.5 million pounds, and a weight upon leaving cruise of 0.9 million pounds. The mid-cruise weight would be 1.2 million pounds.

To determine the density and viscosity in, and indeed the Reynolds number for, the boundary layer we need an appropriate reference temperature, *T'*. This we may determine for a given wall temperature from Peterson.[18] Given this reference temperature, and thereby the freestream Reynolds number for a given flight altitude, we may construct Fig. 3.

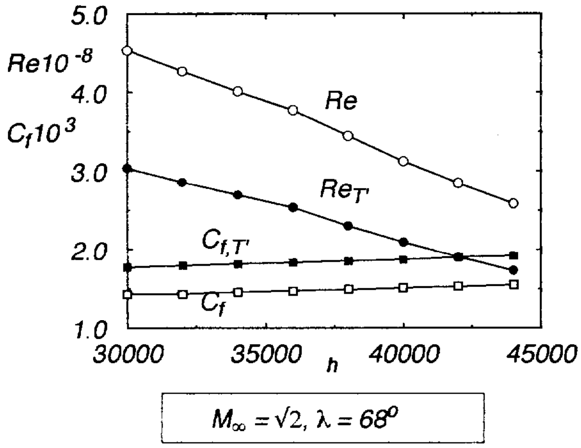

Fig. 3. Freestream and reference temperature Reynolds number and C_f versus flight altitude, h.

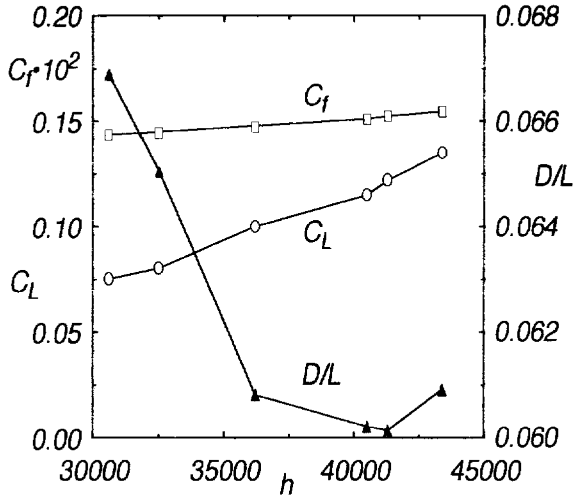

Fig. 4: Lift coefficient, skin friction coefficient and drag-to lift ratio, D/L, as a function of flight altitude.

Knowing C_f as a function of altitude, h, and given the aircraft's weight, say 1.5 million pounds upon entering cruise, we can determine the $C_L(h)$ needed to fly at that altitude. Equation (6) then provides D/L as a function of altitude as shown in Fig. 4. As Fig. 4 depicts, the viscous drag-to-lift ratio is a minimum at 41,300 feet; here $C_L = 0.1221$ and $C_f = 0.001523$.

We then analyze the flow under these conditions to find a new value for the inviscid L/D of 28.42, down from the value of 35.0 depicted in Fig. 2c. Equation (6) then gives us the viscous L/D,

$$\left(\frac{L}{D}\right)_{opt} = \left[\frac{1}{28.42} + \frac{2 \times 0.001523}{0.1221}\right]^{-1}$$

or 16.6 to the accuracy with which we might believe this result. This corresponds to an ML/D of 23.5, which is close to the linear theory optimum of 25.2. Given the flight altitude of 41,300 feet and C_f, the viscous drag is determined to be 37,420 pounds. We have reduced inviscid L/D from its maximum of 35.0 to 28.4 by increasing C_L to fly higher, but we have improved the viscous L/D by nearly 10%.

For nominal conditions we might take the weight to be 1.2 million pounds, the volume to be 85,800 cubic feet and the altitude to be 42,000 feet. Using Eq. (1) and the nominal conditions, we calculate the turbulent skin friction drag on a flat plate, and more directly, the other terms to conclude that the drag in pounds is:

$D = 73.5 * 10^3 = 73.5 * 10^3$[0.583 (skin friction)+ 0.221 (induced)+ 0.05 (wave-lift)+ 0.147 (wave-volume)].

We see that for this aerodynamic optimum aircraft the viscous drag is 58%, the induced drag is 22%, and the wave drag is 20%, with most of this due to volume. The drag due to lift is 27%. This maximum ML/D is for the untrimmed aircraft. Some drag penalty will be incurred by trimming the aircraft. The L/D of 16.6 is nearly double the maximum L/D for swept wing aircraft of the same swept span and length.

Summary

The simplest and most important components of the drag of aerodynamic optimum supersonic aircraft have been delineated. Knowledge of induced drag and the area rule makes it possible to determine the inviscid drag of the aerodynamically optimum aircraft. Viscous effects may be estimated using flat plate skin friction coefficients. They are important in determining the aircraft's flight altitude and cruise lift coefficient.

For this optimized aircraft nearly 60% of its drag is skin friction drag, just over 20% is induced drag, and just under 20% is wave drag. Less than 30% of the drag is due to lift.

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